SOFTWARE FOR

INTRODUCTION TO SYMMETRY ANALYSIS

This readme file describes several recent changes to the Mathematica package IntroToSymmetry.nb as of May 2010. The changes are designed to improve speed, reduce the memory required, and to correct an error in the formation of the determining equations for a certain class of differential equations that involves derivatives that appear in a denominator.

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Version 1.0 of the package was originally developed in Mathematica version 2.2.2 in Jan-Feb 1998. Several improvements were made in September of 1998. In August 2000 additional features were added and the package was adapted to Mathematica 4.0. In May of 2001 an improved user interface was added to the package. This was Version 2.0 of the package and was included along with a number of sample notebooks on a CD with my book "Introduction to Symmetry Analysis" published in 2002 as part of the series Cambridge Texts in Applied Mathematics.

Updated versions of the package and sample notebooks are available on my website at

www.stanford.edu/~cantwell/.

Since it was published the package has worked quite well except for cases involving Lie-Backlund transformations which, when forming the determining equations, often ran very slowly even for a seemingly modest calculation. The results were correct but the time needed to form the determining equations seemed overly long. I finally got around to addressing this problem in December 2006. The culprit was a potentially large number of calls to the Mathematica function **`Position`** near the end of the call to my function **`FindDeterminingEquations`**. At this point the program is sorting through an array containing the left-hand-sides of the determining equations plus a number of duplicates. The function **`Position`** is used to find the indices of the duplicates and in a couple of steps remove the duplicates. The problem is that the function **`Position`** is called a number of times equal to the number of items in the table containing the determining equations plus duplicates. This can be a very large number equal to a thousand or more. In

the revised package the duplicates are removed in a single step using the Mathematica set function **Union** one line ahead of the step that calls **Position** and that step is now commented out with beginning (* and ending*) along with a number of following steps that manipulated the list of indices that resulted from the multiple calls to **Position**. **FindDeterminingEquations** runs much faster as a result. These changes were made final on December 22, 2006.

Another issue that I encountered in the process of making this correction has to do with the application of extended rules in the case of Lie-Backlund transformations. To begin with it is important to be aware that the package is designed to insert these rules into the array **InvarianceConditionNoRules** without inserting them into the infinitesimals themselves (ie. into the function positions where the independent variables appear in the infinitesimals). So there is no possibility of contaminating the functional dependence of the infinitesimals. Therefore there is no need to use a differentiated form of the input equation when searching for Lie-Backlund groups. However it is important to set up the rules so that the highest derivative in the equation is on the left hand side. Otherwise when the rule is substituted into the invariance condition one of the independent derivatives might be replaced by a dependent derivative typically leading to too many determining equations and fewer groups. For example for the heat equation with **r=1** the required rule is

```
rulesarray={"D[u[x,t],x,x]->D[u[x,t],t]"}
```

not

```
rulesarray={"D[u[x,t],t]->D[u[x,t],x,x]"}.
```

If the latter is substituted into the invariance condition then places where **D[u[x,t],t]** (an independent variable) appears will be replaced by **D[u[x,t],x,x]** (a dependent variable). As a result the determining equations may be too restrictive, there may be too many of them and fewer symmetries may be found.

In addition I have included a number of diagnostics that can be printed during the run to show the current status of the function **FindDeterminingEquations**. The printed line usually gives a byte count and time so the user can monitor the run if desired. At the moment most of these are commented out. The user may wish to un-comment these lines so that the status of **FindDeterminingEquations** can be tracked in some detail. This is particularly useful if the equations being analyzed are long and complicated or if one is searching for Lie-Backlund equations of order greater than one where the invariance condition can be quite long and the number of variables that the infinitesimals depend on is large. These diagnostic lines are in red colored font so the user can easily find them in the package and un-comment them if so desired. These changes constituted version 3.0 of the package dated December 24, 2006.

On December 5, 2009 I got an e-mail from a user pointing out that the package fails to find the symmetries of the equation

Ut+k*x^2*Uxx*Ux^2/(Ux-c*x*Uxx)^2=0.

After examining this case it quickly became clear that the package was not finding the proper set of determining equations when the rule(s) applied to the invariance condition contained a denominator where the expression in the denominator contained more than one term with a derivative. When the rule

Ut->-k*x^2*Uxx*Ux^2/(Ux-c*x*Uxx)^2

is inserted into the invariance condition there will of course be quite a few terms that will be in the form of fractions with various powers of **(Ux-c*x*Uxx)^2** in the denominator. In order to generate the proper invariance condition is is necessary to put the entire invariance condition with rules applied over a common denominator and then remove the denominator. This change was incorporated in the revised package in mid December 2009.

On December 29, 2009 I made several changes to speed up the function **SolveDeterminingEquations** and reduce the amount of memory used. This function searches for a multivariate polynomial solution to the determining equations. The order of the trial polynomials is selected by the user by assigning an integer value to the variable **order** in the package. The variables on which the infinitesimals are assumed to depend include all independent and dependent variables and, in the case of Lie-Backlund transformations, all possible derivatives of the dependent variables up to order **r** specified by the user. The list of variables is contained in the table **zvariables**.

Prior to the change, the procedure for creating the list of powers of **zvariables** that appear in the trial polynomials does a search over all powers up to **z1^order*z2^order*z3^order*...*zn^order** where **n** is the number of variables in the list of **zvariables**, ie, a search that includes powers up to **n*order**. But only powers up to **order** are actually needed and items in the list that involve powers exceeding **order** are set to **Null** and then deleted. For calculations of point symmetries where the number of variables is not too large this does not present much of a problem but for Lie-Backlund symmetries where the number of variables can easily be considerably greater than ten, and where higher order polynomials are required, this intermediate step unnecessarily uses vastly more memory and time than is required. The new procedure sorts through powers of z variables only up to the required order and deletes any redundancies. This saves a considerable amount of time and memory for Lie-Backlund calculations. The package now includes a number of diagnostic lines in red font color including time used and memory used. These can be used to track the function **SolveDeterminingEquations** in considerable detail if a user wishes to do so. These

diagnostics are commented out at present using (* …*). With the December 2009 changes the current package is designated version 4.1 dated December 29, 2009.

In May 2010 I encountered a rather unusual problem when I was using the package to work out the Maxwell equations with currents and charges. When I used the dependent variable names **j1, j2, j3** to denote the currents the package at first seemed to run OK but with some odd replacements. On the first run the array **ztableofrules** had the string **j1** replaced by the string **1**. When run a second time the equation in generic variables had **1** where it should have had **z8** and ran incorrectly. The problem was that in the function **GenerateInvarianceConditionRulesApplied** indices **j1, j2, j3,…** are used, and **j1** ends up with the value **1** which later gets inserted into **ztableofrules** . When the package is run a second time **j1** is still **1** and gets inserted into the input equation when it is expressed in generic variables. To prevent this the command **Clear["j*"]** is used at the end of **GenerateInvarianceConditionRulesApplied** and at the end of **FindDeterminingEquations**.

Finally in May 2010 I added the commands **Unprotect["`*"];** and **ClearAll["`*"];** at the beginning of the package so it can be loaded more than once during a given session.

With the May 2010 changes, the package is designated version 4.2 dated May 20, 2010.