

# Introduction to intensionality

Chris Potts, Ling 230b: Advanced semantics and pragmatics, Spring 2018

May 2

## 1 Belief predicates

### 1.1 Our extensional models are not rich enough

This is the best we can do for *believe* at present:

(1) a.  $\mathbf{believe}_{\text{extensional}} : \langle t, \langle e, t \rangle \rangle$   
b.  $\|\mathbf{believe}_{\text{extensional}}\|^M$  = the function  $F$  such that, for all  $p \in \{T, F\}$ ,  $F(p) =$  the function  $f$  such that, for all  $d \in D_e$ ,  $d$  **believes**  $p$ .  
c. Informally:

$$\|\mathbf{believe}_{\text{extensional}}\|^M = \begin{bmatrix} \langle \text{Lisa}, T \rangle \mapsto T & \langle \text{Bart}, T \rangle \mapsto T \\ \langle \text{Lisa}, F \rangle \mapsto T & \langle \text{Bart}, F \rangle \mapsto F \\ \vdots & \vdots \end{bmatrix}$$

We're forced to say that, if Lisa believes any true predication, she believes all of them. In the above model, Lisa believes every predication, and Bart believes only the true ones.

### 1.2 Intensional models

(2) Intensional types: The smallest set  $\text{Types}_I$  such that

- $e \in \text{Types}_I$
- $t \in \text{Types}_I$
- $s \in \text{Types}_I$
- If  $\sigma, \tau \in \text{Types}_I$ , then  $\langle \sigma, \tau \rangle \in \text{Types}_I$

(3) Intensional denotation domains:

- The domain of type  $e$  is  $D_e$ , the set of entities.
- The domain of type  $t$  is  $D_t = \{T, F\}$
- The domain of type  $s$  is  $D_s$ , the set of all possible worlds.
- The domain of a functional type  $\langle \sigma, \tau \rangle$  is the set of all functions from  $D_\sigma$  into  $D_\tau$ .

We can carry over, from our first handout ('Lambda calculi for semantic theories'), the definition of well-formed expressions, models, assignment functions, and the interpretation function  $\llbracket \cdot \rrbracket^{M,g}$ .

For illustrations, let's designate the actual world with  $\llbracket @ \rrbracket^M = \dagger \in D_s$ .

### 1.3 A properly intensional *believe*

From Hintikka (1969:150):

My basic assumption (slightly oversimplified) is that an attribution of any propositional attitude to the person in question involves a division of all the possible worlds (more precisely, all the possible worlds which we can distinguish in the part of the language we use in making the attribution) into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it.

(4) a.  $\text{Dox}(d, \Theta) = \text{the set of all } \Theta' \in D_s \text{ such that } \Theta' \text{ is consistent with all of } d's \text{ beliefs in } \Theta$ .  
b. **believe** :  $\langle \langle s, t \rangle, \langle e, \langle s, t \rangle \rangle \rangle$   
c.  $\|\text{believe}\|^M = \text{the function } F \text{ such that, for all } p \in D_{\langle s, t \rangle}, d \in D_e \text{ and } \Theta \in D_s,$

$$F(p)(d)(\Theta) = T \text{ iff } \text{Dox}(d, \Theta) \subseteq \{\Theta' : p(\Theta') = T\}$$

(5) a.

$$\text{Dox}\left(\begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array}, \dot{\Theta}\right) = \{\Theta_2, \Theta_3\} \quad \text{Dox}\left(\begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array}, \dot{\Theta}\right) = \{\Theta_3, \Theta_4\}$$

b.  $[\![\text{smart}]\!]^{M,g} = \left[ \begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array} \mapsto \left[ \begin{array}{l} \dot{\Theta} \mapsto F \\ \Theta_2 \mapsto T \\ \Theta_3 \mapsto T \\ \Theta_4 \mapsto F \end{array} \right] \right] \quad \left[ \begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array} \mapsto \left[ \begin{array}{l} \dot{\Theta} \mapsto F \\ \Theta_2 \mapsto F \\ \Theta_3 \mapsto T \\ \Theta_4 \mapsto T \end{array} \right] \right]$

c.  $[\![(\text{believe} (\text{smart lisa})) \text{ lisa}]\!](\dot{\Theta}) = T$

$$\begin{aligned} \text{Dox}\left(\begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array}, \dot{\Theta}\right) &\subseteq \{\Theta : [\![(\text{smart lisa})]\!]^{M,g}(\Theta) = T\} \\ \{\Theta_2, \Theta_3\} &\subseteq \{\Theta_2, \Theta_3\} \end{aligned}$$

d.  $[\![(\text{believe} (\text{smart lisa})) \text{ bart}]\!](\dot{\Theta}) = F$

$$\begin{aligned} \text{Dox}\left(\begin{array}{c} \text{Lisa} \\ \text{Bart} \end{array}, \dot{\Theta}\right) &\not\subseteq \{\Theta : [\![(\text{smart lisa})]\!]^{M,g}(\Theta) = T\} \\ \{\Theta_3, \Theta_4\} &\not\subseteq \{\Theta_2, \Theta_3\} \end{aligned}$$

## 1.4 Intensionalizing the rest of the system

At the least, we have to replace all of the  $t$  types with  $\langle s, t \rangle$  and deal with the semantic consequences:

(6) a.  $\langle e, \langle s, t \rangle \rangle : \{\text{run, walk, student, unicorn}\}$

b.  $e : \{\text{sandy, kim, jesse}\}$

c.  $\langle e, \langle e, \langle s, t \rangle \rangle \rangle : \{\text{find, lose, eat, love, be}\}$

d.  $\langle \langle s, t \rangle, \langle e, \langle s, t \rangle \rangle \rangle : \{\text{believe, deny}\}$

e.  $\langle \langle e, \langle s, t \rangle \rangle, \langle e, \langle s, t \rangle \rangle \rangle : \{\text{rapidly, allegedly, fail\_to}\}$

f.  $\langle \langle s, t \rangle, \langle s, t \rangle \rangle : \{\text{necessarily}\}$

g.  $\langle \langle e, \langle s, t \rangle \rangle, \langle \langle e, \langle s, t \rangle \rangle, \langle s, t \rangle \rangle \rangle : \{\text{the, every, a, most, no}\}$

(7) a.  $\|\text{run}\|^M = \text{the } f \text{ such that, for all } d \in D_e \text{ and } \odot \in D_s, f(d)(w) = T \text{ iff } d \text{ runs in } \odot$

b.  $\|\text{love}\|^M = \text{the } R \text{ such that, for all } d \in D_e, d' \in D_e, \text{ and } \odot \in D_s, R(d)(d')(w) = T \text{ iff } d' \text{ loves } d \text{ in } \odot$

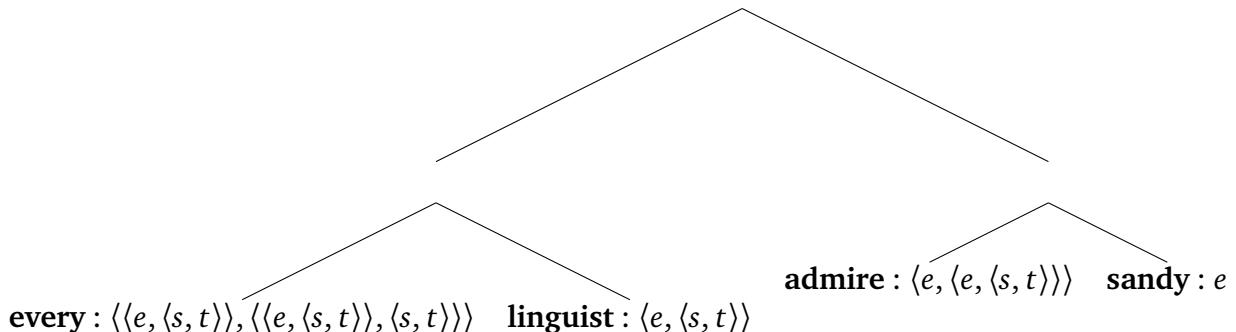
c.  $\|\text{necessarily}\|^M = \text{the } P \text{ such that, for all } p \in D_{\langle s, t \rangle} \text{ and } \odot \in D_s, P(p)(\odot) = T \text{ iff } p(\odot') = T \text{ for all } \odot' \in D_s$

$$\text{necessarily} = \lambda p \lambda w (\forall w' (p w'))$$

d.  $\text{every} = \lambda f \lambda g \lambda w (\forall x (f x w) \rightarrow (g x w))$

e.  $\text{a} = \lambda f \lambda g \lambda w (\exists x (f x w) \wedge (g x w))$

(8)



## 2 Transparent and opaque readings

I'm taking some liberties with functions and their characteristic sets to make this comprehensible:

$\text{Bul}(d, \odot) =$  the set of all  $\odot' \in D_s$  that satisfy all of  $d$ 's desires in  $\odot$

$\|\text{want}\|^M =$  the  $F$  such that  $F(p)(d)(\odot) = T$  iff  $\text{Bul}(d, \odot) \subseteq \{\odot' : p(\odot') = T\}$

$$\begin{aligned} \|\text{inexpensive\_dress}\|^M &= \begin{bmatrix} \odot_1 & \mapsto & \{a\} \\ \odot_2 & \mapsto & \{\} \\ \odot_3 & \mapsto & \{a\} \\ \odot_4 & \mapsto & \{c, d\} \end{bmatrix} \\ \|\text{buy}\|^M &= \begin{bmatrix} \odot_1 & \mapsto & \{\langle m, a \rangle\} \\ \odot_2 & \mapsto & \{\langle m, a \rangle, \langle m, c \rangle\} \\ \odot_3 & \mapsto & \{\langle m, a \rangle, \langle m, d \rangle\} \\ \odot_4 & \mapsto & \{\langle m, b \rangle\} \end{bmatrix} \\ \text{Bul} &= \begin{bmatrix} \langle m, \odot_1 \rangle & \mapsto & \{\odot_2, \odot_3\} \\ \langle m, \odot_2 \rangle & \mapsto & \{\odot_1, \odot_3\} \\ \langle m, \odot_3 \rangle & \mapsto & \{\odot_2, \odot_4\} \\ \langle m, \odot_4 \rangle & \mapsto & \{\odot_2, \odot_3\} \end{bmatrix} \end{aligned}$$

Let  $g(w_i) = \odot_i$  and  $g(w) = g(w') = \odot_1$ , and let  $\varphi_w$  abbreviate  $\lambda x_1 \dots x_n (\varphi x_1 \dots x_n w)$ .

$$(9) \quad \llbracket \lambda w' (\mathbf{m} \text{ buy}_{w'} \mathbf{an} \text{ inexpensive\_dress}_{w'}) \rrbracket^{M,g} =$$

$$(10) \quad \llbracket \lambda w' (\mathbf{m} \text{ buy}_{w'} \mathbf{an} \text{ inexpensive\_dress}_{w_4}) \rrbracket^{M,g} =$$

$$(11) \quad \llbracket \lambda w' (\mathbf{m} \text{ buy}_{w'} \mathbf{an} \text{ inexpensive\_dress}_{w_2}) \rrbracket^{M,g} =$$

$$(12) \quad \llbracket \lambda w (\mathbf{m} \text{ wants}_w (\lambda w' (\mathbf{m} \text{ buy}_{w'} \mathbf{an} \text{ inexpensive\_dress}_{w'}))) \rrbracket^{M,g} =$$

$$(13) \quad \llbracket \lambda w (\mathbf{m} \text{ wants}_w (\lambda w' (\mathbf{m} \text{ buy}_{w'} \mathbf{an} \text{ inexpensive\_dress}_{w'}))) \rrbracket^{M,g} =$$

## 3 Modals

This section is an overview of Kratzer's (1981) theory of modality.

### 3.1 Conversational backgrounds, a.k.a. modal bases

#### 3.1.1 Basics

A conversational background is a function in  $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ , where  $s$  is the type of possible worlds. Intuitively, at each world  $\odot$ , this is giving us a set of propositions. We can regard conversational backgrounds as the basis for the denotations of modal phrases like *what is known (at  $\odot$ )*.

- (14)  $\bigcap$  is the function  $F$  from  $D_{\langle \langle s, t \rangle, t \rangle}$  to  $D_{\langle s, t \rangle}$  such that  $F(P)(w) = T$  iff  $p(w) = T$  for all  $p$  such that  $P(p) = T$ .
- (15) Closure under entailment: For every conversational background  $B$ , world  $\odot$ , and proposition  $p$ , it holds that

$$\text{if } \bigcap B(\odot) \sqsubseteq p, \text{ then } B(\odot)(p) = T$$

Assuming (15), we can reduce conversational backgrounds to functions from worlds into sets of worlds, using  $\bigcap$ . Then it emerges that Dox and Bul are just like conversational backgrounds, but relativized to an entity.

#### 3.1.2 Some conversational backgrounds (modal bases)

- (16) Modal base  $B$  is *realistic* iff  $(\bigcap B(\odot))(\odot) = T$ , for all  $\odot \in D_s$ .
- (17) Modal base  $B$  is *totally realistic* iff  $\{\odot\} = \bigcap B(\odot)$ , for all  $\odot \in D_s$ .
- (18) Epistemic modality: "If we use an epistemic modality, we are interested in what may or must be the case in our world, *given everything we know already*." (Kratzer 1981:52)
- (19) Circumstantial: "And if we use a circumstantial modal, we are interested in what can or must happen, *given circumstances of a certain kind*." (Kratzer 1981:52)

### 3.2 Ordering and ordering sources

(20) Ordering: For all worlds  $\odot, \odot'$  and all sets of propositions  $A$ ,

$$\odot \leqslant_A \odot' \text{ iff } \{p \in A : p(\odot') = \text{T}\} \subseteq \{p \in A : p(\odot) = \text{T}\}$$

(21) Strict ordering:

$$\odot <_A \odot' \text{ iff } \{p \in A : p(\odot') = \text{T}\} \subset \{p \in A : p(\odot) = \text{T}\}$$

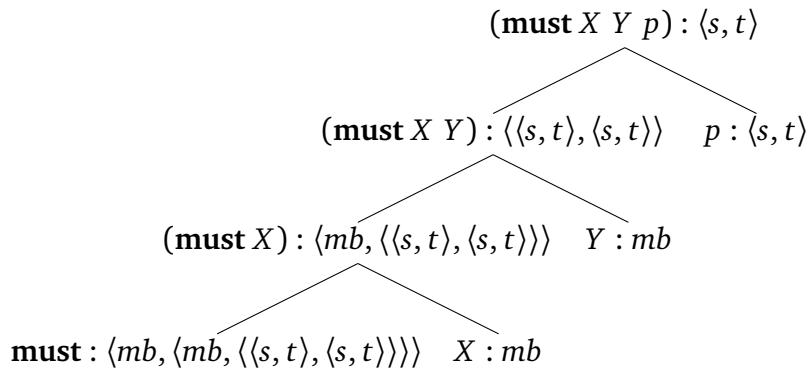
(22) Selection function: A selection function  $\text{best}_p$  grabs the set of all  $<_p$ -best worlds from a set of worlds  $X$ :

$$\text{best}_p(X) = \{\odot \in X : \text{there is no } \odot' \in X \text{ such that } \odot' <_p \odot\}$$

### 3.3 Putting the pieces together

Let  $mb$  abbreviate  $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ .

(23) a.



b.  $\llbracket (\text{must } X Y p) \rrbracket = \{\odot : \text{for all } \odot', \text{if } \odot' \in \text{best}_{\llbracket Y \rrbracket(\odot)}(\bigcap \llbracket X \rrbracket(\odot)), \text{ then } p(\odot')\}$

c. Suppose  $\llbracket X \rrbracket$  is ‘what we know to have happened’.

d. Suppose  $\llbracket Y \rrbracket$  is ‘what is ethical’.

e. Then  $\text{best}_{\llbracket Y \rrbracket(\odot)}(\bigcap \llbracket X \rrbracket(\odot))$  will be what is most ethical in  $\odot$  given what happened in  $\odot$ . (Optimally ethical in light of the facts.)

## Resources on modality

Hacquard, Valentine. 2012. Modality. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of natural language meaning*, vol. 3, Berlin: Mouton de Gruyter.

Hintikka, Jaakko. 1969. Reference and modality. In Leonard Linsky (ed.), *Philosophical logic*, 145–167. Oxford: Oxford University Press.

Kratzer, Angelika. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts*, 38–74. Berlin: Walter de Gruyter.

Kratzer, Angelika. 1991. Modality/conditionals, 639–650. Berlin: Walter de Gruyter.

Portner, Paul. 2009. *Modality*. Oxford University Press.

**For more** <http://web.mit.edu/fintel/lsa220-bibliography.pdf>