Discussion of Jones and Liu, “Growth with Capital-Embodied Technical Change”

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Uzawa (1961) on neoclassical growth model:

\[ Y_t = F(B_t K_t, A_t L_t) \]

- Interior stable factor shares: \( BK \) and \( AL \) must grow at the same rate — balance
- \( K \) accumulates endogenously \( \Rightarrow \) \( K \) inherits \( AL \) trend
- So \( B_t \) must stabilize for balanced growth (or Cobb-Douglas)

- Is a new computer a higher \( B \) or a higher \( A \)?
- But why would \( B \) ever be constant?
Uzawa (continued)

\[ Y_t = F(B_tK_t, A_tL_t) \]

- Acemoglu (2003 JEEA)
  - A 2-dimensional Romer model: entrepreneurs can increase \( A \) or \( B \)
  - Surprise: they endogenously choose to stabilize \( B \) and only increase \( A \)

- However, extremely fragile!
  - Breaks if model is semi-endogenous growth instead of fully endogenous
  - Breaks if any asymmetry in the idea production functions of \( A \) versus \( B \)
Uzawa (continued)

- Grossman, Helpman, Oberfield, Sampson (2017 AER)

\[ Y_t = F((1 - s_t)^\alpha B_t K_t, A_t L_t / (1 - s_t)^\beta) \]

- Add a third factor “schooling” \( s_t \).

- If it enters production in just the right way, you can get a BGP
  - \( \dot{s}_t = \theta (1 - s_t) \): schooling rises, but at a decreasing rate
  - \( 1 - s_t \) falls at a constant exponential rate so \( (1 - s_t)^\alpha B_t \) constant \( \Rightarrow \) satisfies Uzawa

- Aghion-Jones-Jones (2019) and Jones-Liu (2022) have closely-related math, but in a very different economic environment!
Background: Automation

  - Foundational work in this literature, building on Zeira (1998)

- Aghion, B. Jones, and C. Jones (2019) is the direct predecessor to the present paper
  - Many common ingredients
  - Let me show the similarities and then highlight the point of departure
Final good

\[ Y_t = \left( \int_0^1 y_{it}^\frac{\sigma-1}{\sigma} \, di \right)^\frac{\sigma}{\sigma-1} \quad \text{where} \quad \sigma < 1 \]

Tasks

\[ y_{it} = \begin{cases} 
K_{it} & \text{if automated} \quad i \in [0, \beta_t] \\
L_{it} & \text{if not automated} \quad i \in [\beta_t, 1]
\end{cases} \]

Capital accumulation

\[ \dot{K}_t = I_t - \delta K_t \]

Resource constraint (K)

\[ \int_0^1 K_{it} \, di = K_t \]

Resource constraint (L)

\[ \int_0^1 L_{it} \, di = L \]

Resource constraint (Y)

\[ Y_t = C_t + I_t \]

Allocation

\[ I = \bar{s}_K Y \]
Automation and growth

• Combining equations

\[ Y_t = \left[ \beta_t \left( \frac{K_t}{\beta_t} \right)^{\frac{\sigma - 1}{\sigma}} + (1 - \beta_t) \left( \frac{L}{1 - \beta_t} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \]

• How \( \beta \) interacts with \( K \): two effects
  
  ◦ \( \beta \): what fraction of tasks have been automated
  
  ◦ \( \beta \): Dilution as \( K/\beta \Rightarrow K \) spread over more tasks

• Same for labor: \( L/(1 - \beta_t) \) means given \( L \) concentrated on fewer tasks, raising “effective labor”
Rewriting in classic CES form

• Collecting the $\beta$ terms into factor-augmenting form:

$$Y_t = F(B_tK_t, A_tL_t)$$

where

$$B_t = \left( \frac{1}{\beta_t} \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad A_t = \left( \frac{1}{1 - \beta_t} \right)^{\frac{1}{1-\sigma}}$$

• Effect of automation: $\uparrow \beta_t \Rightarrow \downarrow B_t$ and $\uparrow A_t$

*Intuition: dilution effects just get magnified since $\sigma < 1$*
Suppose a constant fraction of non-automated tasks get automated every period:

\[ \dot{\beta}_t = \theta (1 - \beta_t) \]

\[ \Rightarrow \beta_t \to 1 \]

What happens to \( 1 - \beta_t =: m_t \)?

\[ \frac{\dot{m}_t}{m_t} = -\theta \]

*The fraction of labor-tasks falls at a constant exponential rate*
Putting it all together

\[ Y_t = F(B_t K_t, A_t L_t) \] where

\[ B_t = \left( \frac{1}{\beta_t} \right)^{\frac{1}{1-\sigma}} \] and

\[ A_t = \left( \frac{1}{1 - \beta_t} \right)^{\frac{1}{1-\sigma}} \]

- \( \beta_t \to 1 \Rightarrow B_t \to 1 \)

- But \( A_t \) grows at a constant exponential rate!

\[
\frac{\dot{A}_t}{A_t} = -\frac{1}{1 - \sigma} \frac{\dot{m}_t}{m_t} = \frac{\theta}{1 - \sigma}
\]

- When a constant fraction of remaining goods get automated and \( \sigma < 1 \), the automation model features an asymptotic BGP that satisfies Uzawa

\[
\alpha_{Kt} \equiv \frac{F_K K_t}{Y_t} = \beta_t^\frac{1}{\sigma} \left( \frac{K_t}{Y_t} \right)^{\frac{\sigma-1}{\sigma}} \to \left( \frac{\bar{S}_K}{\delta Y + \delta} \right)^{\frac{\sigma-1}{\sigma}} < 1
\]
Intuition for AJJ result

• Why does automation lead to balanced growth and satisfy Uzawa?
  ○ $\beta_t \to 1$ so the KATC piece “ends” eventually
  ○ Labor per task: $L/(1 - \beta_t)$ rises exponentially over time!
  ○ Constant population, but concentrated on an exponentially shrinking set of goods
    $\Rightarrow$ exponential growth in “effective” labor

• Limitation
  ○ An asymptotic result
  ○ Only occurs as $\beta_t \to 1$, so unclear if relevant for U.S. or other modern economies

Interesting question: What fraction of tasks automated today? $\beta_{2022}$
B. Jones and Liu Contribution

- BGP can occur “today” with $\beta_t < 1$, not asymptotically
  - Might describe modern economies like the U.S. / Europe / Japan

- Automation and KATC ($Z_t$) coexist along the BGP
  - The economic environment that achieves this is novel and interesting

- Empirics
Jones-Liu Economic Environment

Final good

\[ Y_t = \left( \int_0^1 y_{it}^{\sigma-1} di \right)^{\frac{\sigma}{\sigma-1}} \text{ where } \sigma < 1 \]

Tasks

\[ y_{it} = \begin{cases} 
  z_{it}^{\frac{1}{1-\sigma}} K_{it} & \text{if automated} \quad i \in [0, \beta_t] \\
  L_{it} & \text{if not automated} \quad i \in [\beta_t, 1]
\end{cases} \]

Familiar

\[ \dot{K}_t = I_t - \delta K_t, \quad \int_0^1 K_{it} di = K_t, \quad \int_0^1 L_{it} di = L \]

Resource constraint (Y)

\[ Y_t = C_t + I_t + \int_0^{\beta_t} d_{it}^v di + \int_{\beta_t}^1 d_{it}^h di \]

Innovation: increasing \( z_i \)

Arrival rate \( q_{it}^v = \zeta^v \left( \frac{z_{it} d_{it}^v}{Y_t} \right)^\alpha \), Step size \( \phi \)

Innovation: automation

Arrival rate \( q_{it}^h = \zeta^h \left( \frac{z_{it} d_{it}^h}{Y_t} \right)^\alpha \), \( z_{it} = \bar{h} \cdot (1 - \beta_t) \) for \( i = \beta_t \)
Combining equations

\[ Y_t = F(B_t K_t, A_t L_t) \]  
where \( B_t = \left( \frac{Z_t}{\beta_t} \right)^{\frac{1}{1-\sigma}} \)  
and \( A_t = \left( \frac{1}{1 - \beta_t} \right)^{\frac{1}{1-\sigma}} \)

and

\[ Z_t \equiv \left( \frac{1}{\beta_t} \int_0^{\beta_t} z_t^{-1} di \right)^{-1} \]  
(harmonic mean)

- Same “engine” of growth as AJJ via \( A_t \)

- Automation: Constant fraction \( q^h \) of remaining goods automated: \( \dot{\beta}_t = q^h (1 - \beta_t) \)
  - But starting productivity of newly automated good is \( z_0 = \bar{h}(1 - \beta_t) \)
  - declines over time (harder to automate goods start out further behind)

- \( \beta_t \to 1 \) as before. What happens with \( Z_t \)?
Understanding $Z_t$

$$Z_t \equiv \left( \frac{1}{\beta_t} \int_0^{\beta_t} z_{it}^{-1} \, dt \right)^{-1} \quad \text{(harmonic mean)}$$

- Already automated goods improve at rate $q^n \phi$ over time, raising $Z_t$
- Newly automated goods come in with very low productivity $z = \bar{h}(1 - \beta_t)$
  - Harmonic mean is dragged down by these low additions
- Surprise! $Z_t$ aggregates as if
  $$\dot{Z}_t = \kappa_t (1 - \beta_t) \quad \text{with} \quad \kappa_t \rightarrow \kappa^*$$
- Just like $\beta_t$!
  $$\Rightarrow Z_t / \beta_t \quad \text{constant along BGP}$$
Remarks

- BGP even with $\beta_t < 1$. Automation and KATC along BGP

- Requires the equivalent of $\dot{Z}_t = \kappa(1 - \beta_t)$
  - Why should this be?
  - On the one hand, standard growth models have $Z$ growing exponentially
  - Cool structure with newly-automated goods having lower productivity in just the right way.
  - But it's a very specific assumption.

- Parallels Acemoglu (2003) in that very special structure required

- Paper should do a better job of clarifying that this is the contribution
Empirics

- What does $\beta_t$ look like over time?
- Two equations in two unknowns

\[ \alpha_{Kt} = \frac{\beta_t}{Z_t} \]

\[ \frac{Y_t}{L_t} = \left(1 - \alpha_{Kt}\right)^{\frac{\sigma}{1 - \sigma}} \left(\frac{1}{1 - \beta_t}\right)^{\frac{1}{1 - \sigma}} \]

- Get $\beta_t$ from labor productivity and $Z_t$ from capital share
Falling Labor Share and Growth Slowdown since 2000
Estimates of $\beta_t$ and $Z_t$

Share automated has risen from 0.5 to 0.75

Rise in capital share since 2000 due to a 25% fall in $Z$?

Would be nice to show $\beta_t$ and $Z_t$ directly
Remarks on Empirics

• Share automated has risen from $\beta_{1950} = 0.5$ to $\beta_{2020} = 0.75$
  ○ Do we believe this? I don’t know. Lots of automation!
  ○ What other evidence? Unclear, but model nicely points to $\alpha_K$ and $Y/L$

• Rise in capital share since 2000 due to a 25% fall in $Z_t$?
  ○ Not a burst of automation b/c automation should increase growth (temporarily)
  ○ Model cannot help us understand a decline in $Z_t$

• Likely other forces contributing to growth that would change the calibration?
  ○ Educational attainment, LATC apart from automation, markups
  ○ Exponential declines in the relative price of information technology
Final Thoughts

Very interesting, provocative, and fun to read!