Benz (2005): Utility and relevance of answers

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Overview

- This handout is a guide to Benz (2005). It provides a bit more empirical background, gathers together the definitions, and uses his examples as exercises for us to do in class.
- This paper comes from the collection Benz et al. 2005, which includes an overview of decision-theoretic and game-theoretic approaches to pragmatics, along with a number of papers addressing topics related to implicature.
- Benz stated goal is to replace the maxim of relevance with the general principle that speakers seek to give answers that maximize shared expected payoffs in context. (One could also think of this as a formalization of relevance, but Benz prefers to call it a principle of utility.)
- The central insight of the model is a game-theoretic one: if the interrogator I poses a question to the expert E, then E should seek to avoid providing information that will mislead I by persuading I to choose a sub-optimal action.
- Benz also provides formal results relating his approach to other formalizations of relevance in this broadly decision-theoretic vein.

1 Answers, resolvedness, and felicity

The general intuition we are working towards is that whether an answer is felicitous and resolving is dependent upon the overarching goals of the discourse participants (Ginzburg 1995a,b, 1996a; Roberts 1996; van Rooy 2003; Benz 2005).

(1) Do you know what time it is?

a. *Context*: Speaker is setting her watch.

b. *Context*: Speaker is checking with addressee to make sure she has what she needs to execute their plan.

(2) Is Lisa in room 10?

a. Clerk A: She's in room 20.

b. Clerk B: No.

(3) Where are you from? (Ginzburg 1995a,b, 1996b)

a. Connecticut.

b. The U. S.

c. Stanford.

d. Planet earth.

(4) Where can we buy supplies? (Beck and Rullmann 1999)

a. *Context*: We're writing a comprehensive guide to the area.

b. *Context*: We're low on food and water.

(5) Who has a light?

a. *Context*: Speaker ensuring that no group member will be stopped by airport security.

b. *Context*: Speaker needs to light her cigar.

(6) What cards do you have? (van Rooy 2003)

a. *Context*: Speaker dealt the cards and noticed that some were missing.

b. *Context*: Speaker folds. He wants to know what beat him.

(7) Are the windows open? (Malamud 2006)

a. *Context*: We're leaving town and want the house secure.

b. *Context*: The sills will be painted.

2 Interrogative denotations (questions) as partition

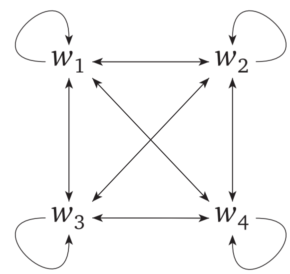
The partition semantics for interrogatives (Groenendijk and Stokhof 1984, 1989) says that questions divide up logical space exhaustively into mutually exclusive cells representing completely resolving answers. In other words, for a given set of possible worlds Ω, an interrogative $?\varphi$ induces an equivalence relation on $\Omega \times \Omega$.

(8) $[\![$*The station has Italian papers*$]\!] = \{w_1, w_2\}$

(9) $[\![$*The palace has Italian papers*$]\!] = \{w_2, w_3\}$

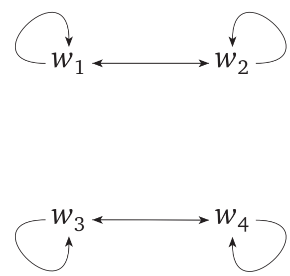
(10) $\Omega = \{w_1, w_2, w_3, w_4\}$

$$\left\{\begin{array}{cccc} \langle w_1,w_1\rangle & \langle w_1,w_2\rangle & \langle w_1,w_3\rangle & \langle w_1,w_4\rangle \\ \langle w_2,w_1\rangle & \langle w_2,w_2\rangle & \langle w_2,w_3\rangle & \langle w_2,w_4\rangle \\ \langle w_3,w_1\rangle & \langle w_3,w_2\rangle & \langle w_3,w_3\rangle & \langle w_3,w_4\rangle \\ \langle w_4,w_1\rangle & \langle w_4,w_2\rangle & \langle w_4,w_3\rangle & \langle w_4,w_4\rangle \end{array}\right\}$$



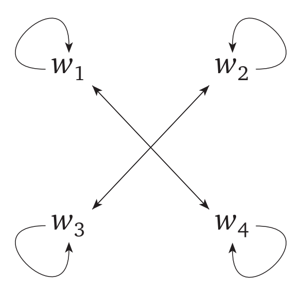
(11) $[\![$*Does the station have Italian papers?*$]\!]$

$$\left\{\begin{array}{cccc} \langle w_1,w_1\rangle & \langle w_1,w_2\rangle & & \\ \langle w_2,w_1\rangle & \langle w_2,w_2\rangle & & \\ & & \langle w_3,w_3\rangle & \langle w_3,w_4\rangle \\ & & \langle w_4,w_3\rangle & \langle w_4,w_4\rangle \end{array}\right\}$$



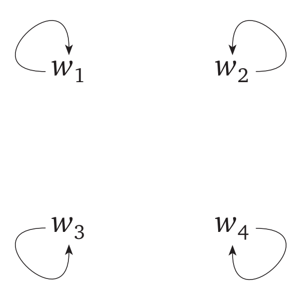
(12) $[\![$*Does the palace have Italian papers?*$]\!]$

$$\left\{\begin{array}{cccc} \langle w_1,w_1\rangle & & & \langle w_1,w_4\rangle \\ & \langle w_2,w_2\rangle & \langle w_2,w_3\rangle & \\ & \langle w_3,w_2\rangle & \langle w_3,w_3\rangle & \\ \langle w_4,w_1\rangle & & & \langle w_4,w_4\rangle \end{array}\right\}$$



(13) $[\![$*Where can I buy an Italian paper?*$]\!]$

$$\left\{\begin{array}{cccc} \langle w_1,w_1\rangle & & & \\ & \langle w_2,w_2\rangle & & \\ & & \langle w_3,w_3\rangle & \\ & & & \langle w_4,w_4\rangle \end{array}\right\}$$



3 Decision problems

Definition 1 (Countable probability space)**.** The pair (Ω, P) is a countable probability space iff Ω is a countable set of worlds and $P : \wp(\Omega) \mapsto \mathbb{R}$ such that

i. for all $w \in \Omega$, $P(w) \geqslant 0$

ii. $\left(\sum_{w \in \Omega} P(w)\right) = 1$

iii. for all $X \subseteq \Omega$, $P(X) = \sum_{w \in X} P(w)$

Definition 2 (Decision problems)**.** A decision problem is a triple $\langle(\Omega, P), A, u\rangle$ where

i. (Ω, P) is a countable probability space

ii. A is finite, non-empty set of actions

iii. u is a utility function in $(\Omega \times A) \mapsto \mathbb{R}$

Definition 3 (Expected utility of an action)**.** The expected utility of an action a relative to a decision problem $\mathbf{D} = \langle(\Omega, P), A, u\rangle$ is

$$EU_{\mathbf{D}}(a) \stackrel{def}{=} \sum_{w \in \Omega} P(w) * u(w, a)$$

Definition 4 (Expected utility of an action given X)**.** Let $\mathbf{D} = \langle(\Omega, P), A, u\rangle$ be a decision problem. The expected utility of action $a \in A$ given $X \subseteq \Omega$ is

$$EU_{\mathbf{D}}(a, X) \stackrel{def}{=} \sum_{w \in \Omega} P(w \mid X) * u(w, a)$$

(14) Example decision problem (assume $[\![\textit{It's raining}]\!] = \{w_1\}$)

a.

	P	carry umbrella	not carry umbrella
w_1	0.1	1.0	−1.0
w_2	0.9	0.5	2.0

b. $EU(\text{carry umbrella}) = (0.1 * 1.0) + (0.9 * 0.5) = 0.55$

c. $EU(\text{not carry umbrella}) =$

d. $EU(\text{carry umbrella} \mid \{w_1\}) =$

e. $EU(\text{carry umbrella} \mid \{w_2\}) =$

4 Support problems

Definition 5 (Support problems)**.** A support problem is a six-tuple $\langle \Omega, P_E, P_I, A, u, B\rangle$ such that

i. (Ω, P_I) and (Ω, P_E) are countable probability spaces

ii. $\langle(\Omega, P_I), A, u\rangle$ is a decision problem

iii. $B : \wp(\Omega) \mapsto A$ is such that $B(X) \in \{a \in A \mid \forall a' \in A,\ EU^I_{\mathbf{S}}(a, X) \geqslant EU^I_{\mathbf{S}}(a', X)\}$

Definition 6 (Well-behaved support problems)**.** A support problem $\mathbf{S} = \langle \Omega, P_E, P_I, A, u, B\rangle$ is well behaved iff

i. for all $X \subseteq \Omega : P_I(X) = 1 \Rightarrow P_E = 1$
ii. for $x \in \{I, E\}$ and all $a \in A$, $\left(\sum_{w\in\Omega} P_x(w) * u(w, a)\right) < \infty$

Definition 7 (I's action)**.** Given a support problem $\mathbf{S} = \langle \Omega, P_E, P_I, A, u, B\rangle$, if E says S, then I chooses action $B(\llbracket S \rrbracket)$.

Definition 8 (Admissible answers; 'quality')**.** Let $\mathbf{S}$ be a support problem $\langle \Omega, P_E, P_I, A, u, B\rangle$. The set of admissible answers is

$$Adm_{\mathbf{S}} \stackrel{def}{=} \{X \subseteq \Omega \mid P_E(A) = 1\}$$

Definition 9 (Expected utility of answers)**.** Let $\mathbf{S}$ be a support problem $\langle \Omega, P_E, P_I, A, u, B\rangle$. The expected utility of an answer is defined as follows:

$$EU^E_{\mathbf{S}}(X) \stackrel{def}{=} \sum_{w\in\Omega} P_E(w) * u(w, B(X))$$

Definition 10 (Optimal answers)**.** Let $\mathbf{S}$ be a support problem $\langle \Omega, P_E, P_I, A, u, B\rangle$. The optimal answers are defined to be the ones that maximize joint expected utility by taking into account how I will respond to new information provided by E:

$$Op_{\mathbf{S}} = \{X \in Adm_{\mathbf{S}} \mid \forall Y \in Adm_{\mathbf{S}},\ EU^E_{\mathbf{S}}(Y) \leqslant EU^E_{\mathbf{S}}(X)\}$$

The system in brief

- The interrogator I poses a question.
- The expert E replies with an optimal answer. This means that she is confined to answers that
  - she knows to be true (admissible; def. 8); and
  - will not lead I to pick a suboptimal action (note the way def. 10 uses the B function)
- Thus, E will appear to have done some planning, to avoid information that will lead I astray. This is very much like Joshi's (1982) view of the maxim of quality.

4.1 Simple example

(15) $[\![\textit{It's raining}]\!] = \{w_1\}$; assume I asked $[\![\textit{Is it raining?}]\!] = \{\{w_1\},\{w_2\}\}$

a.

	P_I	P_E	carry umbrella	not carry umbrella
w_1	0.1	1.0	1.0	−1.0
w_2	0.9	0.0	0.5	2.0

b. Calculations in terms of the support problem

i. $B(\{w_1\}) =$

ii. $B(\{w_2\}) =$

iii. $B(\{w_1, w_2\}) =$

iv. $Adm =$

v. $EU^E(\{w_1\}) =$

vi. $EU^E(\{w_1, w_2\}) =$

vii. $Op =$

4.2 Extended example

Ω	=	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
$[\![$*I am clueless!*$]\!]$	=	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
$[\![$*The palace has foreign papers*$]\!]$	=	w_1	w_2	w_3	w_4	w_5	w_6		
$[\![$*The palace has foreign non-Italian papers*$]\!]$	=					w_5	w_6		
$[\![$*The palace has Italian papers*$]\!]$	=	w_1	w_2	w_3	w_4				
$[\![$*The palace has British papers*$]\!]$	=			w_3	w_4	w_5	w_6		
$[\![$*The station has Italian papers*$]\!]$	=	w_1		w_3		w_5		w_7	

(16) Initial support problem:

	P_I	P_E	go to palace	go to station
w_1	0.125	0.125	1	1
w_2	0.125	0.125	1	0
w_3	0.125	0.125	1	1
w_4	0.125	0.125	1	0
w_5	0.125	0.125	0	1
w_6	0.125	0.125	0	0
w_7	0.125	0.125	0	1
w_8	0.125	0.125	0	0

a. Optimal answers:

(17) Support problem after E has learned ⟦*The palace has Italian papers*⟧:

	P_I	$P_E(\cdot \mid \{w_1, w_2, w_3, w_4\})$	go to palace	go to station
w_1	0.125	0.25	1	1
w_2	0.125	0.25	1	0
w_3	0.125	0.25	1	1
w_4	0.125	0.25	1	0
w_5	0.125	0.00	0	1
w_6	0.125	0.00	0	0
w_7	0.125	0.00	0	1
w_8	0.125	0.00	0	0

a. What is E's expected utility for ⟦*The palace has Italian papers*⟧?

b. What is E's expected utility for ⟦*The palace has foreign papers*⟧?

c. What is E's expected utility for ⟦*The station has Italian papers*⟧?

d. Is ⟦*The station has Italian papers*⟧ an optimal answer?

e. Is ⟦*The palace has Italian papers*⟧ an optimal answer?

(18) Support problem after E has learned ⟦*The palace has foreign papers*⟧:

	P_I	$P_E(\cdot \mid \{w_1, w_2, w_3, w_4, w_5, w_6\})$	go to palace	go to station
w_1	0.125	0.167	1	1
w_2	0.125	0.167	1	0
w_3	0.125	0.167	1	1
w_4	0.125	0.167	1	0
w_5	0.125	0.167	0	1
w_6	0.125	0.167	0	0
w_7	0.125	0.0	0	1
w_8	0.125	0.0	0	0

a. What is E's expected utility for ⟦*The palace has foreign papers*⟧?

b. Show that ⟦*The palace has Italian papers*⟧ is not an optimal answer.

c. Show that ⟦*The palace has British papers*⟧ is not an optimal answer.

d. Is ⟦*I am clueless!*⟧ an optimal answer? Why or why not?

(19) Support problem after E has learned ⟦*The palace has foreign non-Italian papers*⟧:

	P_I	$P_E(\cdot \mid \{w_5, w_6\})$	go to palace	go to station
w_1	0.125	0.0	1	1
w_2	0.125	0.0	1	0
w_3	0.125	0.0	1	1
w_4	0.125	0.0	1	0
w_5	0.125	0.5	0	1
w_6	0.125	0.5	0	0
w_7	0.125	0.0	0	1
w_8	0.125	0.0	0	0

a. Show that ⟦*The palace has foreign papers*⟧ is not an optimal answer.

b. What action would ⟦*The palace has foreign papers*⟧ induce I to undertake?

c. What is the most informative optimal answer and what is its expected utility?

(20) P_I(⟦*The palace has Italian papers*⟧) $= 0.5$
P_I(⟦*The station has Italian papers*⟧) $= 0.667$
P_E(⟦*The palace and the station have Italian papers*⟧) $= 1$

	$P_I(\cdot \mid \{w_1, w_3, w_4, w_5, w_7, w_8\})$	$P_E(\cdot \mid \{w_1, w_3\})$	go to palace	go to station
w_1	0.167	0.5	1	1
w_2	0.0	0.0	1	0
w_3	0.167	0.5	1	1
w_4	0.167	0.0	1	0
w_5	0.167	0.0	0	1
w_6	0.0	0.0	0	0
w_7	0.167	0.0	0	1
w_8	0.167	0.0	0	0

a. What is I's current best action?

b. Is it optimal for E to confirm I's existing best action?

c. Is it optimal for E to push I to another action?

d. What is the exhaustive (mention-all) answer, and is it optimal?

5 Hearer inferences

Benz frames his system in terms of production: the answerer E should pick an utterance that is among the optimal answers given the current support problem. However, if I (qua listener) knows that E is an optimal answerer, then he can make inferences about E's knowledge (i.e., about P_E).

5.1 Lack of exhaustivity inferences

Mention-some answers (when perceived as such) do not implicate exhaustivity or limitations on the speaker's knowledge. This follows from Benz's account; as is evident from example (20), even where the speaker knows the exhaustive answer, the mention-some answers remain optimal.

5.2 Epistemic implicatures of partial answers

(21) I: Where can I buy an Italian newspaper?

E: The palace$_{\mathbf{F}}$ has foreign$_{\mathbf{CT}}$ newspapers.

i. $\Rightarrow P_E(\llbracket \textit{The palace has Italian papers} \rrbracket) \geqslant P_E(\llbracket \textit{The station has Italian papers} \rrbracket)$

ii. $\Rightarrow P_E(\llbracket \textit{The palace has Italian papers} \rrbracket) < 1.0$

Inference (21i) follows We already saw that $\llbracket$*The palace has foreign papers*$\rrbracket$ biases in favor of action 'go to palace' in our model. If we suppose that

$$P_E(\llbracket \textit{The palace has Italian papers} \rrbracket) < P_E(\llbracket \textit{The station has Italian papers} \rrbracket)$$

then the optimal action is 'go to station', thereby contradicting the assumption that E is an optimal answerer. (In this situation, $\llbracket$*The palace has foreign newspapers*$\rrbracket$ is a misleading answer in the sense of Benz's definition 5.1.)

Inference (21ii) does not follow Suppose $P_E(\llbracket \textit{The palace has Italian papers} \rrbracket) = 1.0$. Then, by entailment, $P_E(\llbracket \textit{The palace has foreign papers} \rrbracket) = 1.0$. It remains an optimal answer because it biases in favor of the palace. One might think that we can address this by including in the set of optimal answers only the *strongest* propositions, i.e., those without any proper subsets that are also optimal. However, this recapitulates the partition semantics, i.e., it wrongly predicts mention-some readings to be infelicitous. I propose instead that we derive our notion of exhaustivity from the structure of the decision problem:

Definition 11 (Action propositions; van Rooy 2003)**.** Let **S** be a support problem $\langle \Omega, P_E, P_I, A, u, B \rangle$. The proposition for an action a in **S** is

$$\textit{Prop}_{\mathbf{S}}(a) \stackrel{def}{=} \{w \in \Omega \mid \forall a' \in A,\ u(w,a) \geqslant u(w,a')\}$$

Definition 12 (Action-congruent optimal answers)**.** Let **S** be a support problem $\langle \Omega, P_E, P_I, A, u, B \rangle$. The action-congruent optimal answers are those that favor matches with action propositions:

$$\textit{Op}_{\mathbf{S}}^{max} = \{X \in \textit{Op}_{\mathbf{S}} \mid \neg\exists Y \in \textit{Op}_{\mathbf{S}}, a \in A,\ Y \subseteq X \text{ and } Y = \textit{Prop}_{\mathbf{S}}(a)\}$$

6 Other models of relevance

6.1 Relevance as argumentative force

Merin (1997, 1999) develops a theory of relevance that is grounded in argumentative force: the relevance of a proposition D to a hypothesis H is the degree to which D changes the interrogator's beliefs about H:

$$\log\left(\frac{P_I(D|H)}{P_I(D|\neg H)}\right)$$

Where this is positive, D is positively relevant to (argues for) H. Where it is negative, D is negatively relevant (argues against) H. The equation can be used to pick the most relevant answers to a given hypothesis. Benz's Proposition 1 (p. 215-216) shows that strategic use of the hypothesis H leads to a set-up that selects the same optimal answers as his own.

6.2 Relevance as utility

- van Rooy (2003) suggests that we might associate relevance with information that *changes* I's current plan of action (Benz's 4.8). Information that merely confirms I's plan get a value of 0 on this approach. This is unintuitive because often we want to be reassured that our current plan is the right one.

- van Rooy (2003) also suggests that we might associate relevance with information that alters our expected utilities. This could come from a change in I's action or from additional confirmation that it is correct. However, it still places a premium on surprises; where E can either confirm I's plan or change it, he is compelled to change it. This runs afoul of our intuitions about examples like (20), where confirming I's current plan was intuitively helpful.

- The force of Benz's Theorem 5.3 (p. 214) is that it shows that these non-argumentative theories of relevance are bound to select misleading partial answers in some cases, precisely because they are invariant under a large set of changes to P_E.

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