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Robert Legendre and Henri Werlé: Toward the Elucidation of Three-Dimensional Separation

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■ **Abstract** The description and the physical understanding of three-dimensional separated flows are challenging problems mainly because of the use of inappropriate terms linked to the consideration of two-dimensional flows. This fact was realized in the early 1950s by Robert Legendre, who introduced the basic concepts of the Critical Point Theory to provide a rational definition of separation in three-dimensional flows. In parallel, demonstrative experiments were executed by Henri Werlé in the Onera water tunnel laboratory. From the close cooperation between these two scientists resulted the construction of a powerful theoretical tool allowing the elucidation of the structure of largely separated three-dimensional fields. The importance of their contribution to fluid mechanics is illustrated here by the consideration of basic configurations: flow past wings or elongated bodies, in front of obstacles, and behind a base. For each case, the flow organization is discussed by considering representative water tunnel visualizations and corresponding topological interpretations.

INTRODUCTION

The passage from the familiar two-dimensional to the mysterious three-dimensional requires a complete reconsideration of concepts apparently obvious (separation and reattachment points, separated bubble, recirculation zone, limiting streamline) but inappropriate and even dangerous to use in three-dimensional flows. This fact was recognized by Robert Legendre (who worked at Onera from 1950 until his death in 1994) in the early 1950s when he studied the flow past delta wings and other three-dimensional bodies. He introduced a precise definition of the basic concepts on which we have to reason to describe such a flow; then he applied to these concepts rigorous results inspired from the works of the mathematician Henri Poincaré. Hence, the introduction of notions such as skin friction lines, critical points, separation (or attachment) lines, separation (or attachment)

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surfaces, and topological rules allowed a consistent description of flow fields. The work of Legendre on three-dimensional separation was in great part inspired and confirmed by the visualizations performed, at the same time, by Werlé in his water tunnels. Thus, it is difficult to dissociate the contribution of Legendre's description of three-dimensional separated flows from the experimental work performed in parallel by Werlé, the first giving the inspiration and scientific interpretation, the second executing beautiful experiments that constituted major steps in the physical understanding of complex flows.

Werlé was the head of the Onera Hydrodynamic Visualization Laboratory until he retired in 1988. The first water tunnel he built, in 1951, was a gravity driven facility with a test section of 250×250 mm^2 and a maximum velocity of 0.25 m/s (Werlé 1958). Later, a second larger tunnel was built (test section of 450×450 mm^2, maximum velocity of 1.70 m/s) with the aim of achieving higher Reynolds numbers and of testing more complex and bigger models. A special tunnel, with a very low velocity (0.065 m/s), was also built for helicopter studies (Werlé & Gallon 1982). These facilities were, and still are, dedicated to flow visualizations, and Werlé devoted much effort to developing visualization techniques and obtaining both high quality and informative pictures. Rapidly, he reached a degree of excellence that made him a world-known expert in water tunnel experimentation (Werlé 1974, 1982, 1983, 1986, 1987). Many of the pictures he took during his career were reproduced all around the world and are still regularly published to illustrate text books on fluid mechanics or aerodynamics (Van Dyke 1982). Such an achievement requires highly technical skill, a great expertise in fluid mechanics, and a deep artistic sense. The work of Werlé is a remarkable demonstration of the importance of the visual aspect of phenomena and of the difficulty of grasping the reality of a world with three dimensions. In this case, it is not possible to separate the contributions of the mathematician and of the experimentalist who, per chance, worked in the same place in close contact. This review uses a tiny fraction of the thousands of pictures taken by Werlé during his long career at Onera. Deliberately, we have restricted our choice to visualizations that confirmed the parallel theoretical work of Legendre on three-dimensional separation.

THE NATURE OF THREE-DIMENSIONAL SEPARATION

In many circumstances, three-dimensional separation is such a catastrophic and overwhelming phenomenon that it is nearly independent of the Reynolds number, which is, in fact, the correct scaling parameter in boundary-layer–like situations where viscous effects are confined within thin layers. The Mach number also is not a determining parameter for largely separated flows, the accompanying shock system being in reality an epiphenomenon. Thus, if one excludes exceptional circumstances where the Reynolds number is extremely low, the organization of the flow over a delta wing, downstream of the base of a missile, in front of a blunt obstacle, or past an automobile is basically the same as in a small size water tunnel at a velocity of a few centimeters per second. For this reason, the water tunnel has been an exceptional tool in describing the flow over the wing of the supersonic Concorde (Figure 1), in qualifying vortex breakdown on a combat aircraft wing, in tracking the tip vortices of helicopter rotor blades (Figure 2), and recently in understanding the flow in the base region of the Ariane 5 space launcher (Figure 3). To pretend that, even in these cases, there is no Reynolds number effect would be an exaggeration, but the flow physics does not depend critically on this parameter. If one excludes laminar to turbulent transition, the Reynolds number has its most direct influence on the location of the boundary layer separation. In situations where this separation is imposed by a local singularity, like a sharp edge, then the Reynolds number plays a less important role. This fact explains the important and recognized contribution of Werlé to applied aerodynamics.

The first article by Legendre on three-dimensional separation was published in 1956. At that time it raised a modest echo, the subject seeming too academic to be of use in application, the engineers being then satisfied with the two-dimensional concepts (Legendre 1956). The theoretical approach of Legendre was considered in 1963 by Lighthill to describe the separation of a three-dimensional boundary layer (Lighthill 1963). Later on, Legendre's ideas on three-dimensional separation were reintroduced by Tobak & Peake and others, both in the United States and in Europe (Perry & Fairlie 1974, Hunt et al 1978, Tobak & Peake 1982, Dallmann 1983, Hornung & Perry 1984, Bakker & Winkel 1989, Yates & Chapman 1992).

The present theory of three-dimensional separation is not a predictive theory in that it does not predict separation on an obstacle in given conditions. For this the Navier-Stokes equations must be solved. It is a descriptive theory reasoning on the properties of a given vector field (skin friction or velocity) provided by experimentation or calculation. It is a tool to give a rational description and interpretation of an observation made on a model in a wind tunnel or on the screen of a work station displaying computed results.

The considerations that follow apply to a nonfluctuating flow, a laminar flow—rarely achieved in aeronautical applications—as well as to the time-averaged flow introduced to describe and compute turbulent flows. In this case, as far as the flow organization is concerned, there are no major differences between laminar and turbulent situations (the governing equations are almost the same) except for the characteristic scales. Also, we consider the flow as steady, although the theory can be applied to the instantaneous field of a time-dependent flow.

It is not possible to discuss separation in three-dimensional flows without introducing the Critical Point Theory and it is to the great credit of Robert Legendre to have realized that this mathematical theory is the only rational tool that allows the understanding of the organization—or topology—of three-dimensional separated flows (Legendre 1966, 1965, 1977, 1991). The Critical Point Theory comes from the work of Poincaré on the singular points of systems of differential equations (Poincaré 1882). In general, it applies to a space of n dimensions and leads to major conclusions and results on the behavior of dynamical systems, including the concepts of strange attractors, the notions of ill- and well-posed problems, unpredictibility, and ultimately the chaos theory. In what follows, we restrict ourselves to a very narrow aspect of this theory, which will be sufficient for our purpose and which, in fact, was only considered by Legendre.

Since separation is linked to the existence of viscosity, the effects of which are normally concentrated in the boundary layer, it is natural to look for a definition of separation by considering the properties of the flow on the surface of the vehicle. More precisely, one focuses on the shear stress at the wall, which is a vector field in three-dimensional flows. The trajectories of this field are the skin friction lines, sometimes called limiting streamlines because they are the limit of a streamline when the distance to the wall becomes zero. The set of the skin friction lines covering a body constitutes a skin friction line pattern, and separation is defined from examination of this pattern. However, the sole inspection of the skin friction line pattern is far from being sufficient to define and describe three-dimensional separation. One has to go to the outer field organization, the visualization of which is an indispensable step toward understanding three-dimensional flow physics.

Let us consider the two-dimensional space constituted by the surface of a body and an orthogonal system of coordinates (x, z) on this surface. We denote by $\tau_x(x, z)$ and $\tau_z(x, z)$, the components of the skin friction vector τ_w along x and z, respectively.

For a steady flow the skin friction lines are defined by the following autonomous (time independent) differential system:

$$\frac{dx}{\tau_x(x, z)} = \frac{dz}{\tau_z(x, z)}.$$

These equations define an infinity of solution curves called characteristics lines or trajectories that are associated with the skin friction lines introduced above. In general, one and only one trajectory passes through a point on the surface. The only points that do not satisfy this rule are the singular points of the system where the skin friction vanishes, that is, where we simultaneously have:

$$\tau_x(x, z) = \tau_z(x, z) = 0.$$

To study the shape of the trajectories in the vicinity of a singular point $P_0(x, z)$, a solution is sought by a first order Taylor series expansion. The conditions for a nontrivial solution lead to an eigenvalue problem that is not discussed here.

Introducing the Jacobian matrix,

$$F = \begin{vmatrix} \dfrac{\partial \tau_x}{\partial x} & \dfrac{\partial \tau_z}{\partial x} \\ \dfrac{\partial \tau_x}{\partial z} & \dfrac{\partial \tau_z}{\partial z} \end{vmatrix},$$

and letting $p = -(\text{trace of } F)$ and $q = (\text{determinant of } F)$, the eigenvalues are given by:

$$S_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}.$$

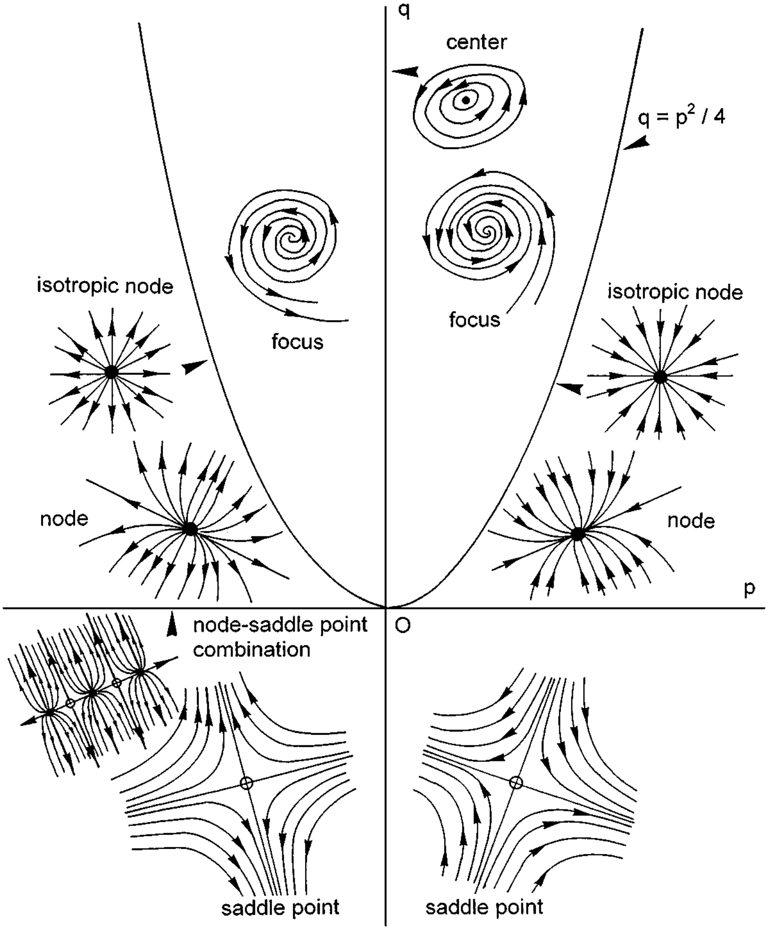


Figure 4 Classification of the critical points in the [p, q] plane.

The behavior of the skin friction lines in the vicinity of the singular or critical point P_0 is determined by the real or imaginary character and the sign of the eigenvalues S_1 and S_2. The different types of behavior obtained according to the eigenvalues are situated in plane [p, q] shown in Figure 4 and the skin friction lines in the vicinity of the critical point are shown in Figure 5. If the two eigenvalues S_1 and S_2 are real and have the same sign, the singular point is a node. If they are distinct, all the trajectories, except one, have a common tangent at P_0 (see Figure 5*a*). If the two eigenvalues are equal (points located on the parabola of equation $q = p^2/4$), all the trajectories have different slopes at P_0. The node is then said to be an isotropic node (see Figure 5*b*). If the eigenvalues are real and have opposite signs, the singular point is a saddle point. Then, only two trajectories

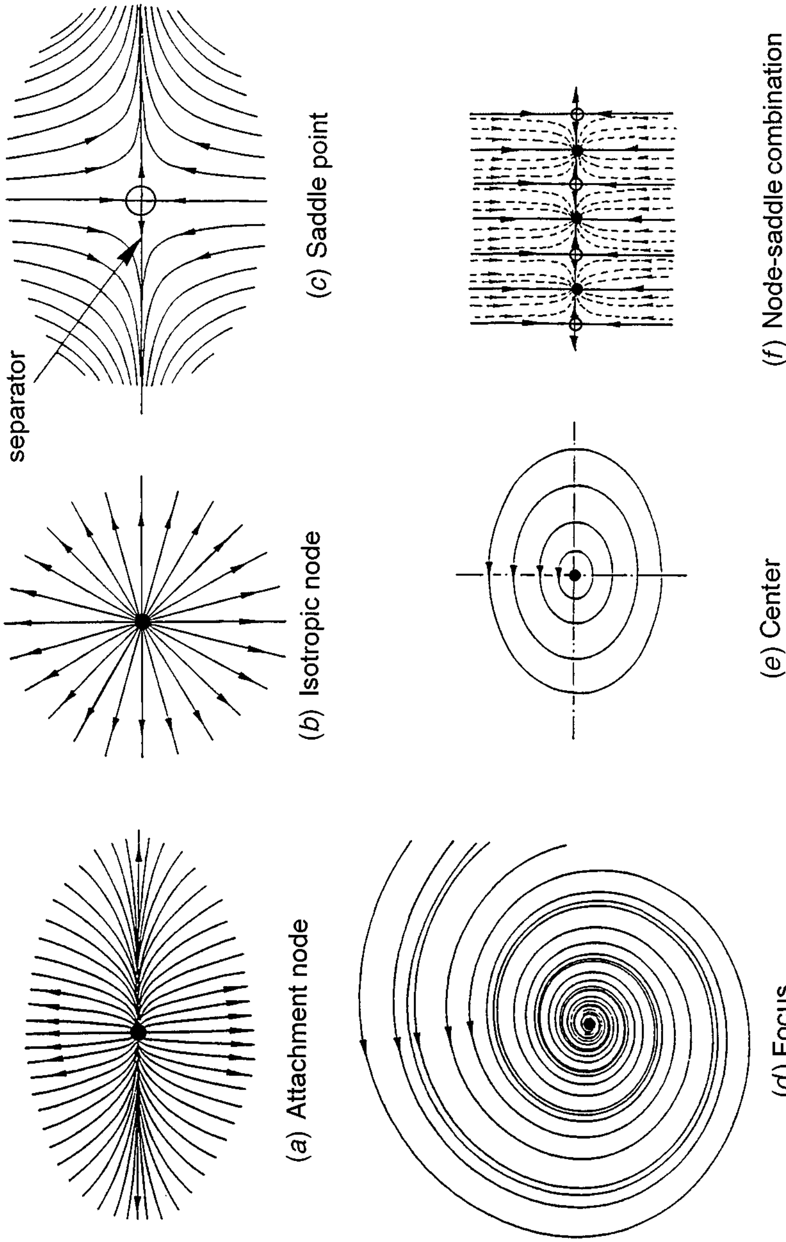


Figure 5 The different critical points.

go through the singular point P_0 whereas the other trajectories avoid P_0 and take on hyperbolic shape, as shown in Figure 5*c*. If the eigenvalues are complex conjugates, the trajectories all end in the singular point and spiral around it to form a focus (see Figure 5*d*). If p is zero for q positive, the singular point is not reached. Then the trajectories are closed curves having the shape of an ellipse (see Figure 5*e*). This singular point is called a center. In the case of what is considered to be a two-dimensional separation, the critical point becomes an infinite set of node-saddle point combinations distributed along a common line (see Figure 5*f*).

The two skin friction lines going through a saddle point are called separators because they separate the skin friction lines into families that can become infinitely close or infinitely far from each other, depending on their orientation and/or flow direction. Saddle points, foci, and separators play a central role in the structure and definition of three-dimensional separated flows. From a physical point of view, a separator can be of the separation or attachment type depending on the behavior of the flow in its vicinity. In a separation process, the skin friction lines tend to converge toward the separation line, while the flow lifts off from the surface. In an attachment process the behavior is opposite, the flow diving in the direction of the surface and the skin friction lines diverging from the attachment line. A separation or attachment line is the trace on the body of a stream surface of the three-dimensional flow acting as a barrier separating the streamlines into two families having different origins, that is, coming from different nodes. The sketch in Figure 6*a* shows the flow organization at the start of a separation surface. Such a surface is constituted by the streamlines emanating from a node N that coincides with the separation saddle point S on the body surface. In the case of an attachment process, as sketched in Figure 6*b*, the streamlines forming the attachment surface end at a node N coincident with the attachment saddle point S.

The following general definition of separation in three-dimensional flow can now be given: A flow past a body is separated if the skin friction line pattern contains at least one saddle point (Tobak & Peake 1982). Hence a separation line exists sustaining a separation surface, the rolling up of which forms the vortical structure, or vortex, typical of three-dimensional separated flows.

The concept of critical points can be generalized to the velocity field of the three-dimensional flow surrounding an obstacle because it can be applied to any continuous vector field. However, because the mathematical formalism is more complicated, only essential results are given without justification. Two singularities are considered for a three-dimensional velocity field:

1. The first is the three-dimensional saddle point, which is associated with a two-dimensional node on the body surface and two-dimensional saddle points in the outer field (see sketch in Figure 7*a*). Depending on the direction of the streamlines and skin friction lines, this structure corresponds either to an attachment point, at the front of an obstacle for example, or to a separation point at its downstream end. The photograph in Figure 7*b* shows the attachment point on the front part of a high-speed

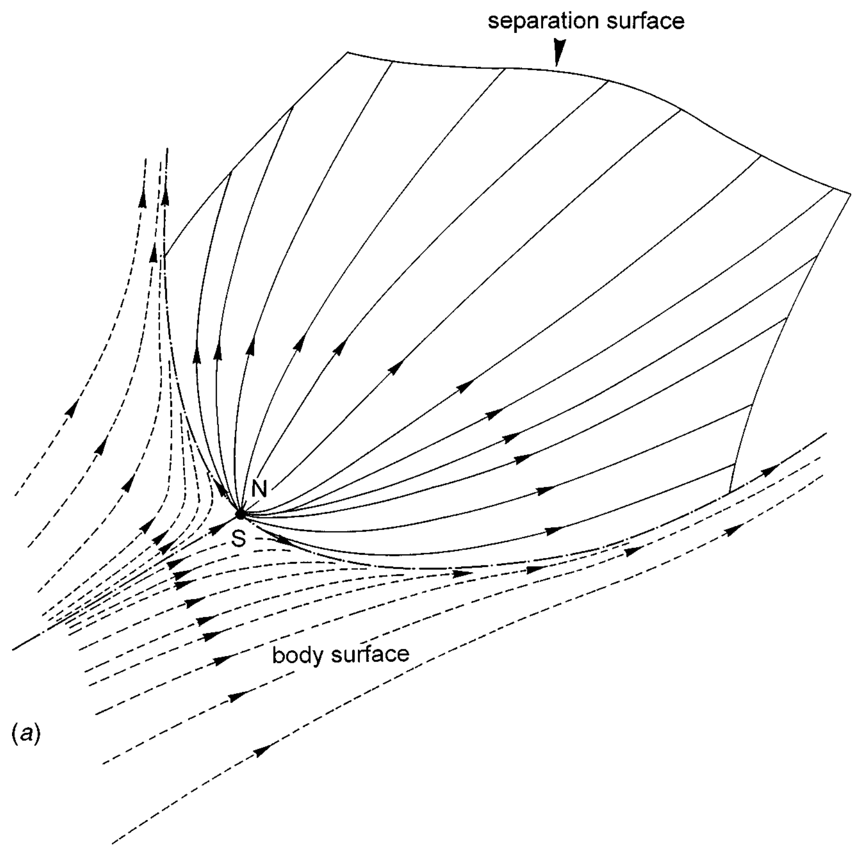


Figure 6 Flow behavior in the vicinity of a separator line: (*a*) separation process, (*b*) attachment process.

train. The visualization, obtained by tiny air bubbles, shows the flow in a plane of the outer field where the singular point has saddle-point behavior.

2. The second is the three-dimensional focus associated with a two-dimensional focus on the obstacle and an isolated line passing through the center of the focus (Figure 8*a*). The other streamlines spiral around this line taking a helical shape. This structure corresponds to a tornado-like vortex. The most spectacular and devastating example of such a structure is the tornado, which forms under certain weather conditions. The visualization in Figure 8*b*, realized with filaments of dye, shows the swallowing by an air intake of a vortex forming on the ground; the swallowing is a result of the suction effect of the air intake, when the plane is still at rest.

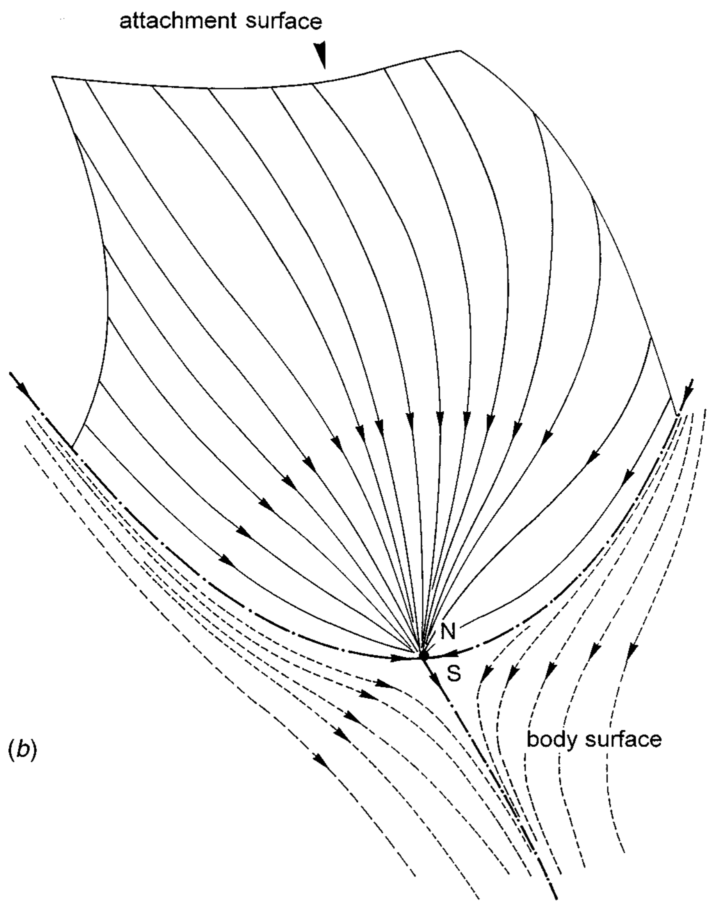
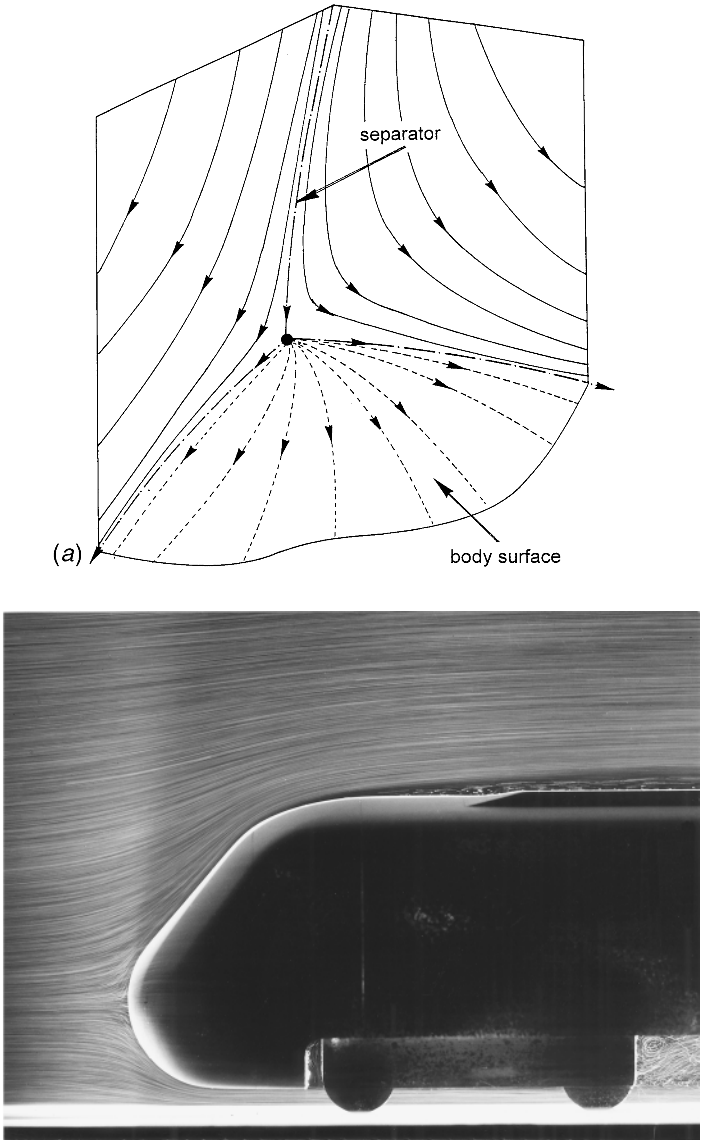


Figure 6 (*continued*)

Considering the flow projected in a plane (P)—an operation frequently done to visualize three-dimensional flows—it is possible to determine the lines of force of the vector field constituted by the velocity projections in (P). The set of these lines can also be interpreted in terms of critical points and separators. However, such a representation of the flow field is not objective because it depends on the chosen plane of projection (P). Except in special situations, a plane of symmetry for example, projected lines of force are not streamlines. To avoid any confusion we call them pseudo-streamlines, which is not entirely satisfactory.

In what follows we examine some of the separated flows studied by Legendre and Werlé. The construction of separated three-dimensional fields is frequently an arduous task, visualizations often being too coarse to reveal all the topological features of the flow. To arrive at a rational construction, the fragmentary information must be complemented by a large dose of imagination, with the guide of logical

(a)



(b)

Figure 7 Three-dimensional saddle point: (*a*) sketch of the flow field topology, (*b*) attachment point on a high-speed train.

rules, and the capacity to see objects in three dimensions. As told to the author by Legendre himself, this exercise may be extremely tedious, requiring both patience and tenacity!

FLOW PAST WINGS

With the design of high-speed airplanes equipped with swept wings, the early 1950s saw a considerable rise of interest in flow past delta wings. It was soon realized that at high incidence an unexpected increase of lift was obtained due to the presence of intense vortices forming above the wing. As explained by Legendre at that time, the origin of these vortices is similar to that of the tip vortices of classical wings: At the leading edge, because of the large pressure difference, the lower-surface flow unfurls like a wave over the wing upper surface (Legendre 1952). At large sweep angles (above 60°), the swirling motion of these vortices is so rapid that they induce a depression on the wing upper surface which is the origin of this extra lift, sometimes called vortical lift. Delta-wing aerodynamics was studied by Legendre who proposed an analytical theory to explain the essential flow features (at that time computers were not available). Nearly at the same time, the first experiments that confirmed the existence of these vortices were made by Werlé in a water tunnel.

The visualization of the delta-wing vortices shown in Figure 9 has been obtained by hydrogen bubbles emitted by electrolytic effect on electric wires placed along the wing leading edges. The two vortices formed by the rolling up of the separation surface emanating from the separation lines, which run along the leading edges, can be clearly seen. The origin of these vortices is at the wing apex or at a point very close to it.

The flow in a plane normal to the wing upper surface is visualized by a sheet of light illuminating bubbles introduced in the settling chamber of the water tunnel. Because of the finite exposure time, each bubble leaves a trace, the length of which represents the projected velocity modulus. The photograph in Figure 10*a* shows this projected field, whereas Figure 10*b* shows its topological interpretation. The flow projected in a normal plane or cross flow reveals a general upward motion. The main features of this cross flow are the two large foci F_1 and F_2, which are the traces in the visualization plane of the main vortical structures. The separation line (S_1) runs through a saddle point S_1 well above the wing; this separates the pseudo-streamlines ending into the foci from those continuing their path to the top of the picture. In this case, two other foci F_3 and F_4 are visible close to the leading edges, under the primary vortices. They correspond to secondary vortices resulting from the separation of the fluid flowing on the wing upper surface from the symmetry plane in the direction of the leading edge. The traces on the wing upper surface of the secondary separation lines are the half saddle point S_3 and S_4.

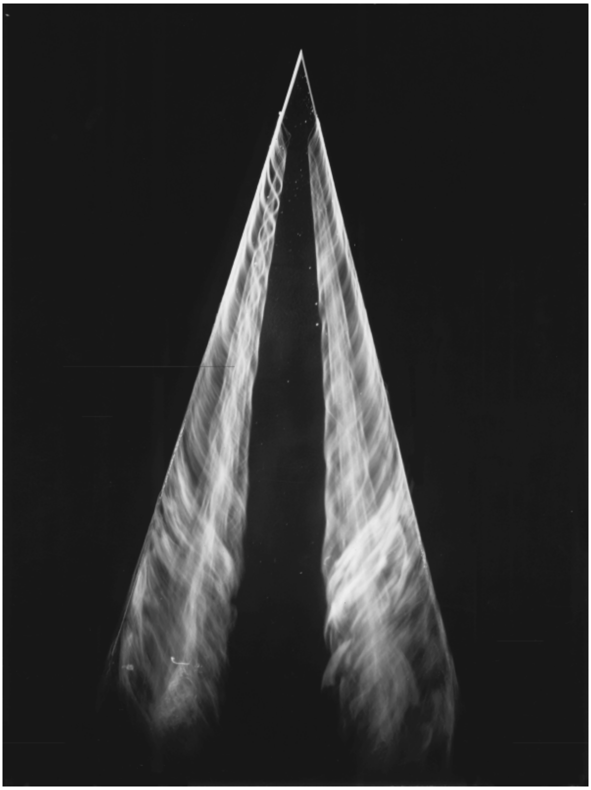


Figure 9 Vortices over a 75°-sweep angle delta wing.

Delta wings with sharp leading edges were most often considered because it was intuitively obvious that the separation at the origin of the vortices was taking place along the leading edge; it was not necessary to predict the location of this separation, which poses a difficult problem. In reality, a sharp leading edge and a pointed apex are singular and unphysical cases leading to difficulties in the topological interpretation. An instructive experiment was performed by Werlé on a thick delta wing with a rounded leading edge. Because of the very low Reynolds number and the more regular geometry, this experiment modeled a blowup of the apex region of the thin delta wing with sharp leading edge. As shown by the photograph in Figure 11*a*, two foci and two saddle points exist on the wing upper surface. The topological interpretation given in Figure 11*b* also shows the attachment node at the nose and an unfolded part of the wing lower surface. The two foci are the traces on the wing of two tornado- like vortices escaping in the outer flowfield. If the thickness of the wing is decreased, its leading edge becoming sharp, one can imagine an evolution of the flow topology toward the organization of the previous experiment. The separation pattern formed by the two

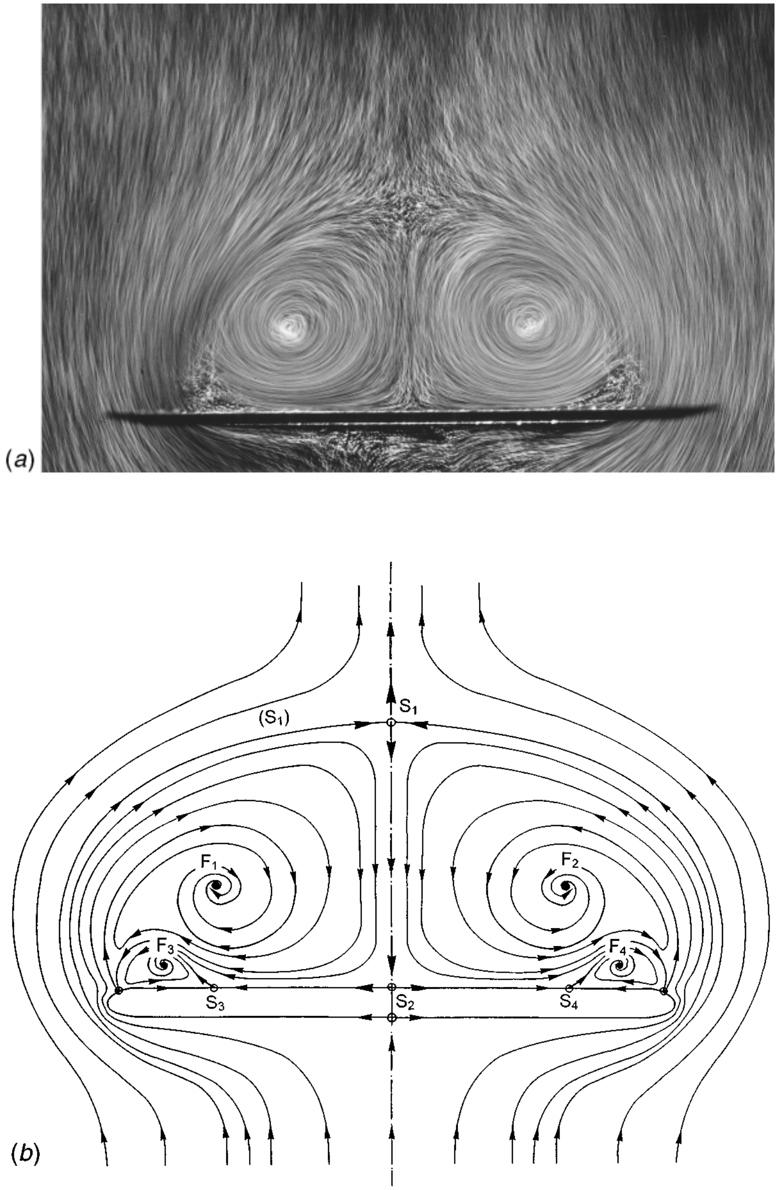


Figure 10 Cross flow over a 75°-sweep angle delta wing: (*a*) air bubbles visualization, (*b*) topological interpretation.

foci and saddle points moves in the direction of the wing apex, and the associated tornado-like vortices having a common origin form a horseshoe vortex. To arrive at a regular topology, we must consider some rounding of the leading edge and blunting of the apex (which in fact always exists at a microscopic scale). The sketch in Figure 12 represents the separation and vortex formation in the apex region: The separation line (S_1) starting from the saddle point S_1 is the trace of the separation surface (Σ_1), which rolls up into a horseshoe vortex. The trace of this vortex in the flow symmetry plane is the focus F_1. The two arms of this vortex constitute the delta wing vortices visible in Figure 9. As they develop from the apex to the trailing edge, their intensity is steadily increased by a continuous shedding of vorticity from the boundary layers of the wing upper and lower surfaces. Two attachment nodes N_1 and N_2 are also present in the apex region. The above change in the flow topology is in great part conjectural; it would be instructive to devise an experiment that reproduces this evolution.

As shown in Figure 13, in the case of the four-vortex system of Figure 10*b*, the topology at the wing apex has two saddle points, S_1 and S_2, through which run the separation lines (S_1) and (S_2) and three attachment nodes, N_1, N_2, and N_3. The attachment line (A_1) must exist between (S_1) and (S_2); the two other attachment lines (A_2) and (A_3) run in the vertical symmetry plane on the wing lower and upper surfaces, respectively. This surface flow topology is consistent with a four-vortex

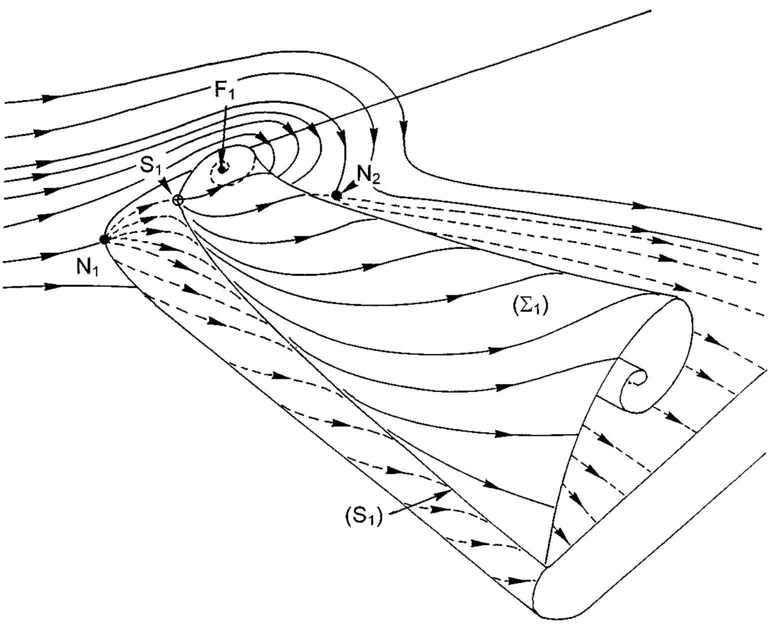


Figure 12 Flow topology at the apex of a thin delta wing. Case of a two-vortex system.

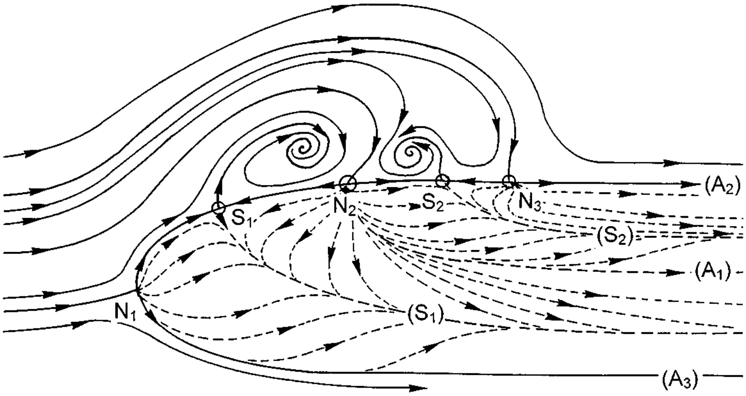


Figure 13 Flow topology at the apex of a thin delta wing. Case of a four-vortex system.

flow over the wing: two intense primary vortices and two secondary much weaker vortices, as represented in Figure 14.

Another topological evolution can be conceived if the shape of the wing is changed from a delta to a trapezoidal or rectangular form. Then, each of these foci moves toward one of the wing tips where they are the origin of the familiar tip vortices of classical wings (Figures 15*a* and 15*b*). These foci are difficult to detect, but their existence was conjectured by Werlé & Roy (1960).

During his studies of delta wings, Werlé discovered a spectacular phenomenon (Werlé 1960). When the wing angle of attack is raised, the vortex swirl velocity increases and the concentrated vortex core suddenly expands, causing the vortex flow to become unsteady (Figure 16). This still mysterious phenomenon, called vortex breakdown, has been the subject of numerous theoretical and experimental investigations (Délery 1994). When the breakdown of the two primary vortices occurs at different locations on each side of the wing, a roll moment results that can compromise the stability of combat aircraft maneuvering at high incidence.

FLOW OVER ELONGATED BODIES

At high incidence, similar vortices form above elongated bodies like aircraft or missile fuselage frontpart. They result from the separation of the flow coming from the windward side of the body when it meets the compression on the leeward side. This type of separation is more difficult to predict because it occurs on a regular surface. For a pointed body (small but finite nose radius) the separation surfaces emanate from separation lines whose origin is close to the model tip, as shown in Figure 17. The rolling up of these surfaces gives rise

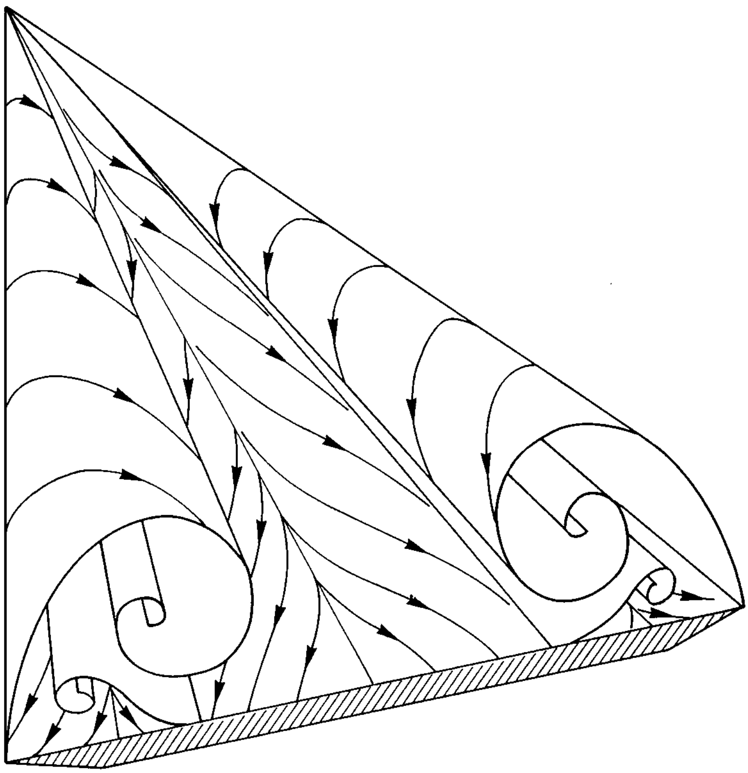


Figure 14 Schematic representation of the separation surfaces over a highly swept delta wing.

to vortices adopting a symmetrical position, as shown by the cross flow visualization in Figure 18*a*). Secondary separations often occur between the model vertical symmetry plane and the primary separation, their number depending on the Reynolds number. Clear visualization of the secondary vortices is often difficult, their swirl velocity being much smaller than that of the primary vortices. In the topological interpretation of Figure 18*b*, it is assumed that only one secondary vortex exists under each primary vortex. The topology of this cross flow is identical to that of the delta wing, in terms of foci, saddle points, and separators.

An interesting feature of this kind of flow (Figure 19*a*) is the establishment of an asymmetrical pattern when the incidence exceeds a certain value, generally close to 50°. The topological interpretation of the cross flow is sketched in Figure 19*b*. This phenomenon results from the interaction between the two primary vortices that are close to each other for this type of body. The wall pressure below the vortex close to the surface is lower than the pressure under the upper vortex, resulting in a side force that is of some concern for missile guidance.

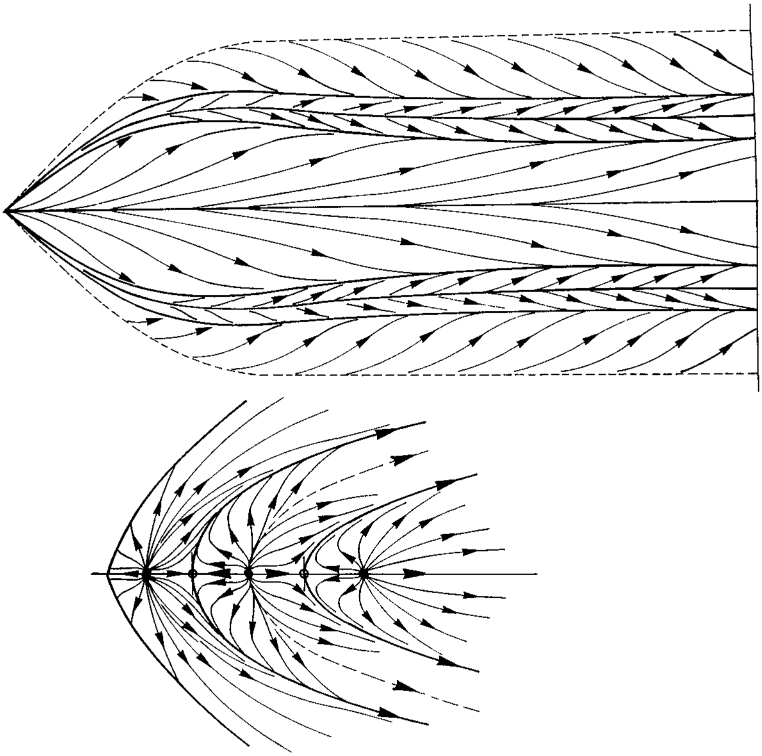


Figure 17 Schematic representation of separation and attachment lines on an elongated body at incidence.

As shown by the photographs in Figure 20, if experiments are performed on an ogive with a larger nose radius and at very low Reynolds number, two foci are present, and it is possible to draw a topological interpretation similar to that of Figure 11. Hence, in this case it is also possible to imagine a displacement toward the ogive apex of the singular points at the origin of the separations when the nose radius is reduced, or the incidence increased.

OBSTACLE-INDUCED SEPARATION

Another situation of great practical interest is the flow induced by an obtacle perpendicular to a surface, like a building, a chimney, or a submarine conning tower. As shown by Figure 21*a*, the incoming flow separates in front of the obstacle along a primary separation line (S_1) passing through the saddle point S_1 indicated in the topological interpretation of Figure 21*b*. An attachment node N_1 must exist in front of the obstacle to feed the surface flow behind the separation line (S_1). The separation surface emanating from (S_1) rolls up to create

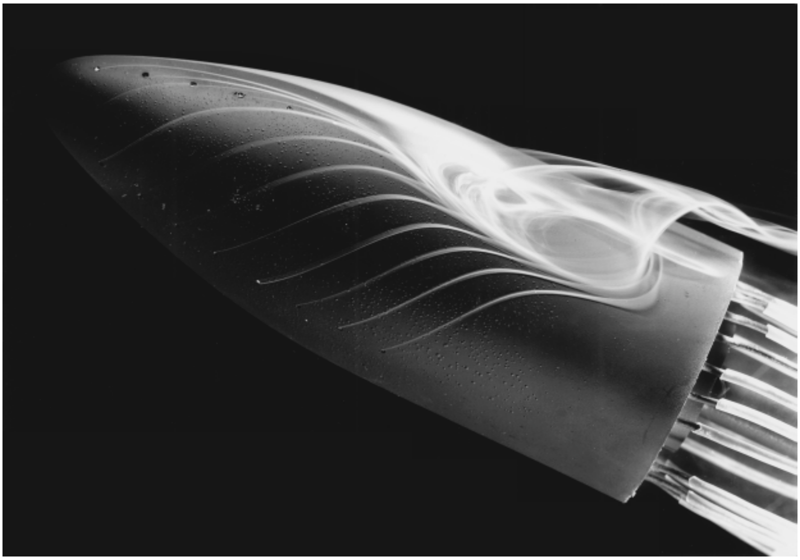


Figure 20 Separation over a blunted body at low Reynolds number.

a horseshoe vortex surrounding the obstacle, as shown in Figure 22. Secondary separations may take place in front of the obstacle, giving rise to a succession of such vortices.

A separated region exists behind the obstacle with separation surfaces starting from each side of the cylinder, as shown in Figure 23. Because of large-scale fluctuations,which blur the photograph, the structure of the mean flow is difficult to visualize so that the interpreter must rely on fragmentary information. To arrive at the consistent topology of the skin-friction-line pattern represented in Figure 24, one has to introduce two foci F_1 and F_2 behind the cylinder (these foci are clearly seen in wind-tunnel experiments using an oil-flow technique to visualize the surface flow; Sedney & Kitchens 1977). At least three saddle points, S_3, S_4, and S_5, must exist along with the separation lines (S_5), (S_6), and (S_7). Two other separation lines (S_3) and (S_4) start from the cylinder base: They are the trace on the plate of the separation surfaces visible in Figure 23. The flow on the obstacle itself will not be examined, because of the lack of clear experimental evidence. One can propose the organization represented in Figure 25 where two tornado-like vortices are present behind the obstacle, in addition to the horseshoe vortices forming ahead of the obstacle. They result from the rolling up of the separation surfaces emanating from the separation lines located on the obstacle, their trace on the plate being the foci F_1 and F_2. If the obstacle has a finite height, the two vortices are bent by the general stream and entrained downstream.

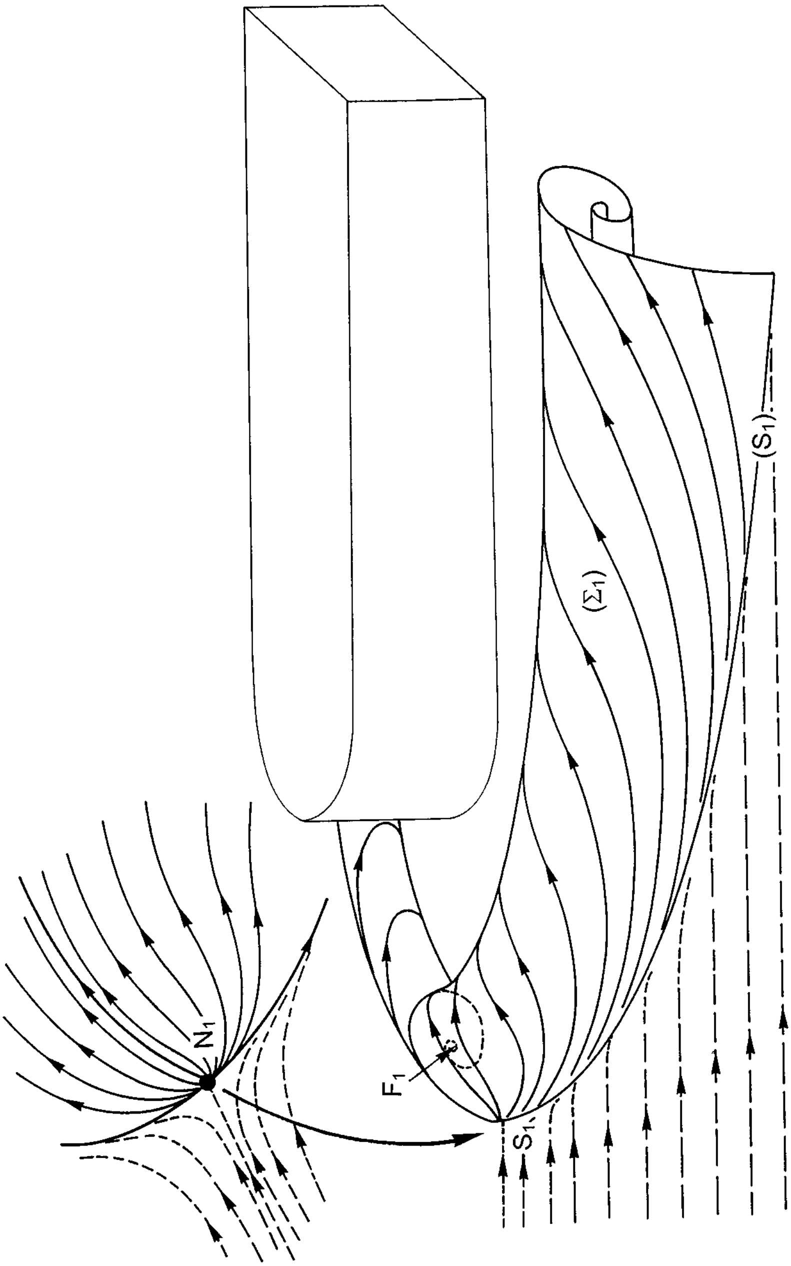


Figure 22 Schematic representation of the horseshoe vortex induced by the obstacle.

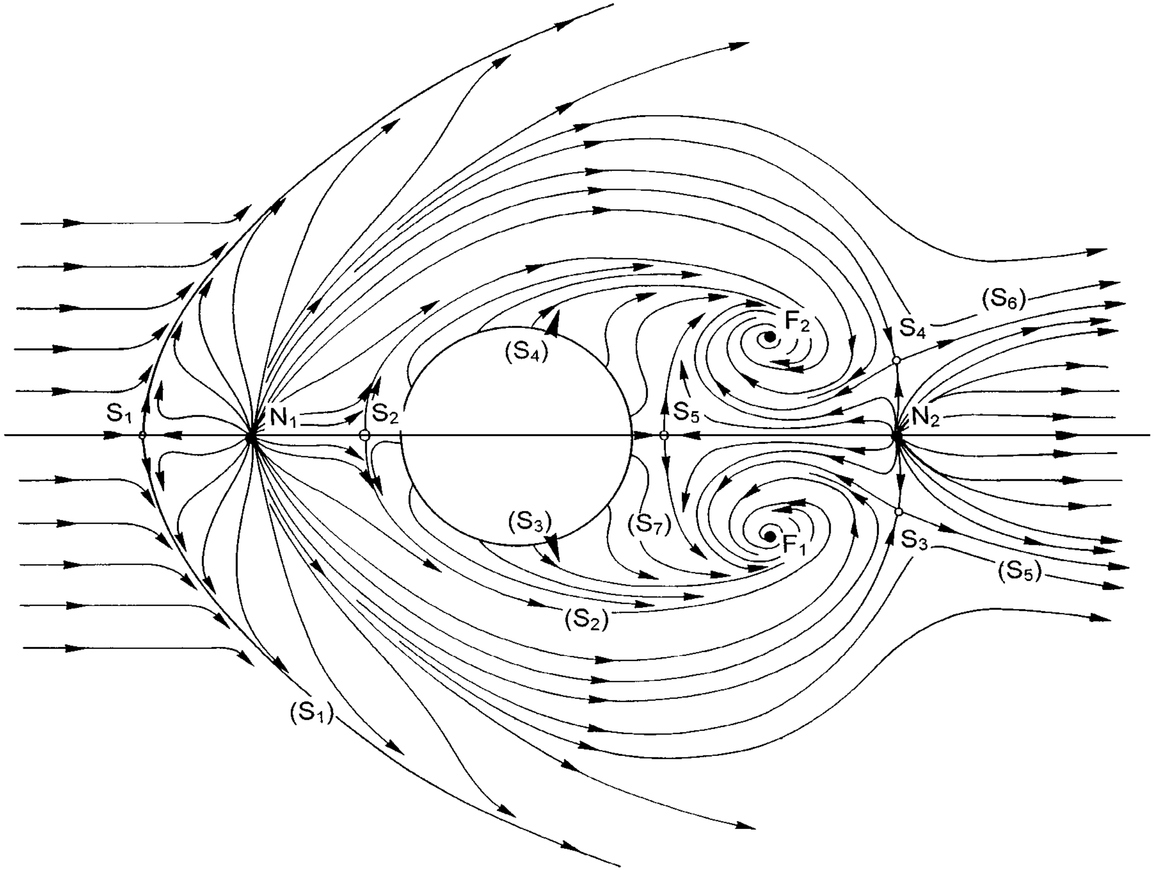


Figure 24 Separation behind a cylindrical obstacle. Topology of the skin-friction-line pattern.

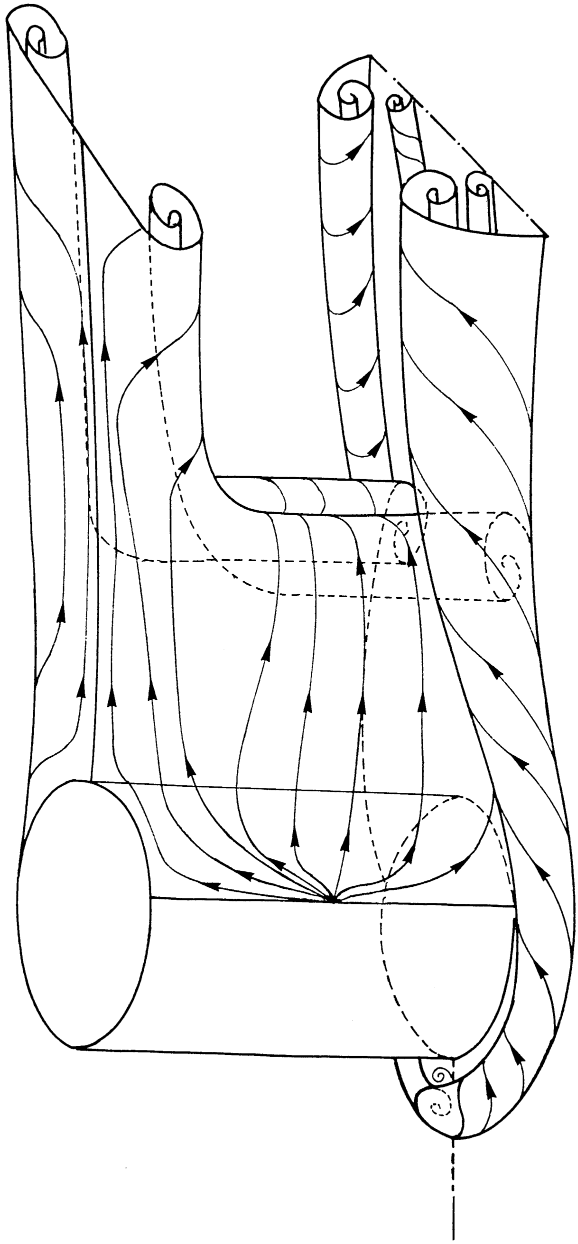


Figure 25 Schematic representation of the vortex system induced by the obstacle.

BASE FLOWS

Base flows have always attracted much attention. The phenomenon takes place in a base region, which is of great importance to the global performance of the vehicle (drag, stability). For missile and space launchers, thermal environment is a major concern because of hot gas coming from the propulsive jets. Even for axisymmetric afterbodies, turbulent base flow prediction remains a challenging problem; none of the existing turbulence models are satisfactory for such flows.

The simplest situation is the axisymmetric afterbody without jet. In this case, the flow visualization shown in Figure 26*a* reveals the existence of two large recirculation bubbles, which are the trace in the visualization plane of a toroidal vortex. As represented in the topological interpretation of Figure 26*b*, if the flow is perfectly axisymmetric, the traces of this vortex are two centers that are special foci (see above). In this case, the streamlines are closed curves rotating around C_1 and C_2; the recirculating flow is separated from the outer stream by the separators (S_1) and (S_2) that go through the saddle point S_4. A perfectly axisymmetric organization being unlikely, it is probable that the true flow topology is that represented in Figure 26*c*, where the centers C_1 and C_2 are replaced by two foci F_1 and F_2. If the flow is very close to axisymmetry, the difference is not detected.

The presence of a central jet changes the flow organization as shown by the photography in Figure 27*a* and the sketch in Figure 27*b*. Now, the toroidal vortex is bounded by a separator (S_1) ending on the base at the half saddle point S_2 (and the symmetric point). The outer flow streamlines flow first around the bubble, then bend rapidly in the downstream direction under the entrainment effect of the jet. A separator must separate the jet flow from the outer stream. For multi-jet arrangements, or afterbodies at incidence, the topology of base flows may consist of complex patterns, which are not discussed here (Délery 1992, 1995).

Many other situations of largely separated flows, which have been studied by Werlé, can exist in reality. We conclude this brief review of his contribution to the physics of fluid motion with the visualization of the unsteady flow behind a plate showing the regular vortex shedding at low Reynolds number, which is one of the most fascinating phenomena of fluid mechanics. The photograph in Figure 28*a* shows the emission lines obtained by injecting dye in the trailing edge of the plate. The sketch in Figure 28*b* shows a topological interpretation of the instantaneous streamline pattern. This picture is representative of the frozen velocity field that would be obtained by the use of the particle image velocimetry (PIV) technique. This illustrates the precautions that must be taken when analyzing unsteady fields, emission lines, trajectories, and streamlines that result in completely different representations of the flow field.

CONCLUSION

As was pointed out by Tobak, the Critical Point Theory and the accompanying concepts of separation, or attachment, lines and of separation, or attachment, surfaces

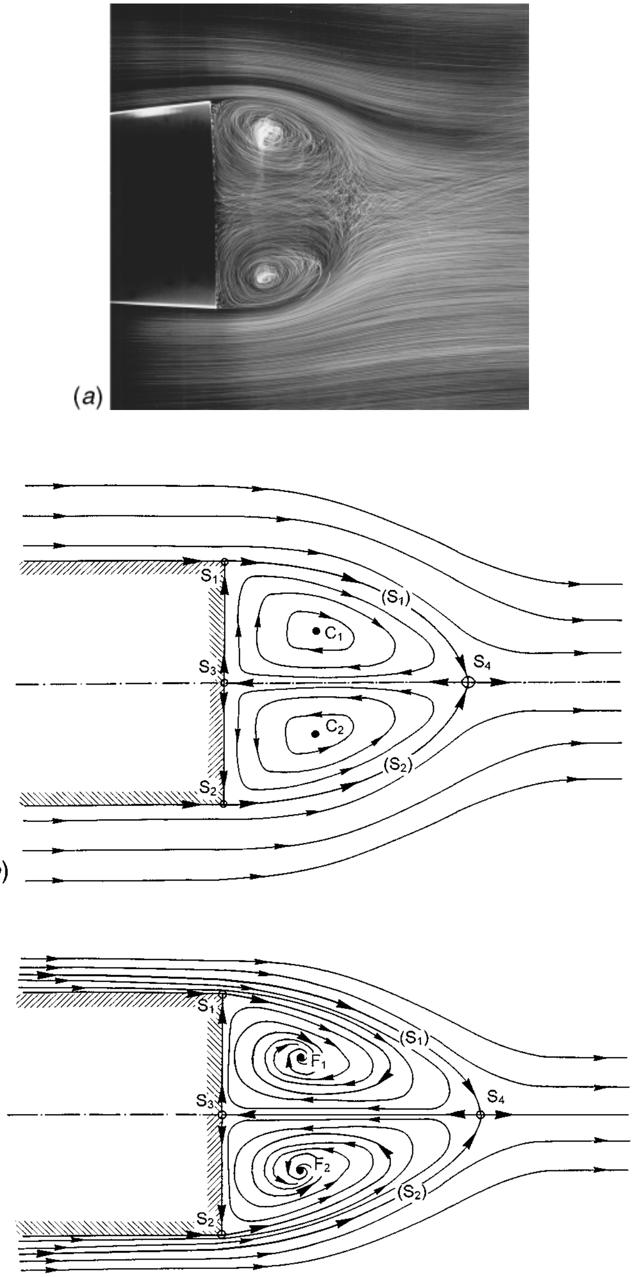


Figure 26 Flow downstream of an axisymmetric afterbody: (*a*) air-bubbles visualization, (*b*) topological interpretation for an axisymmetric field, (*c*) topological interpretation for a slight axisymmetry defect.

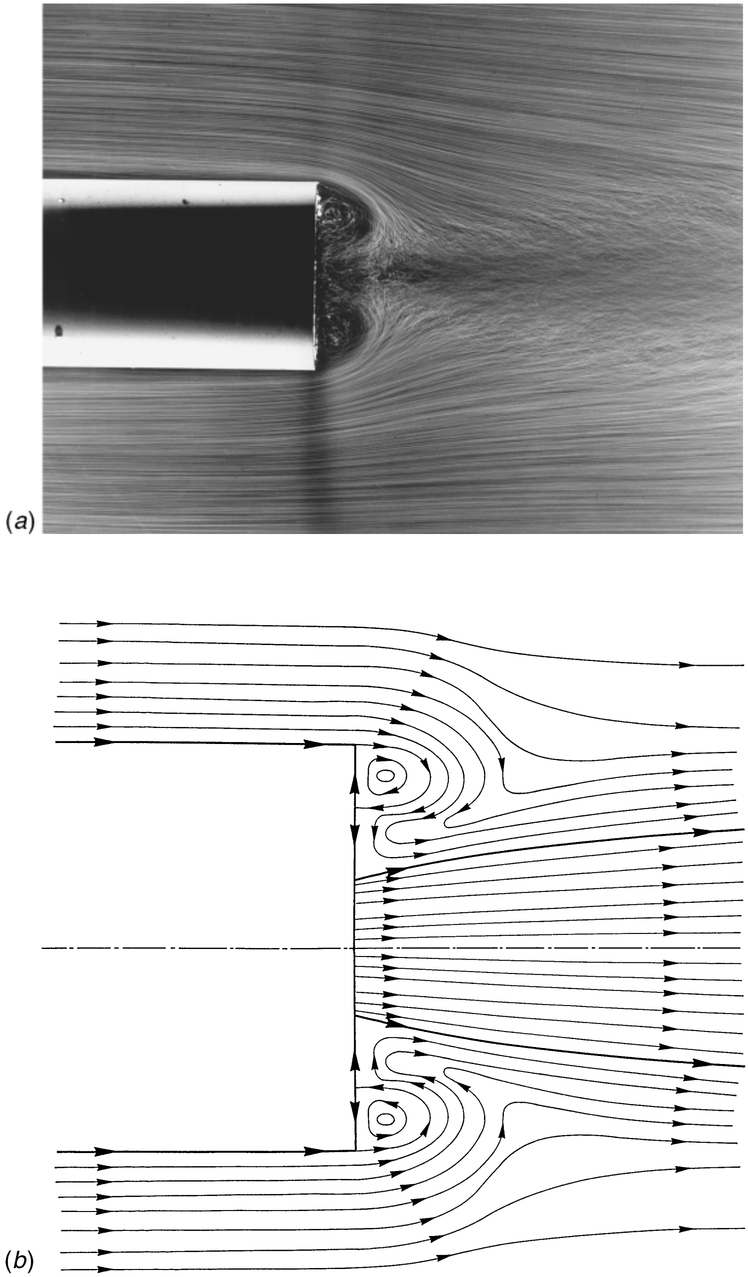


Figure 27 Flow downstream of a cylindrical afterbody with central jet: (*a*) air-bubbles visualization, (*b*) topological interpretation for an axisymmetric field.

constitute a grammar permitting a rational definition of separation and a consistent description of three-dimensional flows. It is to the credit of Legendre, who was both a mathematician and an engineer, to have understood that this theory constitutes the only correct tool to construct the topology of separated three-dimensional flows. Legendre's approach was both rigorous and pragmatic, in the sense that he always remained close to reality, thinking that the operational power of a theory is more important than its mathematical beauty. In fact, the theory of three-dimensional separation represents a small fraction of Legendre's contribution to fluid flow engineering. He followed in the tradition of the great hydraulicians and naval architects of the eighteenth century who founded fluid mechanics by a judicious mixing of empiricism and theoretical rigor.

Legendre's association with Werlé is a remarkable example of the interrelationship that must exist between the theoretician and the experimentalist. An experiment is never innocent, in the sense that it is not a naive contemplation of reality. An experiment is always inspired by a theoretical idea that it either confirms or contradicts; the major scientific breakthroughs result from the contradictions. During his long career at Onera, Werlé realized hundreds of experiments and took thousands of photographs, which contributed to the assessment of sounder ideas about three-dimensional flows, inspired and guided theoretical research, and revealed the beauty of fluid motions to generations of students. His visualizations are an unrivaled treasure and are still far from having been completely exploited.

This review is a modest illustration of the major contribution two great scientists have made to our understanding of three-dimensional flow fields. The results of their work must be seriously considered and studied at a time when the progress in computational fluid dynamics allows taking into account the three-dimensional nature of moving objects. It should not be forgotten that computational capability is not a substitute for physical understanding.

ACKNOWLEDGMENTS

The author is greatly indebted to Marc Gallon, who has been for 24 years the closest collaborator of Werlé and who participated in almost all of the experiments executed in the water tunnel facilities. His contribution should not be forgotten. The author also acknowledges the outstanding help of Claude Quélin and Serge Petit in the preparation of this article.

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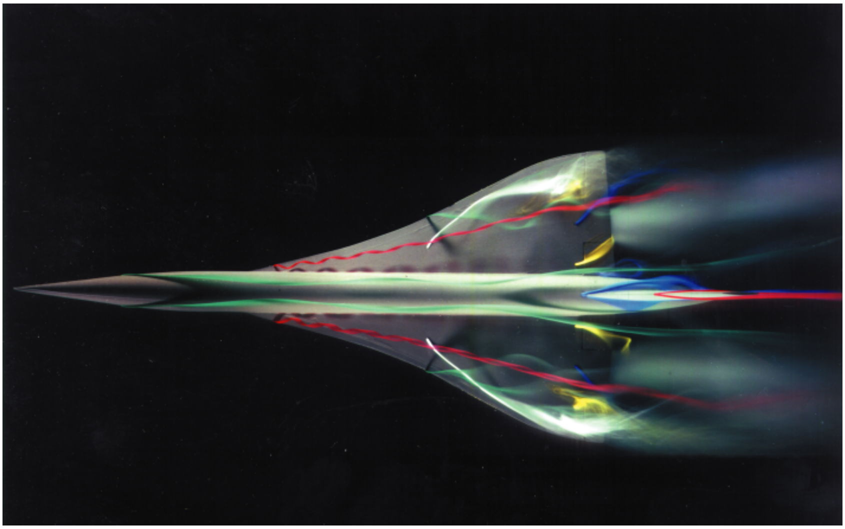
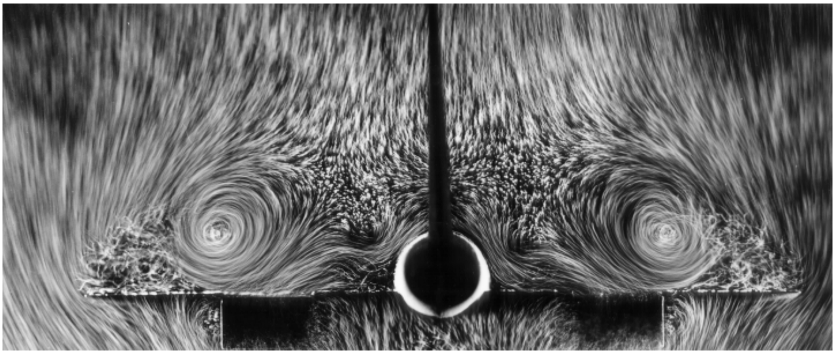


(*a*)



(*b*)

Figure 1 Flow past a Concorde model: (*a*) dye filaments visualization, (*b*) cross flow visualization by air bubbles.

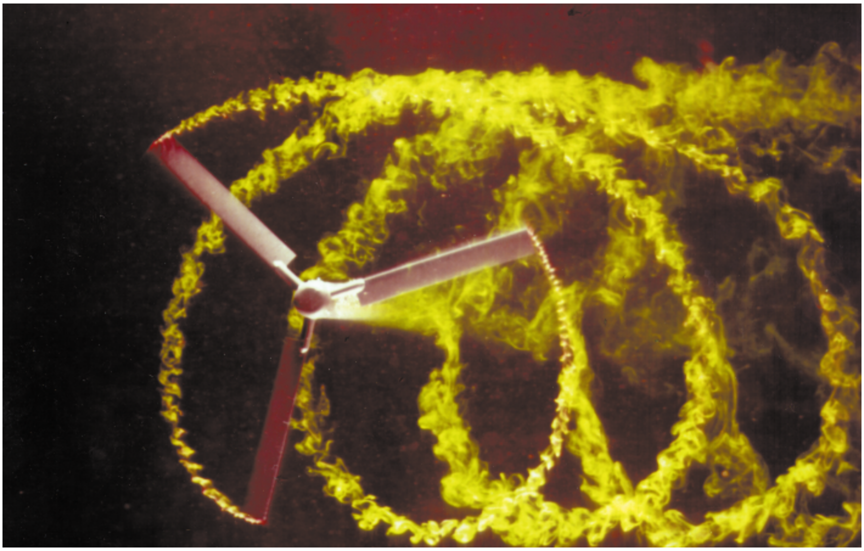
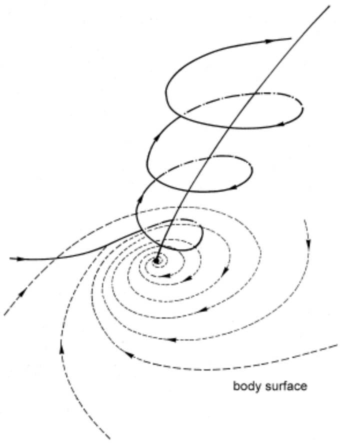
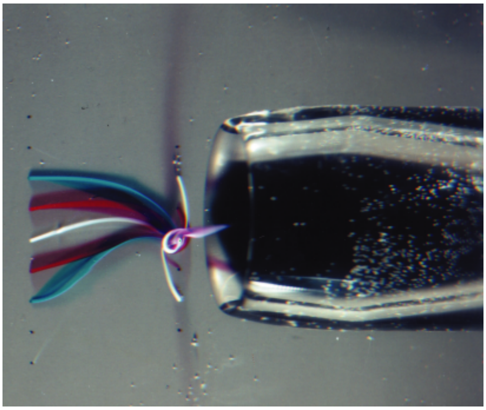


Figure 2 Tip vortices emanating from the blades of a helicopter rotor.

Figure 3 Flow in the base region of the Ariane 5 space launcher.

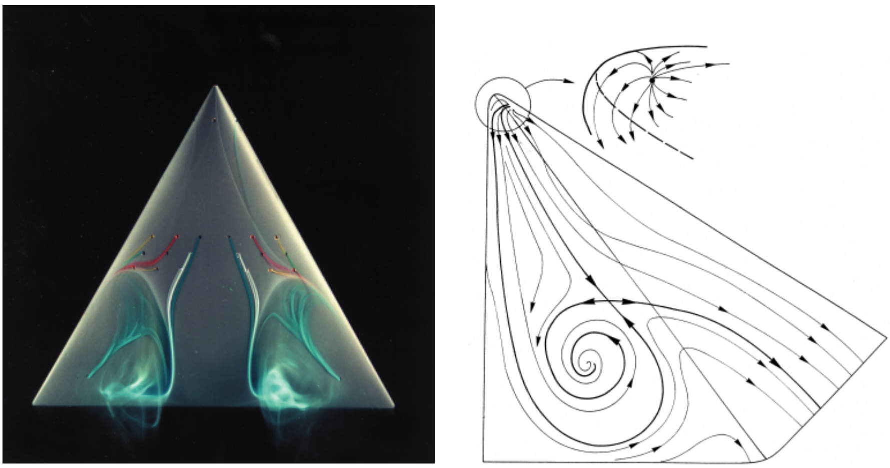


(*a*)



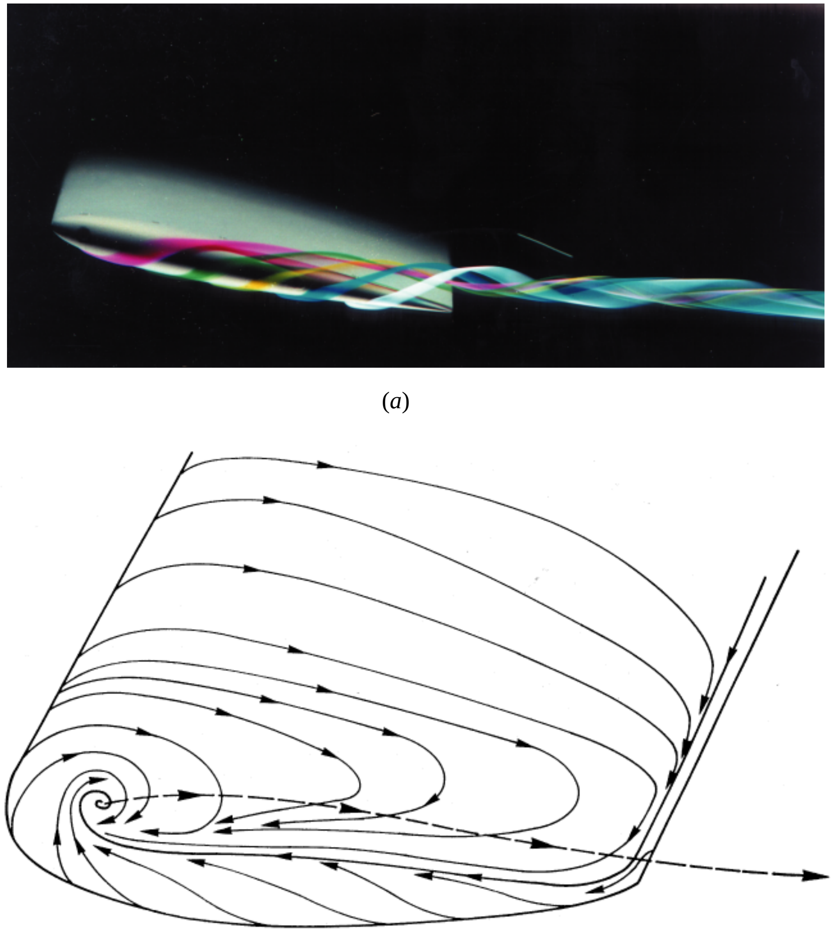
(*b*)

Figure 8 Three-dimensional focus: (*a*) sketch of the flow field topology, (*b*) tornado like vortex swallowed by an air intake.



(*a*) (*b*)

Figure 11 Flow over a thick delta wing at low Reynolds number: (*a*) dye filaments visualization, (*b*) topological interpretation.



(*a*)

(*b*)

Figure 15 Tip vortices for a rectangular wing: (*a*) dye filaments visualization, (*b*) topological interpretation.

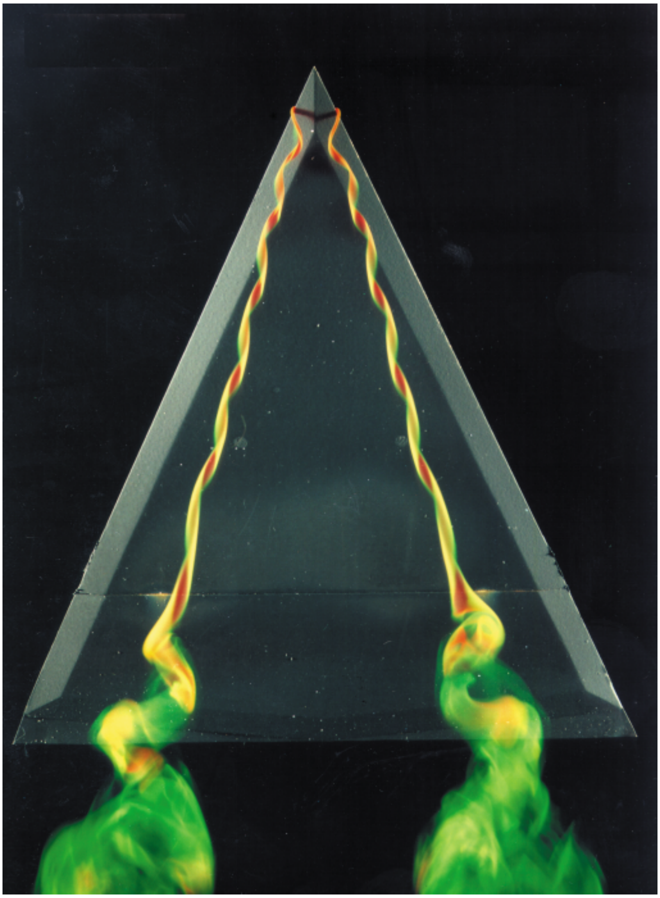
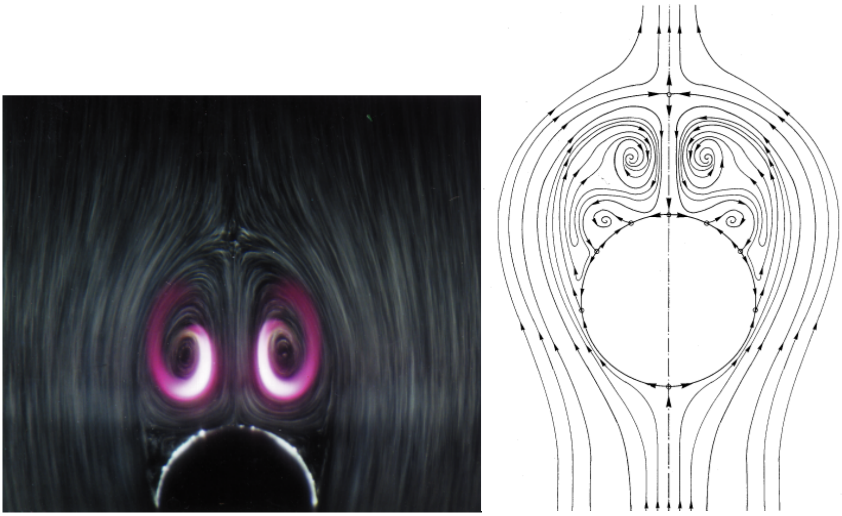
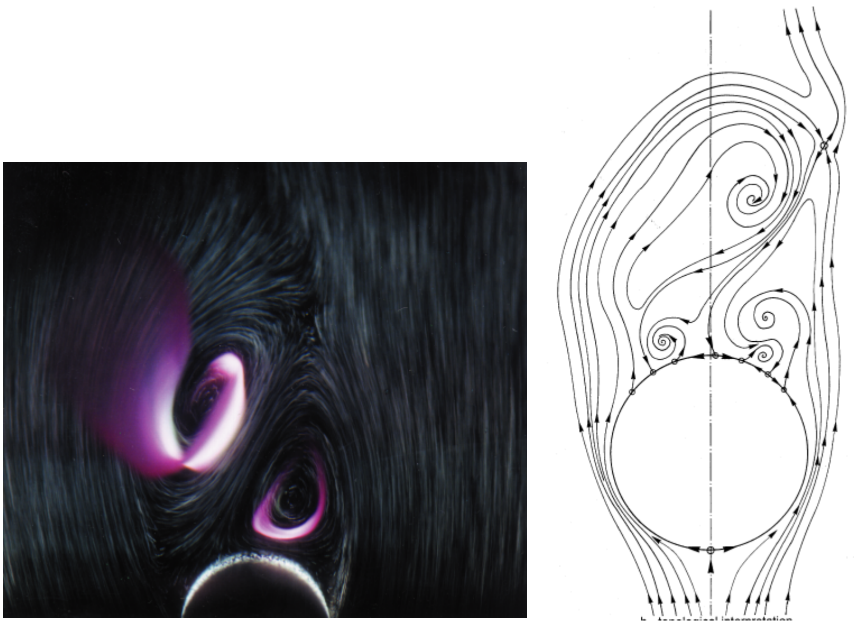


Figure 16 Vortex breakdown over a 65°-sweep angle delta wing.



(*a*) (*b*)

Figure 18 Vortices over an elongated body at incidence. Cross flow organization: (*a*) air-bubbles visualization, (*b*) topological interpretation.



(*a*) (*b*)

Figure 19 Asymmetrical vortices over an elongated body at high incidence. Cross flow organization: (*a*) air-bubbles visualization, (*b*) topological interpretation.

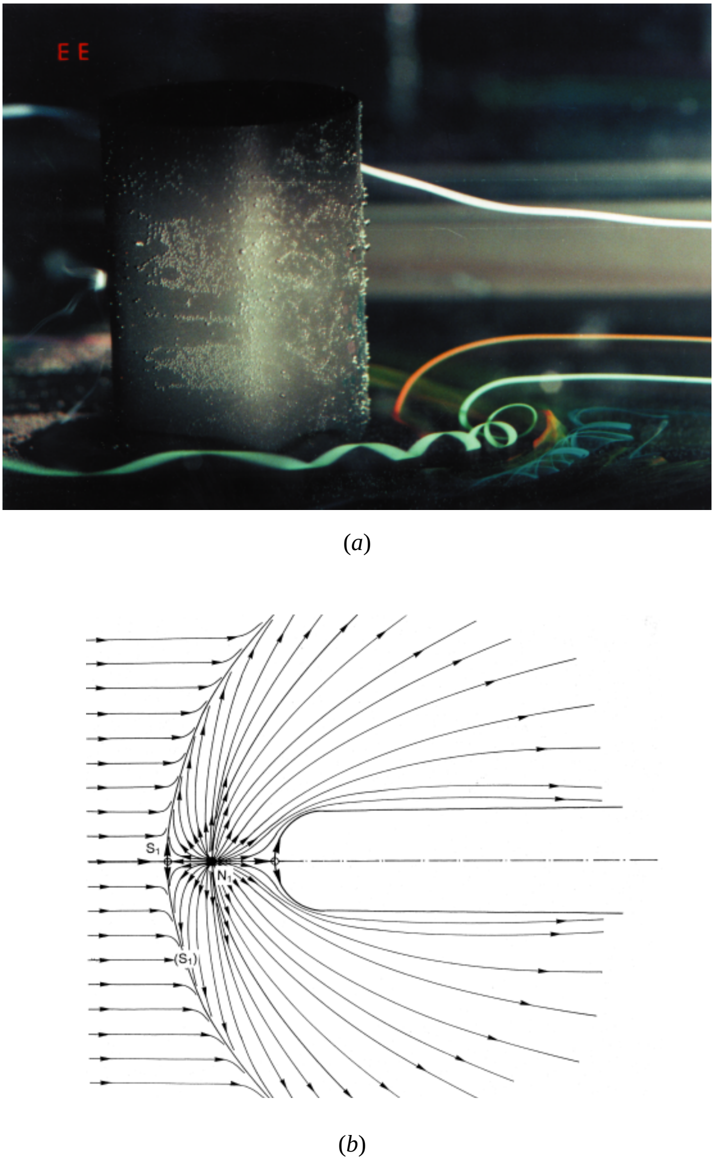


Figure 21 Separation in front of a cylindrical obstacle: (a) dye filaments visualization, (b) topological interpretation.

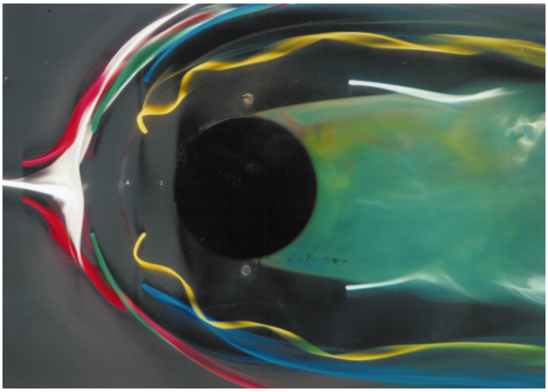
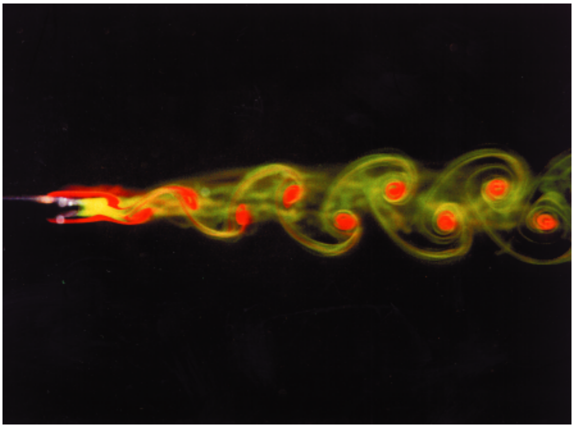
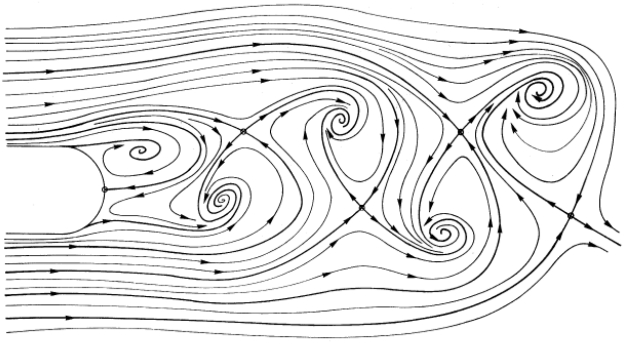


Figure 23 Visualization of separation behind a cylindrical obstacle.



(*a*)



(*b*)

Figure 28 Vortex shedding behind a plate: (*a*) dye filaments visualization, (*b*) topological interpretation of the instantaneous velocity field.

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