Math 19: HW #2

Note: All answers must be justified for full credit. One word answers consisting of only “yes” or “no” or “k = 1” (for example) are insufficient for full credit.

Sec 1.7: #6, #10, #12, #20, #28

Sec 1.8: #2, #64

A1. Sketch the graph of a function \( f(x) \) which satisfies all of the following:

1. \( \lim_{x \to 3^+} f(x) = 4 \)
2. \( \lim_{x \to 3^-} f(x) = 2 \)
3. \( f(3) = 3 \)
4. \( \lim_{x \to -2} f(x) = 2 \)
5. \( f(-2) = 1 \)

A2. Show that the following function is continuous on \((-\infty, \infty)\):

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x < 1 \\
  \sqrt{x} & \text{if } x \geq 1.
\end{cases}
\]

A3. Show that the following function is discontinuous at \( x = 0 \):

\[
g(x) = \begin{cases} 
  e^x & \text{if } x < 0 \\
  x^2 & \text{if } x \geq 0.
\end{cases}
\]

A4. Evaluate the limit, if it exists.

(a) \( \lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5} \)

(b) \( \lim_{h \to 0} \frac{\sqrt{1 + h} - 1}{h} \)

(c) \( \lim_{x \to 4} \frac{5 + \sqrt{x}}{\sqrt{5 + x}} \)

(d) \( \lim_{t \to 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right) \)

A5. Give an example where \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) both do not exist, and yet \( \lim_{x \to a} [f(x) + g(x)] \) does exist.

A6. Show that \( \lim_{x \to 0} x^4 \cos \left( \frac{2}{x} \right) = 0. \)