

# THE BIDDER EXCLUSION EFFECT

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ABSTRACT. We introduce a simple and robust approach to answering several key questions in empirical auction analysis: discriminating between models of entry and quantifying the revenue gains from improving auction design. The approach builds on and brings to empirical work the theoretical results of Bulow and Klemperer (1996) and applies in a broad range of information settings and auction formats without requiring instruments or computation of a complex structural model. We demonstrate the approach using US used-car and timber auction data.

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## 1. INTRODUCTION

Empirical work on auctions often requires estimating a complex structural model relying on strong assumptions about the data generating process. For example, computing revenue under counterfactual auction formats typically involves making simplifying assumptions, such as bidders having independent private values (IPV) or bidders' values being independent of the number of bidders who participate in the auction.<sup>1</sup> The researcher may then estimate the underlying distribution of bidder valuations from bid data, and use the estimated distribution to simulate the counterfactuals of interest. While these structural methods are sometimes necessary, we demonstrate that a variety of key questions in the analysis and design of auctions can be answered with a simple-to-compute tool we refer to as the *bidder exclusion effect*.

The bidder exclusion effect is the decrease in expected auction revenue when a random bidder is excluded from an auction. The effect is not of intrinsic interest but instead serves as a means to an end, providing a simple and powerful tool for empirical auction work. We show that the bidder exclusion effect can be used to test whether the number of bidders varies exogenously across auctions, as an important class of entry models imply, and as some recent methodological advances in structural auctions assume. The bidder exclusion effect can also be used to bound above the revenue gains from optimal auction design, building on and providing a new empirical interpretation of the theoretical work of Bulow and Klemperer (1996).

In contrast to much of the empirical auctions literature, our approach does not involve estimating bidder primitives in a structural model, nor does it rely on exogenous variation in the number of bidders. Computation requires only the calculation of sample means or conditional expectations. The approach also does not

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<sup>1</sup>Paarsch and Hong (2006); Hendricks and Porter (2007); Athey and Haile (2007); and Hickman, Hubbard, and Saglam (2012) describe structural econometric methods for auction data and survey the literature.

require instruments, and even with valid instruments, instrumental variables approaches would not suffice to answer the auction design questions our method addresses. Our approach is also valid under a broad range of information settings and auction formats.

For the intuition behind how the bidder exclusion effect can be estimated, consider the example of an  $n > 2$  bidder ascending auction with private values and no reserve price, where in equilibrium bidders bid their values. If a bidder is excluded at random from the auction, with probability  $\frac{n-2}{n}$  he will be one of the  $n - 2$  lowest bidders, and so his exclusion will not affect revenue. With probability  $\frac{2}{n}$ , he will be one of the two highest bidders, and revenue will drop from the second-highest to the third-highest bid of the  $n$  bidders. The bidder exclusion effect is therefore  $\frac{2}{n}$  times the expected difference between the second and third-highest bids.

This estimator is robust to a variety of modeling frameworks which would normally complicate or prohibit identification in auction models. For example, Athey and Haile (2002) prove that the joint distribution of bidder valuations is not identified in ascending auction models with correlated private values because the researcher never observes the willingness to pay of the highest bidder. The bidder exclusion effect, however, is identified in this setting—and in a broad class of other auction settings considered in the literature—when the econometrician observes the second and third-highest bids and the number of bidders. With these data, the bidder exclusion effect is point identified in ascending auctions with (possibly asymmetric and correlated) private values and auction-level heterogeneity (observed or unobserved).

While it is particularly straightforward to find the bidder exclusion effect in the example above, we show that it can be bounded above in a wide range of other settings, including common values, bidders bidding below their values (which we

refer to throughout as “low bidding”), binding reserve prices, and first price auctions. Bounds can also be obtained when the econometrician only observes the second and third-highest bids and not the number of bidders. Incorporating auction-level covariates parametrically or nonparametrically is also straightforward, as the bounds on the bidder exclusion effect reduce to conditional means in these cases.

In each of these settings, an estimate of the bidder exclusion effect (or an estimate of an upper bound on the effect), sheds light on several important issues in empirical auctions work. First, it is possible to test whether bidders’ valuations are independent of the number of bidders who participate in the auction.<sup>2</sup> This type of independence is central to identification in many recent methodological advances.<sup>3</sup> Knowing whether the number of bidders is independent of valuations is also key in modeling bidders’ entry decisions. Specifically, a prominent distinction in the auction literature is between entry models which have this property, and models of selective bidder entry, which do not.<sup>4</sup> Choosing an inappropriate entry model might result in misleading estimates of bidders’ valuations, or unreliable counterfactual simulations.

Although the assumption that bidders’ values and the number of bidders are independent is widely used in the empirical auctions literature and is critical for modeling entry, it is rarely tested. We propose a simple test of this assumption, which involves comparing the estimated bidder exclusion effect to the observed difference in average revenue between  $n - 1$  and  $n$  bidder auctions. If the

<sup>2</sup>This type of independence has a variety of names in the auctions literature: “exogenous participation” (Athey and Haile 2002, 2007), “valuations are independent of  $N$ ” (Aradillas-López, Gandhi, and Quint 2013a), or an absence of “selective entry” (Roberts and Sweeting 2013a).

<sup>3</sup>See, for example, Haile and Tamer (2003), Aradillas-López, Gandhi, and Quint (2013a), and Sections 5.3 and 5.4 of Athey and Haile (2007).

<sup>4</sup>Bajari and Hortacsu (2003); Bajari, Hong, and Ryan (2010); Athey, Levin, and Seira (2011); Krasnokutskaya and Seim (2011) and Athey, Coey, and Levin (2013) estimate entry models derived from the theoretical work of Levin and Smith (1994). Potential bidders do not observe signals about their values before paying the entry cost, and observe their values afterwards. Entry decisions conditional on observables are governed by mixed strategies, generating exogenous variation in total bidder participation across auctions. By contrast in the selective entry models of Roberts and Sweeting (2013b,a); Bhattacharya, Roberts, and Sweeting (2013) and Gentry and Li (2013), bidders observe a signal about their value before entering, so the entry decision, and consequently the number of entrants, may be correlated with bidder valuations.

two quantities are significantly different, then  $n - 1$  bidder auctions are not just like  $n$  bidder auctions with one bidder randomly removed, and this is evidence against exogeneity of bidder participation. Estimating the bidder exclusion effect can therefore guide the decision of whether to rely on exogenous variation in the number of bidders for identification and, similarly, guide the choice of the most appropriate entry model. In contrast to other methods of discriminating between entry models, this does not require exogenous variation in, or even observing, the number of underlying potential bidders.<sup>5</sup> Nor does it require fully estimating an entry model or other complex structural auction model.<sup>6</sup>

The bidder exclusion effect also allows the analyst to gauge how important it is to set reserve prices optimally. A typical empirical approach to answering this question would rely on assumptions on the distribution of values and the information environment to estimate a detailed model, determine optimal reserve prices using the seller's first-order condition, and finally measure the revenue difference between the optimally designed auction and a no-reserve auction (e.g. Paarsch (1997); Li, Perrigne, and Vuong (2003); Krasnokutskaya (2011)).

We circumvent these steps by relying on a result from the auction theory literature. Under the assumptions of Bulow and Klemperer (1996), sellers raise expected revenue more by adding another bidder than by designing the auction optimally. When revenue is concave in the number of bidders, dropping a bidder has a larger effect on revenue than adding a bidder, and the bidder exclusion effect is an upper bound on the revenue gains from improving the auction mechanism, and, in particular, on setting an optimal reserve price. Calculating the bidder exclusion effect thus allows the researcher to determine whether reserve prices are likely to be important without expending the effort or imposing the assumptions necessary

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<sup>5</sup>The literature uses the term "potential bidders" to refer to bidders who could possibly choose to enter a given auction, in contrast to "participating bidders"—those who actually do enter. Marmer, Shneyerov, and Xu (2013) and Roberts and Sweeting (2013a) propose tests for selective entry based on observing exogenous variation in the number of potential bidders.

<sup>6</sup>Li and Zheng (2009) estimate models with and without selective bidder entry, and select between them on the basis of their predictive power. Aradillas-López, Gandhi, and Quint (2013b) provide an alternative test of selective entry which is somewhat more complex than ours but which has the advantage of not requiring observing multiple order statistics of bids.

to compute a more detailed structural model. Additionally, because the bidder exclusion effect is simple to compute, the magnitude of other estimated effects, such as an experimental change in the auction process, can easily be compared to the important benchmark of optimal mechanism design.

We illustrate the uses of the bidder exclusion effect in two different applications. Our first application uses data from US timber auctions from 1982-1989. We estimate that removing a single bidder at random would decrease revenue by approximately 13% on average. Under conditions discussed in more detail below, this implies that the increase in expected seller revenue from using an optimal reserve price is bounded above by 13%. Similarly, the expected decrease in seller revenue from two random bidders merging or colluding is bounded above by 13%.<sup>7</sup> We test whether variation in the number of bidders is exogenous, that is, whether entry is not selective. We find evidence of selective entry unconditionally, but after controlling for bidder types (loggers vs. mills), the evidence for selective entry appears weaker. This is suggestive that models relying on exogenous variation in the number of bidders may be appropriate in timber auction settings once the bidder type has been accounted for.

Our second application illustrates the use of the bidder exclusion effect in bounding the impact of optimal auction design when minimal data is available. We study wholesale used-car auctions in which the number of bidders varies auction by auction but is unobserved to the econometrician. We estimate an upper bound on the bidder exclusion effect, averaged over the unobserved realizations of the number of bidders. By relying on the Bulow-Klemperer theorem, we obtain an upper bound on the revenue increase from an optimal reserve price. In our setting, the upper bound is approximately \$200. We illustrate how this bound can also be calculated conditional on observable covariates, and we use it as a benchmark for other recently estimated effects at auto auctions (Tadelis and Zettelmeyer

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<sup>7</sup>We also discuss wider bounds which take into account the fact that merging bidders may not have randomly matched.

2011; Lacetera, Larsen, Pope, and Sydnor 2013; Hortaçsu, Matvos, Syverson, and Venkataraman 2013).

In the concluding section, we highlight other potential uses of our approach. The Web Appendix contains more technical discussions of extensions to our approach, including cases of binding reserve price data, first price auctions, and asymmetric bidders, as well as the technical arguments underlying revenue concavity. The Web Appendix also contains Monte Carlo power simulations demonstrating that our proposed test performs well.

In the spirit of Haile and Tamer (2003) and Aradillas-López, Gandhi, and Quint (2013a), our empirical approach does not seek to point identify and estimate the distribution of bidder values. Instead we draw inferences from functions of the value distribution which are point, or partially, identified. To our knowledge, we are the first to point out that a statistic which is straightforward—and in some cases trivial—to compute has implications for modeling entry in auctions or evaluating the revenue impact of optimal mechanism design. More broadly, our approach ties in closely to the recent literature on sufficient statistics for welfare analysis (Chetty 2009; Einav, Finkelstein, and Cullen 2010; Jaffe and Weyl 2013), which focuses on obtaining robust welfare or optimality implications from simple empirical objects without estimating detailed structural models.

## 2. IDENTIFYING AND ESTIMATING THE BIDDER EXCLUSION EFFECT

Our modeling setup is standard, following Athey and Haile (2002). We consider single-unit ascending auctions with risk-neutral bidders throughout.<sup>8</sup> We assume the auctions analyzed took place without a reserve price.<sup>9</sup> Let  $N$  be a random variable denoting the number of auction participants and let  $n$  represent realizations of  $N$ . Let  $W_i = (V_i, S_i)$  denote bidder  $i$ 's value and private signal, and  $B_i$  his bid. With *private values*,  $V_i = S_i$  for all  $i$ . With *common values*, for all  $i$  and  $j$ ,  $V_i$  and  $S_j$  are

<sup>8</sup>Appendix B.2 contains extensions to first price auctions.

<sup>9</sup>Section 4 discusses analysis of counterfactual settings in which reserve prices are used, and Appendix B.1 contains extensions of the paper to cases where the auctions analyzed took place with a binding reserve price.

strictly affiliated conditional on any  $\chi \in \{S_k\}_{k \neq j}$ , but not perfectly correlated. In  $n$  bidder auctions, let  $F^n$  denote the joint distribution of  $W \equiv ((V_i)_{i=1,\dots,n}, (S_i)_{i=1,\dots,n})$ . By *bidder symmetry*, we refer to the case where  $F^n$  is exchangeable with respect to bidder indices, conditional on the information that is common knowledge amongst bidders at the time of bidding.<sup>10</sup> Let  $V^{1:n}, \dots, V^{n:n}$  represent the bidders' valuations ordered from smallest to largest. Similarly, let  $B^{1:n}, \dots, B^{n:n}$  represent their bids ordered from smallest to largest, with realizations of  $B^{k:n}$  denoted  $b^{k:n}$ . Let  $V^{k:m|n}$  and  $B^{k:m|n}$  represent the  $k^{\text{th}}$  smallest values and bids in  $m$  bidder auctions, where the  $m$  bidders are selected uniformly at random from the  $n$  bidders in  $n$  bidder auctions.<sup>11</sup>

We define the bidder exclusion effect in  $n$  bidder auctions with no reserve price,  $\Delta(n)$ , as the expected fall in revenue produced by randomly excluding a bidder from those auctions. In ascending auctions, the bidder exclusion effect is

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1|n}),$$

that is, the expected second-highest bid in  $n$  bidder auctions, minus the expected second-highest bid in  $n - 1$  bidder auctions, where those  $n - 1$  bidder auctions are obtained by dropping a bidder at random from  $n$  bidder auctions.

**2.1. The Naïve Approach.** A naïve approach to estimating the bidder exclusion effect is to compare revenue between  $n$  bidder and  $n - 1$  bidder auctions. This approach is appropriate under two strong assumptions, both of which are frequently employed in the structural auctions literature: First, the number of bidders varies exogenously, and, second, the number of bidders is correctly observed. While these assumptions greatly aid in identification and testing in many structural settings, they may not hold in practice.

<sup>10</sup>Coey, Larsen, and Sweeney (2014) discuss the difference between conditional and unconditional exchangeability.

<sup>11</sup>Note that the distribution of  $V^{k:m|m}$  and  $V^{k:m|n}$  (or  $B^{k:m|m}$  and  $B^{k:m|n}$ ) for  $n > m$  will generally be different, as different kinds of goods may attract different numbers of entrants, and bidders may value goods sold in auctions with  $m$  entrants differently from those sold in auctions with  $n$  entrants.

An appealing alternative is an instrument variables approach.<sup>12</sup> However, even with valid instruments for bidder participation, instrumental variables estimates would only capture the effect on revenue for those auctions in which the instrument causes more bidders to enter (Imbens and Angrist 1994). Our approach, which we turn to next, does not require instruments or variation in the number of bidders, and applies to all auctions, rather than a subset determined by the choice of instrument.

**2.2. The Basic Model: Point Identification.** The following proposition demonstrates how the bidder exclusion effect can be expressed using only data from  $n$  bidder auctions. The proof, and the proofs for all subsequent results, are in Appendix A.

**Proposition 1.** *In ascending auctions with private values and no reserve price where bidders bid their values, for all  $n > 2$  the bidder exclusion effect  $\Delta(n) = \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ .*

This expression for  $\Delta(n)$  follows from the observation that with probability  $\frac{2}{n}$  one of the highest two bidders will be dropped, and revenue will drop to the third-highest bid of the original sample, and with probability  $\frac{n-2}{n}$ , one of the lowest  $n - 2$  bidders will be dropped, and revenue will not change. This result holds with asymmetric and correlated private values.

We define  $\Gamma(n)$  as

$$\Gamma(n) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}). \quad (1)$$

<sup>12</sup>For example, Haile, Hong, and Shum (2003) use the numbers of nearby sawmills and logging firms as instruments for the number of bidders at US timber auctions, in a test for common values.

Estimating the bidder exclusion effect in this setting is trivial, as long as the total number of auction participants and the second and third-highest bids are observed. One can simply form the sample analog of equation (1).<sup>13</sup> Incorporating auction-level observables into estimation is also straightforward. For a vector of auction-level covariates  $X$ , one estimates the sample analog of

$$\Gamma(n|X) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|X) \quad (2)$$

using any standard parametric or nonparametric approach for estimating conditional means.

**2.3. Extensions to the Basic Model.** Upper bounds on the bidder exclusion effect are available with private values and low bidding; with symmetric common values; and in settings where the number of entrants is unobserved but it is known be greater than some lower bound. Upper bounds are more important than lower bounds for our applications. For example, models of selective entry often imply that bidders' valuations are increasing with the total number of entrants (Aradillas-López, Gandhi, and Quint 2013a,b). An upper bound allows us to detect sufficiently large increases in valuations with the number of entrants, in a sense which will be made precise. Similarly, to bound above the impact of optimal mechanism design an upper bound on the bidder exclusion effect is required. Appendices B.1 and B.2 describe further extensions, showing how the bidder exclusion effect can be identified or bounded with binding reserve prices, and in first price auctions.

**2.3.1. Low Bidding.** In the "button auction" model of ascending auctions with private values (Milgrom and Weber (1982)), bidders drop out at their values. As highlighted in Haile and Tamer (2003), in practice bidders' (highest) bids may not equal their values. For example, in English auctions, multiple bidders may attempt to

<sup>13</sup>Alternatively, if all bids are observed, one could estimate the bidder exclusion effect by removing one bid at random from  $n$  bidder auctions and computing the average decrease in the second-highest bid. This second alternative would be subject to more sampling error, as it involves simulating an indicator variable which is 1 with probability  $\frac{2}{n}$  (the indicator for selecting one of the top two bids), rather than using the probability  $\frac{2}{n}$  explicitly.

bid at a certain price but only the first bid the auctioneer sees may be recorded. The researcher may be willing to assume that the bidder with the second-highest valuation bids his value, as there are only two bidders remaining at this stage. However he may only be willing to assume that the remaining bids are lower than the bidders' values.<sup>14</sup>

**Assumption 1.** *Bidders have private values. The second-highest bidder bids his value, and lower bidders' bids are less than or equal to their values.*

Our next proposition shows how this assumption can be used to obtain an upper bound on the bidder exclusion effect.

**Proposition 2.** *If in ascending auctions with no reserve price, Assumption 1 holds, then for all  $n > 2$  the bidder exclusion effect  $\Delta(n) \leq \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ .*

2.3.2. *Symmetric Common Values.* Under the assumptions of Proposition 1, the change in auction revenue when one bidder is excluded can be computed by removing one bidder's bid, and calculating the fall in revenue assuming the other bids remain unchanged. This is not true with common values, as removing a bidder changes the remaining bidders' equilibrium bidding strategies. It follows from Theorem 9 of Athey and Haile (2002) that with symmetric common values, in the button auction model of ascending auctions, the equality of Proposition 1 can be replaced by an inequality.<sup>15</sup> Removing one bidder's bid and assuming other bids remain unchanged overstates the decline in revenue from excluding a random bidder, because it does not account for the increase in bids due to the reduced winner's curse.

The following assumption and proposition summarize our results for common values settings:

<sup>14</sup>Athey and Haile (2002) make this argument, noting, "...for many ascending auctions, a plausible alternative hypothesis is that bids  $B^{n-2:n}$  and below do not always reflect the full willingness to pay of losing bidders, although  $B^{n-1:n}$  does (since only two bidders are active when that bid is placed)."

<sup>15</sup>Theorem 9 is used for a different purpose in Athey and Haile (2002). They show that, assuming total bidder participation varies exogenously across auctions, private value models can be tested against common value models.

**Assumption 2.** *Bidders have symmetric common values, and bidding follows a button auction format.*

**Proposition 3.** *If in ascending auctions with no reserve price, Assumption 2 holds, then for all  $n > 2$  the bidder exclusion effect  $\Delta(n) < \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ .*

2.3.3. *Unobserved Number of Bidders.* In some auctions the number of bidders may not be known to the researcher. In ascending auctions, for example, not all potential bidders may place bids. A lower bound on the number of potential bidders may be known, however, as in our used-car data application in Section 6. In this case the results of Propositions 2–3 extend to yield an upper bound on the average bidder exclusion effect,  $E(\Delta(N))$ , averaged over the unobserved number of bidders  $N$ .

**Corollary 1.** *In ascending auctions with no reserve price, if  $N \geq \underline{n} > 2$  and either Assumption 1 or Assumption 2 holds, then  $E(\Delta(N)) \leq \frac{2}{\underline{n}}E(B^{N-1:N} - B^{N-2:N})$ .*

Estimation in the case where  $n$  is unknown consists of computing the mean gap between second and third order statistics and scaling this quantity by  $2/\underline{n}$ . The expectation of this gap conditional on covariates can be estimated using standard nonparametric or parametric approaches.

### 3. TESTING INDEPENDENCE OF VALUATIONS AND SIGNALS, AND $N$

If bidders' valuations and signals do not vary systematically with the number of auction participants, then  $n - 1$  bidder auctions are just like  $n$  bidder auctions with one bidder removed at random. This suggests a test of whether bidders' valuations and signals are independent of the number of entrants. If the estimated bidder exclusion effect is significantly different from the observed change in revenue between  $n$  and  $n - 1$  bidder auctions, this is evidence against independence of valuations and signals, and the number of entrants. We develop this test in more detail below. Throughout this section, we assume that the econometrician

observes the total number of bidders as well as the second and third-highest bids from all auctions.

Let  $F_m^n$  denote the distribution of values and signals of a random subset of  $m$  bidders, in auctions with  $n$  bidders, where the  $m \leq n$  bidders are drawn uniformly at random from the  $n$  bidders. Define  $\Psi(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1})$ , the difference in expected revenue between  $n$  and  $n - 1$  bidder auctions. Similar to Aradillas-López, Gandhi, and Quint (2013a), we say *valuations and signals are independent of  $N$*  if  $F_m^n = F_m^{n'}$  for any  $m \leq n, n'$ . With private values, we simply say *valuations are independent of  $N$* , as valuations and signals are equal.

When valuations and signals are independent of  $n$ , we have  $F_{n-1}^n = F_{n-1}^{n-1}$ , i.e. the distribution of valuations and signals among  $n - 1$  bidders randomly selected from  $n$  bidder auctions is the same as the distribution of valuations and signals in auctions in which the realized number of participants was indeed  $n - 1$ . Thus if valuations and signals are independent of  $N$ , it follows that  $E(B^{n-2:n-1|n}) = E(B^{n-2:n-1|n-1}) = E(B^{n-2:n-1})$ , and

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1|n}) \quad (3)$$

$$= E(B^{n-1:n}) - E(B^{n-2:n-1}) \equiv \Psi(n). \quad (4)$$

If valuations or signals are not independent of  $N$ , we refer to entry as being *selective*.<sup>16</sup> The previous section discusses estimating the bidder exclusion effect,  $\Delta(n)$ , when it is point identified, and estimating an upper bound on  $\Delta(n)$  when it is not. If the econometrician observes auction revenue and the total number of auction participants, then  $\Psi(n)$  can be estimated by the difference in average revenue between  $n$  and  $n - 1$  bidder auctions. Comparing estimates of  $\Delta(n)$  and  $\Psi(n)$  is the idea behind the test for selective entry.

<sup>16</sup>Aradillas-López, Gandhi, and Quint (2013a) give examples of how models of selective entry lead to valuations being dependent on  $N$ . They prove, for example, that in the entry model of Marmer, Shneyerov, and Xu (2013) where potential bidders observe signals of their values when deciding whether to enter, if the distribution of values and signals is symmetric and affiliated, and a symmetric equilibrium exists in cutoff strategies, then valuations are increasing in  $N$ , in a sense they make precise (Theorem A3 of Aradillas-López, Gandhi, and Quint 2013a).

**3.1. Testing for selective entry with private values.** In private value settings where bidders bid their values, by Proposition 1,  $\Delta(n) = \Gamma(n)$  for  $n > 2$ . If in addition valuations are independent of  $n$ , then  $\Delta(n) = \Psi(n)$ , implying that  $\Gamma(n) = \Psi(n)$ . We define  $T(n)$  as

$$\begin{aligned} T(n) &\equiv \Psi(n) - \Gamma(n) \\ &= \left( E(B^{n-1:n}) - E(B^{n-2:n-1}) \right) - \frac{2}{n} E(B^{n-1:n} - B^{n-2:n}) \\ &= E \left( \frac{n-2}{n} B^{n-1:n} + \frac{2}{n} B^{n-2:n} \right) - E(B^{n-2:n-1}). \end{aligned} \quad (5)$$

The first term in the final expression is the expected revenue in  $n$  bidder auctions when one bidder is dropped at random, and the second term is the expected revenue in  $n - 1$  bidder auctions.<sup>17</sup>

To assess whether valuations are independent of  $N$  (i.e. whether entry is not selective) we test the null hypothesis  $T(n) = 0$  for  $n > 2$ . For any given  $n$  this can be implemented as a simple  $t$ -test.<sup>18</sup> Let  $A_n$  represent the set of auctions with  $n$  entrants. We form the test statistic,  $\hat{T}(n)$ , for this null by replacing expectations by sample averages:

$$\hat{T}(n) = \frac{1}{|A_n|} \sum_{j \in A_n} \left( \frac{n-2}{n} b_j^{n-1:n} + \frac{2}{n} b_j^{n-2:n} \right) - \frac{1}{|A_{n-1}|} \sum_{j \in A_{n-1}} (b_j^{n-2:n-1}). \quad (6)$$

A simple regression-based form of this test is as follows. Let  $y_j = \frac{n-2}{n} b_j^{n-1:n} + \frac{2}{n} b_j^{n-2:n}$  if  $j \in A_n$  and  $y_j = b_j^{n-2:n-1}$  if  $j \in A_{n-1}$ . Regress  $y_j$  on a constant and an indicator  $\mathbf{1}(j \in A_{n-1})$ . The coefficient on the indicator represents  $\hat{T}(n)$ .

<sup>17</sup>Athey and Haile (2002) propose a test of private vs. common values in which they assume valuations and signals are independent of  $N$  and point out that, under this assumption,  $T(n) < 0$  in a common values setting and  $T(n) = 0$  in a private values setting. We discuss generalizations to common values settings in Section 3.2.

<sup>18</sup>Standard techniques, like a Wald test or a Bonferroni correction, can be used to test  $T(n) = 0$  for all  $n$  in some finite set. Note also that this test only uses information on the second and third-highest bids. If more losing bids are available and interpretable as the willingness-to-pay of lower-value bidders, this test could be made more powerful by including information from these losing bids. Intuitively, one could compare the revenue drop which would occur if  $k$  out of  $n$  bidders were dropped at random to the actual revenue difference between  $n$  and  $n - k$  bidder auctions.

If  $\hat{T}(n)$  is significantly different from 0, the test indicates the presence of selective entry. This test is consistent against all forms of selective entry which affect expected revenue, i.e. if  $\Gamma(n) \neq \Psi(n)$  then the test rejects with probability approaching 1 as the number of auctions goes to infinity. Appendix B.3 provides Monte Carlo evidence on the power of this test relative to simply comparing mean values in  $n - 1$  and  $n$  bidder auctions, in a model of selective entry which nests that of Levin and Smith (1994). The bidder exclusion test seems to be a reasonably powerful alternative to the mean comparison test, despite the fact that it uses less data. Moreover, the bidder exclusion test is implementable with ascending auction data whereas the mean comparison test is not.<sup>19</sup>

**3.2. Testing for selective entry with common values or low bidding.** With symmetric common values or low bidding, Propositions 2 and 3 imply that  $\Delta(n) \leq \Gamma(n)$ . If in addition valuations and signals are independent of  $n$ , then  $\Delta(n) = \Psi(n)$ , implying that  $\Psi(n) \leq \Gamma(n)$ . We can test this null using the same statistic as before,  $\hat{T}(n)$ . Unlike private value settings where bidders bid their values, this test only indicates the presence of selective entry if  $\hat{T}(n)$  is significantly greater than zero, and not if it is significantly less than zero. This test is consistent against forms of selective entry in which bidders' values are "sufficiently increasing" with  $N$ : precisely, if  $E(B^{n-2:n-1|n}) - E(B^{n-2:n-1}) > \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}) - (E(B^{n-1:n}) - E(B^{n-2:n-1|n}))$ .<sup>20</sup>

**3.3. Incorporating covariates.** Testing is also possible if valuations and signals are assumed independent of  $n$  conditional on a set of observable auction characteristics  $X$  rather than unconditionally. Under this assumption the null hypothesis in private values auctions where bidders bid their values is  $T(n|X) = 0$ , where

<sup>19</sup>The bidder exclusion test uses the second and third-highest values in  $n$  bidder auctions and the second-highest value in  $n - 1$  bidder auctions, while the mean comparison test uses *all*  $n$  values in  $n$  bidder auctions and *all*  $n - 1$  values in  $n - 1$  bidder auctions. The mean comparison test cannot be implemented in ascending auctions because the highest valuation is never observed.

<sup>20</sup>This feature is shared by the test proposed in Aradillas-López, Gandhi, and Quint (2013b), which the authors explain, "has power against a fairly wide class of 'typical' violations of [valuations being independent of  $N$ ]."

$T(n|X)$  is defined as

$$T(n|X) = E\left(\frac{n-2}{n}B^{n-1:n} + \frac{2}{n}B^{n-2:n}\middle|X\right) - E(B^{n-2:n-1}|X). \quad (7)$$

With symmetric common values or low bidding, the null is  $T(n|X) < 0$ . This test can be performed nonparametrically, without assuming any particular form for the conditional means. Chetverikov (2011); Andrews and Shi (2013) and Chernozhukov, Lee, and Rosen (2013) develop inference procedures which apply to this setting.

A simple parametric version of this test is as follows. For a fixed  $n$ , specify the bidding equation for bidders in auction  $j$  as

$$b_j = \alpha_n + \beta X_j + \varepsilon_{nj}, \quad (8)$$

where  $X_j$  represents a vector of observable characteristics of auction  $j$  and  $\varepsilon_{nj} \perp\!\!\!\perp X_j$ .

Then

$$\frac{n-2}{n}b_j^{n-1:n} + \frac{2}{n}b_j^{n-2:n} = a_1 + \beta X_j + e_{1j}, \quad j \in A_n \quad (9)$$

$$b_j^{n-2:n-1} = a_2 + \beta X_j + e_{2j}, \quad j \in A_{n-1} \quad (10)$$

where  $a_1 = \alpha_n + E(\frac{n-2}{n}\varepsilon_{nj}^{n-1:n} + \frac{2}{n}\varepsilon_{nj}^{n-2:n})$ ,  $a_2 = \alpha_{n-1} + E(\varepsilon_{(n-1)j}^{n-2:n-1})$ ,  $E(e_{1j}) = E(e_{2j}) = 0$  and  $e_{1j}, e_{2j} \perp\!\!\!\perp X_j$ .

After controlling for observables,  $a_1$  determines the expected second order statistic (i.e. seller's revenue) when a bidder is removed at random from  $n$  bidder auctions and  $a_2$  determines the expected second order statistic in  $n - 1$  bidder auctions when the actual number of bidders is indeed  $n - 1$ . In a private values framework, testing the null hypothesis of equation (7) amounts to testing the null of  $a_1 = a_2$ . With common values or low bidding, the null is  $a_1 < a_2$ .

We combine (9) and (10) as follows:

$$y_j = a_1 + (a_2 - a_1)\mathbf{1}(j \in A_{n-1}) + \beta X_j + e_{3j}, \quad j \in A_n \cup A_{n-1}, \quad (11)$$

where if  $j \in A_n$ , then  $y_j = \frac{n-2}{n}b_j^{n-1:n} + \frac{2}{n}b_j^{n-2:n}$  and  $e_{3j} = e_{1j}$ , and if  $j \in A_{n-1}$ , then  $y_j = b_j^{n-2:n-1}$  and  $e_{3j} = e_{2j}$ . This allows for a convenient regression-based test of the null hypothesis that valuations (or valuations and signals, in the common values case) are independent of  $N$ . When  $\beta = 0$ , this test nests the regression-based test described in Section 3.1.

**3.4. Bidder asymmetries.** The approach to testing for selective entry with private values described above does not require bidder symmetry. When bidders are symmetric, the assumption of valuations being independent of  $N$  may be quite natural. If bidders are asymmetric, it is less clear what this assumption entails, or what it might mean if a test rejected it. Intuitively, values may fail to be independent of  $N$  either because different bidders are more likely to enter depending on  $N$ , or because the same bidders enter but the value of the goods sold varies by  $N$ . We formalize and prove this statement in Appendix B.4, following the setup of Coey, Larsen, and Sweeney (2014).

#### 4. BOUNDING THE IMPACT OF OPTIMAL RESERVE PRICES

The celebrated theorem of Bulow and Klemperer (1996) relates bidder entry to optimal mechanism design.<sup>21</sup> Under their assumptions, with symmetric bidders and independent signals, an English auction with no reserve price and  $n + 1$  bidders is more profitable in expectation than *any* mechanism with  $n$  bidders. When bidders have affiliated signals, Bulow and Klemperer (1996) show that an auction

<sup>21</sup>Following Bulow and Klemperer (1996) and most of the auction theory literature, we use “optimal” to mean optimal given a fixed set of participants. If entry is endogenous, then the mechanism’s design may affect the number of participants. Optimal reserve prices for fixed and for endogenous entry may be different (McAfee and McMillan (1987); Levin and Smith (1994)).

with  $n + 1$  bidders and no reserve price still outperforms any “standard” mechanism with  $n$  bidders.<sup>22</sup> On these grounds, they suggest that sellers may be better off trying to induce more entry than trying to implement a better mechanism. As they acknowledge, this interpretation may be problematic if the new bidders are weaker than the bidders who would have entered anyway (for example, if increased marketing efforts induce lower-value bidders to enter the auction).

We propose an alternative interpretation of their theorem, namely that it can be used in empirical work to easily obtain upper bounds on the effect of improving mechanism design, without having to fully estimate a detailed structural model. Below we use the term “increasing marginal revenue” as it is defined in Bulow and Klemperer (1996).<sup>23</sup> We use the term “optimal reserve price” to refer to a take-it-or-leave-it offer made to the final bidder—the last remaining bidder after the auction

<sup>22</sup>A “standard” mechanism in this context is one in which 1) losers pay nothing, 2) the bidder with the highest signal wins (if anyone) and pays an amount which increases in his own signal given any realization of other bidders’ signals. Bulow and Klemperer (1996) highlight a result of Lopomo’s (1995), which shows that an optimal mechanism in this class is an English auction followed by a final, take-it-or-leave-it offer to the high bidder (a reserve price). When bidders have correlated values, Crémer and McLean (1988), McAfee, McMillan, and Reny (1989), and McAfee and Reny (1992) have provided examples of non-standard mechanisms which extract all bidder surplus and outperform an auction with a reserve price.

<sup>23</sup>Let  $(S_1, \dots, S_n)$  denote the private signals of the  $n$  bidders in  $n$  bidder auctions. The  $n + 1^{\text{th}}$  bidder, were he to enter, has a private signal denoted  $S_{n+1}$ . We denote the marginal distribution of  $S_j$  by  $F_j^n$ , and the corresponding density by  $f_j^n$ . Define  $\mathbf{S} = (S_1, \dots, S_{n+1})$ , and  $\bar{\mathbf{S}} = \mathbf{S}_{-(N+1)}$ . Let  $F_j^n(S_j|\mathbf{S}_{-j})$  and  $f_j^n(S_j|\mathbf{S}_{-j})$  represent the distribution and density of bidder  $j$ ’s signal conditional on competitors’ signals. Let  $v_j(\mathbf{S})$  represent the value to bidder  $j$  given all signals, and define  $\bar{v}_j(\bar{\mathbf{S}}) = E_{S_{N+1}} v_j(\mathbf{S})$ . Define  $MR_j(\mathbf{S})$  and  $\overline{MR}_j(\bar{\mathbf{S}})$  as

$$MR_j(\mathbf{S}) = \frac{-1}{f_j^n(S_j|\mathbf{S}_{-j})} \frac{d}{dS_j} \left( v_j(\mathbf{S})(1 - F_j^n(S_j|\mathbf{S}_{-j})) \right) \quad (12)$$

$$\overline{MR}_j(\bar{\mathbf{S}}) = \frac{-1}{f_j^n(S_j|\bar{\mathbf{S}}_{-j})} \frac{d}{dS_j} \left( \bar{v}_j(\bar{\mathbf{S}})(1 - F_j^n(S_j|\bar{\mathbf{S}}_{-j})) \right) \quad (13)$$

We say bidders have “increasing marginal revenue” (as a function of their private signals) if  $S_j > S_i \Rightarrow MR_j(\mathbf{S}) > MR_i(\mathbf{S})$  and  $\overline{MR}_j(\bar{\mathbf{S}}) > \overline{MR}_i(\bar{\mathbf{S}})$ . Equivalently, bidders have decreasing marginal revenue, when marginal revenue is considered to be a function of bidder “quantity” (i.e.  $(1 - F_j^n(S_j|\mathbf{S}_{-j}))$  and  $(1 - F_j^n(S_j|\bar{\mathbf{S}}_{-j}))$ ) rather than of their private signals. In the independent private values case, this assumption simplifies to the function  $MR_j(\mathbf{S}) = \overline{MR}_j(\bar{\mathbf{S}}) = S_j - \frac{1 - F_j^n(S_j)}{f_j^n(S_j)}$  being increasing in  $S_j$ . For more on the interpretation of bidders’ marginal revenue, see Bulow and Roberts (1989).

ends—which maximizes the expected payment of that bidder.<sup>24</sup> Throughout we assume sellers are risk-neutral.

We make the following assumption before stating our main revenue-bounding result:

**Assumption 3.** *Expected revenue is concave in the number of bidders.*

**Proposition 4.** *In ascending auctions with no reserve price, if i) either Assumption 1 or Assumption 2 holds, ii) Assumption 3 holds, and iii) bidders are symmetric and have increasing marginal revenue, then for all  $n > 2$  the increase in expected revenue from using the optimal reserve price is less than  $\Gamma(n) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ .*

The idea behind the proof is as follows. By the Bulow-Klemperer theorem, the increase in expected revenue from the optimal reserve price is less than the increase in expected revenue from adding another bidder, which by revenue concavity, is in turn less than the bidder exclusion effect. The result then follows from Proposition 2, if Assumption 1 holds, and from Proposition 3, if Assumption 2 holds. Assumption 3, revenue being concave in the number of bidders, is natural in many contexts, although it will not hold in all environments.<sup>25</sup> In Appendix B.5

<sup>24</sup>As discussed in Bulow and Klemperer (1996), with correlated bidder signals, expected revenue will generally be greater when the reserve is set optimally conditional on the observed bids rather than being set before the auction takes place. Note that Bulow and Klemperer (1996) use the term “optimal reserve price” to refer to a take-it-or-leave-it offer made to the final bidder which maximizes the seller’s expected profit (i.e. the expected payment of the bidder if the offer is accepted and the seller’s value of retaining the good if the offer is not accepted), and the authors focus on cases where the seller always values the good less than buyers. We focus instead on expected bidder payments in order to generalize to cases in which the seller’s value lies above the lower bound of the support of buyer valuations and where the mechanism designer is not the seller but rather an intermediary (an auction house or platform) who is paid based on payments raised from bidders. See Section 6. Additionally, data on seller valuations is uncommon.

<sup>25</sup>Dughmi, Roughgarden, and Sundararajan (2012) provide a counterexample where all bidders have iid values, which are 1 with probability  $p$  and 0 otherwise. For sufficiently small  $p$ , the revenue increase from one to two bidders is smaller than from two to three bidders. It can be proved that, for any exchangeable value distribution  $F^{n+1}$ , a necessary and sufficient condition for the revenue change from  $n - 1$  to  $n$  bidders to be larger than from  $n$  to  $n + 1$  bidders is  $3(E(V^{n:n+1}) - E(V^{n-1:n+1})) > E(V^{n+1:n+1}) - E(V^{n:n+1})$ .

we demonstrate that with independent private values (or conditionally independent private values), symmetry and increasing marginal revenue (or conditionally increasing marginal revenue) are sufficient to guarantee Assumption 3.<sup>26</sup>

Proposition 4 implies that an estimate of the bidder exclusion effect in  $n$  bidder auctions provides an upper bound on the revenue increase from using an optimal reserve price in these auctions. As pointed out by Bulow and Klemperer (1996), even when signals are affiliated, an ascending auction with a optimal reserve yields at least as much expected revenue as any standard mechanism (Lopomo (1995)). Thus the increase in expected revenue from using any standard mechanism is also less than  $\Gamma(n)$ .<sup>27</sup>

The estimated bidder exclusion effect provides a bound on the impact of optimal reserve prices for a fixed number of bidders, even if the data comes from auctions with selective entry, as computation of the bidder exclusion effect (or its upper bound) does not require exogenous variation in the number of bidders. When the number of bidders is unknown, an upper bound on the average bidder exclusion effect, as described in Section 2.3.3, provides an upper bound on the benefit to improving auction design averaged over the (unobserved) realized values of  $N$ . Section 6 illustrates this empirical application.

## 5. APPLICATION 1: TESTING FOR SELECTIVE ENTRY AT TIMBER AUCTIONS

Our first application uses US timber auction data to illustrate how the bidder exclusion effect can be used to distinguish between models of entry. In doing so, we also present other uses of the bidder exclusion effect, including bounding the

<sup>26</sup>We prove a special case of results already established by Dughmi, Roughgarden, and Sundararajan (2012) (Theorem 3.2). The specialization to our current single item auction setting allows us to use only elementary mathematics, in contrast to Dughmi, Roughgarden, and Sundararajan (2012)'s proof which relies on matroid theory.

<sup>27</sup>In later work, Bulow and Klemperer (2002) highlighted that the assumption of marginal revenues increasing in signals may be more stringent in common values settings than in private values settings, and the authors provided examples of common values settings in which the original Bulow and Klemperer (1996) result does not hold.

impact of optimal auction design and the losses from collusion, which relate to the previous literature.

The Forest Service's timber auction data has been used extensively in the empirical auctions literature, and is a natural context to demonstrate the applications of the bidder exclusion effect. For example, Haile and Tamer (2003) and Aradillas-López, Gandhi, and Quint (2013a) use timber auction data and develop bounds methods relying on the assumption that valuations are independent of  $N$  (i.e. that entry is not selective). Below we test the validity of this assumption. Optimal reserve prices have also been a major focus of timber auction work.<sup>28</sup> By estimating the bidder exclusion effect and relating it to the work of Bulow and Klemperer (1996), we are able to side-step the need for a complex structural model or for restrictive information assumptions and still obtain a simple estimate of the revenue improvement which would arise from an optimal reserve price.

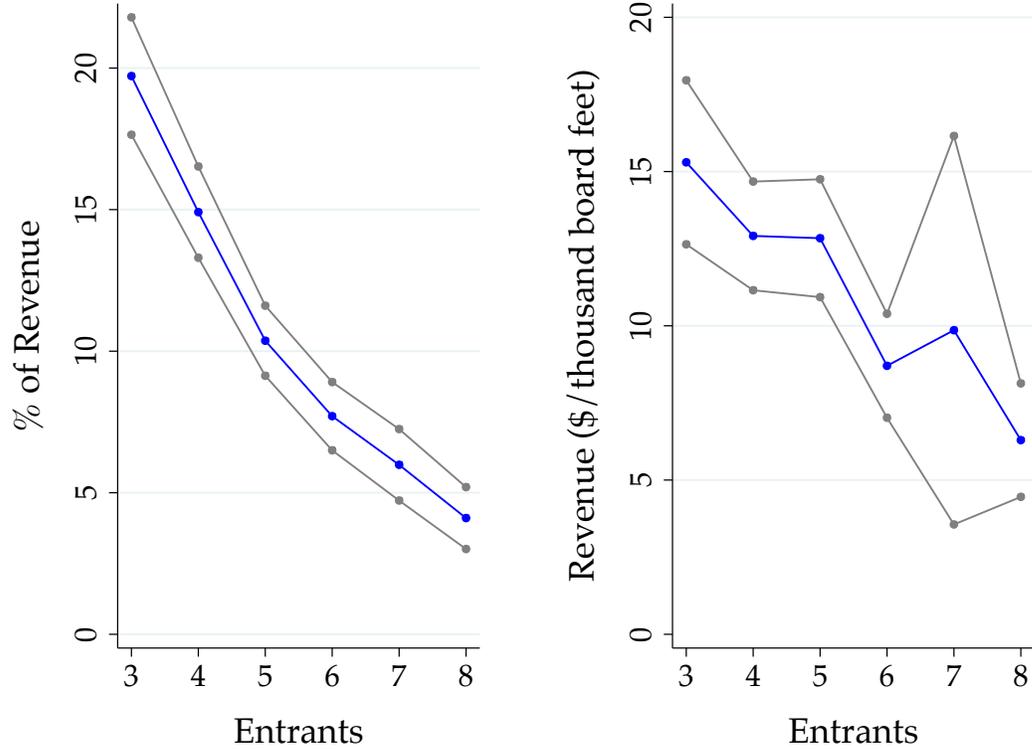
Our data comes from ascending auctions held in California between 1982 and 1989 in which there were at least three entrants. There are 1,086 such auctions. The data contains all bids, as well as information at the auction level, such as appraisal variables, measures of local industry activity, and other sale characteristics.

By Propositions 2 and 3, with private values and low bidding, or in button auctions with symmetric common values, the bidder exclusion effect is smaller than  $\frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ . We estimate the sample analog of this upper bound conditional on the number of entrants in the auction but unconditional on auction characteristics. Figure 1 shows the results, both as a percentage of revenue, and in absolute terms. In our sample we estimate this upper bound on the bidder exclusion effect to be 12% of auction revenue, averaging over all values of  $n$ , with a standard error of 0.4%. Under the conditions of Proposition 4, the average increase in revenue from setting an optimal reserve price is therefore less than around 13%.

We now turn to the test for selective entry. We begin with the simplest version of the test, without controlling for covariates. Table 1 displays the results of this

<sup>28</sup>For example, both Haile and Tamer (2003) and Aradillas-López, Gandhi, and Quint (2013a) study optimal reserve prices.

FIGURE 1. Bounding the Bidder Exclusion Effect



Notes: Graphs show point estimates and 95% pointwise confidence intervals for  $\frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ , for various values of  $n$ , the total number of entrants in the auction. Estimates in the left graph are expressed as a percentage of auction revenue. Estimates in the right graph are expressed in dollars per thousand board feet.

test using the timber auction data. In this table,  $a_1$  represents the expected second order statistic when a bidder is removed at random from  $n$  bidder auctions,  $a_2$  represents the expected second order statistic in  $n - 1$  bidder auctions when the actual number of bidders is indeed  $n - 1$ , and the test statistic is given by  $\hat{T}(n) = a_1 - a_2$ . For most  $n \in \{3, \dots, 8\}$ ,  $\hat{T}(n)$  is insignificant, although at  $n = 3$  and  $n = 5$ , the test statistic is significant and negative, indicating that selective entry may be a concern.

TABLE 1. Unconditional Tests for Selective Entry, All Auctions

Entrants	3	4	5	6	7	8
$a_1$	78.15*** (6.36)	92.34*** (4.62)	119.75*** (4.81)	125.64*** (11.14)	123.16*** (6.69)	147.24*** (9.65)
$a_2$	49.60*** (2.62)	78.37*** (6.43)	89.09*** (3.50)	119.66*** (4.85)	125.02*** (11.15)	130.46*** (12.04)
$T(n) = a_1 - a_2$	28.55*** (6.87)	13.97 (7.92)	30.66*** (5.95)	5.98 (12.15)	-1.86 (13.00)	16.78 (15.43)
Sample Size	497	496	456	350	243	164

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Notes: Table presents results of test for selective entry unconditional on covariates, for various levels of the number of entrants.

Table 2 shows the results of the selective entry test conditional on auction characteristics. The objects  $a_1$ ,  $a_2$ , and  $T(n)$  are as in Table 1, but after controlling for covariates (following the parametric test described in Section 3.3). We control for appraisal variables (quintiles of the reserve price, selling value, manufacturing costs, logging costs, road construction costs, and dummies for missing road costs and missing appraisals), sale characteristics (species Herfindahl index, density of timber, salvage sale or scale sale dummies, deciles of timber volume, and dummies for forest, year, and primary species), and local industry activity (number of logging companies in the county, sawmills in the county, small firms active in the forest-district in the last year, and big firms active in the forest-district in the last year).

There is somewhat stronger evidence for selective entry when controlling for auction characteristics than in the unconditional case. Conditional on auction characteristics, average revenue when a bidder is removed at random from  $n$  bidder auctions is higher than average revenue is  $n - 1$  bidder auctions when  $n \in \{3, 4, 5, 7\}$ , and this difference is significant at the 95% level. This is quite strong evidence of selective entry, and in particular supporting positive selection: bidders' valuations appear to be higher in auctions with more entrants. With  $n \in \{6, 8\}$  the

difference is negative and insignificant, consistent with a setting where valuations are independent of  $N$ . The joint null hypothesis of no selective entry across all  $n \in \{3, 4, 5, 6, 7, 8\}$  can be rejected at the 99.9% level.

TABLE 2. Conditional Tests for Selective Entry, All Auctions

Entrants	3	4	5	6	7	8
$a_1$	-44.52 (59.38)	-120.64 (74.72)	3.34 (24.72)	-95.81** (32.93)	-111.39 (71.81)	-172.47 (159.06)
$a_2$	-64.84 (59.39)	-137.66 (75.24)	-10.47 (25.01)	-89.90** (31.83)	-124.05 (72.75)	-167.05 (145.70)
$T(n) = a_1 - a_2$	20.32*** (5.33)	17.02*** (4.21)	13.81*** (3.63)	-5.91 (4.75)	12.66* (5.78)	-5.42 (17.94)
Sample Size	497	496	456	350	243	164

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Notes: Table presents results of test for selective entry conditional on covariates described in Section 3.3, for various levels of the number of entrants.

TABLE 3. Testing for Selective Entry, Auctions with Only Loggers

Entrants	3	4	5	6
$a_1$	-158.28 (108.96)	-183.18 (115.53)	26.03 (42.50)	-285.34** (115.06)
$a_2$	-172.28 (105.59)	-222.73 (128.23)	8.53 (43.13)	-266.30 (111.34)
$T(n) = a_1 - a_2$	14.00 (16.65)	39.54* (19.29)	17.51 (9.92)	-19.04 (25.66)
Sample Size	149	138	109	76

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Notes: Table presents results of test for selective entry conditional on covariates described in Section 3.3, for various levels of the number of entrants, and only for auctions in which all entrants are loggers.

Some bidders in timber auctions may be stronger than others. One common distinction in the literature is between mills, who have the capacity to process the

timber, and loggers, who do not. Mills typically have higher valuations than loggers (e.g. Athey, Levin, and Seira 2011; Athey, Coey, and Levin 2013; Roberts and Sweeting 2013a). The evidence for selective entry in Table 2 may reflect more mill entry in auctions with many participants. We examine this question by restricting attention to auctions in which only loggers enter.

Table 3 presents these results. The sample size is significantly smaller when restricting to auctions in which all entrants are loggers. At  $n = 4$ , the test still rejects the null hypothesis that valuations are independent of  $N$ . However, the evidence on the whole is much weaker in the loggers-only sample: at  $n \in \{3, 5, 6\}$  the difference is much smaller and insignificant, although the smaller sample size may play a role. Taken together the results from Tables 2 and 3 are suggestive that accounting selective entry may be an issue at timber auctions but might be reasonably controlled for by accounting for bidder types.

## 6. APPLICATION 2: BOUNDING THE IMPACT OF OPTIMAL AUCTION DESIGN AT AUTO AUCTIONS

Our second application uses wholesale used-car auction data to illustrate how the bidder exclusion effect can be used to bound the revenue impact of optimal auction design. Recent studies report causal effects of various interventions at used-car auctions, and our approach allows us to judge the economic relevance of these effects by benchmarking them against the effect of optimal auction design, a question to which much of the auction literature—both theoretical and empirical—is dedicated. This setting also provides an illustration of bounding the bidder exclusion effect under minimal assumptions and minimal data requirements. Specifically, records from used-car auctions do not contain the number of bidders and this number can vary from auction to auction, posing challenges for many structural auctions approaches but not to our approach.

Our dataset contains the second and third-highest bids from 6,003 sales of used cars. The data also records car characteristics. Summary statistics for these auction

sales are shown in Table 4. These cars are mostly late-model (two years old on average), low-mileage cars (33,369 miles on average). The market thickness measure comes from a larger, nationwide sample of auto auction sales from which our final sample is taken, and represents the total number of sales for a given make-by-model-by-age combination.<sup>29</sup> Table 4 shows that, on average, a given make-by-model-by-age combination sold over 1,000 times in the nationwide sample.

Multiple auctions take place in different lanes within the auction house simultaneously, and it is impossible to determine the number of participating bidders (who may be physically present at the auction house or watching online) for any given sale. The data does record the number of online bidders who have logged in to the online console for a given lane at some point during the sale day, which is 35 bidders on average. However, many of these online bidders may not actually be participating bidders for a given sale. Based on personal observations at these auctions, we choose  $\underline{n} = 5$  in the following exercises as a lower bound on the number of bidders present.

Table 4 also displays summary statistics for the average second order statistic (the highest observed bid), third order statistic (one bid increment beyond where the third-final bidder dropped out), and the gap between the two. Following Section 2.3.3, when the number of bidders is unobserved, an upper bound on the bidder exclusion effect, averaged over the unobserved values of  $n$ , is given by the mean of the gap between second and third order statistics scaled by  $2/\underline{n}$ . This implies an unconditional estimate of the bidder exclusion effect upper bound of  $(2/5) \times 425 = \$170$ . This provides an upper bound on the average revenue increase from implementing an optimal reserve price.

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<sup>29</sup>This larger, nationwide sample contains 901,338 auction sales from 27 auction houses nationwide between 2007 and 2010. For each auction in this larger sample, we observe a complete bid log of all bids submitted. However, bid logs at wholesale auto auctions do not contain identities of floor bidders (those who are physically present at the auction); the log simply records the identity as "Floor" for any floor bidder. Therefore, we are only able to identify the third order statistic of bids in cases where at least two of the last three bidders to place bids were online bidders (whose identities are always recorded). This leaves a sample of 8,005 auction sales. We drop recreational vehicles (including boats, motorhomes, motorcycles) or observations lying outside the first or ninety-ninth percentiles of mileage, age, or the number of online bidders. This leaves 6,003 records.

TABLE 4. Wholesale auto auction data descriptive statistics

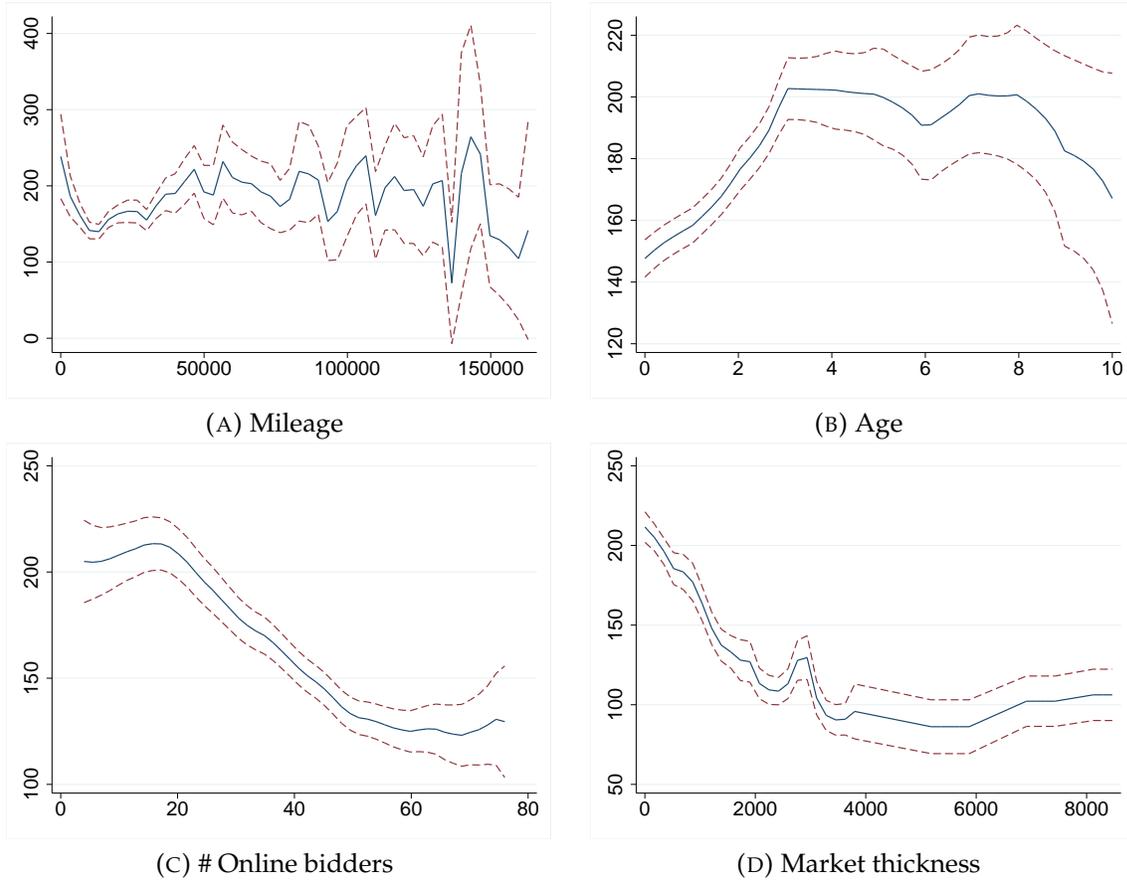
	Mean	s.d.	Min.	Max.
Mileage	33,369.17	29,601.36	13	163,036
Age	1.95	2.18	0	10
# Online bidders	34.71	16.71	4	76
Market thickness	1,196.01	1,648.60	2	8,465
$b^{n-1:n}$	15,429.02	7,733.44	300	55,100
$b^{n-2:n}$	15,003.96	7,648.20	225	53,400
$b^{n-1:n} - b^{n-2:n}$	425.06	493.77	50	6650
Sample size	6,003			

Age is in years. # Online bidders is the number of bidders who had logged into the lane the car was sold in. Market thickness is the number of cars of the same make-by-model-by-age combination as a given car.  $b^{n-1:n}$  is the final auction price (second order statistic),  $b^{n-2:n}$  is the highest bid of the third-final bidder (third order statistic).

Next, following Section 2.3.3, we estimate the upper bound on the average bidder exclusion effect conditional on covariates  $X$ , approximating the conditional expectation using a kernel regression.<sup>30</sup> Figure 2 displays the estimates with  $X$  being the vehicle's mileage, the vehicle's age, the number of online bidders, or the market thickness measure. Panels A and B suggest that auctions of cars which are lower in mileage or younger in age exhibit a lower bidder exclusion effect. That is, excluding a random bidder from the auction does less damage to revenue in these auctions than in auctions for older, higher mileage cars. This is consistent with sales of newer, lower mileage cars being cases where demand is high and many bidders participate, and hence optimal auction design is less important and a no-reserve auction is likely to perform well. Similarly, Panels C and D suggest

<sup>30</sup>We use an Epanechnikov kernel and rule-of-thumb bandwidth.

FIGURE 2. Bidder Exclusion Effect



Notes: Vertical axis in each panel represents the bidder exclusion effect in dollars. Solid line represents bidder exclusion effect conditional on mileage (A), age (B), number of online bidders (C), and market thickness (D). Estimates come from kernel regression with Epanechnikov kernel and rule-of-thumb bandwidth. Dashed lines represent pointwise 95% confidence bands.

that in cases where more bidders have logged in online or where the car is of a frequently sold make-by-model-by-age combination, the bidder exclusion effect and the impact of an optimal reserve price are small.

The bidder exclusion effect also leads to bounds on the revenue from bidders which could be expected in any auction, ranging from an efficient, no-reserve auction to an auction with an optimal reserve price, as a function of sale characteristics. For each observation  $j$ , let lower and upper bounds on revenue conditional

on observables  $X_j$  be denoted  $\underline{\pi}_j(X_j)$  and  $\bar{\pi}_j(X_j)$ , given by

$$\underline{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j) \quad (14)$$

$$\bar{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j) + \frac{2}{n} \left( b_j^{n_j-1:n_j}(X_j) - b_j^{n_j-2:n_j}(X_j) \right) \quad (15)$$

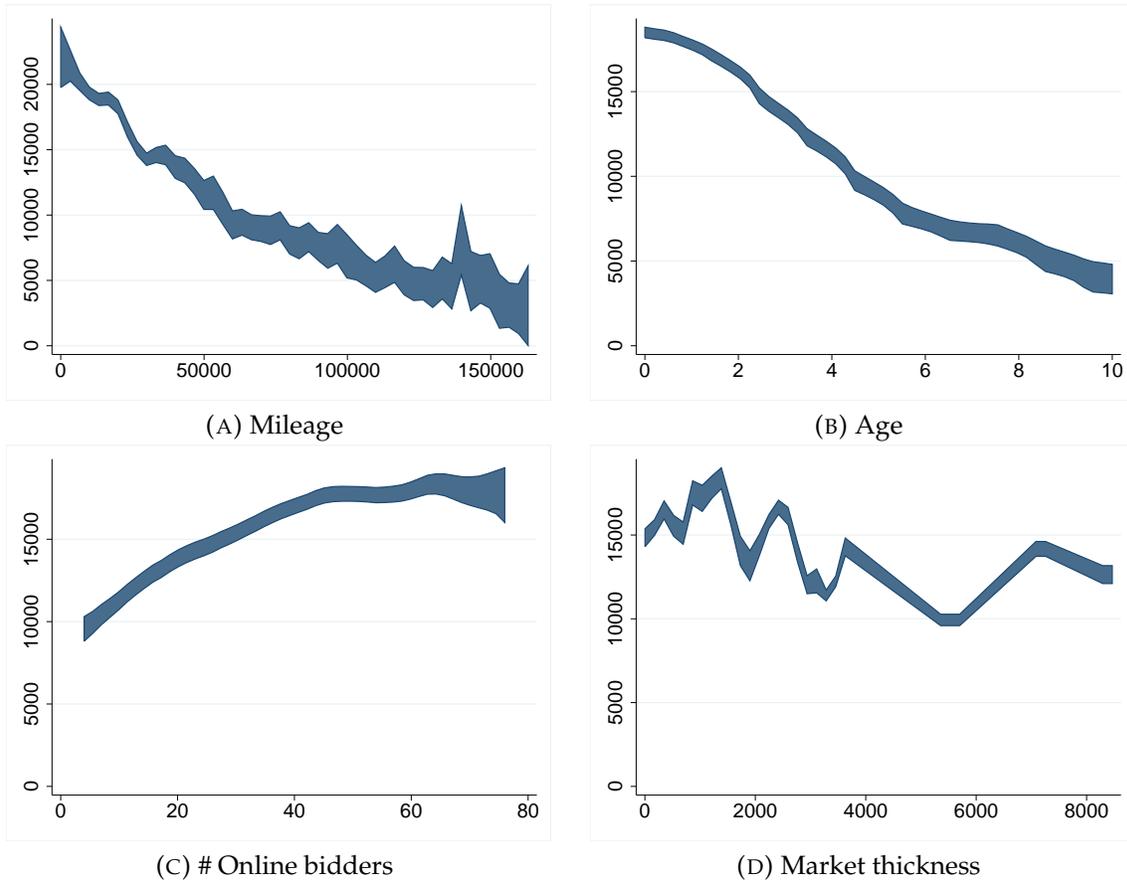
The lower bound is the observed second order statistic (the no-reserve revenue) and the upper bound is the observed second order statistic plus the scaled gap between second and third order statistics (the expectation of this upper bound is an upper bound on the optimal reserve revenue, under the conditions of Proposition 4). The shaded region in Figure 3 shows the area between the upper 95% confidence band for  $\hat{E}(\bar{\pi}_j|X_j)$  and the lower 95% confidence band for  $\hat{E}(\underline{\pi}_j|X_j)$ , where each conditional expectation is estimated using a kernel regression as above.

Panels A and B of Figure 3 demonstrate that the expected revenue is tightly bounded, regardless of the seller's choice of reserve price, for newer, high-mileage cars. Similarly, Panels C and D show that the fraction of expected revenue which could be manipulated through the use of reserve prices is much smaller for auctions of more popular cars or auctions in lanes where more online bidders have logged in.

These estimates provide a benchmark for evaluating the relative economic significance of other effects measured in this industry. For example, Tadelis and Zettelmeyer (2011), through a field experiment at a wholesale auto auction, found that revealing information about the quality of the car through standardized condition reports led to a difference in revenue of \$643.<sup>31</sup> Similarly, Lacetera, Larsen, Pope, and Sydnor (2013) studied auctioneers at wholesale auto auctions and found

<sup>31</sup>Tables 2 and 3 of Tadelis and Zettelmeyer (2011) show that the probability of sale and expected price conditional on sale for the treatment group, for which condition reports were posted, were 0.455 and \$8,738.90, respectively, vs. 0.392 and \$8,502.20 in the control group.

FIGURE 3. Expected Revenue



Notes: Vertical axis in each plot represents expected revenue in dollars. Lower boundary of shaded region is given by lower 95% confidence band about the conditional expectation of the second order statistic. Upper boundary of shaded region is given by upper 95% confidence band about the conditional expectation of the sum of the second order statistic and the scaled gap between second and third order statistics (scaled by  $2/\sqrt{n}$ ). Estimates are conditional on mileage (A), age (B), number of online bidders (C), and market thickness (D), and come from kernel regression with Epanechnikov kernel and rule-of-thumb bandwidth.

that a one-standard-deviation improvement in auctioneer performance raised revenue by \$348.<sup>32</sup> Hortaçsu, Matvos, Syverson, and Venkataraman (2013) used wholesale auto auction data to study the impact of auto firms' financial distress on prices, and found that a 1,000 point increase in credit default swap spreads decreased

<sup>32</sup>Table 2, row 8 of Lacetera, Larsen, Pope, and Sydnor (2013) shows that one standard deviation of the probability of sale among auctioneers is 0.023. Table 1 shows that the average price on cars conditional on sale is \$15,141. The product of these two numbers represents the increase in revenue which would be expected by employing an auctioneer who performs at one standard deviation above the mean, all else equal.

wholesale auction prices by approximately \$68.<sup>33</sup> The cars studied in these papers have average mileage and age values which correspond to a bidder exclusion effect of approximately \$200 in Figure 3.<sup>34</sup> Therefore, a comparison to the bidder exclusion effect suggests that information disclosing reports or high-performing auctioneers do more to increase revenues than would the use of an optimal reserve price. Similarly, swings in auction prices resulting from changes in firms' perceived financial stability are at least one-third of the size of price changes which the adoption of optimal reserve prices could generate.

## 7. CONCLUSION

We developed a simple procedure for estimating the causal effect of removing a random bidder on auction revenue—the bidder exclusion effect—without requiring instruments, a detailed structural model, or exogenous variation in the number of bidders. Our approach is robust to a wide range of auction settings. The bidder exclusion effect is useful in testing the independence of bidders' valuations and the number of bidders participating, allowing the research to distinguish between models of entry. Furthermore, we introduced a new empirical use for the theoretical results of Bulow and Klemperer (1996), demonstrating that the bidder exclusion effect can be used to bound the revenue improvements achievable through optimal auction design.

We believe that given the robustness and computational simplicity of the bidder exclusion effect it will prove a useful tool for empirical work in other settings as well. For example, under certain assumptions, the bidder exclusion effect provides a bound on the revenue loss a seller would face if bidders were to merge or

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<sup>33</sup>This number represents the price conditional on sale, but Hortaçsu, Matvos, Syverson, and Venkataraman (2013) focus primarily on cars which sell with very high probability (fleet/lease cars). Therefore, the effect on the price conditional on sale is a good approximation for overall effect of financial distress on revenue from bidders.

<sup>34</sup>The average odometer reading and age are 75,959 miles and five years old in Table 16 of Tadelis and Zettelmeyer (2011); 56,237 miles and 4.4. years in Table 1 of Lacetera, Larsen, Pope, and Sydnor (2013); and 44,270 miles in Table 4 of Hortaçsu, Matvos, Syverson, and Venkataraman (2013) (which does not report age).

collude. Appendix B.6 provides a brief discussion of this setting. The bidder exclusion effect also provides a simple specification check of whether bidders have independent private values. Briefly, under the assumption of IPV, one can invert the second-order statistic distribution to obtain an estimate of the underlying distribution of buyer valuations (Athey and Haile 2007) and simulate the revenue increase under an optimal reserve price. If the simulated revenue increase exceeds the bidder exclusion effect, the validity of either the assumption of independence or the assumption of private values—or both—is in question.

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## APPENDIX A. PROOFS

**A.1. Proof of Proposition 1.** Let  $(V_1, \dots, V_n)$  be a random vector of values distributed according to  $F^n$ . Let  $(\tilde{V}_1, \dots, \tilde{V}_{n-1})$  be a random vector obtained by dropping a value uniformly at random from  $(V_1, \dots, V_n)$ . With probability  $\frac{n-2}{n}$ ,  $\tilde{V}^{n-2:n-1} = V^{n-1:n}$  and revenue is unchanged, and with probability  $\frac{2}{n}$ ,  $\tilde{V}^{n-2:n-1} = V^{n-2:n}$  and revenue falls by  $V^{n-1:n} - V^{n-2:n}$ . The expected fall in revenue from dropping a bidder at random is therefore  $\frac{2}{n}E(V^{n-1:n} - V^{n-2:n}) = \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ .

**A.2. Proof of Proposition 2.**

$$\begin{aligned} \Delta(n) &= E(B^{n-1:n}) - E(B^{n-2:n-1}|n) \\ &= E(V^{n-1:n}) - E(V^{n-2:n-1}|n) \\ &= \frac{2}{n}E(V^{n-1:n} - V^{n-2:n}) \\ &\leq \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}). \end{aligned}$$

The first equality is true by definition of  $\Delta(n)$ . The second follows by Assumption 1. The third follows because with probability  $\frac{2}{n}$  dropping a value from  $(V_1, \dots, V_n)$  uniformly at random will cause the second-highest value to drop from  $V^{n-1:n}$  to  $V^{n-2:n}$ , and otherwise the second-highest value will be unchanged. The final equality holds by Assumption 1.

**A.3. Proof of Proposition 3.** Athey and Haile (2002), Theorem 9 proves that for button auction ascending auctions with symmetric common values, regardless of

the equilibrium played,  $E(B^{n-2:n-1|n}) > \frac{2}{n}E(B^{n-2:n}) + \frac{n-2}{n}E(B^{n-1:n})$ . This implies  $E(B^{n-1:n}) - E(B^{n-2:n-1|n}) < \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ , as required.

#### A.4. Proof of Corollary 1.

$$\begin{aligned} E(\Delta(N)) &\leq E\left(\frac{2}{N}(B^{N-1:N} - B^{N-2:N})\right) \\ &\leq \frac{2}{\underline{n}}E(B^{N-1:N} - B^{N-2:N}), \end{aligned}$$

where the first inequality follows by Proposition 2 if Assumption 1 holds and from Proposition 3 if Assumption 2 holds, and the second by  $N \geq \underline{n}$ .

**A.5. Proof of Proposition 4.** Theorem 1 of Bulow and Klemperer (1996) implies that the increase in expected revenue from using an optimal reserve price is less than the increase in expected revenue from adding another bidder, which by Assumption 3, is less than the bidder exclusion effect. The result follows from Proposition 2 if Assumption 1 holds and from Proposition 3 if Assumption 2 holds.

# FOR ONLINE PUBLICATION

## APPENDIX B. BINDING RESERVE PRICES, FIRST PRICES AUCTIONS, MONTE CARLO POWER SIMULATIONS, ASYMMETRIC BIDDERS, REVENUE CONCAVITY

**B.1. Binding Reserve Prices.** We consider ascending auctions with private values where bidders bid their values, and where there is a reserve price below which bids are not observed. We modify our notation accordingly:  $\Delta(n, r)$  denotes the fall in expected revenue produced by randomly excluding a bidder from  $n$  bidder auctions, when the reserve price is  $r$ .

**Proposition 5.** *In ascending auctions with private values and a reserve price of  $r$  where bidders bid their value, for all  $n > 2$  the bidder exclusion effect  $\Delta(n, r) = \frac{2}{n}E(B^{n-1:n} - \max(B^{n-2:n}, r)|r \leq B^{n-1:n}) \Pr(r \leq B^{n-1:n}) + \frac{1}{n}r \Pr(B^{n-1:n} < r \leq B^{n:n})$ .*

*Proof.* If  $r \leq B^{n-1:n}$ , then with probability  $\frac{2}{n}$  dropping a bidder at random will cause revenue to fall from  $B^{n-1:n}$  to  $\max(B^{n-2:n}, r)$ , so that in expectation revenue falls by  $\frac{2}{n}E(B^{n-1:n} - \max(B^{n-2:n}, r)|r \leq B^{n-1:n})$ . If  $B^{n-1:n} < r \leq B^{n:n}$ , then with probability  $\frac{1}{n}$  dropping a bidder at random will cause revenue to fall from  $r$  to 0. If  $B^{n:n} < r$ , then dropping a bidder at random will not change revenue. These observations imply the result.  $\square$

This expression for  $\Delta(n, r)$  can be estimated given observed data, as it does not depend on knowing the value of bids lower than the reserve price.

**B.1.1. Applications of the Bidder Exclusion Effect with Binding Reserve Prices.** When the reserve price equals  $r$  in both  $n$  and  $n - 1$  bidder auctions, the expected revenue difference between those auctions is

$$\begin{aligned} & E(\max(B^{n-1:n}, r)|r \leq B^{n:n}) \Pr(r \leq B^{n:n}) \\ & - E(\max(B^{n-2:n-1}, r)|r \leq B^{n-1:n-1}) \Pr(r \leq B^{n-1:n-1}). \end{aligned} \quad (16)$$

If valuations are independent of  $N$ , then  $F_{n-1}^n = F_{n-1}^{n-1} = F^{n-1}$  and hence expression (16) equals the expression for  $\Delta(n, r)$  of Proposition 5. As in Section 3.1, we can test this hypothesis with a  $t$ -test, where the test statistic is formed by replacing expectations by sample averages. This test is consistent against forms of selective entry which affect expected revenue, i.e. such that  $E(\max(B^{n-2:n-1|n}, r) | r \leq B^{n-1:n-1|n}) \Pr(r \leq B^{n-1:n-1|n}) \neq E(\max(B^{n-2:n-1}, r) | r \leq B^{n-1:n-1}) \Pr(r \leq B^{n-1:n-1})$ .

This test can be adapted to incorporate covariates. The null hypothesis is:

$$\begin{aligned} E \left( \mathbf{1}(r \leq B^{n-1:n}) \left( \frac{n-2}{n} B^{n-1:n} + \frac{2}{n} \max\{B^{n-2:n}, r\} \right) + \mathbf{1}(B^{n-1:n} < r \leq B^{n:n}) \frac{n-1}{n} r \middle| X \right) \\ = E(\mathbf{1}(r \leq B^{n-1:n-1}) \max\{B^{n-2:n-1}, r\} | X). \end{aligned} \quad (17)$$

This states that, conditional on covariates, revenue in  $n$  bidder auctions when one bidder is dropped at random equals revenue in  $n - 1$  bidder auctions. The regression-based test of Section 5 can be modified to test this restriction.

For the application to optimal mechanism design, we require an upper bound on  $\Delta(n, 0)$ . Using the fact that bids are non-negative, we have

$$\begin{aligned} \Delta(n, 0) &= \frac{2}{n} E(B^{n-1:n} - B^{n-2:n} | r \leq B^{n-2:n}) \Pr(r \leq B^{n-2:n}) \\ &\quad + \frac{2}{n} E(B^{n-1:n} - B^{n-2:n} | B^{n-2:n} < r \leq B^{n-1:n}) \Pr(B^{n-2:n} < r \leq B^{n-1:n}) \\ &\quad + \frac{2}{n} E(B^{n-1:n} - B^{n-2:n} | B^{n-1:n} < r) \Pr(B^{n-1:n} < r) \end{aligned} \quad (18)$$

$$\begin{aligned} &\leq \frac{2}{n} E(B^{n-1:n} - B^{n-2:n} | r \leq B^{n-2:n}) \Pr(r \leq B^{n-2:n}) \\ &\quad + \frac{2}{n} E(B^{n-1:n} | B^{n-2:n} < r \leq B^{n-1:n}) \Pr(B^{n-2:n} < r \leq B^{n-1:n}) \\ &\quad + \frac{2}{n} r \Pr(B^{n-1:n} < r) \end{aligned} \quad (19)$$

The terms in (19) do not depend on knowing the value of bids lower than the reserve price, and can be estimated given observed data.

**B.2. First Price Auctions.** We now give upper and lower bounds on the bidder exclusion effect in first price auctions with symmetric IPV, and symmetric conditionally independent private values (CIPV). Let  $b(V_i, F^n)$  denote bidder  $i$ 's equilibrium bid, as a function of his value,  $V_i$ , and the distribution of bidders' valuations,  $F^n$ . We assume  $V_i$  is continuously distributed on some interval  $[0, u]$ . In this section we use subscripts to make explicit the distribution with respect to which expectations are taken, e.g. expected revenue with no reserve price is  $E_{F^n}(b(V^{n:n}, F^n))$  in  $n$  bidder auctions and is  $E_{F_{n-1}^n}(b(V^{n-1:n-1}, F_{n-1}^n))$  when one of the  $n$  bidders is randomly excluded. The bidder exclusion effect is  $\Delta(n) \equiv E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n}(b(V^{n-1:n-1}, F_{n-1}^n))$ .

**Proposition 6.** *In first price auctions without a reserve price if i) bidders have symmetric independent private values, or ii) there is a random variable  $U$  common knowledge to bidders such that bidders have symmetric independent private values conditional on  $U$ , then  $E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n}(b(V^{n-1:n-1}, F^n)) < \Delta(n) < E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n}(b(V^{n-2:n-1}, F^n))$ .*

*Proof.* We first consider the case of symmetric independent private values. For the lower bound, note that in symmetric independent private values settings, equilibrium bids are strictly increasing in  $n$ :  $b(v_i, F^n) > b(v_i, F_{n-1}^n)$  (see, for example, Krishna (2009)). This implies  $E_{F_{n-1}^n} b(V^{n-1:n-1}, F^n) > E_{F_{n-1}^n} b(V^{n-1:n-1}, F_{n-1}^n)$ , and therefore  $E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n} b(V^{n-1:n-1}, F^n) < \Delta(n)$ .

For the upper bound, we have

$$E_{F_{n-1}^n}(b(V^{n-2:n-1}, F^n)) < E_{F_{n-1}^n}(V^{n-2:n-1}) \quad (20)$$

$$= E_{F_{n-1}^n}(b(V^{n-1:n-1}, F_{n-1}^n)). \quad (21)$$

The inequality holds because equilibrium bids are strictly less than values. The equality holds by revenue equivalence of first and second price auctions with symmetric independent private values. It follows that  $\Delta(n) < E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n}(b(V^{n-2:n-1}, F^n))$ .

If values are symmetric and CIPV, then because  $U$  is common knowledge to bidders these lower and upper bounds hold conditional on every realization of  $U$ , and therefore hold unconditionally, taking expectations with respect to  $U$ . The bounds thus extend to the conditionally independent private values case.  $\square$

The lower bound above is the expected fall in revenue in  $n$  bidder auctions when one bid is removed at random, assuming the good will be sold at a price equal to the highest of the remaining bids. The upper bound is the expected fall in revenue in  $n$  bidder auctions when one bid is removed at random, assuming the good will be sold at a price equal to the *second* highest of the remaining bids. The following corollary characterizes these bounds more explicitly in terms of the bids from  $n$  bidder auctions.

**Corollary 2.** *In first price auctions without a reserve price if i) bidders have symmetric independent private values, or ii) there is a random variable  $U$  common knowledge to bidders such that bidders have symmetric independent private values conditional on  $U$ , then*

$$\frac{1}{n} \left( E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-1:n}, F^n)) \right) < \Delta(n) \quad (22)$$

and

$$\Delta(n) < \frac{n-2}{n} \left( E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-1:n}, F^n)) \right) + \frac{2}{n} \left( E_{F^n}(b(V^{n:n}, F^n)) - E_{F^n}(b(V^{n-2:n}, F^n)) \right). \quad (23)$$

*Proof.* For the lower bound, note that with probability  $\frac{n-1}{n}$  dropping a bid at random will not change the highest bid, and with probability  $\frac{1}{n}$  the highest bid will drop from  $b(V^{n:n}, F^n)$  to  $b(V^{n-1:n}, F^n)$ . For the upper bound, note that with probability  $\frac{n-2}{n}$  the difference between the highest bid in the original sample and the second-highest bid after one bid has been dropped at random is  $b(V^{n:n}, F^n) - b(V^{n-1:n}, F^n)$ , and with probability  $\frac{2}{n}$  it is  $b(V^{n:n}, F^n) - b(V^{n-2:n}, F^n)$ .  $\square$

Several remarks on these bounds are in order. The lower bound holds under symmetric correlated private values too, as long as equilibrium bids are strictly increasing in  $n$ .<sup>35</sup> The upper bound holds if bidders are risk-averse, as first price auctions raise more revenue than ascending auctions, with symmetric risk-averse bidders in IPV environments (Riley and Samuelson 1981). In the CIPV case, if  $U$  is not common knowledge amongst bidders, then bidders' private information is correlated conditional on what they know at the time of bidding. This affects equilibrium bidding behavior and the argument of Proposition 6 does not hold. Finally, the upper bound of Proposition 6 can be replaced by  $E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^{n'}}(b(V^{n-2:n-1}, F^{n'}))$  for any  $n' > n - 1$ , as bids are below values in  $n'$  bidder auctions too. Consequently,  $\Delta(n) \leq E_{F^n}(b(V^{n:n}, F^n)) - \sup_{n'} E_{F_{n-1}^{n'}}(b(V^{n-2:n-1}, F^{n'}))$ .

**B.2.1. Applications of the Bidder Exclusion Effect in First Price Auctions.** As with ascending auctions, the bidder exclusion effect can be used to test for selective entry in first price auctions. Under the null hypothesis that entry is not selective, for all  $n \geq 2$ ,  $F_{n-1}^n = F_{n-1}^{n-1} = F^{n-1}$ . This implies that the bidder exclusion effect  $\Delta(n) \equiv E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^n}(b(V^{n-1:n-1}, F_{n-1}^n))$  equals  $E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^{n-1}}(b(V^{n-1:n-1}, F^{n-1}))$ . If the sample analog of  $E_{F^n}(b(V^{n:n}, F^n)) - E_{F_{n-1}^{n-1}}(b(V^{n-1:n-1}, F^{n-1}))$ —which is simply average revenue in  $n$  bidder auctions minus average revenue in  $n - 1$  bidder auctions—lies outside the sample analogs of the lower or upper bounds of Corollary 2, this is evidence against the null hypothesis. This test is consistent against violations of the null when values are “sufficiently” decreasing or increasing with  $n$ . Precisely, this is the case if  $E_{F_{n-1}^{n-1}}(b(V^{n-1:n-1}, F^{n-1})) > E_{F_{n-1}^n}(b(V^{n-1:n-1}, F^n))$  or  $E_{F_{n-1}^{n-1}}(b(V^{n-1:n-1}, F^{n-1})) < E_{F_{n-1}^n}(b(V^{n-2:n-1}, F^n))$ . Again, the regression-based test of Section 5 can be modified to test that the null hypothesis holds conditional on observable covariates, rather than unconditionally.

The application to optimal mechanism design also works for first price auctions. The Bulow-Klemperer theorem is stated for ascending auctions, but by revenue equivalence also applies to first price auctions when bidders have symmetric IPV

<sup>35</sup>Pinkse and Tan (2005) give conditions for this to hold.

(or symmetric CIPV). Thus Proposition 4 extends to first price auctions, where the upper bound on the effect on expected revenue of improving mechanism design is given by Corollary 2.

**B.3. Monte Carlo Power Simulations.** For some evidence on how powerful our test of selective entry is, we compare it to another test, which simply compares bidders' mean values in  $n$  and  $n + 1$  bidder auctions. This latter test requires the econometrician to observe all bidders' values. Relative to our test based on the bidder exclusion effect, it requires more data, and does not allow for low bidding. Furthermore, this mean comparison test is not actually feasible in ascending auctions in practice given that the highest bid is never observed.

In our simulation, there are 10 potential bidders, who have iid lognormal private values drawn from  $\ln N(\theta, 1)$ , where  $\theta$  is itself a random variable. All potential bidders see a common signal  $\delta = \theta + \epsilon$ , and Bayes update on the value of  $\theta$  given their observation of  $\delta$ . The random variables  $(\delta, \theta, \epsilon)$  are jointly normally distributed:

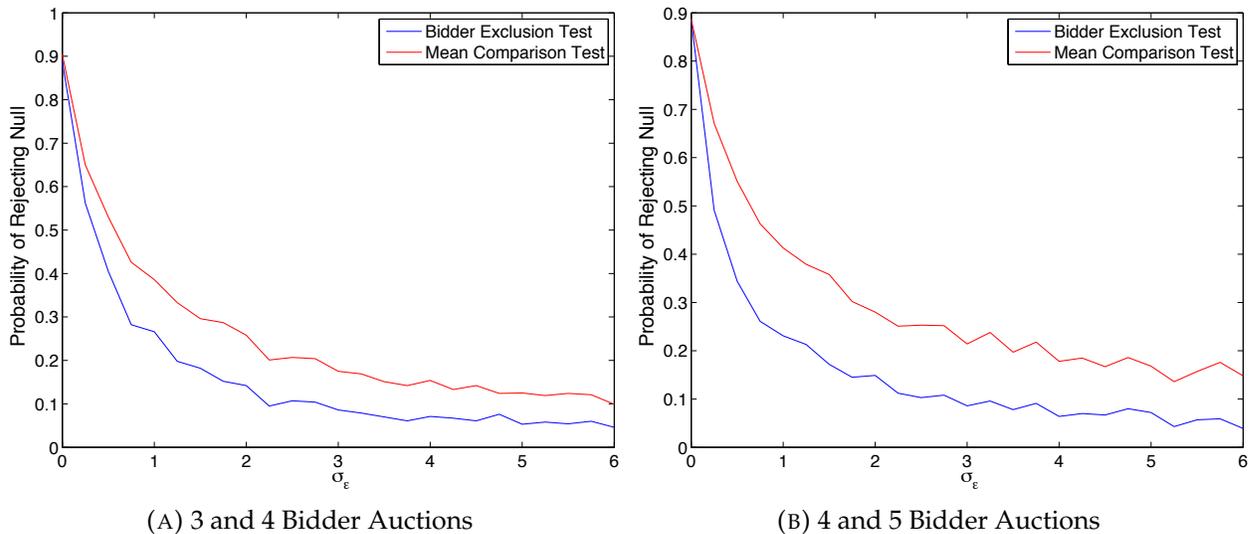
$$\begin{pmatrix} \delta \\ \theta \\ \epsilon \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 + \sigma_\epsilon^2 & 1 & \sigma_\epsilon^2 \\ 1 & 1 & 0 \\ \sigma_\epsilon^2 & 0 & \sigma_\epsilon^2 \end{pmatrix} \right). \quad (24)$$

As  $\sigma_\epsilon$  increases, the ratio of noise to signal increases, and the variable  $\delta$  becomes less informative about the variable  $\theta$ . To learn their value and bid in the ascending auction, potential bidders must pay an entry cost of 0.5. They play mixed entry strategies, entering with a probability  $p$  that depends on  $\delta$ . In the limit as  $\sigma_\epsilon \rightarrow \infty$ , the signal  $\delta$  is uninformative about  $\theta$  and the entry probability  $p$  no longer varies with  $\delta$ . This limiting case corresponds to the entry model of Levin and Smith (1994).

For each  $\sigma_\epsilon \in \{1, 1.25, 1.5, \dots, 7\}$ , and for  $n \in \{3, 4\}$ , we generate 1,000 datasets with auctions in which  $n$  or  $n + 1$  bidders choose to enter. Each dataset contains 500  $n$  bidder auctions and 500  $n + 1$  bidder auctions. We calculate the probability of rejecting the null hypothesis of no selective entry at the 5% level over the 1,000

datasets, for each value of  $\sigma_\epsilon$  and  $n$ , and for both bidder exclusion test, and the comparison of means test. Figure 4 shows the rejection probabilities as a function of  $\sigma_\epsilon$ . The comparison of means uses more data (in the case of Panel (A), all three bids from  $n = 3$  auctions and all four bids from  $n = 4$  auctions; and, in the case of Panel (B), all four bids from  $n = 4$  auctions and all five bids from  $n = 5$  auctions), and is more powerful. The simulation results suggest that when not all bidders' values are observed and the comparison of means test is infeasible (as in ascending auctions), the bidder exclusion based test is a reasonably powerful alternative, especially when entry is more selective (corresponding in this model to low values of  $\sigma_\epsilon$ .)

FIGURE 4. Monte Carlo Power Comparison



Notes: Figures show the simulated probability of rejecting the null hypothesis of no selective entry for various levels of entry selectiveness (with greater  $\sigma_\epsilon$  corresponding to less selective entry), for two tests: one based on a comparison of means between  $n$  and  $n + 1$  bidder auctions, and one based on the bidder exclusion effect computed on  $n$  and  $n + 1$  bidder auctions. The left panel shows the case of  $n = 3$ , and the right panel shows the case of  $n = 4$ .

**B.4. Asymmetric Bidders.** We give sufficient conditions for valuations to be independent of  $n$  with asymmetric bidders and private values, following the setup of Coey, Larsen, and Sweeney (2014). Let  $\mathbb{N}$  be the full set of potential bidders.

Let  $\mathcal{P}$  be a random vector representing the identities of bidders participating in an auction, with realizations  $P \subset \mathbb{N}$ . Let  $N$  be a random variable representing the number of bidders participating in an auction, with realizations  $n \in \mathbb{N}$ . When necessary to clarify the number of bidders in a set of participating bidders, we let  $P_n$  denote an arbitrary set of  $n$  participating bidders. Define  $F^P$  to be the joint distribution of  $(V_i)_{i \in P}$  when  $P$  is the set of participating bidders.<sup>36</sup> As before,  $F^n$  represents the joint distribution of values conditional on there being  $n$  entrants, but unconditional on the set of participants. Therefore,  $F^n(v_1 \dots v_n) = \sum_{P_n \subset \mathbb{N}} \Pr(\mathcal{P} = P_n | N = n) F^{P_n}(v_1 \dots v_n)$ . For  $P' \subset P$ , let  $F^{P'|P}$  denote the joint distribution of  $(V_i)_{i \in P'}$  in auctions where  $P$  is the set of participants. Finally, let  $F_m^P$  denote the joint distribution of values of  $m$  bidders drawn uniformly at random without replacement from  $P$ , when the set  $P$  enters.

**Definition 1.** *Valuations are independent of supersets if for all  $P' \subset P$ ,  $F^{P'|P} = F^{P'}$ .*

**Definition 2.** *Bidder identities are independent of  $N$  if, for all  $P_n$ ,  $\Pr(\mathcal{P} = P_n | N = n) = \frac{1}{n+1} \sum_{P_{n+1} \supset P_n} \Pr(\mathcal{P} = P_{n+1} | N = n+1)$ .*

These definitions describe different kinds of exogeneity. Definition 1 requires that conditional on some set of bidders participating, those bidders' values are independent of which other bidders participate (what Athey and Haile (2002) refer to as exogenous participation). Definition 2 requires that the distribution of participating bidder identities in  $n$  bidder auctions is just like the distribution of participating bidder identities in  $n+1$  bidder auctions, with one bidder randomly removed.<sup>37</sup> It restricts who participates, but not what their values are. The following proposition shows that together these conditions imply that valuations are independent of  $N$ . Consequently, evidence of selective entry suggests either that

<sup>36</sup>We adopt the convention that bidders are ordered according to their identities, i.e. if  $P = \{2, 5, 12\}$  then  $F^P$  is the joint distribution of  $(V_2, V_5, V_{12})$ , rather than, for example, the joint distribution of  $(V_5, V_2, V_{12})$ .

<sup>37</sup>To see this, fix  $P_n$  and note that for each  $P_{n+1} \supset P_n$ ,  $P_n$  is obtained by dropping the bidder  $P_{n+1} \setminus P_n$  from  $P_{n+1}$ . When bidders are dropped uniformly at random, this occurs with probability  $\frac{1}{n+1}$ .

valuations are not independent of supersets, or that bidder identities are not independent of  $N$ .

**Proposition 7.** *If valuations are independent of supersets and bidder identities are independent of  $N$ , then valuations are independent of  $N$ .*

*Proof.* The proof follows Coey, Larsen, and Sweeney (2014), Lemma 3. It suffices to prove that  $F_m^n = F_m^{n+1}$  for any  $n \geq m$ .

$$\begin{aligned}
F_m^n(v) &= \sum_{P_n} \Pr(\mathcal{P} = P_n | N = n) F_m^{P_n}(v) \\
&= \sum_{P_n} \sum_{P_{n+1} \supset P_n} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n}(v) \\
&= \sum_{P_{n+1}} \sum_{P_n \subset P_{n+1}} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n}(v) \\
&= \sum_{P_{n+1}} \sum_{P_n \subset P_{n+1}} \frac{1}{n+1} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_n | P_{n+1}}(v) \\
&= \sum_{P_{n+1}} \Pr(\mathcal{P} = P_{n+1} | N = n+1) F_m^{P_{n+1}}(v) \\
&= F_m^{n+1}(v)
\end{aligned}$$

The second equality follows because bidder identities are independent of  $N$ . The fourth equality follows because  $F^{P_n} = F^{P_n | P_{n+1}}$ , as valuations are independent of supersets. The fifth equality follows because randomly selecting  $m$  bidders from  $n+1$  bidders is the same as randomly selecting  $n$  bidders from  $n+1$  bidders, and then randomly selecting  $m$  bidders from those  $n$  bidders.  $\square$

**B.5. Revenue Concavity.** In this section we give conditions for concavity of expected revenue in symmetric IPV (and CIPV) auctions. In the symmetric IPV case, a sufficient condition for concavity of expected revenue is for bidders marginal revenue to be increasing in their values. This is a special case of a result established by Dughmi, Roughgarden, and Sundararajan (2012).

**Proposition 8.** *In ascending auctions with symmetric independent private values and no reserve price, if bidders' marginal revenue is increasing in their values then expected revenue is concave in the number of bidders.*

*Proof.* We first prove that if  $Z_1, \dots, Z_{n+1}$  are iid random variables,

$$\begin{aligned} E(\max\{Z_1, \dots, Z_n\}) - E(\max\{Z_1, \dots, Z_{n-1}\}) &\geq \\ E(\max\{Z_1, \dots, Z_{n+1}\}) - E(\max\{Z_1, \dots, Z_n\}). \end{aligned} \quad (25)$$

For any  $z_1, \dots, z_{n+1} \in \mathbb{R}^{n+1}$ ,

$$\max\{0, z_{n+1} - \max\{z_1, \dots, z_{n-1}\}\} \geq \max\{0, z_{n+1} - \max\{z_1, \dots, z_n\}\}, \quad (26)$$

implying

$$\max\{z_1, \dots, z_{n-1}, z_{n+1}\} - \max\{z_1, \dots, z_{n-1}\} \geq \max\{z_1, \dots, z_{n+1}\} - \max\{z_1, \dots, z_n\}. \quad (27)$$

Consequently for any random variables  $Z_1, \dots, Z_{n+1}$ ,

$$\begin{aligned} E(\max\{Z_1, \dots, Z_{n-1}, Z_{n+1}\}) - E(\max\{Z_1, \dots, Z_{n-1}\}) &\geq \\ E(\max\{Z_1, \dots, Z_{n+1}\}) - E(\max\{Z_1, \dots, Z_n\}), \end{aligned} \quad (28)$$

because (27) holds for every realization  $z_1, \dots, z_{n+1}$  of  $Z_1, \dots, Z_{n+1}$ . If the  $Z_i$  are iid, then  $E(\max\{Z_1, \dots, Z_{n-1}, Z_{n+1}\}) = E(\max\{Z_1, \dots, Z_n\})$ , yielding (25).

The expected revenue from any mechanism is the expected marginal revenue of the winning bidder (Myerson 1981). Ascending auctions assign the good to the bidder with the highest valuation, and therefore highest marginal revenue, because marginal revenue is increasing in valuations. It follows that expected revenue with  $n$  bidders is  $E(\max\{MR(V_1), \dots, MR(V_n)\})$ . As  $MR(V_1), \dots, MR(V_{n+1})$  are iid random variables, we have

$$\begin{aligned} E(\max\{MR(V_1), \dots, MR(V_n)\}) - E(\max\{MR(V_1), \dots, MR(V_{n-1})\}) &\geq \\ E(\max\{MR(V_1), \dots, MR(V_{n+1})\}) - E(\max\{MR(V_1), \dots, MR(V_n)\}), \end{aligned} \quad (29)$$

implying that expected revenue is concave in the number of bidders.  $\square$

When there exists a random variable  $U$  such that bidder values  $V_1, \dots, V_n$  are iid conditional on  $U$ , if marginal revenue is increasing in values conditional on each realization of  $U$ , then Proposition 8 applies conditional on each realization of  $U$ . Taking expectations over  $U$ , it follows that expected revenue is concave in CIPV environments if bidders' marginal revenue curves are increasing in values conditional on each value of  $U$ .

**B.6. Bidder Collusion and Mergers.** The bidder exclusion effect may also be used to bound above the expected fall in revenue resulting from mergers or collusion between bidders. Consider first the case of two random bidders forming a bidding ring.<sup>38</sup> This bidding ring excludes one bidder from the auction. If the bidder which is excluded is chosen randomly between the bidders in the ring, and if the bidders were randomly matched when forming a ring, the setting is equivalent to one in which a bidder is randomly—and legally—excluded from the auction. Under the conditions of Proposition 1, the seller's losses are given by  $\Delta(n) = \Gamma(n) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$ . In the case of low bidding or symmetric common values, the seller's losses are bounded above by  $\Gamma(n)$ .

The bidder exclusion effect also provides a bound on revenue losses due to mergers between bidders with private values. This argument requires additional steps given that mergers may result in increased production efficiencies and hence an increased willingness to pay of the merged entity. If the merger does not decrease bidders' values, revenue should be at least as great after a merger between two random bidders as after excluding a random bidder. We state the result for the general case of asymmetric bidders. In auctions where  $P_n \subset \mathbb{N}$  is the set of participants, let  $(V_i)_{i \in P_n}$  denote values before a merger. When bidders  $i$  and  $j$  merge, let  $M_k$  denote the value of bidder  $k \neq i, j$ , and  $M_{i,j}$  denote the value of the merged entity.

<sup>38</sup>A bidding ring is a group of bidders in an auction who collude in order to keep prices down.

**Assumption 4.** *Mergers do not decrease values: When bidders  $i$  and  $j$  merge,  $M_k \geq V_k$  for all  $k \neq i, j$ , and  $M_{i,j} \geq \max\{V_i, V_j\}$ .*

**Proposition 9.** *In ascending auctions with private values and no reserve price where Assumptions 1 and 4 hold, then for all  $n > 2$ ,  $\Delta(n)$  is greater than or equal to the decrease in expected revenue from two randomly selected bidders merging.*

*Proof.* We follow the notation from Appendix B.4. In addition, in auctions where  $P_n$  is the set of entrants, if  $i, j \in P_n$  were to merge we let  $F_{n-2:n-1}^{P_n^{i \sim j}}$  denote the distribution of the second-highest value of the unmerged and merged entities, i.e. of  $\{M_k\}_{k \neq i,j} \cup \{M_{i,j}\}$ .

Consider an  $n$  bidder auction with participants  $P_n$ . For any  $v \in \mathbb{R}$ ,

$$\begin{aligned} F_{n-2:n-1}^{P_n}(v) &= \frac{1}{n} \sum_{i \in P_n} F_{n-2:n-1}^{P_n \setminus \{i\} | P_n}(v) \\ &\geq \frac{1}{n} \sum_{i \in P_n} \frac{1}{n-1} \sum_{j \in P_n \setminus i} F_{n-2:n-1}^{P_n^{i \sim j}}(v) \end{aligned}$$

The first equality follows from the definition of  $F_{n-2:n-1}^{P_n}$ . The second equality follows because by the assumption that mergers do not decrease values, for every  $j \neq i$ , the second-highest term in  $\{V_k\}_{k \neq i}$  is less than the second-highest term in  $\{M_k\}_{k \neq i,j} \cup \{M_{i,j}\}$ . Therefore the distribution of the second-highest value after dropping one of the  $n$  bidders in  $P_n$  uniformly at random is first order stochastically dominated by the distribution of the second-highest value when two bidders are selected to merge at random. It follows that the bidder exclusion effect is greater than the decrease in expected revenue if two randomly selected bidders merge. This holds conditional on each set of entrants  $P_n$ , so it also holds unconditionally, averaging over all  $P_n$ , yielding the result.  $\square$

In practice it is likely that bidders are not randomly matched to merge or to form bidding rings, and in these cases the loss in expected revenue may exceed than the bidder exclusion effect.<sup>39</sup> However, the expected loss in seller revenue

<sup>39</sup>Consider the example of bidders 1,2 and 3, with values 0,1, and 1. Assume bidders 2 and 3 merge, forming a bidder with value 1. The drop in revenue from the merger is 1, which is larger

between a no-reserve auction with no collusion or mergers and one with collusion or a merger of two non-random bidders is bounded above by the expectation of the gap between the second and third order statistics of bids,  $E(B^{n-1:n} - B^{n-2:n})$ . If the number of bidders is unobserved, the average revenue loss due can be bounded by averaging over all (unobserved) realizations of  $N$ .<sup>40</sup>

In the timber auctions example of Section 5—a setting in which Baldwin, Marshall, and Richard (1997) point out that accusations of collusion are historically quite common—a bound on the fall in revenue from two random bidders colluding or merging is given by the bidder exclusion effect estimate of 13%. A bound on the loss in seller revenue when instead two non-random bidders form a bidding ring can be recovered simply by scaling the values in Figure 1 by a factor of  $n/2$ , yielding the expected gap between the second and third order statistics.

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than the bidder exclusion effect of  $\frac{2}{3}$ . Similarly, if bidders know their valuations prior to forming a bidding ring, bidders among the  $n - 2$  lowest values would have little incentive to collude among themselves. Collusion among the two highest bidders, on the other hand, would lower the auction price to the third order statistic.

<sup>40</sup>This approach to bounding the loss due to collusion or mergers does not allow one to estimate the loss due to collusive actions or mergers which have already occurred, but rather counterfactual impacts which *additional* collusion or mergers would entail. Also, the above approach extends easily to bidding rings or mergers of size larger than two.