The Efficiency of Real-World Bargaining:
Evidence from Wholesale Used-Auto Auctions

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Abstract

This study quantifies the efficiency of a real-world bargaining game with two-sided incomplete information. Myerson and Satterthwaite (1983) and Williams (1987) derived the theoretical ex-ante efficient frontier for bilateral trade under two-sided uncertainty and demonstrated that it falls short of ex-post efficiency, but little is known about how well bargaining performs in practice relative to the frontier. Using about 270,000 sequences of a game of alternating-offer bargaining following an ascending auction in the wholesale used-car industry, this study estimates (or bounds) distributions of buyer and seller valuations and estimates where realized bargaining outcomes lie relative to efficient outcomes. Results demonstrate that the ex-ante and ex-post efficient outcomes are close to one another, but that the real bargaining falls short of the efficient frontier, suggesting that the bargaining is indeed inefficient but that this inefficiency is not largely due to the information constraints highlighted in Myerson and Satterthwaite (1983).

Keywords: Bargaining, incomplete information, Myerson-Satterthwaite Theorem, efficiency, empirical market design, alternating-offers

JEL Classification: C57, C78, D44, D47, D82

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1 Introduction

Whether haggling in an open-street market, deciding upon prices between an upstream supplier and downstream producer, or negotiating a corporate takeover deal, bargaining between a buyer and seller is one of the oldest and most common forms of transaction. When both parties have incomplete information, it is known that equilibrium outcomes are difficult to characterize.\footnote{Fudenberg and Tirole (1991) stated, “The theory of bargaining under incomplete information is currently more a series of examples than a coherent set of results. This is unfortunate because bargaining derives much of its interest from incomplete information.” Fudenberg, Levine, and Tirole (1985) similarly commented “We fear that in this case [of two-sided incomplete information], few generalizations will be possible, and that even for convenient specifications of the functional form of the distribution of valuations, the problem of characterizing the equilibria will be quite difficult.” Very little work—theoretical or empirical—on bargaining with two-sided uncertainty and continuous valuations has been published before or after this time.} Myerson and Satterthwaite (1983) demonstrated that ex-post efficiency is not possible, and theoretical ex-ante efficiency bounds are derived in Myerson and Satterthwaite (1983) and Williams (1987), but it is unknown how well real-world bargaining performs relative to these bounds. Williams (1987) emphasized that “little is known about whether or not these limits can be achieved with ‘realistic’ bargaining procedures.” This paper is the first attempt to bring data to this question in order to quantify the efficiency of real-world bargaining with two-sided incomplete information. I develop a framework to estimate distributions of private valuations of both buyers and sellers who participate in bargaining in the wholesale used-auto market. I then map these primitives into results from the theoretical mechanism design literature in order to measure the efficiency of bargaining relative to the ex-post and ex-ante efficient frontiers.

The recent surge in empirical bargaining research has focused almost exclusively on complete-information, non-strategic bargaining (e.g. some form of Nash bargaining), which imposes the structure that the buyer and seller each know with exactness the reservation value of the opposing party. This is in stark contrast with the empirical auctions literature, which for the past several decades has focused on studying questions in settings of incomplete information and strategic behavior of players. In this paper I extend ideas behind existing tools used in the empirical auctions literature to enable a structural study of strategic bargaining with incomplete information. I take advantage of a setting—the wholesale used-car market—where alternating-offer bargaining takes place after an auction, as in many business-to-business settings.\footnote{Huh and Park (2010) explained that an auction followed by bargaining is common in many other business-to-business transactions, enumerating a variety of examples. Auctions followed by bargaining have also been noted in the earlier auto industry work of Genesove 1991, as well as in spectrum auctions (Lenard, White, and Riso 2009), corporate takeovers (Burkart 1995; Huh and Park 2010), procurement (Wang 2000; An and Tang 2016; Huh and Park 2010), timber sales (Elyakime, Laffont, Loisel, and Vuong 1997), and elsewhere. Indeed, many business-to-business settings (e.g. procuring subcontractors), government settings (e.g. procuring services), or private settings (e.g. selling a home), consist of one party collecting initial bids from a number of different bidders in an auction-like stage and then selecting a single bidder with whom to negotiate a final deal.} While the alternating-offer bargaining observed in this market is clearly not representative of all bargaining situations observed in practice, it is arguably an example of the “realistic bargaining procedures” to which Williams (1987) referred.

The assumption of non-strategic, complete-information bargaining has allowed the existing empirical bargaining literature to make a variety of novel contributions studying difficult questions. One question that has not been addressed, however, is whether bargaining is actually efficient in practice; e.g. whether
parties agree whenever the buyer values the good or service more than the seller, as would be the case in a first-best world, or, if not, whether the bargaining outcome at least comes close to the ex-ante efficiency bound, as it should in a second-best world. These questions cannot be addressed well using a standard non-strategic, complete-information framework, as this framework presumes knowledge a priori that bargaining is perfectly efficient: in such a world, given that the buyer and seller know one another’s values, it is assumed that bargaining is never even attempted unless agreement is the efficient outcome. Treating bargaining as efficient, if it is in fact not, can result in incorrect market design recommendations or misleading calculations for welfare, pricing, or bargaining power. The data and methodology used in this paper allow me to study whether or not bargaining is actually efficient, rather than assuming it to be so.

Although the question of whether real-world bargaining is efficient has not been addressed in the empirical literature, it is indeed an empirical question—one that theory alone cannot address. Theoretical work by Chatterjee and Samuelson (1983), Satterthwaite and Williams (1989), Ausubel and Deneckere (1993), and Ausubel, Cranton, and Deneckere (2002) demonstrated that certain knife-edge or limiting cases of theoretical bargaining games may reach the efficient frontier, but is unknown whether this efficiency is achieved in practice. Also, while the large theoretical literature on incomplete-information strategic bargaining has yielded valuable insights, it has done so primarily through focusing on special cases rather than the full, two-sided incomplete information setting with a continuum of buyer/seller values (see Appendix Table A1 for a summary of this literature). Only two previous theoretical papers study extensive-form bargaining games with two-sided incomplete information about players’ valuations where both parties can make offers; these papers are Perry (1986) and Cranton (1992), both of which focus on a particular equilibrium or game format that does not match behavior in my data (in the former the game ends immediately in equilibrium and in the latter the game starts with a war of attrition). No canonical model exists of the general setting examined in this paper.

To overcome the theoretical difficulties posed by incomplete-information bargaining, and to avoid the assumption of non-strategic, complete-information bargaining used elsewhere in the empirical literature, I take advantage of a unique, new dataset, consisting of several hundred thousand sequences of back-and-forth bargaining offers between buyers and sellers at wholesale used-car auctions, a large market of business-to-business transactions where new and used-car dealers buy vehicles from other dealers as well as from rental companies and banks. This industry represents the backbone of the supply side of the US used-car market, with 15 million cars annually passing through its lanes, totaling $80 billion in sales. Throughout much of the industry, auction houses employ the following mechanism: a secret reserve price set by the seller followed by an ascending price auction, which, when the secret reserve price is not met, is followed by alternating-offer bargaining, with each offer recorded by the auction house. If the auction price exceeds the reserve price (which only occurs 11–33% of the time), the trade automatically goes through; it is in the bargaining stage of the game that trade can fail, and thus understanding the efficiency of the bilateral bargaining is key to understanding the efficiency of the overall market. Industry wide, about 40% of sales attempts result in no trade. This large portion of failed trades in this market motivates the question of why these trades fail; these could be cases where the seller values the good less than the buyer, and hence no trade should occur even in a fully efficient world; these could be cases where gains from trade do exist, but trade fails due to the
information constraints highlighted by Myerson and Satterthwaite (1983); or these may be cases where trade fails because of the particular bargaining protocol employed or the particular equilibrium played. These questions are the focus of this paper.

The fact that the bargaining takes place after an auction is not an unfortunate characteristic of the data, but is instead a useful feature aiding in the study of whether bargaining is efficient. Indeed, it is quite difficult to make any progress studying incomplete-information bargaining empirically without some kind of special lever. Specifically, given that no canonical model or even characterization of equilibria exists for two-sided incomplete-information bargaining games, theory provides no obvious mapping from observables to primitives in incomplete-information settings. In my setting, however, the distribution of buyer valuations can be estimated using the auction data, and bounds on the distribution of seller valuations can be obtained using sellers’ responses to the first bargaining offer (the auction price). Each of these approaches imposes only minimal assumptions on the structure or equilibrium of the bargaining game, and then bargaining outcomes observed in the data can be compared to efficient outcomes that are simulated from the estimated valuation distributions. This feature of not explicitly solving for or assuming a specific theoretical equilibrium of the bargaining game is particularly useful because it is well known that multiple (qualitatively different) equilibria often exist in two-sided incomplete-information games, some of which can be very efficient and some of which can be very inefficient (see Ausubel and Deneckere 1993). Assuming a given equilibrium for the bargaining game would likely impose a particular level of efficiency, just as would be the case in assuming complete-information bargaining, making it difficult to let the data speak as to whether bargaining is efficient. The precise effect of the auction on the bilateral bargaining game is twofold: first, the auction leads to a truncation of the lower bound of the support of types who bargain. Thus, the game I study is analogous to a setting of bargaining alone where the lower bound of the support of the types in the bargaining game differs across realizations of the game in a tractable manner determined by the realization of the auction price, and the results herein average over these realizations.\(^3\) Second, the auction price provides a lower bound on achievable prices in the bargaining game, similar to how a list price would provide an upper bound in many other real-world bargaining games, such as haggling over a car at a retail outlet.

After a brief introduction to the industry, Section 2 discusses the data. The data comes from several different auction houses from 2007 to 2010, containing detailed information on each car as well as the actions taken by sellers and buyers in each stage of the game for over 270,000 games. The data is unique in being the first bargaining dataset of this volume and detail to be analyzed in the literature, containing not only final negotiated prices on consummated deals, as most empirical bargaining datasets likely would, but also all of the back-and-forth bargaining offers between negotiating parties, and even all cases where bargaining failed to yield an agreement. The data is broken down into two main samples: cars sold by used and new-car dealers (which I refer to as the dealers sample), and cars sold by large institutions, such as rental companies,\(^3\) This would not be the case if the auction were a first price auction instead of an ascending auction. If the pre-bargaining stage were a first price auction, the auction would be affected by the presence of the bargaining in an intractable manner and vice versa (see discussion in Elyakime, Laffont, Loisel, and Vuong 1997). Therefore, most existing empirical studies of first price auctions with a post-auction bargaining element (Elyakime, Laffont, Loisel, and Vuong 1997; An and Tang 2016) adopt a hybrid model where the auction takes place under incomplete information but the bargaining takes place under complete information. See Larsen and Zhang (2017) for an alternative approach for such settings.
banks, and fleet companies (which I refer to as the fleet/lease sample).

I lay out a simple model in Section 3 that describes the game played between buyers and sellers in this industry. I do not explicitly solve for or estimate all features of an equilibrium of this game; indeed, doing so would be infeasible given the fact that, first, unlike many settings of an auction alone, no clear mapping exists here between observable outcomes and underlying theoretical primitives; and, second, empirical methods for dynamic games are not optimized for handling the type of persistent asymmetric information introduced by players' valuations in an alternating-offer bargaining game (which would require the econometrician to keep track of the full history of player actions). I use this model to demonstrate several theoretical properties that aid in estimating model primitives.

These model properties are, first, that game-level heterogeneity affects the game’s outcomes in a tractable manner. Second, the auction price—in addition to playing the role of the first offer in bargaining—will affect the lower bound of the support of the types who bargain in a tractable fashion, allowing me to isolate the bargaining game from the auction. Third, bidders will not have incentive to bid untruthfully in the auction, yielding an argument for identification of the distribution of buyer valuations. Fourth, the distribution of seller valuations is partially identified by revealed preference arguments. This approach to bounding the distribution of seller valuations is similar in spirit to Haile and Tamer (2003), using inequalities implied by very basic assumptions about players’ rationality to learn about model primitives without imposing a complete model of the game or solving for an equilibrium. The bargaining setting is more complicated than the auction setting in Haile and Tamer (2003), however, in that it is not necessarily the case that an upper and lower bound on the valuation is observed for each individual observation in the data; instead, I obtain conditional probability statements that bound the whole distribution of valuations. This methodology is new to the empirical bargaining literature, and can be applied in alternating-offer bargaining settings, regardless of whether the bargaining follows an auction, when the econometrician observes the first offer and the response to that offer.

In order to compute expected gains from trade to measure efficiency in the real-world mechanism, it is necessary to know not only the distributions of valuations but also which player types trade and which do not. While it is infeasible to solve explicitly for an equilibrium of the game, I demonstrate that the direct-revelation mechanism corresponding to this unspecified equilibrium of the real-world game is identified in the data. This argument relies on the Revelation Principle, which has been exploited widely in the theoretical mechanism design literature. Applying this concept to my empirical setting allows me to avoid solving for or characterizing the actual equilibrium of the game and instead work with the direct mechanism corresponding to this game as implied by the data.

Section 4 describes each step of my estimation approach, which closely parallels the model properties described above. After controlling for observable heterogeneity, I use a likelihood approach to deconvolve unobserved game-level heterogeneity and estimate buyer valuations using an order statistic inversion. I then estimate bounds on seller valuations, exploiting revealed preferences inequalities. I estimate the mapping between auction prices and the lower bounds of the support of buyer and seller types in the bargaining game as well as the mapping corresponding to the direct revelation mechanism of the game. These mappings and the seller valuation bounds can each be estimated using flexible spline approximations within a constrained
least squares framework.

After estimating these structural objects, Section 5 describes how I compute welfare under counterfactual
efficient bargaining mechanisms. Here I apply results from the mechanism design theoretical literature. The
counterfactual efficient bargaining mechanisms I compute are solutions to linear programs that are functions
of the distributions of buyer and seller valuations. Therefore, the estimated distributions from Section 4 are
crucial in solving for these mechanisms. I also derive useful monotonicity properties that allow me to obtain
bounds on welfare measures using bounds on the distribution of seller valuations.

In Section 6 I then compare outcomes under efficient bargaining to those under the real bargaining to
measure the relative efficiency. The first type of efficiency loss I measure is the loss due solely to incomplete
information. Ideally, a buyer and seller should trade whenever the buyer values the good more than the seller
(ex-post efficient trade). However, the celebrated Myerson and Satterthwaite (1983) Theorem demonstrated
that, when the supports of buyer and seller types overlap, there does not exist any incentive-compatible,
individually rational bargaining mechanism that is ex-post efficient and that also satisfies an ex-ante balanced
budget. Myerson and Satterthwaite (1983) went on to derive the second-best mechanism that places equal
weight on buyer and seller surplus and that is ex-ante efficient and satisfies incentive compatibility, individual
rationality, and ex-ante budget balance. Williams (1987) then derived the entire ex-ante efficient frontier,
for any relative weights on buyer and seller surplus. The gap between this ex-ante efficient frontier and the
ex-post efficient frontier represents an efficiency loss due to the presence of incomplete information. Using
the estimated distributions, I find that incomplete information per se need not be a huge problem in this
market: The second-best mechanism achieves about the same range of expected surplus as the first-best
mechanism, and any trades that the second-best mechanism fails to capture in this market appear to be
low-surplus trades.

The second type of efficiency loss I measure is to compare the real-world bargaining to the ex-ante efficient
frontier. The real bargaining may fall short of this frontier for several reasons. First, unlike the mechanisms
discussed in Myerson and Satterthwaite (1983) and Williams (1987), real-world bargaining—such as the
alternating-offer bargaining examined here—with two-sided uncertainty has no clear equilibrium predictions
due to signaling by both parties, and many qualitatively different equilibria exist. The equilibrium play
observed in the data may correspond to a particularly inefficient equilibrium. Second, it may the case that
the alternating-offer protocol used in this market is inefficient regardless of the equilibrium played; it may
indeed be the case that a more efficient, practical protocol exists. Third, it may be that the real bargaining
falls short of the theoretically efficient benchmark because that benchmark fails to satisfy other constraints
that real-world bargaining satisfies, such as that of being a mechanism that is not defined in terms of players'
beliefs (i.e. a mechanism that is detail-free in the sense of Chung and Ely 2007 and the related literature)
and the implementation of which does not rely on strong common knowledge assumptions, as advocated by
the influential Wilson doctrine (Wilson 1986). Because I place very little structure on the bargaining game,
my analysis allows for any of these three cases to occur. Any of these cases would lead to a gap between the
real outcome and that of an ex-ante efficient mechanism.

\footnote{Following the taxonomy of Holmström and Myerson (1983), the term \textit{ex-ante} refers to before the players learn their values and before the outcome of the bargaining is realized, and the term \textit{ex-post} refers to after the valuations and bargaining outcomes are realized.}
My findings indicate that the real bargaining falls short of the ex-ante efficient frontier by $152–476 for cars sold by dealers and by $197–609 for cars sold by large fleet or lease institutions. The losses of the real-world mechanism compared to the ex-post efficient frontier are similar in magnitude. These losses represent 11–16% of the first-best gains from trade for cars sold by dealers and 10–18% for cars sold by large institutions. In terms of the probability of trade, the real-world bargaining falls short of the ex-post efficient outcome by 0.083–0.141 for cars sold by dealers and by 0.092–0.139 for cars sold by fleet and lease sellers. This implies that about 8–14% of feasible trades (cases where the buyer indeed values the good more than the seller) fail. The key takeaway of my analysis is that the real-world bargaining is indeed inefficient and that this inefficiency is largely due to factors other than the information constraints highlighted in Myerson and Satterthwaite (1983).

To my knowledge, this paper is the first to bring data to the framework of Myerson and Satterthwaite (1983). As highlighted above, a number of recent papers in the Industrial Organization literature have modeled bargaining in business-to-business settings under the assumption of complete information, including Crawford and Yurukoglu (2012) and others. These papers have yielded valuable insights into bilateral oligopoly settings with complete information, whereas this paper focuses instead on a setting of bilateral monopoly while allowing for strategic behavior and two-sided incomplete information. A number of other interesting contributions to the empirical bargaining literature also assume complete information, such as Merlo and Tang (2012), who provided methodologies for structural estimation in stochastic bargaining games of complete information.

There is very little existing empirical work studying incomplete-information bargaining. Several contemporaneous papers offer reduced-form analysis of implications of incomplete information in bargaining, including Backus, Blake, Larsen, and Tadelis (2018), who studied alternating-offer bargaining data in an online marketplace, and Grennan and Swanson (2016), who studied the effects of information disclosure in hospital-supplier bargaining relationships. Previous work estimating structural models of incomplete-information bargaining is rare, and these studies typically focus on settings of one-sided incomplete information. These include Sieg (2000) and Silveira (2017), who focused on trial settings and modeled bargaining as a take-it-or-leave-it offer game, and Ambrus, Chaney, and Salitsky (2018), who studied ransom negotiations and modeled bargaining following the theoretical work of Fudenberg, Levine, and Tirole (1985).

Structural empirical work that highlights a role for two-sided uncertainty in bargaining includes Genesove (1991), who discussed briefly the bargaining that takes place at wholesale auto auctions. Lacking detailed data on bargaining, he tested several parametric specifications for buyer and seller distributions and found that these assumptions performed poorly in explaining when bargaining occurred or when it was successful. The study most closely related to mine is Keniston (2011). The author collected data on back-and-forth bargaining offers for autorickshaw rides in India and estimated a structural model with two-sided incomplete information to compare welfare under alternating-offer bargaining to welfare under a fixed-price mechanism. Like Keniston (2011), my paper is one of the first to analyze data on actual back-and-forth bargaining offers and to estimate a model of bargaining with two-sided incomplete information. The approach developed in my paper can be applied to other settings with alternating-offer data to identify and estimate bounds on the distribution of valuations for the player who responds to the first offer. Larsen and Zhang (2017) present an
approach than can be used to instead obtain the distribution of valuations for the player who makes (rather than responds to) the first offer.

2 The Wholesale Used-Car Industry and the Data

The wholesale used-auto auction industry provides liquidity to the supply side of the US used-car market. Each year approximately 40 million used cars are sold in the United States, 15 million of which pass through a wholesale auction house. About 60% of these cars sell, with an average price between $8,000 and $9,000, totaling to over $80 billion in revenue (NAAA 2009). The industry consists of approximately 320 auction houses scattered across the country. Throughout the industry, the majority of auction house revenue comes from fees paid by the buyer and seller when trade occurs. Buyers attending wholesale auto auctions are used-car dealers. Sellers may be car dealers (whom I will refer to as “dealers”) selling off extra inventory, or they may be large institutions, such as banks, manufacturers, or rental companies (whom I will refer to as “fleet/lease”) selling repossessed, off-lease, lease-buy-back, or old fleet vehicles.

Sellers bring their cars to the auction house, usually several days before the sale, and establish a secret reserve price. In the days preceding the sale, potential buyers may view car details and pictures online, including a condition report for cars sold by fleet/lease sellers, or may visit the auction house to inspect and test drive cars (although very few visit prior to the day of sale). The auction sale takes place in a large, warehouse-like room with 8–16 lanes running through it. In each lane there is a separate auctioneer, and lanes run simultaneously. A car is driven to the front of the lane and the auctioneer calls out bids, raising the price until only one bidder remains.

If the auction price exceeds the secret reserve price, the car is awarded to the high bidder. If the auction price is below the secret reserve price, the high bidder is given the option to enter into bargaining with the seller. If the high bidder opts to bargain, the auction house will contact the seller by phone (or in person, if the seller is present at the sale), at which point the seller can accept the auction price, end the negotiations, or propose some counteroffer higher than the auction price. If the seller counts, the auction house calls the buyer. Bargaining continues in this fashion until one party accepts or terminates negotiations (with the typical time between calls being 2-3 hours). It is this bilateral bargaining that is the focus on this paper.

The dataset used in this paper is new to the literature. The data come from six auction houses owned by one company, each maintaining a large market share in the region in which it operates. The sample period is from January 2007 to March 2010. An observation in the dataset represents a run of the vehicle, that is, a distinct attempt to sell the vehicle through the mechanism. For a given run, the data records the date, time, auction house location, and auction lane, as well as the seller’s secret reserve price, the auction price,

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5 At these auctions, sellers do not have the choice of whether to make the reserve price public or secret; the auction house mandates that it be secret. Public reserve prices are optimal for the seller in most standard auction environments, but secret reserve prices may be preferred by the auction house if the auction house is an intermediary who is paid only when trade occurs (see Elyakime, Laffont, Loisel, and Vuong 1994). The question of when prefer public vs. secret reserve prices should be preferred in general is still unresolved. Appendix D.3 contains a discussion of secret reserve prices compared to public reserve prices using the estimated distributions from Section 4.

6 If the seller is not present and the auctioneer observes that the auction price and the reserve price are far enough apart that phone bargaining is very unlikely to succeed, the auctioneer may choose to reject the auction price on behalf of the seller.
and, when bargaining occurs over the phone, the full sequence of buyer and seller actions (accept, quit, or counter), and the amounts of any offers/counteroffers.

I drop observations with no recorded auction house blue book estimate; cars less than one year or greater than 16 years old; cars with less than 100 miles or greater than 300,000 miles on the odometer; observations in which the auction sale timestamp is missing; and observations for which the following variables lie outside their respective 0.01 and 0.99 percentiles: auction price, reserve price, and blue book price. I drop observations for which fewer than ten vehicles were observed at a given make-model-year-trim-age combination or days in which fewer than 100 cars were offered for sale at a given auction house. I drop observations where the auction price or reserve price is missing or is equal to zero and incomplete bargaining sequences (see Appendix C.1). In the end, I am left with 136,146 runs of cars offered for sale by used-car dealers (which I will refer to as the dealers sample), and 133,249 offered for sale by fleet/lease sellers (which I will refer to as the fleet/lease sample).

Descriptive statistics are displayed in Table 1 (with additional descriptive statistics shown in Appendix Table A2). The probability of trade is 0.71 in the dealers sample and slightly higher in the fleet/lease sample. In the dealers sample, the average auction price is over $1,000 below the average reserve price and about $600 below the average blue book price. Dealer cars are on average seven years old and have nearly 100,000 miles on the odometer. Fleet/lease cars tend to be newer (three years old and 57,000 miles), higher priced, and have a smaller gap between the reserve and auction prices. Also, unlike dealer cars, in fleet/lease cars the reserve price does not exceed the blue book price on average. All of these descriptive statistics are consistent with conversations with industry participants: Dealer cars tend to be older cars with more aggressive reserve prices and tend to be less likely to sell.

Table 1 also shows information on the number of bidders participating in the auction. A precise measure of the number of bidders is difficult to obtain at these auctions, as many sales take place simultaneously in different auction lanes and bidders are not required to register for the sale of a specific car. However, for some auction sales, the company offers live video streaming and a web-based portal for remote bidding, and for these sales a lower bound on the number of bidders can be obtained from bid logs. These bid logs record each bid and the identity of the bidder if the bidder participated online. If the bidder was instead physically present on the auction house floor, the bid log only records the amount of the bid and an indicator, “floor”, rather than an identity. A lower bound on the number of distinct bidders is given by the number of distinct online identities who placed bids plus 1 if the log records any floor bids or plus 2 if the log records two consecutive floor bids (assuming no bidder bids against himself). This lower bound rarely falls below 2 (this occurs in 0.37% of observations in the dealers sample and 1.76% of observations in the fleet/lease sample). The mean of this lower bound conditional on it being at least 2 is 2.924 in the dealers sample and 2.974 in the fleet/lease sample.

Table 2 displays the characteristics of observations in the dealers sample separated by the period of the game in which the observation ends. For each period of the game, the columns display the number of observations ending in that period, the percent of the total sample that this number represents, the percent of these observations that end in trade, as well as the unconditional reserve price and auction price (whether or not trade occurred) and the reserve price, auction price, and final price estimated only using cases that
did end in trade. Period 1 is the auction. Observations ending in period 1 represent cases that ended with auction price exceeding the reserve price or with the auction price falling short of the reserve price and the buyer opting out of bargaining. The remaining periods are labeled with even numbers for seller turns and odd numbers for buyer turns. Period 2 is the seller’s first turn in the bargaining game, and observations ending in this period represent cases in which the seller accepts the auction price or quits. Period 3 is the buyer’s first turn in bargaining, and is reached only if the seller chooses to counter in period 2. Play continues back and forth between the buyer and seller until one party accepts or quits.

Table 2 demonstrates that in 10.66% of the dealers sample the game ends at the auction, and in these cases the final price when trade happens (which occurs 88.62% of the time) is naturally the auction price. The remainder of the time, the buyer opts out of bargaining. Observations ending in trade in the second period also have the final price equal to the auction price (as the auction price is the first bargaining offer). Consider now the fifth period of the game. Only 1.24% of the full sample reaches this period, but this still consists of nearly 1,700 observations. In the fifth period, when trade does occur, it occurs at an average final price of $7,806, which is over $600 above the average auction price ($7,188), but still does not reach as high as the average reserve price ($8,653). Overall, Table 2 suggests that observations ending in later periods had somewhat higher reserve prices than those ending in earlier periods, consistent with Coasian dynamics (see Fudenberg and Tirole 1991), where lower-value sellers trade earlier in the bargaining sequence. Only one buyer-seller pair endured ten periods of the game, coming to agreement in the end, at a price $2,600 above the auction price. Appendix Table A3 displays similar patterns for the fleet/lease sample. In the fleet/lease sample, the game ends at the auction 33.38% of the time. Thus, in both the dealers and fleet/lease samples, what happens after the auction plays a major role in the market.

3 Model

This section presents a model of the game played in wholesale used-car markets. Prior to stating the assumptions of the model, I first restate the timing of the game, which is as follows:

1. Seller sets a secret reserve price.

2. \(N\) bidders bid in an ascending auction.

3. If the auction price exceeds the secret reserve price, the high bidder wins the item.

4. If the auction price does not exceed the secret reserve price, the high bidder is given the opportunity to walk away, or to enter into bargaining with the seller.

5. If the high bidder chooses to enter bargaining, the auction price becomes the first bargaining offer, and the high bidder and seller enter an alternating-offer bargaining game, mediated by the auction house.

Throughout I maintain the following assumptions:

Assumptions.
(A1) \( N \geq 2 \) risk-neutral bidders participate in an ascending button auction with zero participation costs. For \( i = 1, \ldots, N \), each buyer \( i \) has a private valuation \( \tilde{B}_i = W + B_i \), with \( B_i \sim F_B \) and \( W \sim F_W \), and with \((W, N, \{B_i\}_{i=1}^N)\) mutually independent.

(A2) A risk-neutral seller has a private valuation \( \tilde{S} = W + S \), with \( S \sim F_S \) and with \( S \) independent of \((W, N, \{B_i\}_{i=1}^N)\).

(A3) The bargaining lasts for up to \( T < \infty \) periods; buyers incur a common bargaining cost, \( c_B > 0 \), for each offer made; and sellers incur a common bargaining cost, \( c_S > 0 \), for each offer made.

The random variable \( W \) is observed by all buyers and the seller and represents game-level heterogeneity. Conditional on \( W \), buyers and sellers have independent private values, but unconditional on \( W \) valuations are correlated. The motivation for this framework is that, according to market participants, buyers—as well as dealer-type sellers—have valuations arising primarily from their local demand and inventory needs.\(^7\) Also, seller valuations can depend on the value at which the car was assessed as a trade-in, or, for a bank or leasing company, valuations can arise from the size of the defaulted loan.\(^8\) The button auction assumption simplifies the analysis of the auction, but is also not an unreasonable approximation, as it is the auctioneer in this market who raises the price and not the bidders (unlike in oral English auction). The assumption of symmetric buyers is not restrictive in this setting given that the high bidder’s identity is not known to the seller during bargaining and given that, in a private values ascending auction, bidders’ auction strategies will not depend on the identities of other participants. The assumption that \( N \) is independent of buyer valuations rules out endogenous entry. In Appendix D.2, I document some evidence supporting this assumption, following the intuition derived in Aradillas-López, Gandhi, and Quint (2016).

The form of bargaining costs in Assumption A3 is found elsewhere in the theoretical bargaining literature (e.g. Perry 1986 and Cramton 1991), and prevents players from continuing to bargain even when no surplus is to be had. The cap on the number of periods \( T \) simplifies the proofs of many of the model properties. \( T \) is assumed to be known to the players but not necessarily to the econometrician (and similarly for \( c_B \) and \( c_S \)).

I further assume \( F_B, F_S, F_W \) have corresponding densities \( f_B, f_S, f_W \) with supports \([b, \bar{b}], [s, \bar{s}], [w, \bar{w}]\) and that these densities are strictly positive over their supports. I also make a location normalization, \( E[W] = 0 \), without loss of generality. Finally, I assume that \( s \geq b \), given that the seller will be guaranteed a price of at least \( b \) from the auction. This latter assumption plays no role in the theoretical model, but can aid in pinning down \( s \) in the estimation.

\(^7\)While there is likely some common values component to wholesale auto auctions, accounting for this in estimation would be beyond the state of the methodological literature (positive identification results do not exist for valuations at common values ascending auctions; see Athey and Haile 2007). In conversations with market participants, buyers claim to decide upon their willingness to pay before bidding begins, sometimes having a specific retail customer lined up for a particular car, also suggesting a strong private component to valuations (see also discussions on the popular industry blog, thetruthaboutcars.com, Lang 2011). Studying similar auto auctions in Korea, Roberts (2013) and Kim and Lee (2014) provided evidence that private values models fit bidder behavior well in these settings.

\(^8\)These explanations for seller values are due to conversations with industry professionals. Note also that adverse selection from the seller possessing more knowledge about car quality than the buyer is likely small because of auction house information revelation requirements and because sellers are not previous owners/drivers of the vehicles.
For the next several subsections, I will discuss properties of the game conditional on a realization of \( W \), and thus I will omit \( W \) for notational simplicity and return to it when I discuss game-level heterogeneity in Proposition 5. I ignore auction house fees in this theoretical analysis but account for their existence in the empirical analysis in Section 4.

3.1 Payoffs, Strategies, and Equilibrium

I model the game as follows. In period \( t = 0 \), the seller chooses her secret reserve price, \( R = \rho(S) \), knowing only her type \( S \). This choice of reserve price is not revealed to buyers, before or after the auction. In period \( t = 1 \), the ascending auction takes place. Let \( \beta_i \) denote bidder \( i \)'s auction strategy (a price at which bidder \( i \) drops out of the auction), and let the final auction price be denoted \( P^A \). If \( P^A \geq R \), the high bidder wins the car and the game ends. If \( P^A < R \), the high bidder is given the opportunity to walk away, which ends the game, or to enter into bargaining with the seller.

When the auction price is \( P^A \), a high bidder of type \( B \) chooses to enter into bargaining rather than walk away whenever the payoff from entering into bargaining is non-negative. If the buyer chooses not to walk away, the buyer enters an alternating-offer bargaining game with the seller. In doing so the buyer immediately incurs a bargaining cost, \( c_B > 0 \), and this \( c_B \) will be incurred by the buyer at every offer he makes. The seller will incur a bargaining cost, \( c_S > 0 \), at each offer she makes. The first offer of the bargaining game is \( P^A \). The game moves to period 2 of the game, in which the seller chooses \( D^S_2 \in \{A,Q,C\} \)—a choice to accept (A), quit (Q), or counter (C). If the seller chooses Q or A the game ends. If the seller chooses C, the seller specifies a counteroffer \( P^S_2 \), and play continues to period 3, with the buyer choosing \( D^B_3 \in \{A,Q,C\} \), and so on up to period \( T \). If period \( T \) is reached, the player whose turn it is can only choose to accept or quit.

Throughout the game, bargaining offers must be weakly greater than the auction price. In practice, this is an understood norm at the auction house, and it is supported in the data (bargaining prices lie above the auction price nearly 100% of the time). This feature means that the auction price plays a similar role for the seller that a list price would play for a buyer in many other real-world haggling scenarios, where a seller and buyer may haggle over prices in a range below the list price but the buyer may at any point choose to end the game by returning to the list price and accepting it. In such a haggling setting, the list price can be thought of the first bargaining offer, just as the auction price is here. In the auto auction setting, this ability of the seller to accept the auction price can either be modeled as an additional action available to the seller at any of her turns, or can be modeled as a rule enforced by the auction house that all bargaining offers must lie weakly above the auction price. I follow the latter convention.

The payoffs in the game are as follows. If a buyer of type \( B \) and a seller of type \( S \) agree to trade at a price \( P \), the buyer’s payoff is \( B - P \) less the per-offer bargaining costs the buyer has incurred up to that point. If trade occurs in round 1 of the game (i.e., at the auction), the buyer’s payoff will be \( B - P \), with \( P = P^A \), the auction price. Similarly, if the buyer and seller agree to trade at a price of \( P \), the seller’s payoff is \( P \), less any bargaining costs incurred by the seller up to that point. When disagreement occurs, the buyer receives a payoff of zero, less any costs incurred, and the seller receives a payoff of \( S \), her valuation, less any costs incurred.
In what follows, I will focus on pure strategy Bayesian Nash equilibria, which I will refer to as a BNE. A BNE of the game is as follows. Let $H_t$ represent the history of offers, including the auction price, up through period $t-1$ of the bargaining game. The strategy of a buyer of type $b_i$ is a function $\sigma_B(b_i) = \{\beta_i, \{D^{B}_t|H_t\}, \{P^{B}_t|H_t\}\}$, where the decisions $D^{B}_t$ and offers $P^{B}_t$ included are those for periods in which it is the buyer’s turn.\(^9\) The seller’s strategy is a function $\sigma^S(s) = \{\rho, \{D^{S}_t|H_t\}, \{P^{S}_t|H_t\}\}$, where the decisions and offers are those for periods in which it is the seller’s turn. A set of strategies $\sigma^{B*}(b_i)$ for all buyers and $\sigma^{S*}(s)$ for the seller constitutes a BNE of this game if, for each player, his or her strategy is a best response to opponents’ strategies.

I adopt a BNE framework rather than a perfect Bayesian Nash framework because my identification and estimation approach places no restrictions on off-equilibrium behavior (for example, I do not require that all off-equilibrium behavior be credible). Previous work examining equilibrium refinements of BNE in similar bargaining games with incomplete information and with continuous types (as in my setting) has found that these restrictions either lead to predictions of immediate trade or immediate disagreement (Perry 1986) or that equilibria satisfying proposed refinements often fail to exist.\(^10\) In spite of the independence assumptions in A1–A2, and the simple bargaining structure imposed in Assumption A3, this game is difficult to analyze, both theoretically and empirically, as two-sided bargaining problems (i.e., games where both parties can make offers) with two-sided uncertainty (where both parties have private information) are known to have a multiplicity of equilibria that can be qualitatively quite dissimilar, and no known characterization of these equilibria exists (see discussion in Section 5 of Ausubel, Cramton, and Deneckere 2002). The approach I take in this paper will circumvent this problem by only assuming that participants play some BNE of the game, without attempting to fully characterize or solve for that equilibrium.

In my treatment of BNE, I do impose one additional weak condition on the bargaining subgame:

**(E1) Strategies of the bargaining subgame are continuous in the auction price.**

This is a technical condition required for the differentiability of the seller’s payoff, exploited in Proposition 3 below. It is simple to derive a multiplicity of equilibria of the bargaining game satisfying E1. One such example is an equilibrium in which the seller simply accepts or quits in period 2, and the buyer rejects all (off-equilibrium) offers in period 3. A second example is an equilibrium in which the seller makes an uninformative offer in period 2, the buyer counters in period 3, the seller only accepts or quits (doesn’t counter) in period 4, and the buyer rejects all (off-equilibrium) offers in period 5. A third example is an equilibrium in which all offers and counteroffers must lie within a particular set of possible values, and in the (off-equilibrium) case in which any player deviates from these offers, the opponent responds by quitting. Ausubel and Deneckere

\(^9\)This discussion ignores the possibility of buyers conditioning their auction strategies on information observed during the auction (such as the points at which opponents drop out); as shown in Proposition 1 below, bidders would not gain from conditioning on such information.

\(^10\)See the discussion in Sections 3.1.2 and 5 of Ausubel, Cramton, and Deneckere (2002) of equilibrium refinements of incomplete-information bargaining games proposed in Cramton (1984), Rubinstein (1985), Grossman and Perry (1986), Cho (1990), and Bikhchandani (1992). The bargaining game described in Perry (1986) is nearly identical to the setup described herein, but the author focuses on Sequential Equilibria rather than BNE and demonstrates that the only such equilibrium involves immediate trade or immediate disagreement. Cramton (1991) discussed how the Perry (1986) result can be overturned by allowing for a small amount of time discounting.
(1993) provided a discussion of other partial-pooling equilibria for a similar bargaining game but with one-sided offers, and Ausubel, Cranton, and Deneckere (2002) suggested that such arguments can be extended to two-sided offer games as well.

### 3.2 Mechanism Design Framework for Evaluating Bargaining Efficiency

Prior to deriving the properties of BNE of this game, I describe the mechanism design framework I use to assess efficiency of bargaining, as it is the motivation for deriving some of the game’s properties. By the Revelation Principle (Myerson 1979), any Bayesian Nash equilibrium of an incomplete-information trading game has a corresponding, payoff-equivalent, direct-revelation mechanism. In a direct mechanism, a buyer of type $b$ and seller of type $s$ report their true types to the mechanism designer and then trade occurs with probability $x(s, b)$ (the allocation function), where this allocation function is determined so that players receive the same expected outcomes as in the original game.

The allocation function corresponding to ex-post efficient trade is simply $x^*(s, b) \equiv 1\{s \leq b\}$. The allocation function corresponding to a given point along the ex-ante efficient frontier, on the other hand, will maximize a convex combination of the buyer’s and seller’s ex-ante expected gains from trade, with weight $\eta$ given to the seller’s gains and weight $1 - \eta$ given to the buyer’s. I will use the notation $x^\eta(\cdot)$, for a given $\eta \in [0, 1]$, to denote the allocation function corresponding to a point on the ex-ante efficient frontier. Computing $x^\eta(\cdot)$ boils down to solving a linear programming problem, described in Section 5 below. The direct mechanism corresponding to the real-world bargaining, which I denote $x^{RW}(\cdot)$, can be estimated directly from the data, as described in Section 4.

Computing the efficiency of the real-world mechanism, $x^{RW}(\cdot)$, relative to the ex-ante and ex-post efficient mechanisms, $x^\eta(\cdot)$ and $x^*(\cdot)$, requires estimates of $F_B$ and $F_S$, the distributions of buyer and seller valuations. Thus, a key focus of this paper is on estimating these buyer and seller valuation distributions without imposing a priori any restrictions on how efficient the real-world bargaining is relative to these efficient benchmarks.

### 3.3 Model Properties

I now describe a number of properties of this game that will hold in any equilibrium. These properties will then be exploited in the following section to estimate the distributions of buyer and seller valuations, the support of types who enter the bargaining game, and the allocation function corresponding to the real-world mechanism. Appendix B discusses an extension of this model in which sellers have some uncertainty about the distribution of buyer valuations when choosing the reserve price, which addresses explicitly why sellers may accept offers below their secret reserve price. Appendix B also provides several other explanations of this phenomenon.

#### 3.3.1 Bidding Behavior

This section demonstrates that players do not receive any positive benefit from deviating from truth-telling at the auction stage of the game. In the auction stage, each bidder’s strategy is a price at which he will stop
Proposition 1. If Assumption A1 holds then bidders receive no positive benefit from deviating from a strategy of dropping out of the bidding when, and only when, the auction price reaches their valuations.

The intuition behind the proposition is that, as in a standard ascending button auction, a bidder will not find it optimal to drop out before the current price reaches his value because doing so would make the bidder miss out on a chance to win the auction. A bidder will also not find it optimal to remain in the auction once the current price passes his value because doing so will yield a negative payoff if the bidder does end up winning. One implication of the proposition is that a bidder will not gain from conditioning his auction strategy on any information revealed during the auction, such as other bidders’ drop-out points.

The proof of Proposition 1 is greatly simplified by the rule that the bargained price cannot be below the auction price; if this were not the case, proving the proposition would require ruling out the possibility of buyers bidding above their valuations and then attempting to bargain down to a lower final price afterward. It is possible to show that such behavior cannot occur in equilibrium, but the proof is more involved. See Lemma 2 in Appendix A.

There can exist BNE of this game in which bidders are indifferent between bidding truthfully and not. For example, one such BNE would be where the seller sets a very high reserve price, all bidders drop out at zero, and the seller immediately rejects any (off-equilibrium) positive bid; bidders receive a payoff of zero, but would receive no less by bidding truthfully. I break such indifference cases by assuming bidders bid truthfully when indifferent between doing so and not doing so.

When bidders drop out at their valuations, the distribution of the auction price, $F_{P^A}$, is equivalent to the distribution of the second order statistic of valuations. Given the conditions of Assumption A1—symmetric, conditionally independent private values, with valuations independent of the number of bidders—this second order statistic distribution can be inverted to recover the underlying distribution of buyer valuations, as discussed in Section 4.

3.3.2 The Seller’s Choice to Accept the Auction Price or Quit

I now demonstrate that bounds on the distribution of seller valuations can be achieved by an argument similar to the Haile and Tamer (2003) bounds in English auction settings. The argument differs from Haile and Tamer (2003), however, in that it is not possible to construct both an upper and lower bound on the seller’s value for each individual realization of the game. This is because, as shown below, a lower bound on a seller’s valuation is only observed when the seller chooses to quit. Therefore, rather than observation-level bounds, I will obtain bounds on the distribution of seller values relying on probability statements formed from observations of many sellers’ decisions to accept or walk away from an offer on the table.

Let $D^S = A$, without a $t$ subscript (to distinguish this from the period-specific action described in Section 3.1), represent the event in which the seller takes an action in period 1 or 2 that results in the game ending in agreement at the auction price. This event occurs either when 1) the auction price exceeds the reserve price or 2) the auction price fails to meet the reserve price, the high bidder does not opt out of bargaining, and the seller accepts the auction price on her first bargaining turn. Similarly, let $D^S = Q$ represent the
event in which the seller takes an action in period 2 that results in the game ending in disagreement at the auction price. This event happens when the auction price fails to meet the reserve price, the high bidder does not opt out of bargaining, and the seller quits on her first bargaining turn rather than accepting the auction price or making a counteroffer.

I exploit the following conditions:

(i) The seller never accepts an auction price below her value.

(ii) The seller never walks away from (quits at) an auction price above her value.

These conditions will be satisfied in any BNE of the game, as any violation of these conditions would be suboptimal for the seller. These conditions imply that, if the realized auction price is \( p^A \) and the seller accepts, it must be the case that the seller values the good less than \( p^A \). Similarly, if the seller quits when the auction price is \( p^A \), it must be the case that the seller values keeping the car herself more than \( p^A \). These conditions imply bounds on distribution of \( S \):

\[
\Pr(D^S = A | P^A = p^A) \leq \Pr(S \leq p^A) = F_S(p^A)
\]

\[
\Pr(D^S = Q | P^A = p^A) \leq \Pr(S \geq p^A) = 1 - F_S(p^A) \Rightarrow \Pr(D^S \neq Q | P^A = p^A) \geq F_S(p^A)
\]

where \( \Pr(D^S = A | P^A = p^A) \) represents the probability of the even \( D^S = A \) conditional on \( P^A = p^A \), and similarly for the other conditional probability statements. These bounds rely only on conditions (i) and (ii) and on Assumption A2, that is, that buyer and seller valuations are independent. I state these bounds as the following proposition:

**Proposition 2.** Under Assumption A2 and conditions (i) and (ii), for any \( v \in [s, \bar{s}] \), \( F_S(v) \in [\Pr(D^S = A | P^A = v), \Pr(D^S \neq Q | P^A = v)] \).

Note that these bounds are not necessarily sharp given the available data and the other maintained assumptions of the paper. Proving sharpness of these bounds is difficult within a Bayesian Nash, two-sided incomplete-information framework because, as highlighted above, no full characterization of equilibria exists for such settings in the current theory literature, and hence it is difficult to determine an equilibrium in which the bounds hold with equality. However, relaxing either the assumptions or restricting the available data can lead to sharpness.

For example, if the assumption of BNE were to be relaxed and the maintained assumptions were instead only conditions (i) and (ii) and Assumption A2, then these bounds would indeed be sharp. In particular, the lower bound would correspond to a setting where the seller’s actions are such that she accepts the auction price whenever it is weakly above her valuation. Similarly, the upper bound would correspond to a setting where the seller’s actions are such that she quits whenever the auction price falls below her value. It is indeed possible for such strategies to constitute a BNE, for instance, if the buyer were to commit to never bargain (in which case the seller’s best response would always be to only accept or reject the auction price). In this case the bounds would hold with equality, but such an equilibrium—where buyers never bargain—would be unlikely to describe the data well. In light of these arguments, the bounds in Proposition 2 are conservative.
These bounds can be applied to other alternating-offer bargaining settings, independent of whether the bargaining follows an auction. In such cases, the decisions $D^S = A$ or $D^S = Q$ would represent the first action taken by the player in the bargaining game who responds to the first offer. It is interesting to note that the width of these bounds will be determined by the frequency with which this player chooses to make a counteroffer in response to the first offer. Specifically, the bounds can be re-written

$$\Pr(D^S = A | P^A = p^A) \leq \Pr(D^S = A | P^A = p^A) + \Pr(D^S \neq A \cap D^S \neq Q | P^A = p^A).$$

The object $\Pr(D^S \neq A \cap D^S \neq Q | P^A = p^A)$ is the probability that the seller makes a counteroffer in response to the first offer, $P^A$. If, at a given $P^A = p^A$, the seller never makes a counteroffer, this probability will be zero, and the bounds will collapse to a point equal to the probability of acceptance at that $p^A$, $\Pr(D^S = A | P^A = p^A)$.

### 3.3.3 The Lower Support of Buyer and Seller Types Who Bargain

One advantage of studying bargaining following an ascending auction is that the auction outcome affects the bargaining game in a very tractable manner, allowing me to isolate the bargaining game from the auction. Recall that the high bidder will end up in bargaining when $P^A < R$ and when the high bidder’s expected payoff from entering bargaining is not negative. Let $\pi^B(p^A, b)$ represent the buyer’s expected payoff from entering into bargaining conditional on his value $b$ and the realization of the auction price. Let $\chi(b)$ be defined by $\pi^B(\chi(b), b) = 0$. The object $\chi^{-1}(p^A)$ then represents the buyer type that would be indifferent between bargaining and not bargaining when the realized auction price is $p^A$. Recall also that $\rho(\cdot)$ is the seller’s secret reserve price strategy; that is, $R = \rho(S)$. The following is true:

**Proposition 3.** If Assumptions A1–A3 hold, then in any BNE satisfying E1, conditional on an auction price $P^A = p^A$ and conditional on bargaining occurring, the support of seller types in the bargaining game is $[\beta(p^A), \bar{s}]$ and the support of buyer types is $[\beta(p^A), \bar{b}]$, where $\beta(\cdot) \equiv \rho^{-1}(\cdot)$ and $\bar{\beta}(\cdot) \equiv \chi^{-1}(\cdot)$. Moreover, $\rho(\cdot)$ and $\chi(\cdot)$ are strictly increasing, with $\rho(s) \geq s$ and $\chi^{-1}(p^A) > p^A$.

The intuition behind this result is as follows. When the auction price is $p^A$ and bargaining occurs, it will be common knowledge among the two bargaining parties that the seller’s type $s$ satisfies $\rho(s) \geq p^A$ (i.e. the reserve price is below the auction price), implying $s \in [\rho^{-1}(p^A), \bar{s}]$. Similarly, bargaining occurring means the buyer did not opt out, so $\chi(b) \geq p^A$, implying $b \in [\chi^{-1}(p^A), \bar{b}]$.

The approach for analyzing bargaining efficiency described in Section 5 relies on this result. Note that a clean relationship between the auction and the bargaining game would not be feasible in a game of a pay-your-bid (e.g. first price) auction followed by bargaining, because the presence of post-auction bargaining would create a ratchet effect in the auction bidding strategies, as noted in Elyakime, Laffont, Loisel, and Vuong (1997). Also note that the proof of strict monotonicity of $\rho(\cdot)$ relies on a monotone comparative statics result from Edlin and Shannon (1998), a special case of Topkis’s Theorem. Condition E1, continuity of the equilibrium of the bargaining subgame in the auction price, is required to prove differentiability of the seller’s payoff in order to apply the Edlin and Shannon (1998) result.
3.3.4 The Real-World Mechanism

This section describes the allocation function corresponding to the real-world mechanism. This allocation function, which I denote \( x^{RW} \), will in general depend on the realization of the auction price, as this changes the lower bound of the support of types who bargain. I prove the following result:

**Proposition 4.** Under Assumptions A1–A3, in any BNE satisfying E1, the allocation function \( x^{RW} \) can be written as

\[
 x^{RW}(r, b; p^A) \equiv 1 \{ b \geq g(r, p^A) \}
\]

(1)

where \( g(r, p^A) \) is an unknown function that is strictly increasing in \( r \).

Proposition 4 demonstrates that \( x^{RW} \) depends on a cutoff function defining the boundary between those types who trade and those who do not. The proof relies directly on an argument presented in Storms (2015). Proposition 4 also exploits the strict monotonicity of \( \rho(\cdot) \) proved in Proposition 3, which makes it possible to model the allocation conditional on a realization of the reserve price, \( R = r \), rather than conditional on the seller’s type. This is particularly useful in that it allows for the allocation function for the real-world bargaining to be evaluated without knowing where the true distribution of seller valuations lies within the bounds from Proposition 2.

3.3.5 Game-level Heterogeneity

Sections 3.3.1–3.3.4 derived results conditional on a given realization of game-level heterogeneity. This section demonstrates that, given the additively separable structure of buyer and seller valuations in the common component \( W \), continuous actions of the game (reserve prices, auction prices, and bargaining offers) will also be additively separable in \( W \). Choice probabilities for discrete actions (accepting, declining, or countering in response to an offer) will be unaffected by the value of \( W \).

**Proposition 5.** Suppose, when \( W = 0 \), the equilibrium is such that the reserve price is \( r \); the auction price is \( p_A \); the lowest buyer type who would choose to bargain is \( \chi^{-1}(p^A) \); and, for each period \( t \) at which the game arrives, the offer is given by \( P_t = p_t \), and the decision to accept, quit, or counter is given by \( D_t = d_t \). Then, under Assumptions A1–A3, when \( W = w \), the equilibrium will be such that the reserve price is \( \tilde{r} = r + w \); the auction price is \( \tilde{p}_A = p_A + w \); the lowest buyer type who would choose to bargain is \( \chi^{-1}(\tilde{p}^A - w) + w \); the period \( t \) offer is \( p_t + w \); and the period \( t \) decision is \( d_t \).

Proposition 5 is similar to results used elsewhere in the empirical auctions literature (Haile, Hong, and Shum 2003; Asker 2010) but is a generalization specific to this setting of a secret reserve price auction followed by bargaining. I will exploit this proposition in controlling for both observed and unobserved game-level heterogeneity. An immediate implication of Proposition 5 is that the allocation function is invariant to game-level heterogeneity; that is, \( x^{RW}(r + w, b + w; p^A + w) = x^{RW}(r, b; p^A) \).
4 Estimating Valuations and the Bargaining Mechanism

In this section, I exploit the model properties derived above in order to estimate the distribution of buyer and seller valuations and the bargaining mechanism. The estimation consists of several steps:

1. Controlling for observable game-level heterogeneity
2. Controlling for unobserved game-level heterogeneity (observed by players but not the econometrician)
3. Estimating the distribution of buyer valuations
4. Estimating bounds on the distribution of seller valuations
5. Estimating the lower support of types who bargain
6. Estimating the direct mechanism corresponding to the game

To perform estimation, I make the following additional assumptions on the data:

Assumptions.

(A4) Observations of random variables \((S, B_i, W, N)\) across instances of the game are identically and independently distributed.

(A5) All observations in the data are generated by the same equilibrium.

Assumption A4 is common in the empirical games literature, and it abstracts away from inter-auction dynamics. For example, it ignores the possibility of a buyer’s or seller’s valuation for a given car being related to future transaction opportunities. This is discussed further in Section 6.5.

Assumption A5 is not required for steps 1–4 above but is required for steps 5–6. For example, even if different equilibria of the bargaining subgame are played in different observations of the data, the distribution of buyer valuations can still be estimated (step 3) using the distribution of auction prices, as described below. Similarly, the revealed preference arguments used to bound the distribution of seller valuations (step 4) will still hold even if A5 fails. Steps 5–6, however, require inverting policy functions that will depend on the equilibrium of the game. Fortunately, none of the steps above, including 5–6, require fully specifying or solving for the equilibrium. Like Assumption A4, Assumption A5 is also common in the structural literature.

The typical approach in the literature to handling cases where particular subsamples of the date are believed to exhibit different equilibrium behavior is to estimate the model separately in these subsamples. In line with this, throughout the estimation, I treat the dealers and fleet/lease samples separately because, according to conversations with industry professionals, this is likely the most important division of the data in which behavior may differ. I also perform other subsample analyses in Section 6.5.

Prior to executing the steps outlined above, I first adjust auction prices upward to account for the fact that, if the auction price were to be the transaction price, a buyer would be required to pay the auction price as well as the auction house fee. The reserve price can similarly be adjusted downward. Let these adjusted prices be given by \(R^h \equiv R^{raw} - h^S\) and \(P^{A,h} \equiv P^{A,raw} + h^B\), where \(h^S\) and \(h^B\) represent auction house fees.
and $R^\text{raw}$ and $P^A,\text{raw}$ represent the raw, unadjusted random variables. Let realizations of these random variables for car $j$ be denoted $r^h_j$ and $p^A,h_j$. In practice, auction house fees consist of both a fixed fee and a percentage commission, but the latter makes up only a small portion of the overall fee. I therefore simply use the average seller fee for $h^S$ and the average buyer fee for $h^B$.

### 4.1 Accounting for Observed Heterogeneity Empirically

To account for game-level characteristics that are observed to the econometrician as well as the players, I apply Proposition 5. As above, let $W$ be a random variable representing unobserved game-level heterogeneity. Let $X$ be a random variable representing game-level heterogeneity that is instead observed (by the econometrician as well as the players), with $X$ independent of $W$, $S$, $B$, $N$. Let realizations of $X$ and $W$ be given by $x_j$ and $w_j$ for game $j$. I specify the total game-level heterogeneity (observed plus unobserved) for observation $j$ to be $x'_j\gamma + w_j$, where $\gamma$ is a vector of parameters to be estimated.

Proposition 5 implies that auction prices and reserve prices can be “homogenized” (Haile, Hong, and Shum 2003) by estimating the following joint regression of reserve prices and auction prices on observables:

$$
\begin{bmatrix}
  r^h_j \\
  p^A,h_j
\end{bmatrix} =
\begin{bmatrix}
  x'_j\gamma \\
  x'_j\gamma
\end{bmatrix} +
\begin{bmatrix}
  \tilde{r}_j \\
  \tilde{p}_j^A
\end{bmatrix},
$$

where $\tilde{r}_j = r_j + w_j$, $\tilde{p}_j^A = p^A_j + w_j$.

In the vector $x_j$ I include fifth-order polynomial terms (all degrees of the polynomial from one through five) in the auction houses’ blue-book estimate and the odometer reading. $x_j$ also contains the number of previous attempts to sell the car; the number of pictures displayed online; a dummy for whether or not the odometer reading is considered accurate, and the interaction of this dummy with the odometer reading; the interaction of the odometer reading with car-make dummies; dummies for each make-model-year-trim-age combination (where age refers to the age of the vehicle in years); dummies for condition report grade (ranging from 1-5, observed only for fleet/lease vehicles); dummies for the year-month combination and for auction house location interacted with hour of sale; dummies for 32 different vehicle damage categories recorded by the auction house; dummies for each seller who appears in at least 500 observations; dummies for discrete odometer bins; and several measures of the thickness of the market during a given sale and of the order the cars were run (see Appendix C.1). The adjusted $R^2$ from this first-stage regression is 0.96 in the fleet/lease sample and 0.94 in the dealers sample, implying that most of the variation in auction prices and reserve prices is explained by observables.

An estimate of $\tilde{r}_j$ is then given by subtracting $x'_j\hat{\gamma}$ from $r^h_j$, and similarly for $\tilde{p}_j^A$. Variation in these two quantities is then attributed to unobserved game-level heterogeneity and to players’ private valuations, as detailed below.

---

11In Section 4.4, when estimating the distribution of seller valuations using the seller’s decision to accept or reject the auction price, the auction price is instead adjusted by subtracting the seller’s auction house fee, as there it represents a price the seller would receive.
4.2 Accounting for Unobserved Heterogeneity Empirically

To account for heterogeneity $W$ in the game that is observed by the players but not by the econometrician, I apply a result due to Kotlarski (1967), which implies that observations of $\tilde{R} = R + W$ and $\tilde{P}^A = P^A + W$ (which are additively separable by Proposition 5) are sufficient to recover the densities $f_W$, $f_R$, and $f_{P^A}$. This result has been applied elsewhere in empirical auction work (e.g., Li, Perrigne, and Vuong 2000; Krasnokutskaya 2011). I estimate these densities using a flexible maximum likelihood approach (as in Athey, Levin, and Seira 2011), where the likelihood of the joint density of $(\tilde{R}, \tilde{P}^A)$ is given by

$$
\mathcal{L} = \prod_j \left[ \int f_{P^A}(\tilde{p}^A_j - w) f_R(\tilde{r}_j - w) f_W(w) dw \right]
$$

(2)

For each random variable $Y \in \{W, R, P^A\}$, the density of $Y$ is approximated by normalized orthogonal Hermite polynomials, $f_Y(y) \approx \frac{1}{\sigma_Y} \left( \sum_{k=0}^{K} \theta^Y_k H^k \left( \frac{y - \mu_Y}{\sigma_Y} \right) \right)^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{y - \mu_Y}{\sigma_Y} \right)^2}$, where $K$ is the number of elements in $\theta_Y$; $\theta_Y$, $\mu_Y$, and $\sigma_Y$ are parameters to be estimated for each $Y \in \{W, R, P^A\}$; and $H^k$ are Hermite polynomials, defined recursively by $H^1(y) = 1$, $H^2(y) = y$, and $H^k(y) = \frac{1}{\sqrt{k}} [y H^{k-1}(y) - \sqrt{k-1} H^{k-2}(y)]$ for $k > 2$. This Hermite polynomial approximation for the densities has the advantage of being flexible and parsimonious, and also yields an approximation of the CDFs $F_W$, $F_R$, and $F_{P^A}$ as functions of the same parameters as the densities.

I maximize the likelihood in (2) subject to the constraint that, for each $Y \in \{W, R, P^A\}$, $\sum_{i=1}^{K} (\theta^Y_i)^2 = 1$, which ensures that each approximated function is indeed a density function, and also subject to the constraint $E[W] = 0$. I choose to include $K = 5$ terms.\(^{12}\) The location and scale parameters $\{\mu_Y, \sigma_Y\}_{Y \in \{W, R, P^A\}}$ are not required but improve the performance of the estimator and are standard in estimation with Hermite polynomials. I estimate these parameters in an initial step, maximizing (2) with each density $f_Y$ being approximated by a $N(\mu_Y, \sigma_Y)$. I then plug in the estimated values of $\{\hat{\mu}_Y, \hat{\sigma}_Y\}_{Y \in \{W, R, P^A\}}$ into (2) and maximize the likelihood to obtain consistent estimates of $\{\theta^Y\}_{Y \in \{W, R, P^A\}}$. Numerical integration of (2), and of the other integrals in the paper, is discussed in Appendix C.2.

4.3 Estimating the Distribution of Buyer Valuations

The underlying distribution of buyer valuations, $F_B$, can be recovered from the distribution of auction prices, $F_{P^A}$, which, by Proposition 1, will coincide with the distribution of the second order statistic of buyer valuations. The relationship of $F_{P^A}$ and $\Pr(N = n)$ (the distribution of the number of bidders) to $F_B$ is as follows:

$$
F_{P^A}(v) = \sum_n \Pr(N = n) \left[ n F_B(v)^{n-1} - (n - 1) F_B(v)^n \right]
$$

(3)

The right-hand side of (3) is strictly monotonic in $F_B(\cdot)$. See, for example, Athey and Haile (2007).

To estimate $F_B(\cdot)$, I first replace $F_{P^A}(v)$ with the maximum likelihood estimate $\hat{F}_{P^A}(v)$ from (2), evaluated on a grid of values for $v$. I then estimate $\hat{F}(N = n)$ using the empirical probability mass function of

\(^{12}\) The above framework can be treated as a semi-nonparametric maximum likelihood setting, letting $K$ grow appropriately with the sample size and choosing $K$ through cross-validation. I instead fix $K = 5$, treating this as a flexible parametric approximation, as suggested in Kim and Lee (2014). I find that choosing $K$ larger than 5 does not affect estimates noticeably.
the lower bound of the number of bidders from the bid logs subsample discussed in Section 2. This treats the distribution of the lower bound as though it is the true distribution of the number of bidders; I confirmed that this approach seems reasonable by physically attending over 200 auction sales and recording the number of bidders (see Appendix D.1). It turns out, however, that the choice of Pr(N = n) is not critical: it affects the estimate of the full underlying buyer distribution, \( F_B \), but has a negligible effect on the transformation of \( F_B \) used in evaluating welfare, which is the distribution of the high bidder’s value conditional on the auction price integrated against the auction price density (i.e. the maximum order statistic distribution). In Appendix D.1, I demonstrate numerically that this latter object is not sensitive to the choice of \( \Pr(N = n) \) (Poisson distributions with mean \( \lambda \)), conditional on \( \hat{F}_{PA} \), the inferred maximum order statistic distribution will not vary with the choice of \( \lambda \).

With \( \hat{\Pr}(N = n) \) and \( \hat{F}_{PA}(v) \) in hand, I then obtain \( \hat{F}_B(v) \) by numerically solving (via a bisection method) for the value \( u \) such that

\[
0 = \hat{F}_{PA}(v) - \sum_n \hat{\Pr}(N = n) \left[ nu^{n-1} - (n-1)u^n \right]
\]

(4)

Consistency of this type of order statistics inversion estimator is shown in Menzel and Morganti (2013). Differentiating (3) yields an expression for consistently estimating the density:

\[
f_B(v) = \frac{f_{PA}(v)}{\sum_n \Pr(N = n) [n(n-1)F_B(v)^{n-2}(1 - F_B(p^A))]}\]

(5)

Thus, \( \hat{f}_B \) can be computed using the estimates of \( \hat{f}_{PA} \) from (2), estimates of \( \hat{F}_B \) from (4), and the empirical frequency estimate \( \hat{\Pr}(N = n) \).

4.4 Estimating Bounds on the Distribution of Seller Valuations

Proposition 2 demonstrates that, absent unobserved game-level heterogeneity, the objects \( \Pr(D^S = A|P^A = v) \) and \( \Pr(D^S \neq Q|P^A = v) \) will provide bounds on \( F_S(v) \). Importantly, note that by the definition of \( D^S \) from Section 3.3.2, realizations of the events \( D^S = A \) or \( D^S \neq A \), and \( D^S = Q \) or \( D^S \neq Q \), are observed for each instance of the game in the data, not just the subsample in which bargaining occurs.

I now discuss how game-level heterogeneity (\( W \)) can be introduced to these bounds. The same revealed preference arguments used in Section 3.3.2—conditions (i) and (ii)—immediately provide bounds on the distribution of \( \hat{S} \equiv S + W \) conditional on \( \hat{P}^A \equiv P^A + W \). Let this conditional distribution be given by \( F_{S|P^A}(\hat{s}|\hat{p}^A) \). These bounds are given by

\[
\Pr(D^S = A|\hat{P}^A = \hat{p}^A) \leq F_{S|P^A}(\hat{p}^A|\hat{p}^A) \leq \Pr(D^S \neq Q|\hat{P}^A = \hat{p}^A)
\]

(6)

These bounds can then be used to provide bounds on \( F_S(\cdot) \), which is the object required to evaluate bargaining efficiency. Let the density of \( \hat{P}^A \) be denoted \( f_{PA} \). In stating these bounds, I will also incorporate information from the fact that \( R \geq S \) (proved in Proposition 3). Let \( \mathcal{F} \) represent the space of all possible \( F_S \). For any \( v \in [s, \bar{s}] \), \( F_S(v) \in [F^L_S(v), F^U_S(v)] \), where \( F^L_S(v) \) and \( F^U_S(v) \) are defined by

\[
F^L_S(v) \equiv \inf \left\{ F_S(v) : F_S \in \mathcal{F}, F_S(v) \geq F_R(v), \right\}
\]
\[
\Pr(D^S = A|\hat{P}^A = v) \leq \frac{1}{\int f_{P^A}(v)dw} \int F_S(v-w)f_{P^A}(v-w)F_W(w)dw
\]

(7)

\[
F^U_{S}(v) \equiv \sup \left\{ F_S(v) : F_S \in \mathcal{F}, \right\}
\]

\[
\Pr(D^S \neq Q|\hat{P}^A = v) \geq \frac{1}{\int f_{P^A}(v)dw} \int F_S(v-w)f_{P^A}(v-w)F_W(w)dw
\]

(8)

To estimate these bounds, I parameterize \(F^L_{S}(\cdot)\) and \(F^U_{S}(\cdot)\) using piecewise linear splines. A spline approximation—or other linear sieve approximation—to an unknown function has the advantage of being linear in parameters while remaining very flexible in fitting the function. Let \(\{v^S_k\}_{k=1}^{K_S}\) represent a fixed vector of \(K_S\) knots on the support of \(\hat{P}^A\), and \(LS_S(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^{K_S}\) be the piecewise linear spline transformation for this vector of knots. An approximation to \(F_S\) is then given by \(F_S(\cdot; \theta^S) \equiv LS_S(v)\theta^S\) and an approximation to the density is given by \(f_S(v; \theta^S) \equiv LS_S'(v)\theta^S\), where \(\theta^S\) is a \(K_S\)-by-1 vector of parameters.\(^{13}\) I choose the knots \(\{v^S_k\}_{k=1}^{K_S}\) to be uniformly spaced between the 0.001–0.999 quantiles of \(\hat{A}\) and \(\hat{I}\), and I choose \(K_S = 200\) as the number of knots for each bound (\(K_S = 250\) or \(K_S = 300\) yield similar results, as shown in Panels A and B of Appendix Figure A4).

The lower and upper bounds can then be denoted \(F^L_{S}(v) = LS_S(v)\theta^S.L\) and \(F^U_{S}(v) = LS_S(v)\theta^S.U\), where \((\theta^S.L, \theta^S.U)\) are each \(K_S\)-by-1 vectors of parameters to be estimated. I estimate these parameters by solving the following constrained least squares problem:

\[
\min_{\theta^S.L, \theta^S.U} \sum_{k=1}^{K_S} \left\{ \left[ \hat{P}(D^S = A|\hat{P}^A = v^S_k)\hat{f}_{P^A}(v^S_k) - LS_S(v^S_k)\theta^S.L \right]^2 + \left[ (\hat{P}(D^S \neq Q|\hat{P}^A = v^S_k))\hat{f}_{P^A}(v^S_k) - LS_S(v^S_k)\theta^S.U \right]^2 \right\}
\]

(9)

where \(LS_S(v^S_k)\) is a \(K_S\)-by-1 vector with the \(\ell\)th element given by \(\int (LS_S(v^S_k - w)e(\ell))\hat{f}_{P^A}(v^S_k - w)\hat{f}_W(w)dw\) and \(e(\ell)\) is a \(K_S\)-by-1 selection vector with 1 in the \(\ell\)th spot and zeros elsewhere. An advantage, again, of the spline approximation is that this object \(LS_S(v^S_k)\) can be computed outside the optimization problem. I impose several constraints on the minimum distance problem in (9): (i) \(F^L_{S}\) lies graphically above the CDF of secret reserve prices, (ii) \(F^U_{S}\) lies graphically below \(F^U_{S}\), (iii) the lower and upper bounds lie in \([0, 1]\), and (iv), the lower and upper bounds are weakly increasing (so they will correspond to proper distribution functions).

Computing the objective function in (9) requires estimates of \(\hat{F}_R, \hat{f}_{P^A},\) and \(\hat{f}_W\), as well as \(\hat{f}_{P^A}, \hat{P}(D^S = A|\hat{P}^A = \hat{P}^A)\), and \(\hat{P}(D^S \neq Q|\hat{P}^A = \hat{P}^A)\). Estimate of the first three objects come from the maximum likelihood procedure in (2). I estimate \(\hat{f}_{P^A}\) using a kernel density with a Gaussian kernel and Silverman’s rule-of-thumb bandwidth. I estimate \(\hat{P}(D^S = A|\hat{P}^A = \hat{P}^A)\) using a local linear regression of the event

\[^{13}\text{Specifically, let } m_k(v) \equiv (v - v^S_k)/(v^S_{k+1} - v^S_k) \text{ and let } v \in [v^S_k, v^S_{k+1}] \text{ for some } k \in \{1, \ldots, K_S - 1\}. \text{ Then } LS_S(v) \text{ returns a } K_S \text{-by-1 vector with } (1 - m_k(v)) \text{ as the } k \text{th element, } m_k(v) \text{ as the } (k+1) \text{th element, and zeros elsewhere. Thus, letting } \Delta_k \equiv (\theta^S_{k+1} - \theta^S_k)/(v^S_{k+1} - v^S_k), \text{ then } F_S(v; \theta^S) \text{ is given by } \theta^S_k + (v - v^S_k)\Delta_k. \text{ For the density, } LS'_S(v) \text{ returns the derivative of the vector } LS_S(v) \text{ with respect to } v, \text{ and thus } f_S(v; \theta^S) = \Delta_k.\]
1\{D^S = A\} on realizations of \( \hat{P}^A = \hat{\rho}^A \). I estimate \( \hat{\Pr}(D^S \neq Q|\hat{P}^A = \hat{\rho}^A) \) analogously. In (9) the combination of a nonparametric first-stage estimate of a nuisance vector (e.g. \( \hat{\Pr}(D^S = A|\hat{P}^A = v^U_k) \)) followed by a parametric second stage estimated through minimum distance falls into the class of two-step semiparametric GMM estimators, discussed in Ackerberg, Chen, Hahn, and Liao (2014) and references therein, and is consistent for estimating \( \hat{\theta}^{S,L} \) and \( \hat{\theta}^{S,U} \).

I now discuss briefly the surjectivity of these bounds in this context as well as in other alternating-offer settings. As explained in Section 3.3.2, the revealed preference bounds proposed in this paper can be used in other alternating-offer settings to identify bounds on the valuations of the player who responds to the first bargaining offer, whom I will refer to as the first responder. In order for these bounds to be surjective (i.e. provide bounds for the whole range of the CDF from \([0, 1]\)) when evaluated on a given range of knots, the data must contain some realizations of the first bargaining offer that are extreme enough that the first responder accepts with probability close to one and some realizations of the first bargaining offer such that the first responder rejects (quits) with probability close to one. If this is not the case in a given dataset, specific institutional details may aid in identifying the support of the bounds, as I now describe.

In examining the surjectivity of the bounds in the current application, I find that, in the dealers sample, there is sufficient variation in the data for both the upper bound and lower bound on \( F_S \) to be surjective over the range of chosen knots. In the fleet/lease sample, the estimate of \( F^U_S \) is surjective, and the estimate of \( F^U_L \) reaches a value of 1, but does not fully reach a value of 0; in particular, in the fleet/lease sample I find an estimate of \( \hat{\theta}^{S,U}_1 \) (the lowest spline parameter) of approximately 0.19, meaning that the estimate of \( F^U_S \) does not rule out the possibility that 19% of sellers have values less than or equal to \( v^S_1 \), the lowest knot, which, as explained above, is equal to the 0.001 quantile (approximately the minimum) of the auction price distribution. I therefore treat \( F^U_S \) in the fleet/lease sample as having a mass point of 0.19 at \( v^S_1 \). This mass point could be placed at any arbitrary point weakly below \( v^S_1 \). I choose to place it at \( v^S_2 \) based on the assumption that \( \bar{s} \geq \bar{b} \) as discussed in Section 3.15

### 4.5 Estimating the Lower Support of Bargaining Types

This section describes how I estimate \( \bar{b}(\cdot) \equiv \chi^{-1}(\cdot) \), and \( \bar{g}(\cdot) \equiv \rho^{-1}(\cdot) \), which, by Proposition 3, are both increasing functions, and are the lower support of buyer and seller types who enter the bargaining game. Let \( D^B \in \{0, 1\} \) represent the buyer’s decision to walk away or not when informed that the high bid does not exceed the buyer’s reservation value.

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14 For this local linear regression, as well as the local linear regressions run in Sections 4.5 and 4.6 below, I use a Gaussian kernel with the rule-of-thumb bandwidth presented in Ruppert, Sheather, and Wand (1995), who suggested computing the bandwidth on the sample lying between the \( \alpha \) and \( 1 - \alpha \) quantiles for some \( \alpha \in [0, 1] \) in order to reduce sensitivity to data in the tails. I use \( \alpha = .001 \).

15 Surjective estimates of the bounds correspond to the case where \( \hat{\theta}^{S,U}_1 \) and \( \hat{\theta}^{S,L}_1 \) are close to 0 and the estimates of \( \hat{\theta}^{S,U}_K \) and \( \hat{\theta}^{S,L}_K \) are close to one. If the bounds are not surjective, the following arguments provide a complete description of \( F^L_S(v) \) and \( F^U_S(v) \) for points \( v \) that lie outside of the support of the chosen knots: (i) for \( v > v^S_K \), \( F^L_S(v) \) is given by a horizontal line at \( \hat{\theta}^{S,L}_K \) from \( v^S_K \) to \( F^{-1}(\hat{\theta}^{S,L}_K) \) and by \( F_R(v) \) above that point; (ii) for \( v > v^S_K \), \( F^U_S(v) = 1 \); (iii) for \( v < v^S_1 \), \( F^U_S(v) = F_R(v) \); (iv) for \( v < v^S_1 \), \( F^U_S(v) = 0 \) (or, if the assumption that \( \bar{s} \geq \bar{b} \) were to be relaxed, \( F^U_S(v) \) for \( v < v^S_1 \) would be given by a horizontal line at \( \hat{\theta}^{S,L}_L \)). If the estimates of \( F^L_S \) and \( F^U_S \) are surjective, then specifying conditions (i)–(iv) is unnecessary, as I find to be the case in the dealers sample and, for the most part, in the fleet/lease sample. The only condition that matters in practice is condition (iv) for the upper bound in the fleet/lease sample, where (iv) is binding.
meet the reserve price. A buyer enters the bargaining game \((D_B^I = 0)\) if and only if his type \(B \geq \chi^{-1}(P^A)\), and the probability of this event conditional on a realization of \(\tilde{P}^A\) is given by

\[
\Pr(D_B^I = 0|\tilde{P}^A = \tilde{p}^A, \tilde{P}^A < \tilde{R}) = \frac{1}{M_x(\tilde{p}^A)} \int \frac{1 - F_B(\chi^{-1}(\tilde{p}^A - w))}{1 - F_B(\tilde{p}^A - w)} f_{P^A}(\tilde{p}^A - w)(1 - F_R(\tilde{p}^A - w)) f_W(w) dw
\]

(10)

where \(M_x(\tilde{p}^A) = \int f_{P^A}(\tilde{p}^A - w)(1 - F_R(\tilde{p}^A - w)) f_W(w) dw\).

I approximate \(\chi^{-1}(\cdot)\) as a flexible piecewise linear spline, denoted \(\chi^{-1}(\cdot; \theta^x) \equiv LS_x(\cdot; \theta^x)\), where \(\theta^x\) is a vector of spline parameters and \(LS_x(\cdot) : \mathbb{R} \mapsto \mathbb{R}^{K_x}\) is the linear spline transformation for a fixed vector of knots \(\{v^x_k\}_{k=1}^{K_x}\), defined analogously to the linear spline transformation described in Section 4.4 for estimating seller valuation bounds. I choose the knots \(\{v^x_k\}_{k=1}^{K_x}\) to be uniformly spaced between the 0.001–0.999 quantiles of \(\tilde{P}^A\), and set \(\chi^{-1}(v) = v\) for any \(v\) outside of these knots. I choose \(K_x = 25\). I estimate \(\theta^x\) by solving the following constrained nonlinear least squares problem:

\[
\min_{\theta^x} \sum_{k=1}^{K_x} \frac{1}{\hat{\sigma}^2(v^x_k)} \left[ \frac{\Pr(D_B^I = 0|\tilde{P}^A = v^x_k, \tilde{P}^A < \tilde{R})}{\hat{M}_x(v^x_k)} - \int \frac{1 - \hat{F}_B(\chi^{-1}(v^x_k - w; \theta^x))}{1 - \hat{F}_B(v^x_k - w)} f_{P^A}(v^x_k - w)(1 - \hat{F}_R(v^x_k - w)) f_W(w) dw \right]^2
\]

(11)

subject to the constraints that \(\chi^{-1}(\cdot; \theta^x)\) is increasing and \(\chi^{-1}(P^A; \theta^x) \geq P^A\). Computing the objective function in (11) requires the estimates of \(\hat{F}_B\), \(\hat{F}_R\), \(\hat{f}_{P^A}\), and \(\hat{f}_W\) from above. The object \(\hat{\sigma}^2(\cdot)\) comes from evaluating \(M_x(\cdot)\) using these estimated distributions. To estimate \(\hat{\Pr}(D_B^I = 0|\tilde{P}^A = \tilde{p}^A, \tilde{P}^A < \tilde{R})\), I use a local linear regression of the event \(|D_B^I = 0|\) on realizations of \(\tilde{P}^A = \tilde{p}^A\) using observations where the auction price falls below the reserve \((\tilde{P}^A < \tilde{R})\). The weight \(\hat{\sigma}^2(v^x_k)\) in (11) is the estimated variance of this local linear regression estimate, computed following Fan and Gijbels (1996). As with the estimator described in (9), the estimator in (11) falls into the class of two-step semiparametric GMM estimators and will be consistent for \(\hat{\theta}^x\).

To estimate the \(\rho(\cdot)\) function, I exploit the property that, for any \(F_S(\cdot)\) satisfying Proposition 2, \(\rho(\cdot)\) is related to \(F_S(\cdot)\) and \(F_R(\cdot)\) as follows:

\[
F_S(s) = \Pr(S \leq s) = \Pr(\rho(S) \leq \rho(s)) = F_R(\rho(s)) \\
\Rightarrow \rho(s) = F_R^{-1}(F_S(s))
\]

(12)

Below, when constructing bounds on welfare measures, I will evaluate the function in (12) using the estimated \(\hat{F}_R\) and using the function \(F_S(\cdot)\) lying in the estimated bounds \([\hat{F}_S^L(\cdot), \hat{F}_S^U(\cdot)]\) that maximizes or minimizes a given welfare measure, as described in Section 5.

4.6 Estimating the Direct Mechanism Corresponding to Real-World Bargaining

Proposition 4 demonstrates that the allocation function corresponding to the real-world mechanism can be written as \(x_{RW}(r, b; P^A) = 1 \{b \geq g(r, P^A)\}\) for some unknown function \(g(\cdot)\). The empirical object that can be used to identify this function \(g(\cdot)\) is the probability of trade conditional on a realization of \(\tilde{R}\) and \(\tilde{P}^A\).
Let \( A \in \{0, 1\} \) be a random variable indicating whether or not trade occurs in a given instance of the game. Then the probability of trade, conditional on realizations of \( \tilde{R} \) and \( \tilde{P}^{A} \), is given by

\[
\Pr(A = 1|\tilde{R} = \tilde{r}, \tilde{P}^{A} = \tilde{p}^{A}) = \frac{1}{M_{g}(\tilde{r}, \tilde{p}^{A})} \int \frac{1 - F_{B}(g(\tilde{r} - w, \tilde{p}^{A} - w))}{1 - F_{B}(\tilde{p}^{A} - w)} f_{R}(\tilde{r} - w)f_{P^{A}}(\tilde{p}^{A} - w)f_{W}(w)dw
\]

(13)

where \( M_{g}(\tilde{r}, \tilde{p}^{A}) \equiv \int f_{R}(\tilde{r} - w)f_{P^{A}}(\tilde{p}^{A} - w)f_{W}(w)dw \).

To obtain an estimate of \( g(\cdot) \), I first estimate \( \hat{\Pr}(A = 1|\tilde{R} = \tilde{r}, \tilde{P}^{A} = \tilde{p}^{A}) \) using a bivariate local linear regression. I then evaluate this function on a two-dimensional grid for \( \tilde{r} \) and \( \tilde{p}^{A} \), with \( K_{A} = 25 \) grid points in each dimension. Let these grid points be denoted \( \{v^{R}_{k}\}_{k=1}^{K_{A}} \) and \( \{v^{P^{A}}_{j}\}_{j=1}^{K_{A}} \). Let \( \hat{\sigma}^{2}(v^{R}_{k}, v^{P^{A}}_{j}) \) be the estimated variance of this local linear regression estimate at a given point. I then approximate \( g(\cdot) \) flexibly using the tensor product of two univariate cubic b-spline functions, with fifteen knots in each dimension (uniformly spaced between the 0.001 and 0.999 quantiles of \( \tilde{R} \) and \( \tilde{P}^{A} \), respectively), yielding \( K_{g} = 121 \) parameters to be estimated.\(^{16}\) I denote this approximation \( g(r, p^{A}; \theta^{g}) \equiv BS(r, p^{A}/\theta^{g}) \), where \( BS(\cdot) : \mathbb{R}^{2} \rightarrow \mathbb{R}^{K_{g}} \). I estimate \( \theta^{g} \) by solving the following nonlinear constrained least squares problem:

\[
\min_{\theta^{g}} \sum_{k=1}^{K_{A}} \sum_{j=1}^{K_{A}} \frac{1}{\hat{\sigma}^{2}(v^{R}_{k}, v^{P^{A}}_{j})} \left[ \hat{\Pr}(A|\tilde{R} = v^{R}_{k}, \tilde{P}^{A} = v^{P^{A}}_{j}) - \int \frac{1 - \hat{F}_{B}(BS(v^{R}_{k} - w, v^{P^{A}}_{j} - w)/\theta^{g})}{1 - \hat{F}_{B}(v^{P^{A}}_{j} - w)} \hat{f}_{R}(v^{R}_{k} - w)\hat{f}_{P^{A}}(v^{P^{A}}_{j} - w)\hat{f}_{W}(w)dw \right]^{2}
\]

(14)

subject to the constraints that \( g(r, \cdot; \theta^{g}) \) is increasing in \( r \), \( g(\cdot, p^{A}; \theta^{g}) \geq p^{A} \), and that \( g(r, \cdot; \theta^{g}) \geq \rho^{-1}(r) \). I enforce this latter constraint by evaluating \( \rho^{-1}(r) \) as in (12) at \( F_{S} = F_{T}^{L} \). This ensures that the estimated real-world mechanism is consistent with the estimated valuation distributions in that it does not imply trade occurring for cases where \( S > B \). In (14), the objects \( \hat{F}_{B}, \hat{f}_{R}, \hat{f}_{P^{A}}, \) and \( \hat{f}_{W} \) are those estimated above, and \( \hat{M}_{g}(\cdot) \) comes from evaluating \( M_{g}(\cdot) \) using these estimated distributions. As with (9) and (11), this estimator falls into the class of two-step semiparametric GMM estimators, which will be consistent for estimating \( \theta^{g} \).

4.7 Summary of Identification

The distributions \( F_{R}, F_{P^{A}}, \) and \( F_{W} \), along with their corresponding densities, are nonparametrically identified from the joint distribution of \( (\tilde{R}, \tilde{P}^{A}) \). The underlying buyer distribution, \( F_{B} \), is nonparametrically identified by the probability mass function, \( \Pr(N = n) \), and by the marginal distribution of auction prices, \( F_{P^{A}} \). Absent unobserved game-level heterogeneity, following Proposition 2, bounds on the distribution of seller valuations are nonparametrically identified by probabilities of \( D^{A} \) and \( D^{S} \) conditional on \( P^{A} \). When

\(^{16}\)Given a set of \( I \) knots, \( \{t_{i}\}_{i=1}^{I-1} \), a univariate b-spline transformation of degree \( d \) evaluated at a point \( v \) is defined as follows.

\( BS_{0}(v) \) returns an \( (I - d - 1) \)-by-1 vector with 1 as its \( i^{th} \) element if \( t_{i} \leq v < t_{i+1} \) and 0 otherwise. \( BS_{k}(v) \), for \( k \in \{2, ..., d\} \), is defined recursively, returning a vector with the \( i^{th} \) element given by \( BS_{k}(v) = \frac{v - t_{i}}{t_{i+k} - t_{i}} BS_{k-1}(v) + \frac{t_{i+k+1} - v}{t_{i+k+1} - t_{i+1}} BS_{k+1}(v) \).

A cubic b-spline is given by \( BS_{3}(v) \), and with \( I = 15 \) knots, yields \( I - d - 1 = 11 \) parameters to be estimated. The tensor product of two such univariate b-splines thus yields 121 parameters to be estimated. Throughout this section I use cubic b-splines and suppress the cubic subscript on the b-spline function.
unobserved heterogeneity is present, a parametric approximation is required for identification of $F_S^L$ and $F_S^U$, but fortunately this parametric specification can be extremely flexible given the large dataset. The same is true for the objects $\chi^{-1}(\cdot)$ and $g(\cdot)$; in the absence of unobserved heterogeneity, each is nonparametrically identified by conditional probabilities of observables, and, when unobserved heterogeneity is present, each is parametrically identified and each admits a very flexible parametric specification.

5 Computing Bargaining Efficiency

This section describes how I bring together the estimated distributions of buyer and seller valuations, the supports of the types who bargain, and the allocation function for the real-world mechanism in order to compute welfare measures for the bargaining game and for counterfactual, efficient, theoretical bargaining mechanisms. I describe the approach for measuring efficiency and, in particular, computing bounds on these efficiency measures using the bounds on the seller valuation distribution.

The focus of this paper is the efficiency of the bilateral bargaining between the seller and the high bidder. To analyze this, I integrate welfare measures over the support of buyer and seller types who bargain in a given instance of the game, $([b(p^A), b], [s(p^A), s])$, and then integrate over realizations of the auction price $p^A$. Because valuations and transfers are additively separable in game-level heterogeneity, $W$ will play no role in evaluating welfare measures.

5.1 Mechanisms

The mechanisms I consider in this paper are the real-world bargaining, first-best (ex-post efficient) bargaining, and the mechanisms corresponding to the ex-ante efficient, information-constrained Pareto frontier. As described in Section 3.2, each bargaining mechanism has a corresponding allocation function. For the real-world game, this allocation function is the object $x^{RW}$ estimated in Section 4.6. For the first-best, this allocation function is simply an indicator yielding trade whenever the buyer values the good more than the seller, corresponding to $x^*$ in the notation from Section 3.2. For the mechanisms along the ex-ante efficient frontier, these allocation functions are $x^\eta$, for $\eta \in [0, 1]$, discussed in Section 3.2. Of particular interest are $x^1$, the allocation function corresponding to the seller-optimal mechanism, equivalent to a take-it-or-leave-it offer by the seller; $x^0$, the allocation function corresponding to the buyer-optimal mechanism, equivalent to a take-it-or-leave-it offer by the buyer; and $x^{1/2}$, the equal-weighted ex-ante efficient mechanism (referred to herein as the second-best).

Computing the mechanisms along the Pareto frontier is very involved for two reasons. First, I must evaluate these mechanisms at each value of the lower bound of the support of types (i.e. at each value of the auction price), and, second, the valuation distributions I estimate are not necessarily regular. $F_B$ and $F_S$ are referred to as regular if $b - \frac{1 - F_B(b)}{f_B(b)}$ and $s + \frac{F_S(s)}{f_S(s)}$, the virtual valuations of buyers and sellers, are increasing. Myerson and Satterthwaite (1983) and Williams (1987) derived convenient solutions for the mechanisms along the ex-ante efficient frontier under the assumption of regularity. Without regularity, I am forced to solve a linear programming problem for $x^\eta$, maximizing an $\eta$-weighted welfare function subject to a number of constraints described in Myerson and Satterthwaite (1983) and Williams (1987). Let the scalar
q represent the expected utility of the lowest seller type (q). For a given realization of the auction price, pA, let \( f_S(s|pA) = \frac{f_S(s)}{f_S(pA)} \) and \( f_B(b|pA) = \frac{f_B(b)}{f_B(pA)} \) represent the densities of seller and buyer valuations condition on pA, and let \( F_S(s|pA) \) and \( F_B(s|pA) \) represent their corresponding distributions. The allocation function \( x^0 \) is the solution to the linear programming problem,

\[
\max_{x} \left\{ \eta[q + \bar{U}_S(x)] + (1 - \eta)[\Gamma(x) - q + \bar{U}_B(x)] \right\}
\]

subject to \( x(s, b; pA) \in [0, 1] \) \( \forall (s, b) \in [a(pA), \bar{s}] \times [b(pA), \bar{b}] \)

\[
\bar{U}_S(x) = \int_{b(pA)}^{\bar{b}} \int_{a(pA)}^{\bar{s}} x(s, b; pA)f_B(b|pA)F_S(s|pA)dsdb
\]

\[
\bar{U}_B(x) = \int_{b(pA)}^{\bar{b}} \int_{a(pA)}^{\bar{s}} x(s, b; pA)(1 - F_B(b|pA))f_S(s|pA)dsdb
\]

The objects \( \Gamma(x) \) and q are given by

\[
\Gamma(x) = \int_{b(pA)}^{\bar{b}} \int_{a(pA)}^{\bar{s}} (b - s)x(s, b; pA)f_S(s|pA)f_B(b|pA)dsdb - [\bar{U}_S(x) + \bar{U}_B(x)] \quad \text{and} \quad q = \begin{cases} 0 & \text{if } \eta \leq \frac{1}{2} \\ \Gamma(x) & \text{if } \eta > \frac{1}{2} \end{cases}
\]

5.2 Welfare and Efficiency Measures at a Given \( F_S \)

I consider several welfare measures, each of which will depend on the allocation function \( x \) and the distribution of buyer and seller valuations. I will use the notation \( \mathcal{W}(x, F_S) \) to refer to any welfare measure under a given allocation function \( x \) and at a given seller valuation distribution \( F_S \) (suppressing dependence on \( F_B \) given that I have point estimates of \( F_B \)). I will add specific subscripts to \( \mathcal{W} \) to denote a particular type of welfare measure and specific superscripts to \( x \) to denote a particular allocation function.

To begin, the first welfare measure I consider is the expected gains from trade (\( \mathcal{W}_{EG}(x, F_S) \)), given by

\[
\mathcal{W}_{EG}(x, F_S) = \int_{b(pA)}^{\bar{b}} \left[ \int_{b(pA)}^{\bar{b}} \int_{a(pA)}^{\bar{s}} (b - s)x(s, b; pA)f_S(s|pA)f_B(b|pA)dsdb \right] f_{pA}(pA)dpA
\]

This welfare measure, along with the others I consider, is integrated over realizations of the lower bound of the support of buyer and seller types.
The second welfare measure is the expected gains from trade for the buyer alone, denoted $W_B(x, F_S)$. For the ex-ante efficient mechanisms, this object is $U_B$ from (17). For the real-world mechanism, this object is given by replacing $(b - s)$ in (18) with $b$, less the expected buyer transfer paid by the buyer. This transfer is simply the expectation of the observed final prices from the game. The third welfare measure is the expected gains from trade for the seller alone, denoted $W_S(x, F_S)$. This is given by $U_S$ from (16) for ex-ante efficient mechanisms and, for the real-world mechanism, by replacing $(b - s)$ in (18) with the expected transfer to the seller less $s$. The fourth welfare measure is the expected probability of trade, denoted $W_{Pr}(x, F_S)$ and given by replacing $(b - s)$ in (18) with a value of 1.

In addition to computing $W_{EG}(x, F_S)$, $W_B(x, F_S)$, $W_S(x, F_S)$, and $W_{Pr}(x, F_S)$ for a given mechanism $x$, I also compute the gap between the expected gains from trade for the first-best vs. second-best mechanisms, represented by the following notation:

$$W_{EG}(x^* - x^\frac{1}{2}, F_S) \equiv W_{EG}(x^*, F_S) - W_{EG}(x^\frac{1}{2}, F_S)$$

This measures the deadweight loss due to incomplete information. A similar gap measures the difference between the probability of trade in the first-best and second-best mechanisms. I also compute the gap between the expected gains from trade in the second-best mechanism and the real-world mechanism, denoted by

$$W_{EG}(x^\frac{1}{2} - x^{RW}, F_S) \equiv W_{EG}(x^\frac{1}{2}, F_S) - W_{EG}(x^{RW}, F_S)$$

This gap can be viewed as the inefficiency due to the choice of bargaining used in practice and the particular equilibrium of the game that is played. A similar gap measures the difference between the probability of trade for the second-best and real-world mechanisms. Finally, I compute the gap between the first-best and real-world mechanisms, denoted by

$$W_{EG}(x^* - x^{RW}, F_S) \equiv W_{EG}(x^*, F_S) - W_{EG}(x^{RW}, F_S)$$

This gap combines the two different sources of inefficiency. A similar gap measures the difference between the probability of trade for the first-best and real-world mechanisms.

### 5.3 Computing Bounds on Welfare

Each of the above welfare measures can be computed by plugging in the corresponding empirical estimates from Section 4 and using any seller valuation CDF lying between the bounds computed in Section 4.4. Let the space of such CDFs be given by $F^* = \{F_S \in F : F_S(v) \in [F^L_S(v), F^U_S(v)] \mid v \in [s, \bar{s}]\}$. In the spirit of Reguant (2016), bounds on any welfare measure $W(x, F_S)$, denoted $[\underline{W}(x), \overline{W}(x)]$, can be computed by

$$\left[ \min_{F_S \in F^*} W(x, F_S), \max_{F_S \in F^*} W(x, F_S) \right]$$

Numerically computing these bounds is extremely burdensome, as it requires searching over a high-dimensional parameter vector $F_S$ and solving a linear programming problem at each realization of the lower.

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17I ignore bargaining costs in this subsection and discuss how they are incorporated in the following subsection.
bound of the support of types at each guess of $F_S$. To reduce this computational complexity, in computing bounds on welfare I reduce the number of knots in the spline approximation to $K'_S = 25$, with these knots uniformly spaced over the range of the original $K_S = 200$ knots used in Section 4.4. Panels C and D of Appendix Figure A4 demonstrate that this number of knots still approximates the bounds well. Let the subset of these uniformly spaced indices be denoted $K$ and let the vector $\hat{\theta}^{S,L} = \{\hat{\theta}^{S,L}_{k'} : k' \in K\}$ and the vector $\hat{\theta}^{S,U} = \{\hat{\theta}^{S,U}_{k'} : k' \in K\}$; that is, these vectors are subsets of the estimated spline coefficients from Section 4.4 corresponding to the uniformly spaced $K'_S$ knots. The search to find bounds on welfare can then be performed over the set $\Theta^{S} \equiv \{\theta^{S} : \theta^{S}_{k} \in [\hat{\theta}^{S,L}_{k}, \hat{\theta}^{S,U}_{k}] \text{ and } \theta^{S}_{k} \leq \theta^{S}_{k+1} \forall k < K'_S\}$ following (19).

To ease the computational burden further, I demonstrate that many of the welfare measures and mechanisms I consider have the useful property that they are monotonic in a first-order stochastically dominating change in $F_S$. For any welfare measure that satisfies this property, bounds can be obtained by simply evaluating the welfare measure at the lower and upper seller distribution bounds rather than searching numerically over $F_S$. I display these monotonicity properties in Table 3 and state these results as the following proposition:

**Proposition 6.** A first order stochastically dominating change in $F_S$ will lead to the monotonic changes in welfare measures described in Table 3.

Each cell in Table 3 marked with a ↓ signifies that the specified measure will decrease given a first-order stochastically dominating shift in the distribution of seller valuations. Each cell marked with an asterisk indicates that there is no analytic proof of a monotonicity result and that the bounds must be determined numerically using (19). The buyer and seller gains in the column labeled second-best are marked with “–” because, in the analysis below, I will report bounds on these quantities that correspond to the respective player’s surplus under the $F_S$ that leads to the maximum and minimum bounds on the total gains from trade. Similarly, for the buyer-optimal column, bounds on the total gains from trade and seller gains correspond to the $F_S$ that leads to the maximum and minimum bounds on the buyer gains from trade, and similarly for the seller-optimal column (although it turns out that some of these welfare measures also satisfy the monotonicity property, as discussed in the proof of Proposition 6).

For the real-world mechanism, bargaining costs will enter into the computation of the expected gains from trade. For the lower bounds on expected gains from trade in the real-world bargaining ($W_{EG}(x^{RW})$, $W_B(x^{RW})$, and $W_S(x^{RW})$), I incorporate these costs by subtracting an upper bound on expected bargaining costs (described in Appendix C.3). To obtain an upper bound on gains from trade, bargaining costs are set to zero. Auction house fees have already been accounted for in measuring buyer and seller gains from trade for the real-world mechanism through the adjustment described at the beginning of Section 4. For the total gains from trade in the real bargaining, I add these fees back so that the auction house gains will be included in welfare.

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18 To give a rough idea of the computational burden, even with the techniques I introduce here to reduce this burden, it takes about one year of computation time for a single machine to compute all of the estimates reported in the paper. I parallelize these computations on a high-performance computing cluster to reduce this time to less than one week.
6 Putting It All Together: How Efficient Is Bargaining?

6.1 Distribution Estimates

This section presents the distributions of buyer and seller valuations estimated using the procedures described in Sections 4.1–4.4. In each of the figures that follow, monetary values are denoted in units of $1,000. Appendix Figure A1 displays the estimated distributions of secret reserve prices and auction prices before and after the deconvolution accounting for unobserved heterogeneity, and Appendix Figure A2 displays estimates of the functions $\rho^{-1}(\cdot)$, $\chi^{-1}(\cdot)$, and $g(\cdot)$.

In Figure 1, panels A and B display, for the dealers and fleet/lease samples respectively, the distribution of the auction price net of unobserved heterogeneity, $F_{PA}$ (the dashed line), and the estimated underlying distribution of buyer valuations, $F_B$. The distribution of auction prices does not entirely dominate that of the underlying buyer valuations in a first order stochastic dominance sense. This is due to the distribution of the number of bidders, $Pr(N = n)$, having much of its mass at two or three bidders.

Panels C and D of Figure 1 show the distribution of secret reserve prices net of unobserved heterogeneity, $F_R$ (the dashed line), and the estimated upper and lower bounds on the distribution of seller valuations, $F^U_S$ and $F^L_S$, in solid lines. The seller distribution bounds suggest that, for dealer cars (panel C), when the first bargaining offer (the auction price) is about -$1,000 (i.e. $1,000 lower than would be predicted based on car-level heterogeneity), sellers choose to accept this offer or walk away from it with frequencies that imply that the probability that $S$ is less than -$1,000 is in the range [0.56, 0.82]. For the fleet/lease sample (panel D) the corresponding probability inferred from sellers accepting or walking away from an offer of this same magnitude is in the range [0.66, 0.80].

Comparing the top panels of Figure 1 to the bottom panels, it is clear that there is overlap in the support of buyer valuations and seller valuations. This feature illustrates what is referred to in the theoretical bargaining literature as the “no gap” case (i.e., there is no gap between the upper bound of the support of seller valuations and the lower bound of the support of buyer valuations, and hence there is uncertainty as to whether gains from trade actually exist), and is the case motivating Myerson and Satterthwaite (1983) (see Fudenberg and Tirole 1991). However, the actual overlap in terms of mass appears to be small, as most seller values (at least 60% in each sample) lie below zero—in some cases, far below zero—while buyer values are centered about zero and are much less dispersed. This implies that the actual efficiency loss due to incomplete information may be small in this setting. A more precise quantitative analysis of the overlap in buyer and seller mass will be discussed below, taking into account the support of the types who actually end up in the bargaining game.

6.2 Graphical Analysis of Bargaining Efficiency

Using the approach described in Section 5, I compute buyer gains and seller gains in the real-world bargaining mechanism as well as the ex-post and ex-ante efficient frontiers. In this section, I display these results graphically. The performance of the real-world bargaining relative to these theoretical frontiers is displayed in Figure 2. In each panel, the dashed line displays the ex-post efficient frontier in the space of buyer gains (the vertical axis) and seller gains (the horizontal axis). The solid line displays the ex-ante efficient frontier,
computed as described in Section 5 using the results from Williams (1987). The solid dot indicates the expected gains in the real-world mechanism. The top panels in Figure 2 use the seller CDF lower bound and the bottom panels use the upper bound.

Comparing the ex-ante efficient frontier to the ex-post efficient frontier provides an indication of the size of efficiency loss due strictly to incomplete information. In each panel of Figure 2, the ex-ante efficient frontier lies very close to the ex-post efficient frontier, particularly away from the endpoints of the ex-ante frontier. This suggests that, in this market, incomplete information per se may not be leading to large inefficiencies, likely due to the limited overlap in buyer and seller distributions suggested by Figure 1.

Comparing the real-world bargaining outcome to the ex-ante efficient frontier, on the other hand, provides an indication of the size of efficiency loss due to the fact that the mechanism used in practice (alternating-offer bargaining) or its equilibrium may be inefficient. Figure 2 suggests that the real-world outcome lies on the interior of the ex-ante frontier. This, combined with the close proximity of the ex-ante and ex-post frontiers, suggests that any shortfall in efficiency between the real-world bargaining outcome and the ex-post efficient outcome is not due solely to incomplete information.

6.3 Quantitative Analysis of Bargaining Efficiency

The graphical analysis in the preceding section does not capture bounds on the difference in welfare between the different mechanisms; it only evaluates these mechanisms at the upper and lower bounds on seller valuations. This section presents a quantitative analysis of the bounds on welfare measures and the bounds on differences in welfare described in Section 5. Tables 4 and 5 contain numerical values for each of these bounds. Panel A displays the expected gains from trade, buyer and seller gains, and probability of trade for the first-best mechanism, the second-best mechanism, the buyer-optimal mechanism, the seller-optimal mechanism, and the real-world bargaining. Panel B displays the expected gains and probability of trade for the difference between first-best and second-best, second-best and the real-world mechanism, and first-best and the real-world mechanism. Gains are reported in units of $1,000. Tables 4 displays results using the dealers sample and Table 5 displays results using the fleet/lease sample. The estimated bounds are reported in square brackets and confidence sets (computed as described in Appendix C.4) are in parentheses.

I begin by discussing the estimates in the final column of panel A of Table 4, which contains the welfare numbers corresponding to the real-world mechanism. The expected gain from trade in the real-world mechanism ranges from $1,342 to $2,517, with the buyer’s expected gains lying in $820–844 and the seller’s lying in $304–1,456. For buyers (who are all car dealerships), this gain can be interpreted as an expected retail markup above the auction price (plus any costs of remarketing), as a buyer’s valuation should be the amount he expects to eventually resell the car at. For a seller, this gain is the expected profit from selling the car at the auction today as opposed to selling it through her next-best option. This next-best option includes the possibility of taking the car back to the seller’s own lot or leaving it at the auction house to run again in several weeks.

The first through fourth columns of panel A in Table 4 display the welfare measures under the counterfactual mechanisms. The first column demonstrates that, in a full-information world, where ex-post efficiency would be achievable, the gain from trade for the bargaining game would lie in a range $1,532–2,967. The
probability of trade ranges from 0.831–0.890. This latter quantity (or rather, one minus this quantity) serves as a direct measure of the amount (in mass) of overlap between the buyer and seller valuations.

The second column of panel A displays the second-best mechanism—the direct revelation mechanism maximizing the equally weighted expected gains subject to ex-ante budget balance, individual rationality, and incentive compatibility. I find that the range of surplus for this mechanism is only slightly below that of the first-best, suggesting that there is very little loss due solely to incomplete information in this setting. Moving to panel B of Table 4, the results in the first column confirm this finding, where I display bounds on the difference between the first-best and second-best gains from trade and probability of trade. These bounds indicate that the second-best gains from trade fall below the first-best by at most $37. Interestingly, however, the probability of trade in the second-best mechanism can be substantially lower than in the first-best (a lower bound of 0.735 as opposed to 0.831 in panel A, and an upper bound on the first-best probability of trade minus the second-best of 0.122 in panel B). Thus the second-best mechanism can miss out on trades that would be ex-post efficient (i.e. cases where the buyer values the car more than the seller), but these missed trades appear to be cases where the difference in valuations is small, and hence the surplus is still close to first-best.

Relative to the real-world bargaining outcomes, columns 2 and 3 of panel B indicate that the ex-post efficient outcome would achieve an increase in expected surplus of $162–478 per bargaining transaction. This surplus lost in the real bargaining represents 11–16% of the first-best surplus. Efficient bargaining would also yield a higher conversion rate, with an increase in the probability of trade ranging from 0.083–0.141 when moving to the ex-post efficient mechanism, and an increase of up to 0.132 when moving to the ex-ante efficient mechanism. Note that the lower bound for this latter gap is negative (-0.014, with a lower 95% confidence bound of -0.053), implying that I cannot reject the possibility that the real bargaining achieves a similar trade volume to—or even slightly higher than—the ex-ante efficient mechanism, although I can reject the possibility that the real bargaining achieves a surplus level that is as high as the ex-ante efficient outcome (the lower bound on the confidence interval for this surplus gap is positive—$96). This again highlights the feature that the ex-ante efficient mechanism would guarantee an increase in expected surplus relative to the real bargaining by capturing higher-value trades, not necessarily a higher volume of trade. However, industry participants suggest that it is high conversion—a high probability of trade—that is the primary goal of wholesale auto auction houses (see Treece 2013; Lacetera, Larsen, Pope, and Sydnor 2016), and thus real-world the mechanism may be achieving this goal well by achieving a high volume of trade.

The third and fourth columns of panel A display bounds on welfare outcomes under the buyer-optimal and seller-optimal mechanisms. These mechanisms also lie along the ex-ante efficient frontier, but give all of the bargaining power to one party or the other. One interesting feature of these mechanisms is that they are easy to implement; they simply require letting one party make a take-it-or-leave-it offer to the opposing party. I find that the buyer-optimal mechanism would yield a payoff for the buyer that is much higher than his payoff under the currently used mechanism ($1,070–1,925), and could potentially even lead to a higher payoff for the seller ($258–697). The probability of trade, however has the potential to drop as low as 0.307 under the buyer-optimal mechanism (with an upper bound of 0.759). Some of these changes are due to the fact that in this buyer-optimal bargaining the buyer is no longer forced to treat the auction price as a lower
bound on the available bargaining prices.

The seller-optimal mechanism would potentially yield drastic improvements for the seller, with the seller’s expected gains from trade lying in a range from $997–2,335, and the buyer’s gains in this mechanism dropping to $459–585. The probability of trade under the seller-optimal mechanism can be as low as 0.626, not nearly as low as in the buyer-optimal mechanism, although still lower than in the real-world bargaining. The bounds on the total expected gains from trade in the seller-optimal mechanism are similar to those in the second-best mechanism ($1,457–2,920).

The findings for the fleet/lease sample are shown in Table 5 and display similar patterns. The gains from trade overall and separately for both buyers and sellers are generally higher in the fleet/lease sample than in the dealers sample. As in the dealers sample, the second-best mechanism achieves a surplus level that is similar to the first-best in panel A, and panel B indicates that an upper bound on the gap between second-best and first-best gains from trade is $49. The real-world bargaining falls short of the ex-post efficient gains by $200–610, a loss of about 10–18% of the first-best surplus. Panel A indicates that the lower bound on the probability of trade can be lower in the second-best mechanism than in the first-best, although still higher than in the real-world bargaining. This is confirmed in panel B, where the gap in probability of trade between the second-best and real-world bargaining ranges from 0.020–0.136, but between first-best and real-bargaining the gap ranges only from 0.092–0.139. Table 5 suggests that the probability of trade would be lower under the seller-optimal mechanism ([0.592, 0.722]) than under the real-world mechanism (0.750); while this mechanisms is optimal for the seller—it is not optimal for maximizing trade volume.

The estimated probabilities of trade for the real bargaining in Tables 4–5 come from integrating the estimated allocation function $x^{RW}(\rho(S), B; P^A)$ over all three of its arguments, as described in Section 5. The raw probability of trade in the data, for games that include bargaining, can be calculated by combining periods 2 and higher from Table 2 (and from Appendix Table A3) and can be compared to the probability of trade reported in Tables 4–5 for the real-world bargaining. For the dealers sample this number is 0.684 and for the fleet/lease sample this number is 0.656, each of which are somewhat below the estimated probability of trade from Tables 4–5 for the real-world mechanism.

The surplus levels in Tables 4–5 for the real-world bargaining are consistent with other industry estimates. Huang, Luo., and Xia (2015) reported a net profit of $1,150 for the large used-car dealerships (buyers) they study, analogous to (and similar in quantity to) the buyer gains from trade in Tables 4–5. Also, Gavazza, Lizzeri, and Roketskiy (2014) reported that, for popular used cars, the gap between the suggested retail price and trade-in value, as a fraction of the retail price, ranges from 0.15–0.50 for cars ranging from one to ten years old; that is, the gains from trade represent 15–50% of the retail price of the car. Similar measures can be computed here by dividing the expected gains from trade by the sum of the expected gains for the buyer and the mean auction price (together, this latter sum should approximate the retail price). This computation yields 19–35% in the dealers sample and 17–25% in the fleet/lease sample, both lying in a similar range to the estimates of Gavazza, Lizzeri, and Roketskiy (2014).

The results from this section suggest that the bilateral bargaining observed in this market, for both dealers and fleet/lease sales, is indeed inefficient in that it falls short of the ex-ante and ex-post efficient frontiers. However, the ex-post and ex-ante efficient outcomes themselves are close to one another, suggesting that the
inefficiency in this market is not primarily due to incomplete information per se, but rather to the specific bargaining protocol or equilibrium, or to other constraints that a real-world mechanism must satisfy that are not be captured in the information constraints of Myerson and Satterthwaite (1983). This is discussed further in Section 6.6.

6.4 Bargaining Between a Random Buyer and Seller

This section analyzes the estimated expected gains from trade in a setting where bargaining takes place between the seller and a random buyer rather than the high-value bidder. Note that this alters the support of the types in the bargaining game for both the buyer and the seller, and this shifts not only the outcomes achieved in the real-world bargaining, but also the ex-ante and ex-post efficient frontiers. The allocation function for the real-world bargaining in this setting is possible to simulate by evaluating the estimated \( g(r, p^A) \) function at a very small realized value of \( p^A \) (I choose the 0.0001 quantile). At this small realization, the valuation of the buyer who bargains approximately represents a draw from the full support of buyer valuations, \([\bar{b}, \tilde{b}]\) rather than from the truncated support used in the main analysis above, \([\hat{b}(p^A), \tilde{b}]\). Similarly, the valuation of the seller who bargains is a draw from the full support of seller valuations, \([\underline{s}, \bar{s}]\) rather than from \([\underline{s}(p^A), \bar{s}]\).

Table 6 displays the expected gains from trade in the first-best, second-best, and real bargaining between a random buyer and seller. In both samples, the range for the first-best and second-best gains are very similar to one another, but are slightly more divergent than in the main results from Tables 4–5. The overall loss in efficiency between the real bargaining and the first-best setting is $240–878 in the dealers sample, which corresponds to a loss of 19–30% of the first-best surplus. This loss is larger than the percentage loss in the main results in Table 4, suggesting that, in the dealers sample, bargaining between a random buyer and seller is more inefficient than bargaining between the high bidder and the seller. For the fleet/lease sample, the findings are similar. The overall efficiency loss of the real bargaining is $458–1,335, corresponding to a percentage loss of about 20–29% of the first-best surplus, a range in losses that is larger that in the main results in Table 5. The results of this exercise suggest that the auction’s roles of truncating the support of the types who arrive at bargaining and limiting how low the final offer can be do indeed improve the efficiency of the bargaining. The main findings of the paper still hold in this analysis: the real-world bargaining is inefficient, and nearly all of this inefficiency is due to factors other than the information constraints highlighted in Myerson and Satterthwaite (1983).

6.5 Alternative Sample Specifications

Table 7 displays bounds on the expected gains from trade using several different subsamples of the data: cars with below-median blue book value (less-pricey cars), cars with below-median age (newer cars), and cars that are being run through the auction house for the first time by a given seller (as opposed to later sales attempts after a failed trade). The table also displays, in the first row of each panel, the full sample results, equivalent to those in Tables 4 and 5, for comparison.

The estimated gains from trade for the first-best, second-best, and real-world mechanisms are nearly
identical in the full sample and in the sample of less expensive cars. For the sample of newer cars, the range of the gains from trade is substantially lower than in the full sample, for both dealer and fleet/lease sales, and the gap between the first-best and real bargaining is slightly lower than in the full sample. These results suggest that the efficiency loss in bargaining may be lower for newer cars than for older cars. However, the key qualitative finding from the main sample still holds: any efficiency loss appears not to be due to incomplete information per se (as the first-best welfare bounds are very similar to the second-best), but rather to some other feature of the real bargaining such as the specific mechanism or equilibrium played.

One limitation with the analysis in this paper is that it abstracts away from dynamics across instances of the game (Assumption A4). For example, when the game does not result in a trade, the seller may take the car back to her own store or lot, or may leave the car at the auction house where another sales attempt will take place, typically one month later. Through this process, the average car runs through the auction lanes 1.6 times in my data. The main estimates in the paper treat each attempt to sell the car as an independent observation. In the fourth row of Table 7, I instead limit the sample only to the seller’s first attempt to sell the car. This sample restriction results in a slightly higher surplus and a higher gap between the first-best and the real bargaining in the dealers sample, and the opposite result in the fleet/lease sample. However, the main qualitative findings do not differ greatly from those in the main sample.

This last sample specification does not alleviate all concerns of biases that might be introduced by ignoring between-auction dynamics. For example, buyers who fail to acquire a car have the option to later bid on a similar car. This outside option would lead buyers to bid less than their full valuation for a given car (B), and ignoring this outside option in estimation would likely lead to underestimated buyer valuations. On the other hand, the option of sellers to re-run the car later when it doesn’t sell would lead them to demand more than their reservation value for the car (S), and ignoring this feature would likely lead to overestimated seller values. Measures of the total gains from trade, which are functions of B − S, would therefore likely be underestimated when such dynamics are ignored. I ignore these dynamics across games in order to focus on dynamics within the game; studying instead the dynamics across games would be an interesting avenue for future research.

6.6 Discussion of Relative Efficiency

The finding of this paper that the ex-ante and ex-post efficient frontiers are close to one another in this market stands in stark contrast to the result in the most popularly studied theory example of bilateral bargaining, that of symmetric uniform values (where both buyer and seller valuations are uniformly distributed on the interval [0, 1]; see, for example, Chatterjee and Samuelson 1983 and Myerson and Satterthwaite 1983). This case is known to yield a gap between the ex-post and ex-ante efficient probability of trade. The large gap in this special case, however, may have little bearing on the gap to be expected in real-world settings, where the features of the distribution and the extent of asymmetries may diverge far from uniformity and symmetry. Also, as the results above highlight, even in situations where some efficient trades fail to occur, many of these failed trades may be cases where only very small gains from trade exist (i.e. where the buyer’s value is very close to the seller’s), and thus the loss in efficiency due to information constraints need not be large.

Overall, it is not obvious whether the results of this paper should be interpreted as implying that the
real-world bargaining mechanism is relatively efficient or relatively inefficient, particularly given that there are no existing empirical studies of bargaining with two-sided uncertainty to which these results may be compared. Estimating a structural model of one-sided uncertainty, Ambrus, Chaney, and Salitsky (2018) find an efficiency loss of 14% in studying ransom negotiations, and thus the losses I find are very similar to these. Several papers in the experimental literature can also provide an interesting comparison. Bazerman, Gibbons, Thompson, and Valley (1998) argued that real-world bargaining can potentially yield more efficient outcomes than the theoretical second-best due to non-traditional utility functions (where one player’s utility nests the other’s), limits on players’ abilities to mimic other types, and other features of bounded rationality; and Valley, Thompson, Gibbons, and Bazerman (2002) found evidence in lab experiments that communication between players can allow them to outperform the ex-ante efficient frontier. In light of these arguments, the bargaining at wholesale auto auctions might be seen as relatively inefficient given that it falls short of that frontier at all.

As discussed in the introduction, a gap between the outcome of actual bargaining and the efficient frontier can occur for a number of reasons. First, real-world bargaining mechanisms can have multiple equilibria, many of which may be inefficient, and the actions I observe in the data may correspond to one of these inefficient equilibria. While a full characterization of such equilibria does not exist, Satterthwaite and Williams (1989) analyzed a class of bilateral bargaining games (the $k$ double auction) and characterized a continuum of equilibria, with many equilibria having outcomes lying far from the frontier. Second, it may be that this particular bargaining protocol, even if its most efficient equilibrium, falls short of the frontier. Ausubel and Deneckere (1993) demonstrated that, in a bargaining game where players’ distributions have monotone hazard rates and when only one party makes all the offers, some equilibria can come close to the extremes of the ex-ante efficient frontier (where $\eta = 0$ or $\eta = 1$). Ausubel, Cramton, and Deneckere (2002) argued that these results may be extended to alternating-offer games, but no general exposition exists. Third, it may be the case that a gap exists because of a Wilson-doctrine-like argument: the second-best mechanisms can be unwieldy to implement in practice (in particular when $\eta \in (0, 1)$). These mechanisms require that players and the mechanism designer all have knowledge of buyer and seller distributions, and furthermore that the players comprehend that it is indeed incentive compatible for them to truthfully reveal their valuations. Alternating-offer bargaining, on the other hand, is not defined in terms of players’ beliefs (it is detail-free, in the sense of Chung and Ely 2007) and hence may be more simple for the auction house to implement. Such protocols are easy for players to understand, unlike the black box that theoretical mechanisms may appear to be from a player’s perspective. It may indeed be the case, as hypothesized by Wilson (1986) and Ausubel and Deneckere (1993), that “[real-world bargaining mechanisms] survive because they employ trading rules that are efficient for a wide class of environments.” Actual quantitative estimates of real-world bargaining efficiency from other studies will be a welcome addition to the literature in the future for comparison to the estimates in this paper.

The consistent finding in the above analysis is that the ex-ante and ex-post efficient frontiers lie close together in this market, while the real-world bargaining falls short of the efficient frontiers. This suggests that efficiency loss in this market may not be due to incomplete information alone, but to the other aspects of the real-world bargaining described above. It is important to note, however, that these other aspects all
have their roots in incomplete information; if players were to have complete information, many of these other barriers to efficiency might also disappear.

7 Conclusion

This paper examined the efficiency of bargaining from a real-world setting with two-sided incomplete information. I developed a model and strategy for estimating the distributions or bounds on distributions of valuations on both sides of the market while placing very little structure or equilibrium assumptions on the bargaining game itself. I also estimated the revelation mechanism corresponding to the real-world bargaining. I then mapped these distributions into the static, direct-revelation mechanism framework that traces out the efficient frontier derived in Myerson and Satterthwaite (1983) and Williams (1987), averaging over different realizations of the support of types in the bargaining game. I found that the efficiency loss in the bargaining does not appear to be due to incomplete information per se but rather to some other aspect of the mechanism used in practice (such as inefficient equilibria or other constraints faced in a detail-free mechanism). In particular, the ex-ante efficient outcome lies close to the ex-post efficient outcome, but the real-world outcome falls short of both. A fruitful avenue for future research would be to apply a similar bounding methodology or Revelation Principle approach to study the efficiency of bargaining in other settings, potentially exploiting more fully all of the offers observed in alternating-offer bargaining data—a form of data that is becoming increasingly available (e.g. Keniston 2011; Backus, Blake, Larsen, and Tadelis 2018; Bagwell, Staiger, and Yurukoglu 2017).

References


Figure 1: Distribution Estimates

(A) Buyer Values, Dealers

(B) Buyer Values, Fleet/lease

(C) Seller Values, Dealers

(D) Seller Values, Fleet/lease

Notes: Panels A and B display estimated distribution of auctions prices (dashed line) after removing observable and unobservable game-level heterogeneity, and estimated distribution of buyer valuations (solid line). Panels C and D display estimated distribution of reserve prices (dashed line) after removing observable and unobservable game-level heterogeneity, and estimated lower and upper bounds on distribution of seller valuations (solid lines). Panels on left use dealers sample and on right use fleet/lease sample. Units = $1,000.
Figure 2: Bargaining Relative to Ex-ante and Ex-post Efficient Frontiers

(A) Dealers, Using Seller Lower Bound
(B) Fleet/lease, Using Seller Lower Bound
(C) Dealers, Using Seller Upper Bound
(D) Fleet/lease, Using Seller Upper Bound

Notes: Figure displays estimated expected seller and buyer gains on ex-post efficient frontier (dashed line), on ex-ante efficient frontier (solid line), and in real-world bargaining (solid dot). Top panels use seller distribution lower bound and bottom panels use seller distribution upper bound. Panels on left use dealers sample and on right use fleet/lease sample. Units = $1,000.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Dealers</th>
<th>Fleet/lease</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Full Sample</strong></td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Trade</td>
<td>0.706</td>
<td>0.768</td>
</tr>
<tr>
<td>Reserve price</td>
<td>$7,398</td>
<td>$10,385</td>
</tr>
<tr>
<td>Auction price</td>
<td>$6,251</td>
<td>$9,872</td>
</tr>
<tr>
<td>Number of periods</td>
<td>2.095</td>
<td>1.776</td>
</tr>
<tr>
<td>Blue book</td>
<td>$6,816</td>
<td>$11,023</td>
</tr>
<tr>
<td>Age (years)</td>
<td>6.786</td>
<td>3.156</td>
</tr>
<tr>
<td>Mileage</td>
<td>98,030</td>
<td>57,215</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>136,146</td>
<td>133,249</td>
</tr>
<tr>
<td><strong>B. Bid Log Sample</strong></td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Number bidders lower bound</td>
<td>2.924</td>
<td>2.974</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>13,374</td>
<td>103,716</td>
</tr>
</tbody>
</table>

Notes: Mean and standard deviation of variables in dealers and fleet/lease samples. Trade is an indicator for whether trade occurred between the buyer and seller. Number of periods is 1 if game ends through auction price exceeding reserve price or through buyer opting out of bargaining, 2 if seller accepts at her first bargaining turn, etc. Blue book is an estimate of the market value of the car, provided by the auction house. Panel A displays full sample and panel B displays subsample containing bid log records.
Table 2: Outcomes of Game By Period: Dealers Sample

<table>
<thead>
<tr>
<th>Ending period</th>
<th>Player's turn</th>
<th># Obs</th>
<th>% of Sample</th>
<th>% Trade</th>
<th>Full Sample</th>
<th>Conditional on Sale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reserve price</td>
<td>Auction price</td>
</tr>
<tr>
<td>1</td>
<td>(Auction)</td>
<td>14,484</td>
<td>10.655%</td>
<td>88.62%</td>
<td>$5,806</td>
<td>$6,044</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($4,707)</td>
<td>($4,720)</td>
</tr>
<tr>
<td>2</td>
<td>S</td>
<td>101,884</td>
<td>74.947%</td>
<td>77.21%</td>
<td>$7,619</td>
<td>$6,371</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>14,770</td>
<td>10.865%</td>
<td>11.96%</td>
<td>$7,226</td>
<td>$5,534</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($5,048)</td>
<td>($4,607)</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
<td>2,961</td>
<td>2.178%</td>
<td>65.59%</td>
<td>$7,807</td>
<td>$6,351</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($5,164)</td>
<td>($4,829)</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>1,685</td>
<td>1.239%</td>
<td>38.16%</td>
<td>$8,274</td>
<td>$6,683</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($5,292)</td>
<td>($4,942)</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>192</td>
<td>0.141%</td>
<td>76.04%</td>
<td>$8,715</td>
<td>$7,105</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($5,352)</td>
<td>($5,005)</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>143</td>
<td>0.105%</td>
<td>55.24%</td>
<td>$8,415</td>
<td>$6,784</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($5,216)</td>
<td>($4,918)</td>
</tr>
<tr>
<td>8</td>
<td>S</td>
<td>20</td>
<td>0.015%</td>
<td>75.00%</td>
<td>$9,015</td>
<td>$7,570</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($4,883)</td>
<td>($4,891)</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>6</td>
<td>0.004%</td>
<td>66.67%</td>
<td>$7,583</td>
<td>$6,225</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($4,362)</td>
<td>($4,517)</td>
</tr>
<tr>
<td>10</td>
<td>S</td>
<td>1</td>
<td>0.001%</td>
<td>100.00%</td>
<td>$14,500</td>
<td>$11,000</td>
</tr>
</tbody>
</table>

Notes: Dealers sample. For each period (period 1 = auction, period 2 = seller’s first turn in bargaining, period 3 = buyer’s turn, etc.), table reports the number of observations ending in that period, percent of total sample ending in that period, and percent of cases in which trade occurred. Table also reports reserve price and auction price for observations ending in a given period and, for those observations ending in trade, the reserve price, auction price, and final price conditional on trade. Corresponding statistics for the fleet/lease sample are found in Table A3.
Table 3: Monotonicity Results for Welfare Measures

<table>
<thead>
<tr>
<th>A. Levels</th>
<th>First-best</th>
<th>Second-best</th>
<th>Buyer-optimal</th>
<th>Seller-optimal</th>
<th>Real bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gains</td>
<td>↓</td>
<td>↓</td>
<td>–</td>
<td>–</td>
<td>↓</td>
</tr>
<tr>
<td>from trade</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buyer gains</td>
<td>–</td>
<td>↓</td>
<td>–</td>
<td>–</td>
<td>↓</td>
</tr>
<tr>
<td>Seller gains</td>
<td>–</td>
<td>–</td>
<td>↓</td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>Probability of</td>
<td>↓</td>
<td>*</td>
<td>*</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>trade</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Differences</th>
<th>First-best minus second-best</th>
<th>Second-best minus real</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gains</td>
<td>*</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>from trade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of</td>
<td>*</td>
<td>*</td>
<td>↓</td>
</tr>
<tr>
<td>trade</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table displays monotonicity results for welfare measures proved in Proposition 6. Each cell marked with a ↓ signifies that the specified measure will decrease given a first-order stochastically dominating shift in $F_S$. Each cell marked with an asterisk indicates that there is no analytic proof of a monotonicity result and that the bounds must be determined numerically. Cases marked with “…” in the second-best column indicate that I will report bounds on these quantities corresponding to the $F_S$ leading to the maximum and minimum bounds on the total expected gains from trade. Cases marked with “…” in the buyer-optimal column indicate that I will report bounds on these quantities corresponding to the $F_S$ leading to the maximum and minimum bounds on the buyer gains from trade. Cases marked with “…” in the seller-optimal column indicate that I will report bounds on these quantities corresponding to the $F_S$ leading to the maximum and minimum bounds on the seller gains from trade.
Table 4: Bounds on Welfare Measures, Dealers Sample

<table>
<thead>
<tr>
<th>A. Levels</th>
<th>First-best</th>
<th>Second-best</th>
<th>Buyer-optimal</th>
<th>Seller-optimal</th>
<th>Real bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gains from trade</td>
<td>[1.532, 2.967]</td>
<td>[1.522, 2.966]</td>
<td>[1.328, 2.622]</td>
<td>[1.457, 2.920]</td>
<td>[1.342, 2.517]</td>
</tr>
<tr>
<td>Buyer gains</td>
<td>[0.587, 0.758]</td>
<td>[1.070, 1.925]</td>
<td>[0.459, 0.585]</td>
<td>[0.820, 0.844]</td>
<td>(0.476, 1.003)</td>
</tr>
<tr>
<td>Seller gains</td>
<td>[0.934, 2.208]</td>
<td>[0.258, 0.697]</td>
<td>[0.997, 2.335]</td>
<td>[0.304, 1.456]</td>
<td>(0.854, 2.356)</td>
</tr>
<tr>
<td>Probability of trade</td>
<td>[0.831, 0.890]</td>
<td>[0.735, 0.880]</td>
<td>[0.307, 0.759]</td>
<td>[0.626, 0.790]</td>
<td>[0.748, 0.748]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Differences</th>
<th>First-best minus second-best</th>
<th>Second-best minus real</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected gains from trade</td>
<td>[0.001, 0.037]</td>
<td>[0.152, 0.476]</td>
<td>[0.162, 0.478]</td>
</tr>
<tr>
<td>Seller gains</td>
<td>(0.001, 0.064)</td>
<td>(0.096, 0.559)</td>
<td>(0.135, 0.574)</td>
</tr>
<tr>
<td>Probability of trade</td>
<td>[0.008, 0.122]</td>
<td>[-0.014, 0.132]</td>
<td>[0.083, 0.141]</td>
</tr>
<tr>
<td>Buyer gains</td>
<td>(0.003, 0.147)</td>
<td>(-0.053, 0.260)</td>
<td>(0.031, 0.309)</td>
</tr>
</tbody>
</table>

Notes: Dealers sample. Bounds on welfare measures under first-best, second-best, buyer-optimal, and seller-optimal mechanisms compared to real-world mechanism. Panel A displays levels and panel B displays differences. Estimated bounds are in square braces and 95% confidence set is in parentheses. Gains are in $1,000 units.
Table 5: Bounds on Welfare Measures, Fleet/lease Sample

<table>
<thead>
<tr>
<th></th>
<th>First-best</th>
<th>Second-best</th>
<th>Buyer-optimal</th>
<th>Seller-optimal</th>
<th>Real bargaining</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Levels</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected gains from trade</td>
<td>[1.961, 3.366]</td>
<td>[1.958, 3.365]</td>
<td>[1.795, 2.760]</td>
<td>[1.815, 3.248]</td>
<td>[1.722, 2.795]</td>
</tr>
<tr>
<td>(1.527, 3.449)</td>
<td>(1.463, 3.448)</td>
<td>(1.279, 2.983)</td>
<td>(1.334, 3.335)</td>
<td>(1.326, 2.849)</td>
<td></td>
</tr>
<tr>
<td>Buyer gains</td>
<td>[1.118, 1.189]</td>
<td>[1.557, 2.212]</td>
<td>[0.607, 0.752]</td>
<td>[1.245, 1.279]</td>
<td></td>
</tr>
<tr>
<td>(0.701, 1.238)</td>
<td>(0.904, 2.321)</td>
<td>(0.492, 0.768)</td>
<td>(0.869, 1.296)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seller gains</td>
<td>[0.840, 2.176]</td>
<td>[0.238, 0.549]</td>
<td>[1.208, 2.496]</td>
<td>[0.244, 1.283]</td>
<td></td>
</tr>
<tr>
<td>(0.753, 2.224)</td>
<td>(0.220, 0.881)</td>
<td>(0.830, 2.580)</td>
<td>(0.170, 1.345)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of trade</td>
<td>[0.842, 0.889]</td>
<td>[0.770, 0.886]</td>
<td>[0.486, 0.768]</td>
<td>[0.592, 0.722]</td>
<td>[0.750, 0.750]</td>
</tr>
<tr>
<td>(0.781, 0.891)</td>
<td>(0.626, 0.886)</td>
<td>(0.373, 0.777)</td>
<td>(0.467, 0.729)</td>
<td>(0.592, 0.758)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>First-best minus second-best</th>
<th>Second-best minus real</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B. Differences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected gains from trade</td>
<td>[0.001, 0.049]</td>
<td>[0.197, 0.609]</td>
<td>[0.200, 0.610]</td>
</tr>
<tr>
<td>(0.001, 0.084)</td>
<td>(0.080, 0.647)</td>
<td>(0.140, 0.648)</td>
<td></td>
</tr>
<tr>
<td>Probability of trade</td>
<td>[0.003, 0.108]</td>
<td>[0.020, 0.136]</td>
<td>[0.092, 0.139]</td>
</tr>
<tr>
<td>(0.003, 0.164)</td>
<td>(-0.005, 0.175)</td>
<td>(0.078, 0.246)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Fleet/lease sample. Bounds on welfare measures under first-best, second-best, buyer-optimal, and seller-optimal mechanisms compared to real-world mechanism. Panel A displays levels and panel B displays differences. Estimated bounds are in square braces and 95% confidence set is in parentheses. Gains are in $1,000 units.
Table 6: Expected Gains From Trade in Bargaining Between a Random Buyer and Seller

<table>
<thead>
<tr>
<th></th>
<th>First-best</th>
<th>Second-best</th>
<th>Real bargaining</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dealers Sample</td>
<td>[1.280, 2.946]</td>
<td>[1.238, 2.919]</td>
<td>[1.039, 2.068]</td>
<td>[0.240, 0.878]</td>
</tr>
<tr>
<td></td>
<td>(1.248, 3.258)</td>
<td>(1.207, 3.213)</td>
<td>(0.062, 2.554)</td>
<td>(0.051, 2.987)</td>
</tr>
<tr>
<td>Fleet/lease Sample</td>
<td>[2.343, 4.542]</td>
<td>[2.309, 4.509]</td>
<td>[1.885, 3.207]</td>
<td>[0.458, 1.335]</td>
</tr>
<tr>
<td></td>
<td>(2.163, 4.607)</td>
<td>(2.102, 4.562)</td>
<td>(0.657, 4.181)</td>
<td>(0.023, 3.134)</td>
</tr>
</tbody>
</table>

Notes: Bounds on expected gains from trade in first-best, second-best, real-world bargaining, as well as the gap between first-best and real-world bargaining, when a random buyer (rather than the high bidder) bargains with a seller. As explained in Section 6.4, this setting is computed by setting the auction price to a low quantile of the auction price distribution. Units are $1,000. Estimated bounds are in square braces and 95% confidence set is in parentheses.
Table 7: Expected Gains From Trade Using Alternative Sample Restrictions

<table>
<thead>
<tr>
<th>A. Dealers Sample</th>
<th>First-best</th>
<th>Second-best</th>
<th>Real bargaining</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>[1.532, 2.967]</td>
<td>[1.522, 2.966]</td>
<td>[1.342, 2.517]</td>
<td>[0.162, 0.478]</td>
</tr>
<tr>
<td></td>
<td>(1.392, 3.109)</td>
<td>(1.352, 3.107)</td>
<td>(1.200, 2.658)</td>
<td>(0.135, 0.574)</td>
</tr>
<tr>
<td>Below Median Blue Book</td>
<td>[1.531, 2.968]</td>
<td>[1.522, 2.966]</td>
<td>[1.343, 2.517]</td>
<td>[0.161, 0.478]</td>
</tr>
<tr>
<td></td>
<td>(1.384, 3.149)</td>
<td>(1.347, 3.146)</td>
<td>(1.184, 2.677)</td>
<td>(0.143, 0.590)</td>
</tr>
<tr>
<td>Below Median Age</td>
<td>[1.085, 1.737]</td>
<td>[1.057, 1.731]</td>
<td>[0.893, 1.398]</td>
<td>[0.143, 0.388]</td>
</tr>
<tr>
<td></td>
<td>(0.966, 2.240)</td>
<td>(0.943, 2.233)</td>
<td>(0.773, 1.881)</td>
<td>(0.113, 0.424)</td>
</tr>
<tr>
<td>First Run of Car</td>
<td>[1.730, 3.386]</td>
<td>[1.723, 3.385]</td>
<td>[1.386, 2.631]</td>
<td>[0.286, 0.812]</td>
</tr>
<tr>
<td></td>
<td>(1.417, 3.573)</td>
<td>(1.366, 3.572)</td>
<td>(1.151, 2.902)</td>
<td>(0.164, 0.885)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Fleet/lease Sample</th>
<th>First-best</th>
<th>Second-best</th>
<th>Real bargaining</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Sample</td>
<td>[1.961, 3.366]</td>
<td>[1.958, 3.365]</td>
<td>[1.722, 2.795]</td>
<td>[0.200, 0.610]</td>
</tr>
<tr>
<td></td>
<td>(1.527, 3.449)</td>
<td>(1.463, 3.448)</td>
<td>(1.326, 2.849)</td>
<td>(0.140, 0.648)</td>
</tr>
<tr>
<td>Below Median Blue Book</td>
<td>[1.965, 3.324]</td>
<td>[1.962, 3.322]</td>
<td>[1.728, 2.761]</td>
<td>[0.199, 0.602]</td>
</tr>
<tr>
<td></td>
<td>(1.536, 3.458)</td>
<td>(1.479, 3.457)</td>
<td>(1.347, 2.877)</td>
<td>(0.139, 0.649)</td>
</tr>
<tr>
<td>Below Median Age</td>
<td>[1.406, 2.551]</td>
<td>[1.360, 2.538]</td>
<td>[1.208, 2.087]</td>
<td>[0.156, 0.506]</td>
</tr>
<tr>
<td></td>
<td>(1.264, 2.679)</td>
<td>(1.228, 2.665)</td>
<td>(1.044, 2.174)</td>
<td>(0.126, 0.581)</td>
</tr>
<tr>
<td>First Run of Car</td>
<td>[1.835, 3.008]</td>
<td>[1.831, 3.007]</td>
<td>[1.568, 2.507]</td>
<td>[0.192, 0.575]</td>
</tr>
<tr>
<td></td>
<td>(1.405, 3.306)</td>
<td>(1.350, 3.305)</td>
<td>(1.214, 2.733)</td>
<td>(0.136, 0.658)</td>
</tr>
</tbody>
</table>

Notes: Full Sample row reports expected gains using full sample, as in Tables 4–5. Below Median Blue Book row uses the sample of cars with a blue book value below the median. Below Median Age row uses the sample of newer cars. First Run of Car row uses only the first time a given car was attempted to be sold by a given seller. Panel A contains dealer sellers and panel B fleet/lease sellers. Estimated bounds are in square braces and 95% confidence set is in parentheses. Units are $1,000.
A Proofs

Before providing the proofs corresponding to results in the main text, I first introduce some additional notation and state some preliminary lemmas. Let $H_t \equiv \{P_\tau\}_{\tau=1}^{t-1}$ represent the set of offers made from period 1 up through period $t-1$ of the bargaining game. The player whose turn it is at time $t$ has not yet made an offer and so this offer does not enter into $H_t$. Let $D_t^S \in \{A, Q, C\}$ represent the seller’s decision in period $t$, and let $D_{t+1}^B \in \{A, Q, C\}$ represent the buyer’s decision in period $t+1$.

The seller’s payoff at period $t$ of the bargaining game is given by the following. Conditional on a realization of the history $H_t = h_t$, which includes the buyer’s most recent offer ($P_{t-1}$), a seller of type $S = s$, chooses to accept (A), quit (Q), or counter (C), yielding the following payoffs:

A : $P_{t-1}$

Q : $s$

C : $V_{t}^S (s|h_t)$

$$= \max_p \left\{ p \Pr (D_{t+1}^B = A|h_{t+1}, p) + s \Pr (D_{t+1}^B = Q|h_{t+1}, p) \right\}$$

$$+ \Pr (D_{t+1}^B = C|h_{t+1}, p) E_{P_{t+1}} \left\{ \max \left\{ P_{t+1}^B, s, V_{t+2}^S (s|h_{t+1}, p, P_{t+1}^B) \right\} \right\} - c_S$$

where $p$ is the counteroffer chosen by the seller. The seller’s counteroffer payoff takes into account that the buyer may either accept, quit, or return a counteroffer. In the latter case, the seller receives her expected payoff from being faced with the decision in period $t+2$ to accept, quit, or counter.

The buyer’s payoff at period $t+1$ of the bargaining game is defined similarly, with the buyer receiving $b - p$ if he accepts a price $p$, 0 if he quits, and an expected counteroffer payoff if he counters. Conditional on a realization of the history $H_{t+1} = h_{t+1}$, which includes the seller’s most recent offer ($P_{t}^S$), a buyer of type $B = b$ chooses to accept (A), quit (Q), or counter (C), yielding the following payoffs:

A : $b - P_t^S$

Q : 0

C : $V_{t+1}^B (b|h_{t+1})$

$$= \max_p \left\{ (b - p) \Pr (D_{t+2}^S = A|h_{t+1}, p) \right\}$$

$$+ \Pr (D_{t+2}^S = C|h_{t+1}, p) E_{P_{t+2}} \left\{ \max \left\{ b - P_{t+2}^S, 0, V_{t+3}^B (b|h_{t+1}, p, P_{t+2}^S) \right\} \right\} - c_B$$

where $p$ is the counteroffer chosen by the buyer. The buyer’s outside option is normalized to zero.

The expected payoff of a buyer of type $B = b$ in the bargaining subgame, conditional on winning the auction and conditional on entering bargaining when the auction price is $P^A$, is given by

$$\pi^B (P^A, b) = (b - P^A) \Pr (D_2^S = A|P^A)$$

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\[
+ \Pr \left( D_2^S = C | p^A \right) E_{P_2^S} \left[ \max \left\{ b - P_2^S, 0, V_3^B \left( b | \{ p^A, P_2^S \} \right) \right\} \right] p^A, D_2^S = C - c_B
\]

This expression is the payoff to the buyer from stating the auction price as a counteroffer, which is how the bargaining game begins.

**Lemma 1.** If Assumptions A1–A3 are satisfied, then for any finite \( T \) and any realized histories \( h_t \) and \( h_{t+1} \), \( V_t^S(s|h_t) \) is weakly increasing in \( s \) and \( V_{t+1}^B(b|h_{t+1}) \) is weakly increasing in \( b \) for all \( t \leq T \).

**Proof.** The proof proceeds by induction on the number of periods remaining. I prove the result in the case where the buyer moves last; analogous reasoning proves that the result also holds if the seller moves last. Suppose there are \( T \) total periods in the game and there is currently one period remaining; it is the seller’s turn and after her turn the buyer will only be allowed to accept or quit. At a given realization of \( H_{T-1} = h_{T-1} \), the seller’s payoff from counteracting at a price of \( p \) is then

\[
U_t^S(s, p|h_{T-1}) = p \Pr(D_{t}^B = A(h_{T-1}, p)) + s(1 - \Pr(D_{t}^B = A(h_{T-1}, p))) - c_S
\]

Let \( p^*(s|h_{T-1}) = \arg \max_p U_t^S(s, p|h_{T-1}) \). That is, \( V_{T-1}^S(s|h_{T-1}) = U_{T-1}^S(s, p^*(s|h_{T-1})|h_{T-1}) \). Now let \( V_{T-1}(s, s'|h_{T-1}) \) represent the payoff to the seller of type \( s \) who mimics type \( s' < s \) (note that the ability of a seller—or buyer—to mimic another type relies on the i.i.d. properties in Assumptions A1–A2). Clearly \( V_{T-1}(s, s'|h_{T-1}) \geq V_{T-1}(s, s'|h_{T-1}) \) because \( V_{T-1}(s, s'|h_{T-1}) \) is the maximized counteroffer payoff given the seller’s true value, \( s \). It remains to be shown that \( V_{T-1}(s, s'|h_{T-1}) \geq V_{T-1}(s', s'|h_{T-1}) \).

Below, \( p^*(s'|h_{T-1}) \) represents the offer that would be optimal for a seller of type \( s' \) given the realized history \( h_{T-1} \). Observe that

\[
V_{T-1}(s, s'|h_{T-1}) = p^*(s'|h_{T-1}) \Pr(D_{t}^B = A(h_{T-1}, p^*(s'|h_{T-1}))) + s(1 - \Pr(D_{t}^B = A(h_{T-1}, p^*(s'|h_{T-1})))) - c_S,
\]

and

\[
V_{T-1}(s', s'|h_{T-1}) = p^*(s'|h_{T-1}) \Pr(D_{t}^B = A(h_{T-1}, p^*(s'|h_{T-1}))) + s'(1 - \Pr(D_{t}^B = A(h_{T-1}, p^*(s'|h_{T-1})))) - c_S
\]

Thus,

\[
V_{T-1}(s, s'|h_{T-1}) - V_{T-1}(s', s'|h_{T-1}) = (s - s')(1 - \Pr(D_{t}^B = A(h_{T-1}, p^*(s'|h_{T-1})))) \geq 0
\]

Therefore, \( V_{T-1}(s, s|h_{T-1}) \geq V_{T-1}(s', s'|h_{T-1}) \), and the seller’s counteroffer payoff is weakly increasing in her type when there is one period remaining.

To complete the proof by induction, let \( V_{T-(t-1)}^S(s|h_{T-(t-1)}) \) denote the seller’s counteroffer payoff with \( t - 1 \) periods remaining, and suppose \( V_{T-(t-1)}^S(s|h_{T-(t-1)}) \) is weakly increasing in \( s \). Note that, for \( s' < s \), when there are \( t \) periods remaining, \( V_{T-t}(s, s|h_{T-t}) \geq V_{T-t}(s, s'|h_{T-t}) \) by the same argument as above for. It remains to be shown that \( V_{T-t}(s, s'|h_{T-t}) \geq V_{T-t}(s', s'|h_{T-t}) \).
Note that

\[ V_{T-t}(s, s'|h_{T-t}) - V_{T-t-1}(s', s|h_{T-t}) \]

\[ = (s - s') \Pr \left( D^B_{T-(t-1)} = Q\{h_{T-t}, p^*(s'|h_{T-t})\} \right) + \Pr \left( D^B_{T-(t-1)} = C\{h_{T-t}, p^*(s'|h_{T-t})\} \right) \]

\[ \times E_{P_{T-(t-1)}} \left[ \max \left\{ D^B_{T-(t-1)}, s, V^S_{T-(t-1)} \left( s, s'|h_{T-t}, p^*(s'|h_{T-t})\right) \right\} \right] 

\[ - \max \left\{ D^B_{T-(t-1)}, s', V^S_{T-(t-1)} \left( s', s'|h_{T-t}, p^*(s'|h_{T-t})\right) \right\} \right] \right] \]

\[ \geq 0 \]

Therefore, \( V_{T-t}(s, s|h_{T-t}) \geq V_{T-t-1}(s', s'|h_{T-t}) \), completing the proof. The proof that the buyer counteroffer payoff, \( V^B_{t+1}(b|h_{T+1}) \), is increasing in \( b \) follows by the same steps.

\[ \square \]

Lemma 2. If the rules of the game are relaxed such that the bargained price is allowed to be lower than the auction price, then it is still the case that the following cannot occur in equilibrium: Some bidder \( i \) remains in the bidding even after the current bid exceeds \( b_i \) and, if bargaining occurs and if given the chance to counteroffer, the bidder makes a counteroffer less than \( b_i \) and this offer is accepted by the seller.

Proof. Note that a buyer who bids above his value bears a risk of winning the auction at a price that exceeds the reserve price, giving the bidder a negative payoff. Therefore, if some bidder \( i \) wants to bid above his value in the auction, it must be because that bidder hopes to eventually end up in bilateral bargaining with the seller and hopes to have the bargaining game end at a price weakly below his value, and the bidder believes that the chance of this happening is sufficiently high to warrant the risk of bidding above his value. That is, for this behavior to occur in equilibrium, it must be the case that, in the bargaining game, \( i \) makes a counteroffer lower than \( p^A \) and this offer is accepted by some seller (the seller should never make such an offer herself because accepting \( p^A \) would be preferable). Let the term high-low strategy refer to this strategy of remaining in the bidding even after the current bid exceeds \( b_i \) and then, if given the chance to do so, making an offer less than \( b_i \) (and hence also less than \( p^A \)).

In order for the high-low strategy to be optimal for \( b_i \), the following must be true (I will refer to this as the high-low supposition): the seller cannot distinguish between type \( b_i \) and some type \( \tilde{b} > b_i \) whom the seller believes may not be playing the high-low strategy (and hence may not be attempting to later counter below \( p^A \)). This is because, if the seller could distinguish between the two, the seller would simply accept \( p^A \) when faced with the bidder known to be playing the high-low strategy. Note that it cannot be the case that all bidders play the high-low strategy, or else the seller would always immediately accept the auction price and the bidders would all obtain a negative surplus and hence all bidders would have been better off not bidding above their values. Thus, in order for the high-low strategy to be optimal for \( b_i \), it must be the case that the seller cannot distinguish between type \( b_i \) and some type \( \tilde{b} > b_i \) whom the seller believes may not be playing the high-low strategy.

Suppose the high-low supposition is true. Note that, if type \( b_i > p^A \) finds it optimal to play the high-low strategy, so will all types higher than \( b_i \), because \( \pi^B(p^A, b_i) \geq 0 \Rightarrow \pi^B(p^A, \tilde{b}) > 0 \) for all \( \tilde{b} > b_i \) (because \( V^B_{t+1}(\cdot) \) is weakly increasing in \( b_i \) by Lemma 1 and \((b_i - p^A)\) is strictly increasing in \( b_i \), so \( \pi^B(p^A, b_i) \) is strictly
increasing in $b_i$). Therefore, if any buyer finds it optimal to play the high-low strategy, so will buyer type $\bar{b} > b_i$, which contradicts our supposition. Therefore, if any buyer finds it optimal to play the high-low strategy, and if $p^A$ is higher than the lowest buyer type who would find it optimal to play the high-low strategy, then the seller’s best response when facing an auction price of $p^A$ at the beginning of a bargaining game would be to immediately accept $p^A$ if $p^A \geq s$ and quit otherwise. □

**Proof of Proposition 1**

*Proof.* Consider an arbitrary bidder of type $B = b$. A bidder’s strategy is the price at which he stops bidding as a function of his type. Suppose the current price of the ascending button auction is some value $\bar{p}$ and suppose the bidder is one of at least two bidders still remaining in the auction up until the price reaches its current level $\bar{p}$.

If $b > \bar{p}$, it is optimal for the bidder to remain in the auction, as dropping out would yield a payoff of 0 and staying in would yield a non-negative expected payoff because there is some chance that the bidder will win at a price $p^A < b$. If this auction price and reserve price satisfy $p^A \geq R$, the car will sell through the auction and the bidder will receive a positive payoff. If instead $p^A < R$, the bidder will be given the option to enter into bargaining, and will only enter if doing so yields a non-negative expected payoff.

If $b < \bar{p}$, the buyer cannot receive a strictly positive expected payoff from remaining in the auction. To see this, note that if the bidder remains in the auction there is some chance that he will win at some $p^A > b$. If this occurs and the auction price and reserve price satisfy $p^A \geq R$, the car will sell through the auction and the bidder will receive $b - p^A < 0$. If, on the other hand, the bidder wins and $p^A < R$, the bidder’s payoff conditional on entering bargaining will necessarily be negative because the final bargained price must be greater than $p^A$ and hence, in this case, the bidder will opt out of bargaining, receiving a payoff of 0. □

**Proof of Proposition 3**

*Proof.* Note that for $b' > b$, $\pi^B(\chi(b), b') > 0$. This follows because, by Lemma 1, $V^B_s(\cdot)$ is weakly increasing in $b$, and this fact, combined with the term $(b - p^A)$ appearing in $\pi^B(p^A, b)$, which is strictly increasing in $b$, implies that $\pi^B(p^A, b)$ is strictly increasing in $b$. Thus, $\chi(b') > \chi(b)$, and hence $\chi$ is strictly increasing, and $\chi^{-1}$ exists and is also strictly increasing.

The property that $\chi^{-1}(p^A) > p^A$ follows from the following argument. A buyer must pay $c_B > 0$ if he opts to bargain, and the best possible outcome a buyer can expect from bargaining would be to only have to pay $p^A$. Therefore, for any auction price $p^A$, there exists some buyer with type close to $p^A$, say $p^A + \varepsilon$, where $\varepsilon < c_B$, who would prefer to opt out of bargaining rather than receive a payoff of (at most) $\varepsilon - c_B$, which is negative.

Strict monotonicity of $\rho(\cdot)$, along with the fact that $\rho(s) \geq s$, is proven separately in Lemma 3 below.

When the auction price is $p^A$ and bargaining occurs, it will be common knowledge among the two bargaining parties that seller’s type $s$ satisfies $\rho(s) \geq p^A$, and thus $s \in [\rho^{-1}(p^A), \pi]$. Similarly, bargaining occurring means the buyer did not opt out, so $\chi(b) \geq p^A$, implying $b \in [\chi^{-1}(p^A), \bar{b}]$. □

**Lemma 3.** If Assumptions A1–A3 are satisfied, then in any BNE satisfying E1, the seller’s optimal secret reserve price, $\rho^s(s)$, is strictly increasing in $s$ and satisfies $\rho^s(s) \geq s$.  

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Lemma 4. With realizations of the reserve price \( r \) and a cutoff value in any BNE of this game, conditional on a realization of the auction price, \( p \), Theorem 1 of Storms (2015) (included below as Lemma 4, modified to fit this setting) implies that,

\[
E_B \left[ \max_{p_A} \left( P^A \mathbb{1} \{ P^A \geq \rho(s) \} + s \mathbb{1} \{ P^A < \rho(s), \pi^B(P^A, B) < 0 \} \right) \right]
\]

This term consists of three pieces: 1) the auction price, which the seller receives if it exceeds the reserve price; 2) the seller’s type \( s \), which the seller receives if the auction price is below the reserve price and the buyer opts out of bargaining; and 3) the seller’s bargaining payoff, \( \pi^S(P^A, s) = \max \{ P^A, s, V^S_2(s|P^A) \} \), which the seller receives when the price is below the reserve price and bargaining occurs.

Proof. In choosing her secret reserve price, \( \rho(s) \), a seller of type \( S = s \) wishes to maximize her ex-ante payoff, given by

\[
E_B \left[ \max_{p_A} \left( P^A \mathbb{1} \{ P^A \geq \rho(s) \} + s \mathbb{1} \{ P^A < \rho(s), \pi^B(P^A, B) < 0 \} \right) \right]
\]

The seller’s payoff can be re-written as

\[
\int_0^\rho p^A f_{P^A}(p^A)dp^A + \int_{\rho}^\infty \left[ \int_0^{\chi^{-1}(p^A)} s f_B(b)db + \int_{\chi^{-1}(p^A)}^{\infty} \pi^S(p^A, s) f_B(b)db \right] \frac{f_{P^A}(p^A)}{1 - F_B(p^A)} dp^A = \int_0^\rho p^A f_{P^A}(p^A)dp^A + \int_{\rho}^\infty \left[ \int_0^{\chi^{-1}(p^A)} s F_B(\chi^{-1}(p^A)) - F_B(p^A) + \pi^S(p^A, s) (1 - F_B(\chi^{-1}(p^A))) \right] \frac{f_{P^A}(p^A)}{1 - F_B(p^A)} dp^A
\]

Condition E1 implies \( \pi^S(\cdot, s) \) is continuous and thus the payoff is differentiable. Differentiating the above expression using Leibniz Rule yields the following derivative with respect to \( \rho \):

\[
\frac{\partial}{\partial \rho} = -\rho + s \frac{F_B(\chi^{-1}(\rho)) - F_B(\rho)}{1 - F_B(\rho)} + \pi^S(\rho, s) \frac{1 - F_B(\chi^{-1}(\rho))}{1 - F_B(\rho)}
\]

Lemma 1 implies that \( \pi^S(p^A, s) \) is weakly increasing in \( s \). The proof of Proposition 3 demonstrates that \( \chi^{-1}(p^A) > p^A \), and thus \( F_B(\chi^{-1}(\rho)) > F_B(\rho) \). Combining these arguments implies that \( \chi^{-1}(\rho) \) is strictly increasing in \( s \). Given that \( \frac{\partial}{\partial \rho} \) is strictly increasing in \( s \), the Edlin and Shannon (1998) Theorem implies that, as long as the optimal \( \rho^*(s) \) lies on the interior of the support of \( \rho \), \( \rho^*(s) \) will be strictly increasing in \( s \). The support of \( \rho \) is the real line, thus completing the proof of strict monotonicity. Note that without costly bargaining a weak monotonicity result can be obtained using Topkis’s Theorem.

Finally, the fact that \( \rho^*(s) \geq s \) can be seen by noting that the first-order condition above implies that the reserve price is given by a convex combination of \( s \) and a quantity weakly greater than \( s \) (i.e. \( \pi^S(\rho, s) \)).

Proof of Proposition 4

Proof. Theorem 1 of Storms (2015) (included below as Lemma 4, modified to fit this setting) implies that, in any BNE of this game, conditional on a realization of the auction price, \( p_A \), for each seller type \( s \), there is a cutoff value \( g_0(s, p^A) \) such that trade occurs if and only if the buyer’s type \( b \) satisfies \( b \geq g_0(s, p^A) \). Given the strict monotonicity of \( \rho(\cdot) \) (Proposition 3), such a cutoff function also exists with realizations \( s \) replaced with realizations of the reserve price \( r \). Call this cutoff function \( g(r, p^A) \).

Lemma 4. (Due to Storms 2015) If Assumptions A1–A3 are satisfied, then, conditional on any realization of the auction price \( P^A = p^A \), in any BNE of the bargaining subgame satisfying E1, for each seller type \( s \) there is a cutoff value \( g_0(s, p^A) \) such that \( s \) trades with a buyer \( b \) if and only if \( b \geq g_0(s, p^A) \).

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Proof. Fix $P^A = p^A$ throughout this proof. I first prove a preliminary property. Fix any arbitrary BNE. Let $\Pr(A = 1|b, h_t)$ represent the probability of trade for a buyer who mimics the strategy of a buyer of type $b$ when the history so far in the game is $h_t$. Here, $A \in \{0, 1\}$ is a random variable indicating whether or not trade occurs, where, from the buyer’s perspective, the seller’s valuation is unknown. Let $y(b, h_t)$ represent the expected transfer from playing such an action. Also, let $h_t(s, b)$ denote the history of the game in time $t$ when the players’ types are $s, b$ and when they play their equilibrium strategies.

I will discuss properties that must hold on histories that have a positive probability of being played in equilibrium (i.e., histories that at least some buyer and seller pair would play). In such histories, in any BNE, each buyer type must weakly prefer to play his own strategy from any history onward to playing that of another type. Thus, for $b' > b$, we have

$$b \Pr(A = 1|b, h_t) - y(b, h_t) \geq b \Pr(A = 1|b', h_t) - y(b', h_t)$$

$$b' \Pr(A = 1|b', h_t) - y(b', h_t) \geq b' \Pr(A = 1|b, h_t) - y(b, h_t)$$

Combining inequalities demonstrates that, for $b' > b$,

$$\Pr(A = 1|b', h_t) \geq \Pr(A = 1|b, h_t)$$

A similar result holds for $s' < s$,

$$\Pr(A = 1|s', h_t) \geq \Pr(A = 1|s, h_t)$$

Using this property, Lemma 4 can be proved by contradiction. Such a contradiction would be a triple $s, b,$ and $b'$ with $b' > b$ such that $s$ eventually (at some unspecified time period of the game) trades with $b$, but does not at any period of the game reach agreement with a type $b'$. For the sake of clarity, I will give such triples a name, referring to them as Type $A$ triples. Let $h^*_t$ be the longest history of play among all Type $A$ triples such that the strategy for $b$ is the same as that for $b'$ up to time $t$ when the seller’s type is $s$ (that is, $h^*_t = h_t(s, b) = h_t(s, b')$). Throughout the remainder of the proof, let $s, b,$ and $b'$ be a Type $A$ triple at which $h^*_t$ is achieved. The result in (20) implies that $b'$ must trade with weakly greater probability than $b$ from $h_t$ onward. This weak inequality, combined with $(s, b, b')$ being a Type $A$ triple, implies that there must be some seller type $s'$ who reaches history $h_t$ against both $b$ and $b'$ and who trades with $b'$ but not $b$.

Now consider two cases.

1. Case where $s' > s$. Since $s$ does not trade with $b'$ from the history $h_{t+1}(s, b')$, $s$ cannot trade with any types $\tilde{b} < b'$ from $h_{t+1}(s, b')$, or else $(s, \tilde{b}, b')$ would form a counterexample to $h^*_t$ because it would constitute a Type $A$ triple with buyers having $t+1$ periods of identical strategies. But by (21), $s$ must trade more often than $s'$ conditional on the history $h_{t+1}(s, b')$, and hence there must be some type $b'' > b'$ such that $b''$ eventually trades with $s$ but not with $s'$ when the history is $h_{t+1}(s, b')$. The triple $(s, b', b'')$ then gives a contradiction because it constitutes a Type $A$ triple with buyers having $t + 1$ stages of their strategies being identical.

2. Case where $s' < s$. Since $s$ trades with $b$ from the history $h_{t+1}(s, b)$, $s$ must trade with all types $\tilde{b} > b$ from $h_{t+1}(s, b)$, or else $(s, b, \tilde{b})$ would form a counterexample to $h^*_t$ because it would constitute a Type
A triple with buyers having $t+1$ periods of identical strategies. By (21), $s'$ must trade more often than $s$ conditional on the history $h_{t+1}(s, b)$. It follows that there must be some type $b'' < b$ that trades with $s'$ but not $s$. The triple $(s', b, b'')$ then gives a contradiction because it constitutes a Type A triple with buyers having $t+1$ stages of their strategies being identical.

\[\square\]

Proof of Proposition 5

Proof. Given the structure of additive separability in the willingness to pay/sell, the goal is to show that the auction price, players’ bargaining counteroffers, and the seller’s secret reserve price will also be additively separable in the game-level heterogeneity. The buyer’s type is given by $\hat{B} \equiv B + W \sim F_B$, with density $f_B$. The seller’s type is given by $\hat{S} \equiv S + W$. For this proof, let the realization of $W$ be $w$.

That the auction price will be additively separable in $w$ is obvious, given that there is no incentive for bidders to deviate from truthful bidding by Proposition 1. To demonstrate that bargaining offers are also additively separable, the proof proceeds by induction on the number of periods remaining. Suppose there is currently one period remaining in the bargaining game: it is the seller’s turn and after her turn the buyer will only be allowed to accept or quit (I prove the result in the case where the buyer moves last; analogous reasoning proves that the result also holds if the seller moves last). In the final period, a buyer with type $\hat{B} = \hat{b}$ will accept a price, $\hat{p}$, if and only if $\hat{p} \leq \hat{b}$. In period $T - 1$, the seller of type $\hat{S} = \hat{s}$ chooses $\hat{p}^*$ to solve

\[
\hat{p}^* = \arg\max_{\hat{p}} \hat{p}(1 - F_B(\hat{p})) + \hat{s}F_B(\hat{p}) - c_S
\]

\[
= \arg\max_p p(1 - F_B(p)) + sF_B(p) - c_S + w(1 - F_B(p)) + wF_B(p)
\]

\[
= w + \arg\max_p \{p(1 - F_B(p)) + sF_B(p) - c_S\}
\]

Therefore, the penultimate bargaining offer in the heterogeneous setting will be $w$ above the bargaining offer from the homogeneous good setting, and similarly for the seller’s maximized payoff.

To complete the proof by induction, suppose that offers and payoffs in periods $T - (t - 1)$ and $T - (t - 2)$ are $w$ higher than their homogeneous good counterparts and the probability of the buyer accepting, quitting, or countering in period $T - (t - 1)$ will be the same in the heterogeneous good model as in the homogeneous good model. It remains to be shown that the same holds true for the offers and payoffs in period $T - t$. Let all ($\hat{\cdot}$) expressions represent the heterogeneous model expressions. The seller’s payoffs from accepting, quitting, or countering in period $T - t$ can be written as follows:

\[A : \hat{p}^B_{T-(t+1)} = w + \hat{p}^B_{T-(t+1)}\]

\[Q : \hat{s} = w + s\]

\[C : \hat{V}^S_{T-t} \left( \hat{s}|\hat{h}_{T-t} \right)\]

\[= \max_{\hat{p}} \hat{p} \Pr \left( D^B_{T-(t-1)} = A|\{\hat{h}_{T-t}, \hat{p}\} \right) + \hat{s} \Pr \left( D^B_{T-(t-1)} = Q|\{\hat{h}_{T-t}, \hat{p}\} \right) + \Pr \left( D^B_{T-(t-1)} = C|\{\hat{h}_{T-t}, \hat{p}\} \right)\]
heterogeneous good models. It also immediately follows that

\[ \pi \]

And thus the claim is true because the buyer’s bargaining payoffs are the same in the homogeneous good and

\[ \tilde{\pi} \]

price, \( \tilde{\pi} \)

I begin by proving that

Proof.

Proof of Proposition 6

Let

\[ \rho \]

Therefore, the optimal secret reserve price in the heterogeneous setting will be

\[ A \]

\[ B \]

Now consider the seller’s secret reserve price in the setting with game-level heterogeneity \( w \). From the

\[ C \]

And thus the claim is true because the buyer’s bargaining payoffs are the same in the homogeneous good and

heterogeneous good models. It also immediately follows that \( \pi^B(\tilde{\chi}, \tilde{b}) = \pi^B(\tilde{\chi}, b) \) by the above arguments

for the buyer’s bargaining payoff, where \( \tilde{\chi} \) satisfies \( 0 = \pi^B(\tilde{\chi}, \tilde{b}) \).

Now consider the seller’s secret reserve price in the setting with game-level heterogeneity \( w \). From the

\[ \tilde{C} \]

From the proof of Lemma 3, the derivative of the seller’s payoff with respect to the seller’s choice of secret reserve

\[ \tilde{\rho} \]

will be given by

\[ \frac{\partial}{\partial \tilde{\rho}} = -\tilde{\rho} + s \frac{F_B(\chi^{-1}(\tilde{\rho})) - F_B(\tilde{\rho})}{1 - F_B(\tilde{\rho})} + \pi^S(\tilde{\rho}, \tilde{\rho}) \frac{1 - F_B(\chi^{-1}(\tilde{\rho}))}{1 - F_B(\tilde{\rho})} \]

\[ = -\tilde{\rho} + w + s \frac{F_B(\chi^{-1}(\tilde{\rho} - w)) - F_B(\tilde{\rho} - w)}{1 - F_B(\tilde{\rho} - w)} + \pi^S(\tilde{\rho} - w, \tilde{\rho}) \frac{1 - F_B(\chi^{-1}(\tilde{\rho} - w))}{1 - F_B(\tilde{\rho} - w)} \]

Therefore, the optimal secret reserve price in the heterogeneous setting will be \( w \) above the optimal reserve

in the homogeneous setting.

An immediate implication of these results is a generalization of Proposition 4: At a general realization

\[ W = w \]

trade occurs if and only if \( \tilde{b} \geq g(\tilde{\rho}, \tilde{p}^A) \) \( \Rightarrow b \geq g(\tilde{\rho} - w, \tilde{p}^A - w) = g(r, p^A) \).

Proof of Proposition 6

Proof. I begin by proving that \( W_{EG}(x^*, F_S) \) will decrease given a stochastically dominating change in \( F_S \). Let \( W_{EG}(x^*, F_S; p^A) \) be the expected gains from trade in the first-best mechanism conditional on a realization
of the auction price $p^A$. This object can be written as

$$W_{EG}(x^*, F_S; p^A) = \int_{\underline{\Lambda}(p^A)}^{\bar{\Lambda}(p^A)} \Lambda(s) f_S(s|p^A) ds$$  \hspace{1cm} (22)

where $\Lambda(s, p^A)$ is given by

$$\Lambda(s, p^A) = \int_{\underline{\Lambda}(p^A)}^{\bar{\Lambda}(p^A)} (b - s) 1\{b \geq s\} f_B(b|p^A) db $$

Note that this function is weakly decreasing in $s$. Also recall that $f_S(s|p^A) = \frac{f_s(s)}{1 - F_S(\underline{\Lambda}(p^A))}$, and note that the denominator of this term does not vary with $F_S$, because $F_S(\underline{\Lambda}(p^A)) = F_S(\rho^{-1}(p^A)) = F_S(F_S^{-1}(F_R(p^A))) = F_R(p^A)$. It is well known that, for any weakly decreasing $\Lambda(\cdot)$ in an expectation such as (22), replacing $f_S$ with a density corresponding to a distribution that first-order stochastically dominates $f_S$ will lead to a lower value for the evaluated integral $W_{EG}(x^*, F_S; p^A)$. Integrating $W_{EG}(x^*, F_S; p^A)$ over $p^A$ yields the desired result. The monotonicity of $W_{fr}(x^*, F_S)$ follows by the same result.

For the second-best mechanism, the proof of monotonicity of the expected gains from trade is much more involved and is found in recent work by Zhang (2017). It does not yield an analytic proof of the monotonicity of the expected buyer or seller gains or the probability of trade in the second-best mechanism. For the buyer-optimal mechanism, monotonicity of the buyer gains follows from the same line of reasoning as in Zhang (2017), but there is no analytic proof of monotonicity of the total expected gains from trade, seller gains, or probability of trade.

For the seller-optimal mechanism, the allocation function is $x^1 = 1\{b - \frac{1-F_S(b)}{F_S(b)} \geq s\}$. As in the first-best case, this allocation function does not depend on $F_S$, and thus monotonicity of all the welfare measures in the seller-optimal mechanism are guaranteed (even for those marked with “..” in the seller-optimal column of Table 3).

For the real-world mechanism, the proof of monotonicity of the expected gains from trade exploits that I have defined the real-world mechanism as a function of the reserve price, $R$, given that $R$ is a strictly increasing function of $S$, and hence, holding fixed $F_R$, the allocation function, $x^{RW}(\rho(s), b; p^A) \equiv 1\{b \geq g(\rho(s), p^A)\}$, does not depend on the unknown distribution $F_S$. The expected gains from trade in the real-world mechanism can be written

$$W_{EG}(x^{RW}, F_S) = \int_{\underline{\Lambda}}^{\bar{\Lambda}} \left[ \int_{\underline{\Lambda}(p^A)}^{\bar{\Lambda}(p^A)} (b - s) 1\{b \geq g(\rho(s), p^A)\} f_S(s|p^A) f_B(b|p^A) ds \right] f_{p^A}(p^A) dp^A $$

where $\rho(s) \equiv F_R^{-1}(F_S(s))$. Holding $F_R$ fixed, the object $\rho(s)$ would be unchanged by such a first-order stochastically dominating shift in $F_S$. This is because $\rho(s)$ always returns the $F_S(s)$ quantile of $F_R$. To see this, let $\tilde{F}_S$ first order stochastically dominate $F_S$ and let $\tilde{s}$ and $s$ be realizations such that $\tilde{s} > s$ and $\tilde{F}_S(s) = F_S(s)$. Thus, $\tilde{s} = \tilde{F}_S^{-1}(F_S(s))$. Then $\tilde{\rho}(s) \equiv F_R^{-1}(\tilde{F}_S(s)) = F_R^{-1}(\tilde{F}_S(\tilde{F}_S^{-1}(F_S(s)))) = \rho(s)$, proving the result. The monotonicity of $W_B(x^{RW}, F_S), W_S(x^{RW}, F_S)$, and $W_{fr}(x^{RW}, F_S)$ follow by the same argument. Note that these results should not be interpreted as suggesting that the real-world mechanism does not depend on the distribution of seller valuations; these results simply imply that, because reserve prices are one-to-one with seller valuations, after conditioning on $F_R$, the estimation of the object $\rho(\cdot)$ will not depend on the estimate of $F_S$. 

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For the third-to last column in Table 3, there is no analytic proof of monotonicity of the gap between the expected gains from trade in the first-best and second-best mechanisms, \( W_{EG}(x^* - x^1, F_S) \), and similarly for the gap for the probability of trade. For the gap between the expected gains from trade in the second-best mechanism and the real-world mechanism, \( W_{EG}(x^{1/2} - x^{RW}, F_S) \), monotonicity follows from the arguments above (the allocation function \( x^{RW} \) does not depend on \( F_S \), and the expected gains from trade for the second-best mechanism are monotonic). For the gap in the probability of trade there is no analytic proof of monotonicity. For the final column of Table 3, monotonicity of the gap between first-best and real-world expected gains from trade, \( W_{EG}(x^* - x^{RW}, F_S) \), follows by the same arguments as above for the real-world and first-best mechanisms, and similarly for the probability of trade gap.

\[\Box\]

B Sellers Accepting Offers Below the Secret Reserve Price

This section provides a discussion of several explanations rationalizing why some sellers would set a secret reserve price and then, in the bargaining stage, accept an offer below the reserve price. These explanations include a specific equilibrium example where this behavior can occur; an extension to the main model in which seller’s have market-level uncertainty at the time they set their reserve prices; and seller biases due to over-optimism about buyer demand or due to attempts to influence auctioneer effort.

B.1 An Equilibrium Example

This section demonstrates that the model in the body of the paper can contain BNE in which sellers accept offers that lie below their previously set reserve prices.\(^{19}\) For simplicity of exposition, assume the seller has a value of \( S = 0 \) and buyer value \( B \) is uniformly distributed on \([0, 1]\).

Suppose the buyer commits to reject all counteroffers when \( p^A \geq 1/2 \) and the seller refuses to consider any counteroffer after her first counteroffer when \( p^A < 1/2 \). The seller’s optimal secret reserve price must then be at least 1/2. When \( p^A < 1/2 \), let \( y(p^A) \) denote the optimal take-it-or-leave-it offer for the seller at her first chance to counteroffer, given the realized auction price. Suppose also that for each \( p^A < 1/2 \), there is a unique counteroffer as a function of \( p^A \), call it \( z(p^A) \), that the buyer will accept if \( z(p^A) < B \). Define this \( z(p^A) \) to be equal to \( p^A \) at \( p^A = 1/4 \), to be equal to \( y(p^A) \) outside of \([1/4 - \varepsilon, 1/4 + \varepsilon]\) for some small \( \varepsilon > 0 \), and to be the linear interpolation of \( p^A \) and \( y(p^A) \) along this interval:

\[
z(p^A) = \begin{cases} 
\frac{|p^A - 1/4|}{\varepsilon} y(p^A) + \left(1 - \frac{|p^A - 1/4|}{\varepsilon}\right) p^A & \text{if } p^A \in [1/4 - \varepsilon, 1/4 + \varepsilon] \\
y(p^A) & \text{otherwise}.
\end{cases}
\]

For small enough \( \varepsilon \), the seller’s optimal reserve price will be 1/2. Moreover, the seller’s best response counteroffer for \( p^A \in [0, 1/2] \) and \( p^A \notin [1/4 - \varepsilon, 1/4 + \varepsilon] \) is \( y(p^A) \). Now consider the seller’s response when \( p^A \in [1/4 - \varepsilon, 1/4 + \varepsilon] \). Since the buyer will not consider any counteroffer other than \( z(p^A) \), the seller’s only options are to accept the auction price or to counter at \( z(p^A) \). Which option she chooses will depend on whether the price increase from countering is higher than the cost of making a counteroffer. The price

\[^{19}\text{I thank Evan Storms for this example.}\]
increase is \( z(p^A) - p^A \), which is non-negative since \( y(p^A) > p^A \), while the cost of an making an offer is \( c_S \). Since \( y(p^A) - p^A = 0 \) for \( p^A = 1/4 \), for \( p^A \) sufficiently close to 1/4, the seller will accept the auction price, while for \( p^A \) further away from 1/4 she will counter at \( z(p^A) \). This is the seller’s optimal response given that the buyer rejects any counteroffer other than \( z(p^A) \), while the buyer’s strategy of accepting only \( z(p^A) \) and accepting it if and only if \( z(p^A) \) is below his valuation is optimal since the seller never offers a lower price. Thus, in this equilibrium, the seller accepts some auction prices below her reserve price. Note also that this equilibrium example satisfies condition \( E_1 \).

### B.2 Model Extension with Market-Level Uncertainty

This section extends the model of Section 3 to allow sellers to have uncertainty, at the time they set their reserve prices (which is several days in advance of the actual auction day), about what their own valuation and what the distribution of buyer valuations will be on the day the auction will take place. This uncertainty is then resolved once the auction takes place and the seller sees (or learns over the phone through an auction house employee) additional information, such as the level of buyer turnout/interest, which can be affected by weather, financial news, or other shocks. Such uncertainty can rationalize why some sellers would set a secret reserve price and then accept an auction price below the reserve price.

The existence of such uncertainty would not affect certain key results of the model, such as bidding behavior in the auction. It would also not affect the result that the seller’s secret reserve price strategy is strictly increasing, as I demonstrate below.

Let \( \zeta \) be a finite vector parameterizing a seller’s uncertainty about buyer valuations, where \( \zeta \) is independent of buyer and seller valuations. Let \( F_b(:, \zeta), f_b(:, \zeta), f_{PA}(:, \zeta), \chi^{-1}(b; \zeta), \) and \( \pi^S(p^A, s; \zeta) \) be equivalent to the analogous objects in the main model but conditional on \( \zeta \), and let \( u(s; \zeta) \) represent the seller’s valuation conditional on the realization of \( \zeta \). At \( \zeta = 0 \), let each of these functions be equal to its counterpart in the main model (so \( u(s; 0) = s \), \( F_b(:, 0) = F_b(:, \zeta), f_b(:, 0) = f_b(:, \zeta), f_{PA}(:, 0) = f_{PA}(:, \zeta), \chi^{-1}(b; 0) = \chi^{-1}(b), \) and \( \pi^S(p^A, s; 0) = \pi^S(p^A, s) \)). At the time the seller chooses the reserve price, she knows each of these functions but does not know the realization of \( \zeta \). I also assume that \( u(s; \zeta) \) is weakly increasing in \( \zeta \).

The following argument follows the steps of the proof of Lemma 3 and demonstrates that in this model reserve prices would still be increasing in \( s \). The seller’s expected payoff, prior to knowing the realization of \( \zeta \), can be written as

\[
E_{\zeta} \left\{ \int_{\rho}^{\rho} p^A f_{PA}(p^A; \zeta) dp^A \right. \\
\left. + \int_{\zeta}^{\rho} \left[ \int_{p^A}^{\chi^{-1}(p^A; \zeta)} u(s; \zeta) f_{B}(b; \zeta) db + \int_{\chi^{-1}(p^A; \zeta)}^{\rho} \pi^S(p^A, u(s; \zeta); \zeta) f_{B}(b; \zeta) db \right] \frac{f_{PA}(p^A; \zeta)}{1 - F_{B}(p^A; \zeta)} dp^A \right\} \\
= E_{\zeta} \left\{ \int_{\rho}^{\rho} p^A f_{PA}(p^A; \zeta) dp^A \right. \\
\left. + \int_{\zeta}^{\rho} \left[ u(s; \zeta) \left( F_{B}(\chi^{-1}(p^A; \zeta); \zeta) - F_{B}(p^A; \zeta) \right) + \pi^S(p^A, u(s; \zeta); \zeta) \left( 1 - F_{B}(\chi^{-1}(p^A; \zeta); \zeta) \right) \right] \right\}
\]
\[ \times \frac{f_{\rho \chi}(p^A; \zeta)}{1 - F_B(p^A; \zeta)} dp^A \] 

Differentiating the above expression using Leibniz Rule yields the following first-order condition for the reserve price:

\[ \frac{\partial}{\partial \rho} = -\rho E_\zeta [f_{\rho \chi}(\rho; \zeta)] + u(s; \zeta) E_\zeta \left[ \frac{F_B(\chi^{-1}(\rho; \zeta); \zeta) - F_B(\rho; \zeta)}{1 - F_B(\rho; \zeta)} \right] + E_\zeta \left[ \pi^S (\rho, u(s; \zeta); \rho m^{-1} - F_B(\chi^{-1}(\rho; \zeta); \zeta)) \right] \]

Conditional on a realization of \( \zeta \), Lemma 1 applies, and hence \( \pi^S(p^A, u(s; \zeta); \zeta) \) is weakly increasing in \( s \) for any realization of \( \zeta \). Also, conditional on a realization of \( \zeta \), \( \chi^{-1}(p^A; \zeta) > p^A \) by the same arguments as in Proposition 3, and thus \( F_B(\chi^{-1}(\rho; \zeta); \zeta) > F_B(\rho; \zeta) \). Combining these results demonstrates that \( \frac{\partial}{\partial \rho} \) will be strictly increasing in \( s \), and thus the Edlin and Shannon (1998) Theorem implies that the reserve price will be strictly increasing in \( s \) for a given realization of \( \zeta \), and taking expectations over \( \zeta \) yields the desired result.

A special case of this market uncertainty model that would be empirically tractable would be the case where 1) buyer and seller valuations are additively separable in \( \zeta \), such that \( u(s; \zeta) = s + \zeta \), \( F_b(b; \zeta) = F_{b+}(b) \), etc.; and 2) the realization of \( \zeta \) can be estimated in the data by computing a market-level (e.g. auction-location by date-of-sale) fixed effect. In this case, the arguments in Proposition 5 imply that buyer and seller valuations on the day of the sale can be shifted additively just as with the observed and unobserved game-level heterogeneity included in the main model. Also relying on arguments from Proposition 5, it follows immediately that the secret reserve price in the market uncertainty model (which I will denote \( \rho^{m*} \), where \( m \) denotes “market-uncertainty”) is related to the secret reserve price in the main model (\( \rho^* \)) by \( \rho^{m*}(s) = \rho^*(s) + E[\zeta] \). In the market uncertainty model some sellers will set a reserve price that, ex-post, is too high relative to the realized value of \( \zeta \). These sellers can correct for this ex-post by accepting the auction price. Conversely, some sellers will have set a reserve price that is ex-post too low relative to the realized value of \( \zeta \). These latter sellers cannot correct for this occurrence, as the car will already have sold if the auction price exceeds the secret reserve price.

This latter possibility implies that, under the market-uncertainty model, observations in the data in which the auction price exceeds the reserve price (\( p^A + \zeta \geq \rho^{m*}(s) \)) should not all be considered to be cases where the seller’s value is less than the realized auction price (as I do in computing the bounds on seller valuations in the main model). Rather, a correction should be made to the seller’s decision to account for what the seller would have done had she known the realization of \( \zeta \). This corrected decision would imply that the observations that should be considered to be cases in which the seller’s value is less than the realized auction price are those with \( p^A + \zeta \geq \rho^{m*}(s) - E[\zeta] + \zeta \). A similar correction is not needed for cases where the seller rejected the auction price, because this decision of the seller already takes into account the realization of market uncertainty \( \zeta \).

I implement this market uncertainty model by 1) estimating \( \zeta \) using a regression of residual auction prices from Section 4.1 on auction-location by date-of-sale fixed effects, and 2) incorporating the seller’s corrected decision into the estimation of the bounds on the seller’s valuation. I then compute expected gains from trade for the counterfactual and real bargaining using these new estimates for the seller distribution. The results are displayed in Table A4. For both the dealers and fleet/lease samples, I find that the estimated
gains from trade in the real-world and counterfactual mechanism are not substantially different from those in the main model. Note that these gains from trade are computed using the seller’s revised valuation, after learning the realization of \( \zeta \).

**B.3 Optimistic Beliefs and Influencing Auctioneers**

Sellers accepting prices below their secret reserve price can also be rationalized by sellers having overly optimistic beliefs about auction prices prior to the auction taking place (Treece 2013) or by an attempt to influence auctioneers to exert greater effort to achieve higher prices (Lacetera, Larsen, Pope, and Sydnor 2016; Treece 2013). Such situations can be modeled as the sellers choosing a reserve price given by the optimal, un-biased reserve price, \( \rho^*(s) \), plus a bias term, \( h(s) \), that is weakly increasing in \( s \). Under such a weakly increasing bias assumption, the observed reserve prices would be strictly increasing in the underlying seller valuation.

**C Computational Details**

**C.1 Controlling for Game-level Observables**

Some observations in the data record only a secret reserve price or only an auction price but not both. These observations, or observations with incorrectly recorded bargaining sequences (such as a seller acceptance followed by a buyer counteroffer) are not suitable for my final analysis but are still useful in controlling for observable heterogeneity, and therefore I include these observations in the regressions in Section 4.1. Missing secret reserve prices typically occur when the seller chooses not to report a reserve price, either planning to be present at the auction sale to accept or reject the auction price in person or planning to have the auction house call her on the phone rather than determining a reserve price ex-ante. Missing auction prices can occur due to the descending/ascending practice of auctioneers: auctioneers do not start the bidding at zero; they start the bidding high and then lower the price until a bidder indicates a willingness to pay, at which point the ascending auction begins. If bidders are slow to participate, the auctioneer will cease to lower bids and postpone the sale of the vehicle until a later date, leaving no auction price recorded. See Lacetera, Larsen, Pope, and Sydnor (2016).

In the regressions controlling for observable covariates, I compute market thickness measures as follows: for a given car on a given sale date at a given auction house, I compute the number of remaining vehicles still in queue to be sold at the same auction house on the same day lying in the same category as the car in consideration. The six categories I consider are make, make-by-model, make-by-age, make-by-model-by-age, age, or seller identity. I also control for the run numbers, which represents the order in which cars are auctioned. I include fifth-order polynomials for both the run number within an auction-house-by-day combination, and the run number within an auction-house-by-day-by-lane combination. The odometer bins I use are as follows: four equally sized bins for mileage in \( [0, 20000) \); eight equally sized bins for mileage in \( [20000, 80000) \); four equally sized for mileage in \( [100000, 200000) \); one bin for mileage in \( [200000, 250000) \); and one bin for mileage greater than 250000.
For the dealers sample, the right-hand side variables in the regression includes 11,480 make-model-year-trim-age category effects and 329 other covariates, as described in Section 4.1; for the fleet/lease sample the corresponding numbers are 7,431 and 291.

C.2 Numerical Integration

Throughout the paper, for any estimator requiring integration against the density \( f_W \), such as in (2), I perform this integration using Gauss-Hermite quadrature with 10 nodes. For any univariate function \( g(\cdot) \), Gauss-Hermite quadrature is given by

\[
\int_{-\infty}^{\infty} g(v)dv \approx \sum_{k=1}^{K_{GH}} g(x_k)e^{x^2_k}w_k
\]

where \( K_{GH} \) is the number of nodes and \( x_k \) and \( w_k \) are the Gauss-Hermite quadrature nodes and weights described in Judd (1998).

The integrals in the welfare measures described in Section 5.2 involve integration in three dimensions, \( p^A \), \( b \), and \( s \). For integration in the \( p^A \) dimension, this integral is computed using Gauss-Chebyshev quadrature with 25 nodes. For any univariate function \( g(\cdot) \) to be integrated over \([v, \bar{v}]\), Gauss-Chebyshev quadrature is given by

\[
\int_{v}^{\bar{v}} g(v)dv \approx \pi(v-\bar{v})^{1/2}K_{GC}\sum_{k=1}^{K_{GC}} g(x_k)w_k
\]

where \( K_{GC} \) is the number of nodes; \( x_k = (1/2)(z_k+1)(v-\bar{v})+\bar{v}; w_k = (1-z^2_k)^{1/2}; \) and \( z_k = \cos(\pi(2k-1)/(2K_{GC})) \) (see Judd 1998).

The integration in the \( s \) and \( b \) dimension is required not only for evaluating welfare measures but also for solving the linear programming problem in (15); the \( p^A \) dimension, on the other hand, is only involved in computing a simple average. For the \( s \) and \( b \) dimensions, therefore, I choose a larger number of nodes (50 in each dimension) to achieve a high degree of accuracy in solving for the efficient mechanisms. I choose these nodes to be evenly spaced quantiles of \( F_S \) and \( F_B \), which works particularly well here for numerical integration due to the fact that the seller valuation distribution can be multi-modal (corresponding to a CDF that is nearly flat over large portions of the seller support).

C.3 Bounding Bargaining Costs

Bounds on the parameters \( c_B \) and \( c_S \) can be derived from cases in which a player chooses to make a counteroffer. A necessary condition for a party to choose to counter is that the payoff in the state where the opponent accepts with probability one must exceed the player’s payoff from accepting the current offer on the table. That is,

\[
p^S_2 - c_S \geq p^B_1
\]

for a seller offer and

\[
b - p^B_3 - c_S \geq b - p^S_2
\]

for a buyer offer. Rearranging yields

\[
p^S_2 - p^B_1 \geq c_S
\]

(23)

\[
p^2_2 - p^B_3 \geq c_B
\]

(24)

Thus, an upper bound on \( c_S \) is given by the minimum gap between period 2 and period 1 offers and an upper bound on \( c_B \) is given by the minimum gap between period 2 and period 3 offers (in cases where such offers
took place). Rather than use the minimum over all observations, I follow Chernozhukov, Lee, and Rosen (2013) to obtain a bias-corrected, one-sided 95% confidence bound for $c_S$ and $c_B$ and treat these as upper bounds on $c_S$ and $c_B$, which I denote $c_S^*$ and $c_B^*$.20

Let the random variable $T$ be the period in which the game ends. The buyer’s and seller’s ex-ante expected disutility due to bargaining costs are then given $c_B E[\lfloor T/2 \rfloor]$ and $c_S E[\lfloor (T - 1)/2 \rfloor]$, respectively, because by round $t$ of the game the buyer has made a total of $\lfloor t/2 \rfloor$ offers (where $\lfloor \cdot \rfloor$ is the floor function), and similarly for the seller.

Applying this approach yields estimates of an upper bound of $25$ for both $c_B$ and $c_S$ in the dealers sample and $50$ for both buyers and sellers in the fleet/lease sample. The upper bound on the total expected loss due to bargaining costs is $23.3$ for buyers and $4$ for sellers in the dealers sample, and $33.7$ for buyers and $5.1$ for sellers in the fleet/lease sample.

### C.4 Computing Confidence Sets for Welfare Bounds

Confidence sets for the bounds on welfare can be computed by bootstrapping. Specifically, for any estimated bounds on a welfare measure, which constitute an interval, $[\mathcal{W}(x), \overline{\mathcal{W}}(x)]$, the lower 95% bootstrapped confidence band about $\mathcal{W}(x)$ and the upper 95% bootstrap confidence band about $\overline{\mathcal{W}}(x)$ will provide a conservative 95% confidence interval for the set $[\mathcal{W}(x), \overline{\mathcal{W}}(x)]$. This claim follows by a simple Bonferroni inequality argument: For a fixed $\alpha \in [0, 1]$, let $c_{\alpha/2}$ be the lower critical value for the $1-\alpha$ confidence band for $\mathcal{W}(x)$ and $c_{1-\alpha/2}$ be the upper critical value for the $1-\alpha$ confidence band for $\overline{\mathcal{W}}(x)$. Also, let $A$ be the event that $\mathcal{W}(x) \geq c_{\alpha/2}$ and let $\overline{A}$ be the event that $\overline{\mathcal{W}}(x) \leq c_{1-\alpha/2}$. Therefore, $\Pr(A) = \Pr(\overline{A}) = 1-\alpha/2$. Bonferroni inequalities imply

$$\Pr(A \cap \overline{A}) \geq \Pr(A) + \Pr(\overline{A}) - \Pr(A \cup \overline{A}) \geq \Pr(A) + \Pr(\overline{A}) - 1 = 1 - \alpha,$$

thus completing the argument. To compute these confidence sets I use 200 bootstrap replications.

This approach yields very similar results to the method proposed by Chernozhukov, Hong, and Tamer (2007) (CHT) but has the advantage of yielding an asymmetric confidence set that is guaranteed to lie within the range of bootstrapped estimates, i.e., the confidence set will naturally be contained within the minimum and maximum bootstrapped estimates, as it is composed of quantiles that are on the interior of these estimates. This is not the case with the symmetric confidence sent of CHT, for example, where it is possible for one end of the estimated confidence set to lie outside the extremes of the bootstrapped estimates. For example, even if the estimated gap between the expected gains from trade in the second-best and real-world mechanisms is positive in every bootstrap sample, the CHT confidence set can contain zero. Given this feature, and given that the above confidence sets are easier to compute, I adopt the above approach throughout the paper.

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20Chernozhukov, Lee, and Rosen (2013) explained that taking the minimum over all observations using inequalities such as those in those in (23) and (24) will be biased downward, precisely because they are derived from taking a minimum. The authors suggested a bias-corrected $1-\alpha$ confidence bound (which the authors refer to as being half-median unbiased) given, in this case, by taking the $1-\alpha$ quantile of bootstrapped estimates of the minimum. I perform this using 200 bootstrap replications.
D Additional Discussion of Distribution Estimates

D.1 Robustness to Distribution of Number of Bidders

In order to guide the choice of \( \Pr(\mathcal{N} = n) \), the distribution of the number of bidders, I first manually collected additional data by visiting multiple auction house locations and physically observing over 200 auctions. For each auction sale, I recorded the number of bidders who appeared to be actively participating or interested in the car. The mean of these observations, conditional on cases where at least two bidders appeared to be active, was close to the mean of the lower bound on the number of bidders from the bid log sample (2.9 in both the dealers and fleet/lease samples).

However, I also present evidence here that the key estimates in the paper are not sensitive to how \( \Pr(\mathcal{N} = n) \) is specified. In doing so I compare several possibilities for \( \Pr(\mathcal{N} = n) \). The first is that which is used in the body of the paper, which is the empirical probability mass function of the lower bound on the number of bidders in each auction in the bid log sample. Let this random variable be denoted \( \mathcal{N} \). The next is the empirical probability mass function of a (very conservative) upper bound on the number of participants in each auction in the bid log sample. This upper bound comes from adding the total number of bidders who signed in through the online portal for a given lane on a given day (I observe this number in the data, whether or not these signed-in bidders placed any bids) to the total number of floor bids (physically present bids, as described in Section 2); thus, this treats each floor bid as having come from a distinct bidder. Let this random variable be denoted \( \mathcal{N} \). The mean of this upper bound is 15.67 in the dealers bid log subsample and 26.07 in the fleet/lease bid log subsample. An additional possibility is that \( \mathcal{N} \) is drawn from some parametric distribution, such as a Poisson, Negative Binomial, etc. Here I consider cases in which \( \mathcal{N} \) follows a Poisson distribution with mean \( \lambda \in \{3, 7, 10, 20\} \), conditional on \( \mathcal{N} \geq 2 \) (thus, these latter four are truncated Poisson distributions).

I find that the welfare estimates are not sensitive to the approximation chosen for \( \Pr(\mathcal{N} = n) \). This finding is due to the fact that the welfare measures depend primarily on the distribution of the valuation of the highest bidder (the buyer who bargains) conditional on the auction price; this yields the distribution of the maximum order statistic of buyer valuations. To see this, note that the maximum order statistic distribution (averaged over values of \( \mathcal{N} \)) can be computed as

\[
F_{B^{(1)}}(v) = \sum_{n} \Pr(N = n) F_B(v)^n,
\]

or, alternatively, it can be computed from the density of the maximum order statistic conditional on the second order statistic integrated against the density of the second order statistic, given by

\[
F_{B^{(1)}}(v) = \int_{\frac{v}{2}}^{\infty} \int_{\frac{v}{2}}^{y} \frac{f_B(y)}{1 - F_B(p_A)} f_{P^A}(p_A) dp_A dy.
\]

Each of the welfare measures herein depend on an object similar to the latter formulation, although the denominator term is \( 1 - F_B(\chi^{-1}(p_A)) \); this difference is negligible, however, given that \( \chi^{-1}(\cdot) \) is estimated to be very close to the identity function (see Panels C and D of Figure A2).

Figure A3 demonstrates that the distribution of the maximum order statistic is much less sensitive than the underlying distribution \( F_B \) to the choice of \( \Pr(\mathcal{N} = n) \). Panels A and B show the estimates of the
underlying distribution, $F_B$, which differ widely as $\Pr(N = n)$ changes. Panels C and D show the distribution of the maximum order statistic, obtained by using $F_{P_A}(v)$ and $\Pr(N = n)$ to obtain $F_B(v)$ by solving (3) and then using this $F_B(v)$ and $\Pr(N = n)$ to obtain $F_{B(1)}(v)$ using (25) or (26). Panels C and D demonstrate that the distribution of the maximum order statistic is quite insensitive to the choice of $\Pr(N = n)$ used in this procedure.

For certain choices of $\Pr(N = n)$, including the Poisson distribution, it is possible to prove analytically that the computed maximum order statistic distribution will be entirely insensitive to $\Pr(N = n)$. I state this as the following proposition. Below, note that $F_{B(2)}$ denotes the second order statistic distribution (averaged over values of $N$), which is equivalent to $F_{P_A}$ in the body of the paper.

**Proposition 7.** If $\Pr(N = n)$ is given by a Poisson distribution with parameter $\lambda$, and $F_{B(2)}$ is known, then, at any point $v$, the maximum order statistic distribution $F_{B(1)}(v)$ obtained by the following two steps will have zero derivative with respect to $\lambda$: 1) use $F_{B(2)}(v)$ and $\Pr(N = n)$ to obtain $F_B(v)$ by solving (3); 2) use this $F_B(v)$ and $\Pr(N = n)$ to obtain $F_{B(1)}(v)$ using (25) or (26).

**Proof.** When $\Pr(N = n)$ is a Poisson with mean $\lambda$, it can be shown that $F_B(v) = \psi(F_{B(2)}(v); \lambda)$, where $\psi(F_{B(2)}(v); \lambda)$ is defined implicitly as the solution to

$$F_{B(2)}(v) = (1 + \lambda(1 - \psi(F_{B(2)}(v); \lambda)))e^{\lambda(\psi(F_{B(2)}(v); \lambda) - 1)}$$

and the maximum order statistic distribution is given by

$$F_{B(1)}(v) = e^{\lambda(\psi(F_{B(2)}(v); \lambda) - 1)}$$

Holding fixed $v$ and $F_{B(2)}(v)$, implicitly differentiating (27) with respect to $\lambda$ yields

$$0 = \left(1 - \psi(F_{B(2)}(v); \lambda) - \lambda \frac{d\psi(F_{B(2)}(v); \lambda)}{d\lambda}\right) F_{B(1)}(v) + (1 - \lambda(1 - \psi(F_{B(2)}(v); \lambda))) \frac{dF_{B(1)}(v)}{d\lambda}$$

where I take into account that $F_{B(1)}$ will depend on $\lambda$. Note that

$$\frac{dF_{B(1)}(v)}{d\lambda} = e^{\lambda(\psi(F_{B(2)}(v); \lambda) - 1)} \left(\psi(F_{B(2)}(v); \lambda) - 1 + \lambda \frac{d\psi(F_{B(2)}(v); \lambda)}{d\lambda}\right)$$

Plugging (30) into (29) yields

$$0 = \lambda(1 - \psi(F_{B(2)}(v); \lambda)) \frac{dF_{B(1)}(v)}{d\lambda}$$

Recall that $\psi(F_{B(2)}(v); \lambda) = F_B(v)$. This expression in (31) must hold at all $v$, even at $v$ where $F_B(v) \neq 1$. Thus, it must be the case that $\frac{dF_{B(1)}(v)}{d\lambda} = 0$. 

Proposition 7 should not be misinterpreted as an unconditional statement that the distribution of the maximum order statistic does not depend on $\lambda$. Rather, the result demonstrates that, conditional on $F_{B(2)}$, the exercise of inverting $F_{B(2)}$ to obtain $F_B$, and then computing $F_{B(1)}$ from this $F_B$, is invariant to $\lambda$. It may be possible to prove this analytical result for a larger class of $\Pr(N = n)$, for, as demonstrated in Figure A3, even at non-Poisson distributed $N$ (such as the distributions of the upper and lower bounds on the number of bidders or the truncated Poisson distributions), $F_{B(1)}$ is very insensitive to the choice of $\Pr(N = n)$. 

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To my knowledge, Proposition 7 is new to the literature, and may be of some independent interest, as it suggests assumptions under which one can compute the marginal distribution of the maximum order statistic solely from knowledge of the marginal distribution of the second order statistic, without knowing \( N \) or even fully specifying the distribution of \( N \).\footnote{Precisely, these assumptions are symmetric, conditionally independent, private values with valuations independent of \( N \) and with \( N \) distributed according to a Poisson distribution with unknown mean \( \lambda \).} Aradillas-Lopez, Gandhi, and Quint (2013) pointed out that many objects of interest in ascending auctions, such as bidder surplus, seller profits, and optimal reserve prices can be computed if the researcher knows the marginal distributions of the first and second order statistics.

\section*{D.2 Valuations Independent of Number of Bidders}

The estimation of buyer valuations in Section 4.3 relies on the assumption (stated in Assumption A1) that buyer valuations are independent of \( N \), the number of bidders. Here I examine the validity of this assumption.

For any \( k \leq n \) and any \( v \), let

\[ \psi_{k:n}(v) \equiv \frac{n!}{(n-k)!(k-1)!} \int_0^v t^{k-1}(1-t)^{n-k} \, dt \]

Proposition 2 of Aradillas-López, Gandhi, and Quint (2016) demonstrated the following (modified to the environment of this paper). Suppose buyer values (given by the random variable \( \tilde{B} = B + W \)) are non-negatively correlated, as they will be in an environment of conditionally independent private values with independent, additively separable unobserved heterogeneity. Then for any \( v \) and any \( n > n' \), valuations being independent of \( N \) implies

\[ \psi_{n-1:n}(F_{\tilde{B}_{n-1:n}}(v)) \geq \psi_{n'-1:n'}(F_{\tilde{B}_{n'-1:n'}}(v)) \]

(32)

where, for any \( n \), \( F_{\tilde{B}_{n-1:n}} \) represents the distribution of the auction price (including unobservable heterogeneity) when \( n \) bidders are present. Importantly, Aradillas-López, Gandhi, and Quint (2016) demonstrated that standard models of endogenous entry in auctions, such as those of Samuelson (1985) and Levin and Smith (1994), would violate (32).

I apply this result by performing the inversion in (32) for different values of \( n \) in the bid log subsample. For this exercise, I treat the lower bound on the number of bidders in a given auction in this subsample as though it represents the true number of bidders in that auction. Panels E and F of Figure A3 display estimates of \( \psi_{n-1:n}(F_{\tilde{B}_{n-1:n}}(\cdot)) \) for the most prevalent values of \( n \) observed in the bid log subsample. In both the dealers and fleet/lease samples, a pattern emerges consistent with the inequality in (32) and inconsistent with the models of endogenous entry highlighted in Aradillas-López, Gandhi, and Quint (2016).

\section*{D.3 Seller Distribution Estimates Under Secret vs. Public Reserve Prices}

I present here an analysis of public vs. secret reserve prices using the estimated distributions. Consider a modified version of the game consisting of no bargaining and only a public reserve price set optimally by the seller prior to the auction. Such a modified game would be equivalent to an auction with no reserve price followed by a bargaining game in which the seller makes a take-it-or-leave-it offer to the high bidder.
(see Menezes and Ryan 2005). In this modified setup, the seller’s valuation would be related to the public reserve price \( R_P \) and to the distribution of buyer valuations according to

\[
S = R_P - \frac{1 - F_B(R_P)}{f_B(R_P)}
\] (33)

I compare the seller valuations estimated in the body of the paper to those that would be implied from naively treating the secret reserve prices in the data as optimally set public reserve prices, using (33). Such a comparison is possible because the procedure for estimating seller valuations in the body of the paper does not rely on secret reserve prices other than through the inequality \( R \geq S \).

Panels E and F of Figure A4 display the results of this comparison. The results in each panel indicate that the seller valuations estimated in the body of the paper are, for the most part, lower than those that would be inferred from treating reserve prices as optimal public reserve prices. Equivalently, the secret reserve prices observed in the data appear to be for the most part higher than optimal public reserve prices would be (i.e. higher than the public reserve prices that would be optimal given the estimated buyer valuation distribution and estimated bounds on the seller valuation distribution). This may arise because, in the current mechanism, when sellers set a high reserve price and the auction price falls short of the reserve price, sellers can still have the option to accept the auction price (or take other bargaining actions), whereas in the public reserve setting the seller must commit to a non-negotiable reserve price. Thus, the downside to setting a high secret reserve price may be smaller than the downside to setting a high public reserve price. Along these lines, Kim (2013) and others have pointed out that the loss in expected revenue for a seller from setting too high of a public reserve price can be large, and are much larger than the loss from setting a public reserve price too low.

Appendix References


Notes: Panels A–D display distributions of reserve prices and auction prices prior to removing unobserved heterogeneity (dashed lines) and after removing unobserved heterogeneity (solid lines). Panels E and F display the estimated distribution of unobserved heterogeneity. Panels on the left use dealers sample and on the right use fleet/lease sample. Units = $1,000.
Figure A2: Estimates of $\rho^{-1}(\cdot)$, $\chi^{-1}(\cdot)$, and $g(\cdot)$

- (A) $\rho^{-1}$, Dealers
- (B) $\rho^{-1}$, Fleet/lease
- (C) $\chi^{-1}(\cdot)$, Dealers
- (D) $\chi^{-1}(\cdot)$, Fleet/lease
- (E) $g(\cdot)$ bounds, Dealers
- (F) $g(\cdot)$ bounds, Fleet/lease

Notes: Panels A and B display estimates of $\rho^{-1}(R)$ using the upper and lower bound on the distribution of seller values (solid lines) as well as the 45 degree line (dashed line). Panels C and D display the estimates of $\chi^{-1}(P^A)$ (solid line) and the 45 degree line (dashed line). Panels E and F display estimates of $g(\cdot, 0)$; that is, the $g(R, P^A)$ function evaluated at $P^A = 0$. Panels on the left use dealers sample and on the right use fleet/lease sample. Units = $1,000$. 

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Figure A3: Robustness of Buyer Distribution Estimates

(A) $F_B$, Dealers 
(B) $F_B$, Fleet/lease

(C) $F_{B(1)}$, Dealers 
(D) $F_{B(1)}$, Fleet/lease

(E) Order Statistics Inversion of Correlated Buyer Values, Dealers 
(F) Order Statistics Inversion of Correlated Buyer Values, Fleet/lease

Notes: Panels A and B display estimates of $F_B$ under different distributions for the number of bidders. Values for $\lambda$ represent the mean number of bidders under a Poisson distribution. $\mathbb{N}$ and $\mathbb{N}$ represent distribution of the upper bound and lower bound on the number of bidders derived from bid log data. Panels C and D display the distribution of the maximum order statistic, $B^{(1)}$. Panels E and F display estimates of $\psi^{-1}_{-1,n}(F_{Bn-1:n,W}(\cdot))$ for varying $n$, following the logic proposed in Aradillas-López, Gandhi, and Quint (2016) using the bid log subsample. Panels on the left use dealers sample and on the right use fleet/lease sample. Units = $\$1,000.$
Figure A4: Robustness of Seller Distribution Estimates

(A) Alternative Knots, Dealers

(B) Alternative Knots, Fleet/lease

(C) 25 knot approximation, Dealers

(D) 25 knot approximation, Fleet/lease

(E) Inferred $F_S$, Dealers

(F) Inferred $F_S$, Fleet/lease

Notes: Panels A and B display bounds on seller distribution using linear splines with 200 knots (solid lines), as in the main results displayed in Figure 1; 250 knots (dashed lines); 300 knots (dotted lines). Panels C and D display 25 uniformly spaced knots from the main 200 knots and their corresponding estimated coefficients. Panels E and F display the main bounds on $F_S$ (solid lines) along with CDF of seller valuations inferred from naively treating reserve prices as optimal public reserves (dashed line). Panels on the left use dealers sample and on the right use fleet/lease sample. Units = $1,000.
Table A1: Theoretical Incomplete-Information Bargaining Literature

<table>
<thead>
<tr>
<th>One-sided incomplete information</th>
<th>One-sided offers</th>
<th>Alternating offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-sided incomplete information</td>
<td>2-types</td>
<td>Chatterjee and Samuelson (1987, 1988)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Abreu and Gul (2000)</td>
</tr>
</tbody>
</table>

Notes: While by no means exhaustive, this table presents a list of a variety of theoretical papers on incomplete information bargaining settings, demonstrating that most of these papers do not model such games as being cases of two-sided incomplete information with a continuum of buyer/seller valuations, where both parties can make offers. Rather, the literature focuses primarily on settings of one-sided uncertainty (Gul, Sonnenschein, and Wilson 1986; Gul and Sonnenschein 1988; Fudenberg and Tirole 1983; Sobel and Takahashi 1983; Fudenberg, Levine, and Tirole 1985; Ausubel, and Deneckere 1989; Rubinstein 1985, Rubinstein 1985b; Bikhchandani 1992; Grossman and Perry 1986; Admati and Perry 1987; Cramton 1991), settings of one-sided offers (Cramton 1984; Cho 1990; Ausubel and Deneckere 1993; Feinberg and Skrzypacz 2005), settings with two-types rather than a continuum of types (Chatterjee and Samuelson 1988; Compte and Jehiel 2002), or settings with uncertainty not being about valuations (Abreu and Gul 2000; Watson 1998). Two papers that did model bargaining as an alternating-offer game and a continuum of types with two-sided incomplete information, where the incomplete information is about players’ valuations, are Perry (1986), which predicted immediate agreement or disagreement, and Cramton (1992), which modeled the bargaining game as beginning with a war of attrition and consisting of players signaling their valuations through the length of delay between offers, as in Admati and Perry (1987). An additional line of research considers static bargaining games with two-sided incomplete information referred to as $k$ double auctions (see Chatterjee and Samuelson 1983 and Satterthwaite and Williams 1989), discussed in the body of the paper. See Binmore, Osborne, and Rubinstein (1992), Kenan and Wilson (1993), Roth (1995), and Ausubel, Cramton, and Deneckere (2002) for additional surveys of the theoretical and experimental bargaining literature.
Table A2: Additional Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Dealers</th>
<th></th>
<th>Fleet/lease</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td><strong>A. Trade Sample</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Reserve price</td>
<td>$6,964</td>
<td>$4,945</td>
<td>$10,481</td>
<td>$5,962</td>
</tr>
<tr>
<td>Auction price</td>
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<td>$4,692</td>
<td>$10,316</td>
<td>$5,029</td>
</tr>
<tr>
<td>Final price</td>
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<td>$4,700</td>
<td>$10,325</td>
<td>$6,026</td>
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<tr>
<td>Buyer fee</td>
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<td>$36</td>
<td>$209</td>
<td>$45</td>
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<tr>
<td>Seller fee</td>
<td>$146</td>
<td>$55</td>
<td>$94</td>
<td>$15</td>
</tr>
<tr>
<td>Number of periods</td>
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<td>0.587</td>
<td>1.594</td>
<td>0.580</td>
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<td>$4,621</td>
<td>$11,246</td>
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</tr>
<tr>
<td>Age (years)</td>
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<td>3.196</td>
<td>2.529</td>
</tr>
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<td>Mileage</td>
<td>100,995</td>
<td>45,784</td>
<td>55,728</td>
<td>39,533</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>96,091</td>
<td></td>
<td>102,365</td>
<td></td>
</tr>
<tr>
<td><strong>B. No-trade Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reserve price</td>
<td>$8,440</td>
<td>$5,672</td>
<td>$10,068</td>
<td>$5,240</td>
</tr>
<tr>
<td>Auction price</td>
<td>$6,742</td>
<td>$5,321</td>
<td>$8,403</td>
<td>$5,112</td>
</tr>
<tr>
<td>Number of periods</td>
<td>2.426</td>
<td>0.765</td>
<td>2.380</td>
<td>0.681</td>
</tr>
<tr>
<td>Blue book</td>
<td>$7,753</td>
<td>$5,231</td>
<td>$10,282</td>
<td>$5,402</td>
</tr>
<tr>
<td>Age (years)</td>
<td>6.170</td>
<td>3.500</td>
<td>3.026</td>
<td>2.550</td>
</tr>
<tr>
<td>Mileage</td>
<td>90,916</td>
<td>47,433</td>
<td>62,144</td>
<td>42,181</td>
</tr>
<tr>
<td><strong>Sample size</strong></td>
<td>40,055</td>
<td></td>
<td>30,884</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Mean and standard deviation of variables in dealers and fleet/lease samples. Number of periods is 1 if game ends through auction price exceeding reserve, 2 if seller accepts at her first bargaining turn, etc. Blue book is an estimate of the market value of the car, provided by the auction house. Panel A displays subsample where trade occurs and panel B displays subsample where no trade occurs.
<table>
<thead>
<tr>
<th>Ending period</th>
<th>Player's turn</th>
<th># Obs</th>
<th>% of Sample</th>
<th>% Trade</th>
<th>Full Sample</th>
<th>Conditional on Sale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reserve price</td>
<td>Auction price</td>
</tr>
<tr>
<td>1</td>
<td>(Auction)</td>
<td>45,553</td>
<td>33.384%</td>
<td>98.35%</td>
<td>$10,344</td>
<td>$11,101</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td>($6,055)</td>
<td>($6,180)</td>
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<tr>
<td>2</td>
<td>S</td>
<td>75,069</td>
<td>55.015%</td>
<td>73.98%</td>
<td>$10,727</td>
<td>$9,676</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>($5,794)</td>
<td>($5,728)</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>10,686</td>
<td>7.831%</td>
<td>11.05%</td>
<td>$8,447</td>
<td>$6,515</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($4,374)</td>
<td>($3,975)</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
<td>919</td>
<td>0.673%</td>
<td>55.93%</td>
<td>$8,596</td>
<td>$6,923</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($4,658)</td>
<td>($4,279)</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>941</td>
<td>0.690%</td>
<td>31.56%</td>
<td>$8,933</td>
<td>$7,242</td>
</tr>
<tr>
<td>6</td>
<td>S</td>
<td>37</td>
<td>0.027%</td>
<td>54.05%</td>
<td>$8,646</td>
<td>$6,857</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>($4,211)</td>
<td>($4,326)</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>40</td>
<td>0.029%</td>
<td>32.50%</td>
<td>$11,021</td>
<td>$9,216</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($6,385)</td>
<td>($5,920)</td>
</tr>
<tr>
<td>8</td>
<td>S</td>
<td>2</td>
<td>0.001%</td>
<td>0.00%</td>
<td>$16,250</td>
<td>$14,500</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($7,425)</td>
<td>($6,364)</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>2</td>
<td>0.001%</td>
<td>0.00%</td>
<td>$11,750</td>
<td>$9,925</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($8,132)</td>
<td>($7,743)</td>
</tr>
</tbody>
</table>

Notes: Fleet/lease sample. For each period (period 1 = auction, period 2 = seller’s first turn in bargaining, period 3 = buyer’s turn, etc.), table reports the number of observations ending in that period, percent of total sample ending in that period, and percent of cases in which trade occurred. Table also reports reserve price and auction price for observations ending in a given period and, for those observations ending in trade, the reserve price, auction price, and final price conditional on trade. Corresponding statistics for the dealers sample are found in Table 2.
Table A4: Expected Gains in Market-Level Uncertainty Model

<table>
<thead>
<tr>
<th></th>
<th>First-best</th>
<th>Second-best</th>
<th>Real bargaining</th>
<th>First-best minus real</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dealers Sample</strong></td>
<td>[1.527, 3.216]</td>
<td>[1.521, 3.214]</td>
<td>[1.308, 2.654]</td>
<td>[0.192, 0.589]</td>
</tr>
<tr>
<td></td>
<td>(1.303, 3.243)</td>
<td>(1.263, 3.241)</td>
<td>(1.101, 2.690)</td>
<td>(0.156, 0.654)</td>
</tr>
<tr>
<td><strong>Fleet/lease Sample</strong></td>
<td>[1.830, 3.206]</td>
<td>[1.828, 3.204]</td>
<td>[1.568, 2.536]</td>
<td>[0.223, 0.708]</td>
</tr>
<tr>
<td></td>
<td>(1.418, 3.460)</td>
<td>(1.347, 3.459)</td>
<td>(1.186, 2.688)</td>
<td>(0.181, 0.803)</td>
</tr>
</tbody>
</table>

Notes: Bounds on expected gains from trade in first-best, second-best, real-world bargaining, as well as the gap between first-best and real-world bargaining, in market uncertainty model described in Appendix B. Estimated bounds are in square braces and 95% confidence set is in parentheses. Units are $1,000.