Midterm 2-alternate 1
Tuesday, 11/09/2010

• Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

• This is a closed-book exam. No calculators or other electronic aids will be permitted.

• You may have one 8.5 by 11 inch sheet of paper with handwritten notes on it.

• In order to receive full credit, you must show all of your work and justify your answers. Your answer should be clearly labeled.

• If you need extra room, use the back sides of each page. Staple any scratch paper to your exam.

• Please sign the following:

  “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.

Full Name: ______________________________
Circle your lecture time: 9:30 / 11:00

Signature: ______________________________

Please leave the following table blank for the grader.

1. __________ ( /15 points)

2. __________ ( /25 points)

3. __________ ( /15 points)

4. __________ ( /10 points)

5. __________ ( /15 points)

Total. __________ ( /80 points)
1. (15 points) Evaluate each of the following limits. Check two-sided limits, if needed. Only one point per problem is assigned for the actual value of the limit, thus even if you can ‘spot’ a limit, you must provide appropriate justification.

(a) (3 points)
\[ \lim_{x \to (-2)^-} \frac{x(x - 2)}{x + 2}. \]

(b) (3 points)
\[ \lim_{x \to 1} \frac{x^2 - 1}{x^2 - 2x + 1}. \]

(c) (3 points)
\[ \lim_{x \to (2\pi)^-} \cot(x). \]
(d) (3 points) \[ \lim_{x \to +\infty} e^{-7x} \sin(x). \]

(e) (3 points) \[ \lim_{x \to -\infty} \frac{3x^3 + 5}{x - 4}. \]
2. (25 points) Find $\frac{dy}{dx}$ in each of the following. You must solve for $dy/dx$, but you do not need to simplify your answer otherwise. It is okay to do some of this in your head instead of showing every step – BUT if you get it wrong we cannot give you much partial credit. You may use any rule you like.

(a) (5 points) 
\[ y = \pi x + e^x + 7x^2 + 1 \]

(b) (5 points) 
\[ y = \tan(\sec(2x)) \]
(c) (5 points)

\[ y = \left( \frac{x^2 + 1}{x^2 - 1} \right)^3. \]

(d) (5 points)

\[ y = e^{-3x} \cdot \sqrt{2x + 4}. \]
(e) (5 points)

\[ \sin(x) + \cos(y) = \sin(x) \cos(y). \]
3. (15 points) This is a question about tangent lines.

   (a) (5 points) Find the equation of the tangent line to

   \[ x(t) = te^t, \quad y(t) = \sqrt[3]{t}. \]

   at \( t = 1. \)

   (b) (5 points) At what point(s) in the plane is the tangent line to \( y = \sin(x) \) parallel to \( y = 1 - x? \)

   (c) (5 points) Find the equation of the normal line to the parabola \( y = x^2 - 5x + 4 \) that is parallel to the line \( x - 3y = 5. \)
4. (10 points) This question is about the definition of derivative.

(a) (5 points) Using the limit definition of the derivative, compute $f'(3)$ for

$$f(x) = \sqrt{x - 2}.$$

(Note: you can use the rules to check your answer, but no credit will be given for this alone.)

(b) (5 points) Find the equation of the tangent line to $y = \sqrt{x - 2}$ at the point $x = 3$. 
5. Short answer.

(a) (5 points) Suppose that you are told the tangent line to an unknown function $y = f(x)$ at $x = 1$ is $y = -x + 2$. What are the values of $f(1)$ and $f'(1)$? Write “impossible” if it is impossible to tell from this information.

(b) (5 points) Derive a formula for the derivative of $y = \csc x$. (i.e. compute the derivative using techniques from class, showing all steps. Don’t just cite the result from a formula sheet)
(c) (5 points) Consider the function \( y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + \pi. \) Compute the first two derivatives of \( y \) and use this information to sketch the graph of \( f \) — make your graph as accurate as possible.