\[ H^{(a)}(j\omega) \]

\[ H^{(b)}(j\omega) \]

**Figure 5.8** The step responses of the transfer functions in (5.17) and (5.18). Note that \( \| H^{(a)}_{13} \|_{\text{pk,step}} = 1.36 \), and \( \| H^{(b)}_{13} \|_{\text{pk,step}} = 1.40 \).

**Figure 5.9** The magnitudes of the transfer functions in (5.17) and (5.18). Note that \( \| H^{(a)}_{13} \|_{\infty} = 1.47 \), and \( \| H^{(b)}_{13} \|_{\infty} = 3.72 \).
7.2 INTERNAL STABILITY

7.2.1 A Motivating Example

Consider our standard example SASS 1-DOF control system described in section 2.4, with the controller

\[ K(s) = \frac{36 + 33s}{10 - s}. \]

This controller yields the closed-loop I/O transfer function

\[ T(s) = \frac{33s + 36}{s^3 + 10s^2 + 33s + 36} = \frac{33s + 36}{(s + 3)^2(s + 4)}, \]

which is a stable lowpass filter. Thus, we will have \( y_p \approx r \) provided the reference signal \( r \) does not change too rapidly; the controller \( K \) yields good tracking of slowly
where $H_{z\bar{w}}$ is some entry or submatrix of $H$ (c.f. robust stability, which involves the gain bound $\|H_{z\bar{w}}\|_\infty < \infty$).

Throughout this section, we will consider the robustness specification that is formed from the perturbed plant set $\mathcal{P}$ and the RMS gain bound specification

$$\|H_{z\bar{w}}\|_\infty \leq 1. \quad (10.75)$$

We will refer to this robust performance specification as $\mathcal{D}_{\text{rob, perf}}$. We will also assume that the perturbed plant set $\mathcal{P}$ is described by a perturbation feedback form for which the maximum RMS gain of the feedback perturbations is one, i.e., $M = 1$ in (10.54).

The inner approximation of $\mathcal{D}_{\text{rob, perf}}$ is

$$\left\| \begin{bmatrix} H_{\bar{z} \bar{w}} & H_{\bar{z} \bar{p}} \\ H_{\bar{q} \bar{w}} & H_{\bar{q} \bar{p}} \end{bmatrix} \right\|_\infty < 1. \quad (10.76)$$

Like the inner approximation (10.57-10.60) of the robust stability specification $\mathcal{D}_{\text{rob, stab}}$, we can interpret (10.76) as limiting the size of $H_{\bar{z} \bar{p}}, H_{\bar{q} \bar{w}},$ and $H_{\bar{q} \bar{p}}$.

Let us show that (10.76) implies that the specification (10.75) holds robustly, i.e.,

$$\|H_{z\bar{w}} + H_{z\bar{p}}(I - H_{q\bar{p}}\Delta)^{-1} H_{q\bar{w}}\|_\infty \leq 1 \text{ for all } \Delta \in \Delta. \quad (10.77)$$

Assume that (10.76) holds, so that for any signals $\bar{w}$ and $p$ we have

$$\left\| \begin{bmatrix} \bar{z} \\ q \end{bmatrix} \right\|_{\text{rms}} < \left\| \begin{bmatrix} \bar{w} \\ p \end{bmatrix} \right\|_{\text{rms}}, \quad (10.78)$$

where

$$\begin{bmatrix} \bar{z} \\ q \end{bmatrix} = \begin{bmatrix} H_{\bar{z} \bar{w}} & H_{\bar{z} \bar{p}} \\ H_{\bar{q} \bar{w}} & H_{\bar{q} \bar{p}} \end{bmatrix} \begin{bmatrix} \bar{w} \\ p \end{bmatrix}.$$
11.1 I/O Specifications

Figure 11.2 The step responses from the reference input, $r$, to plant output, $y_p$, for the closed-loop transfer matrices $H^{(a)}$, $H^{(b)}$, and $H^{(c)}$.

Figure 11.3 Level curves of the step response settling time, from the reference $r$ to $y_p$, given by (11.1).