

# Lambda calculi for semantic theories

Chris Potts, Ling 230b: Advanced semantics and pragmatics, Spring 2018

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## 1 Fragment

**Definition 1** (Semantic types). The smallest set *Types* such that

- i.  $e \in \text{Types}$
- ii.  $t \in \text{Types}$
- iii. If  $\sigma, \tau \in \text{Types}$ , then  $\langle \sigma, \tau \rangle \in \text{Types}$

**Definition 2** (Well-formed expressions). The smallest set *WFF* such that ( $\alpha : \sigma$  as ' $\alpha \in \text{WFF}$ , has type  $\sigma$ ')

- i. Every constant is in *WFF*:
  - a.  $\langle e, t \rangle : \{\text{run, walk, student, unicorn}\}$
  - b.  $e : \{\text{sandy, kim, jesse}\}$
  - c.  $\langle e, \langle e, t \rangle \rangle : \{\text{find, lose, eat, love, be}\}$
  - d.  $\langle t, \langle e, t \rangle \rangle : \{\text{believe, deny}\}$
  - e.  $\langle \langle e, t \rangle, \langle e, t \rangle \rangle : \{\text{not, rapidly, allegedly, fail\_to}\}$
  - f.  $\langle t, t \rangle : \{\text{necessarily}\}$
  - g.  $\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle : \{\text{the, every, a, most, no}\}$
- ii. For every type  $\tau$ , we have an infinite stock of variables of type  $\tau$ . Variables are in *WFF*.
- iii. If  $\alpha : \langle \sigma, \tau \rangle$  and  $\beta : \sigma$ , then  $(\alpha \beta) : \tau$
- iv. If  $\alpha : \tau$  and  $\chi$  is an variable of type  $\sigma$ , then  $(\lambda \chi \alpha) : \langle \sigma, \tau \rangle$

**Definition 3** (Denotation domains). Each semantic type has a corresponding denotation domain:

- i. The domain of type  $e$  is  $D_e$ , the set of entities.
- ii. The domain of type  $t$  is  $D_t = \{T, F\} \wp(W \times J)$ , ~~the powerset of the set of all world-time pairs~~
- iii. The domain of a functional type  $\langle \sigma, \tau \rangle$  is the set of all functions from  $D_\sigma$  into  $D_\tau$ .

**Definition 4** (Models). A model is a pair  $\mathcal{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$ , where  $\mathbf{D}$  is the infinite hierarchy of domains defined in def. 3, and  $\|\cdot\|^{\mathbf{M}}$  is a valuation function interpreting the constants of the language, constrained so that  $\|\alpha\|^{\mathbf{M}} \in D_\sigma$  iff  $\alpha$  is of type  $\sigma$ .

**Definition 5** (Assignment functions). Given a model  $\mathcal{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$  and well-formed expressions *WFF*, a function  $g$  is an assignment function iff  $g$  is a total mapping from the set of variables in *WFF* into  $\mathbf{D}$  where  $g(\chi) \in D_\sigma$  iff  $\chi$  is of type  $\sigma$ .

**Definition 6** (Interpretation). The interpretation function given model  $\mathcal{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$  and assignment function  $g$  is  $\llbracket \cdot \rrbracket^{\mathbf{M}, g} : \text{WFF} \mapsto \mathbf{D}$ .

- $\llbracket \varphi \rrbracket^{\mathbf{M}, g} = g(\varphi)$  if  $\varphi$  is a variable.
- $\llbracket \varphi \rrbracket^{\mathbf{M}, g} = \|\varphi\|^{\mathbf{M}}$  if  $\varphi$  is a constant.
- $\llbracket (\varphi \psi) \rrbracket^{\mathbf{M}, g} = \llbracket \varphi \rrbracket^{\mathbf{M}, g}(\llbracket \psi \rrbracket^{\mathbf{M}, g})$
- For  $(\lambda \chi \varphi) : \langle \sigma, \tau \rangle$ ,  $\llbracket (\lambda \chi \varphi) \rrbracket^{\mathbf{M}, g}$  = the  $f$  from  $D_\sigma$  into  $D_\tau$  such that for all  $\odot \in D_\sigma$ ,  $f(\odot) = \llbracket \varphi \rrbracket^{\mathbf{M}, g[\chi \mapsto \odot]}$

## 2 Denotations for (some of) the constants

(1)  $\|\text{student}\|^M =$

(2)  $\|\text{not}\|^M =$

(3)  $\|\text{fail\_to}\|^M =$

(4)  $\|\text{necessarily}\|^M =$

(5)  $\|\text{every}\|^M =$

(6)  $\|\text{a}\|^M =$

(7)  $\|\text{most}\|^M =$

(8)  $\|\text{no}\|^M =$

(9)  $\|\text{the}\|^M =$

### 3 Axioms

The expression  $\varphi[x \triangleright a]$  says ‘ $\varphi$  with all occurrences of  $x$  turned into  $a$ .’ I assume that  $\varphi[x \triangleright a]$  has no effect if  $x$  is not a variable, and that  $\varphi[x \triangleright a]$  is permitted iff it does not result in accidental binding, that is, a term in which there is a  $\lambda$  that binds more variables than it did prior to substitution. For the full definition, see Carpenter 1997:44.

**Definition 7** ( $\alpha$  conversion).  $(\lambda x \varphi) \xRightarrow{\alpha} (\lambda y \varphi[x \triangleright y])$  (permitted iff  $\varphi[x \triangleright y]$  is permitted)

**Definition 8** ( $\beta$  conversion).  $((\lambda \chi \varphi) \psi) \xRightarrow{\beta} \varphi[\chi \triangleright \psi]$  (permitted iff  $\varphi[\chi \triangleright \psi]$  is permitted)

**Definition 9** ( $\eta$  conversion).  $(\lambda \chi (\varphi \chi)) \xRightarrow{\eta} \varphi$  (permitted iff  $\chi$  is not free in  $\varphi$ )

The axioms all preserve meaning in the sense of the models described above. The special provisions are designed to ward off meaning changes.

(10)  $\alpha$  conversion of  $x$ ?

- a.  $\lambda x$  (**see**  $x$ )
- b.  $\lambda x ((\text{see } x) y)$

(11)  $\beta$  conversion?

- a.  $((\lambda x (\text{happy } x)) \text{kim})$
- b.  $((\lambda x (\lambda y ((\text{see } x) y))) (\text{friend-of } y))$

(12)  $\eta$  conversion?

- a.  $(\lambda x (\text{dog } x))$
- b.  $(\lambda x ((\text{see } x) x))$

### 4 Reasoning

**Definition 10** (Generalized entailment for meanings). For all domains  $\mathbf{D}$  and meanings  $a, b \in \mathbf{D}$ :

- i. If  $a, b \in D_e$ , then  $a \sqsubseteq b$  iff  $a = b$
- ii. If  $a, b \in D_s$ , then  $a \sqsubseteq b$  iff  $a = b$
- iii. If  $a, b \in D_t$ , then  $a \sqsubseteq b$  iff  $a = \text{F}$  or  $b = \text{T}$
- iv. If  $a, b \in D_{\langle \sigma, \tau \rangle}$ ,  $a \sqsubseteq b$  iff for all  $d \in D_\sigma$ ,  $a(d) \sqsubseteq b(d)$

**Definition 11** (Generalized entailment for forms). For all models  $\mathbf{M}$ , assignments  $g$ , and expressions  $\alpha$  and  $\beta$ ,  $\alpha \Rightarrow \beta$  iff  $\llbracket \alpha \rrbracket^{\mathbf{M}, g} \sqsubseteq \llbracket \beta \rrbracket^{\mathbf{M}, g}$ .

**Definition 12** (Non-entailment,  $\nRightarrow$ ). If it's false that  $\alpha \Rightarrow \beta$ , then  $\alpha \nRightarrow \beta$

**Definition 13** (Synonymy,  $\equiv$ ). If  $\alpha \Rightarrow \beta$  and  $\beta \Rightarrow \alpha$ , then  $\alpha \equiv \beta$ .

**Definition 14** (Contradiction,  $|$ ).  $\alpha | \beta$  iff  $\alpha \Rightarrow \neg \beta$

**Definition 15** (Antonymy). [not sure ... very tricky]

## 5 Compositionality

Compositionality is often attributed to Frege, but, as Janssen (1997:420) points out, Frege explicitly endorsed a ‘principle of contextuality’, which says “One should ask for the meaning of a word only in the context of a sentence, and not in isolation”. This is roughly the contrary of compositionality. Janssen does concede that compositionality might be “in the spirit of his [Frege’s —CP] later writings” (p. 421). See also Szabó 2012.

**Definition 16** (Informal compositionality; Partee 1984:281). The meaning of an expression is a function of the meanings of its parts and of the way they are syntactically combined.

**Partee (1996:15–16) on ‘unconstrained compositionality’:**

Montague’s paper ‘Universal Grammar’ [UG] [...] contains the most general statement of Montague’s formal framework for the description of language. The central idea is that anything that should count as a grammar should be able to be cast in the following form: the syntax is an algebra, the semantics is an algebra, and there is a homomorphism mapping elements of the syntactic algebra onto elements of the semantic algebra. This very general definition leaves a great deal of freedom as to what sorts of things the elements and the operations of these algebras are. [...]

It is the homomorphism requirement, which is in effect the compositionality requirement, that provides the most important constraint on UG in Montague’s sense [...].

(13) Propositional logic interpretation as a homomorphism:

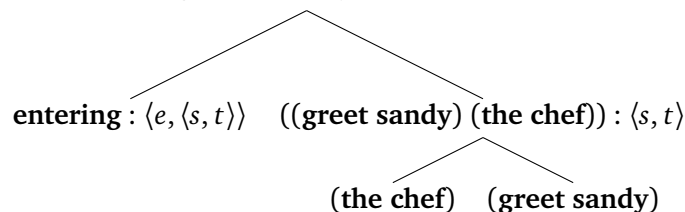
- a. Syntax:  $\langle P, \neg, \vee, \wedge \rangle$
- b. Semantics:  $\langle \{\emptyset, \{\emptyset\}\}, -, \cup, \cap \rangle$
- c. The interpretation function  $\llbracket \cdot \rrbracket$  is a homomorphism:
  - i.  $\llbracket p \rrbracket \in \{\emptyset, \{\emptyset\}\}$  for all  $p \in P$
  - ii.  $\llbracket \neg \varphi \rrbracket = \{\emptyset, \{\emptyset\}\} - \llbracket \varphi \rrbracket$
  - iii.  $\llbracket \varphi \vee \psi \rrbracket = \llbracket \varphi \rrbracket \cup \llbracket \psi \rrbracket$
  - iv.  $\llbracket \varphi \wedge \psi \rrbracket = \llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket$

**Dowty (2007) on ‘context-free compositionality’:**

When a rule  $f$  combines  $\alpha, \beta(\dots)$  to form  $\gamma$ , the corresponding semantic rule  $g$  that produces the meaning  $\gamma'$  of  $\gamma$ , from  $\alpha'$  and  $\beta'$ , may depend only on  $\alpha'$  “as a whole”, it may not depend on any meanings from which  $\alpha'$  was formed compositionally by earlier derivational steps (similarly for  $\beta$ )."

(14) Consider the prescriptive rule saying that the implicit argument of fronted participial construction must be the subject of the matrix clause. This rule is not context-free compositional:

- a. Entering the restaurant, the chef greeted Sandy.
- b.



- c. Assuming context-free compositionality, the rule cannot be right.
- d. What’s more, essentially no speakers maintain the rule, even those who advocate it: <http://arnoldzwicki.wordpress.com/category/danglers/>

## 6 Meaning postulates

Carlson (1977):

Foreword

In the spring of 1976, Terry Parsons and Barbara Partee taught a course on Montague grammar, which I attended. On the second to the final day of class, Terry went around the room asking the students if there were any questions at all that remained unanswered, and promised to answer them on the last day of class. I asked if he really meant ANY question at all, which he emphatically said that he meant. As I had encountered a few questions in my lifetime that remained at least partially unresolved, I decided to ask one of them. What is life? What is the meaning of life? After all, Barbara and Terry had promised to provide answers to any question at all.

On the final day of class Barbara wore her Montague grammar T-shirt, and she and Terry busied themselves answering our questions. At long last, they came to my question. I anticipated a protracted and involved answer, but their reply was crisp and succinct. First Barbara, chalk in hand, showed me the meaning of life.

‘<sup>^</sup>life’

Terry then stepped up and showed me what life really is.

‘<sup>^</sup><sup>^</sup>life’

As we were asked to show on a homework assignment earlier in the year, this is equivalent to: life’.

Leaving me astounded that I had been living in such darkness for all these years, the class then turned to the much stickier problem of pronouns.

Thomason (1974:48-49), cited by Lepore (1983:180):

The problems of a semantic theory should be distinguished from those of lexicography [...]. A central goal of (semantics) is to explain how different kinds of meanings attach to different syntactic categories; another is to explain how the meanings of phrases depend on those of their components. [...] But we should not expect a semantic theory to furnish an account of how any two expressions belonging to the same syntactic category differ in meaning. “Walk” and “run,” for instance, and “unicorn” and “zebra” certainly do differ in meaning, and we require a dictionary of English to tell us how. But the making of a dictionary demands considerable knowledge of the word.

In theories of the sort described here, lexical semantics is done via meaning postulates, which seek to ensure the needed entailments of lexical items and capture relationships between them:

(15) For all models  $\mathbf{M} = \langle \mathbf{D}, \|\cdot\|^{\mathbf{M}} \rangle$  appropriate for English:

- a.  $\|\text{dog}\|^{\mathbf{M}} \subseteq \|\text{mammal}\|^{\mathbf{M}}$
- b.  $\|\text{dog}\|^{\mathbf{M}} \subseteq \|\neg\text{cat}\|^{\mathbf{M}}$
- c.  $\|\text{couch}\|^{\mathbf{M}} = \|\text{sofa}\|^{\mathbf{M}}$
- d. For all  $P \in D_{\langle e, \{s, t\} \rangle}$  and  $a \in D_e$ ,  $\|\text{find}\|^{\mathbf{M}}(P)(a) \subseteq \|\text{exist}\|^{\mathbf{M}}(P)$  (cf. seek)

For discussion, Zimmermann 1999; Lepore 1983. For serious lexical semantics, see Levin & Rappaport Hovav 1995; Levin 1993; Levin & Rappaport Hovav 2005.

## 7 Foundational assumptions

The term *Montague grammar* refers primarily to the three papers Montague 1970a (EFL), Montague 1970b (UG), and Montague 1973 (PTQ), listed here in increasing order of influence. Sometimes the term is applied to all the language-related papers in Montague 1974. And of course it is often used as a broad label for any kind of formal semantics.

### Montague (1970b:373):

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory. On this point I differ from a number of philosophers, but agree, I believe, with Chomsky and his associates.

### Barwise & Cooper (1981:204):

It is here that Montague made his biggest contribution. To most logicians (like the first author) trained in model-theoretic semantics, natural language was an anathema, impossibly vague and incoherent. To us, *the* revolutionary idea in Montague's PTQ paper (and earlier papers) is the claim that natural language is not impossibly incoherent, as his teacher Tarski had led us to believe, but that large portions of its semantics can be treated by combining known tools from logic, tools like functions of finite type, the  $\lambda$ -calculus, generalized quantifiers, tense and modal logic, and all the rest.

### Montague (1970b:373–374):

The basic aim of semantics is to characterize the notions of a true sentence (under a given interpretation) and of entailment, while that of syntax is to characterize the various syntactical categories, especially the set of declarative sentences. It is to be expected, then, that the aim of syntax could be realized in many different ways, only some of which would provide a suitable basis for semantics. It appears to me that the syntactical analyses of particular fragmentary languages that have been suggested by transformational grammarians, even if successful in correctly characterizing the declarative sentences of those languages, will prove to lack semantic relevance; and I fail to see any great interest in syntax except as a preliminary to semantics.

### Lewis (1970:18):

My proposals regarding the nature of meanings will not conform to the expectations of those linguists who conceive of semantic interpretation as the assignment to sentences and their constituents of compounds of 'semantic markers' or the like. (Katz and Postal, 1964, for instance.) Semantic markers are *symbols*: items in the vocabulary of an artificial language we may call *Semantic Markerese*. Semantic interpretation by means of them amounts merely to a translation algorithm from the object language to the auxiliary language Markerese. But we can know the Markerese translation of an English sentence without knowing the first thing about the meaning of the English sentence: namely, the conditions under which it would be true. Semantics with no treatment of truth conditions is not semantics.

### Partee (1997:9):

I believe linguists did presuppose tacit competence in Markerese and moreover took it to represent a hypothesis about a universal and innate representation, what Jerry (J.A.) Fodor later dubbed the Language of Thought (e.g., Fodor (1975)), and therefore not in need of further interpretation (see Jackendoff 1996 for a contemporary defense of a similar view).

**Lewis (1970:19):**

My proposals will also not conform to the expectations of those who, in analyzing meaning, turn immediately to the psychology and sociology of language users: to intentions, sense-experience, and mental ideas, or to social rules, conventions, and regularities. I distinguish two topics: first, the description of possible languages or grammars as abstract semantic systems whereby symbols are associated with aspects of the world; and second, the description of the psychological and sociological facts whereby a particular one of these abstract semantic systems is the one used by a person or population. Only confusion comes of mixing these two topics. This paper deals almost entirely with the first. (I discuss the second elsewhere: Lewis, 1968b and 1969, Chapter V.) [These works are cited here as Lewis 1969 and Lewis 1975 —CP.] (p. 19)

**Partee (1997:18, fn. 11):**

When I once mentioned to Montague the linguist's preferred conception of universal grammar as the characterization of all and only the possible human languages, his reaction was to express surprise that linguists should wish to disqualify themselves on principle from being the relevant scientists to call on if some extraterrestrial beings turn out to have some kind of language.

**Partee (1980):**

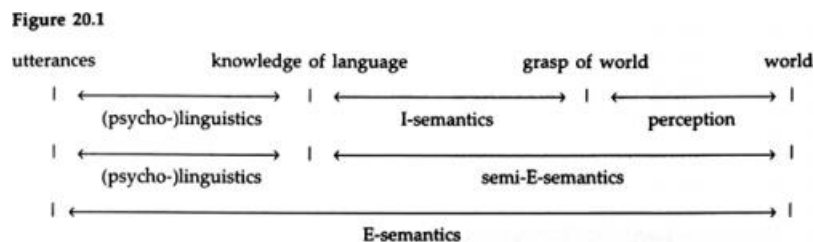
So I don't see how we can get a correct account of propositional attitudes without bringing psychology into the picture, but I also don't see how we can get along *with* it. The relevant psychological factors are ones which vary from speaker to speaker and moment to moment. No one can infallibly recognize logical equivalence, but there is no general way of determining who will recognize which equivalence when. The psychological correlates of word intensions are similarly variable across speakers and times. These were the very reasons why Frege suggested that if we want propositions to stand in a close relation both to language and to truth, we must not equate them with ideas.

**Katz (1972):**

"The arbitrariness of the distinction between form and matter reveals itself [...]"

**Jackendoff (1996:543):**

In order to treat semantics as an issue about the structure of the human organism, it is necessary to be careful about basic goals of the enterprise. In particular, the traditional preoccupation with explicating the notion of the truth or falsity of a sentence must be re-evaluated. For there is no longer a direct relation between an utterance and the world that renders the utterance true or false; there is instead the sequence of three relations diagrammed in the upper line of Figure 20.1.



See also Partee 1981; Katz 1996; Jackendoff 1992, 1997.

## 8 Some consequences

- (16) It should drive you crazy to see  $\lambda x$  (**redundant**  $x$ ). Why?
- (17) Representational details in the lambda calculus matter only if they impact denotations. Why?
- (18) Meaning and reference: all meaning is reference in the extended sense of identifying objects in the domains.
- (19) Confluence: the order of  $\beta$ -reductions does not matter. All licit reduction paths lead to the same formula and hence to the same meaning. Why?
- (20) Other consequences?

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