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Design of l_1 Optimal Controllers

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ABSTRACT

The l_1 norm of a discrete-time system determines its *peak gain*, whereas its H_∞ norm determines its *RMS gain*. In many cases it may be desirable to make a certain map small in the peak gain sense; this is l_1 optimal controller design. This overview talk covers the basic results in this area.

INTRODUCTION

This paper discusses the solution of a long standing problem of interest to control engineers. In many control systems the major objective is to keep errors small. Error is defined as the difference between a desired response and the actual response of the system to inputs, some of which are reference inputs and some of which are disturbance inputs.

In frequency domain design, errors are controlled by controlling the magnitude of loop gains in the frequency ranges where the inputs are expected to lie. An appeal to Parseval's theorem will establish that this approach corresponds to trying to obtain small integral square errors when the system inputs have finite integral square values, i.e. finite L_2 norms. The H_∞ theory introduced recently by Zames establishes a systematic approach to error minimization in the sense that the maximum L_2 norm of the error is minimized over the class of L_2 inputs. The induced norm in this case is the H_∞ norm and the control problem is posed as minimization of this norm over the class of stabilizing controllers.

In LQG problems, which form a special class of H_2 optimization problems, the system inputs are white noise and the objective is to minimize the RMS value of the error.

In neither of the above approaches is the magnitude of the system error controlled directly. In order to avoid unsatisfactory system performance caused by saturation of nonlinear system components it is of considerable interest to be able to control the peak values of the system errors and other selected signals in the control loop. This has long been a problem of interest to control engineers and is where l_1 optimal control can be used.

This formulation was introduced recently by Vidyasagar who posed the problem of minimizing the maximum L_∞ norm of the system error when the inputs are in L_∞, i.e. bounded but persistent. This is perhaps a more realistic class of inputs than L_2 inputs and white noise inputs. The induced norm is the L_1 norm of the system impulse response (l_1 norm of the system pulse response in the discrete-time case).

This paper discusses the current state of affairs in this area. We will concentrate on the discrete-time problem since rational controllers are obtained when the plant and weightings are rational. This is not the case with the continuous-time problem. For a certain class of MIMO problems, the solution is obtained in two steps, first solution of a linear programming problem to obtain the minimum value of the performance index and next, solution of a set of linear equations to obtain the optimal transfer function.

Examples will be presented to demonstrate the results available with this theory.