Math 19: HW #5

Sec 3.7: #14, #16, #24

Sec 3.9: #14, #16, #18

Ch 3 Review Exercises and Problems: #106(b)

A1. Find the derivative of the given function.
   (a) \( f(x) = x \cos x \)
   (b) \( g(x) = (\sin x)^{\ln x} \)
   (c) \( h(x) = \sqrt{x} e^{x^2} (x^2 + 1)^{10} \)

A2. Consider the curve \( x^2 y^2 + xy = 2 \).
   (a) Find all points on the curve, if there are any, where the tangent line has slope \(-1\).
   (b) Are there any points on the curve where the tangent line is vertical? Justify your answer.

A3. Dominic drove 120 miles on a highway. The speed limit on this highway varies between 55 mph and 75 mph. He got on the highway 12:00 pm and got off the highway at 1:30 pm. Prove that Dominic was speeding at some point in his trip.

A4. Prove that if \( f \) and \( g \) are differentiable functions such that \( f'(x) = g'(x) \) for all \( x \), then \( f(x) = g(x) + C \) for some constant \( C \).

A5. Prove that if \( k \) is a differentiable function with \( k'(x) = k(x) \) for all \( x \), then \( k(x) = Ce^x \) for some constant \( C \).

Hints

Hint for A4: Begin by considering the function \( h(x) = f(x) - g(x) \). What can you say about its derivative?

Hint for A5: Begin by considering the function \( k(x)/e^x \). What can you say about its derivative?