Prof. Chad Jones
Econ 202b
Spring 2007

Problem Set #10

Due Thursday, April 12, 2007

1. *Deriving the Log-Linearized Capital Accumulation Equation.* (Campbell, 1994). Consider a simple version of the RBC model, where labor is exogenously fixed (there is no labor-leisure decision) and there is no government. Output is produced with the Cobb-Douglas production function $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$. The equation of motion for capital accumulation:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t.$$

   (a) Show that the capital accumulation equation can be rewritten as

$$\log\left[e^{\Delta \log K_{t+1}} - (1-\delta)\right] = \log Y_t - \log K_t + \log\left[1 - e^{\log C_t - \log Y_t}\right]. \quad (1)$$

   (b) Treat the left-hand-side of equation (1) as a function of $\Delta \log K_{t+1}$ and linearize the left-hand-side around the balanced growth path. Use the fact that $K_{t+1}/K_t \approx G \equiv 1+g$ along the balanced growth path.

   (c) Linearize the right-hand-side of equation (1) around the balanced growth path.

   (d) Show that in terms of deviations from the (trend) balanced growth path, the log-linearized capital accumulation equation can be written as

$$\tilde{K}_{t+1} = \lambda_1 \tilde{K}_t + \lambda_2 \tilde{A}_t + (1-\lambda_1-\lambda_2)\tilde{C}_t.$$

   What are the values for λ_1 and λ_2?

   (e) Why do the coefficients on $\tilde{K}$, $\tilde{A}$, and $\tilde{C}$ sum to one?

2. *An RBC Model with Taste Shocks* (from Blanchard-Fischer).

Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual gets utility according to the expected value of

$$\sum_{t=0}^{\infty}(1+\rho)^{-t}u(C_t), \rho > 0.$$

The utility kernel is $u(C_t) = C_t - \theta(C_t+\eta_t)^2$. η_t is a mean zero, i.i.d. shock that is normally distributed with variance σ_η^2. Also, η_t is realized before any choices are made at time t. Assume that C is always in the range where $u'(C) > 0$, and $\theta > 0$.

Output is linear in capital:

$$Y_t = AK_t,$$

where A is the (constant) marginal product of capital. Assume $A = \rho$ for simplicity. Capital accumulates with no depreciation:

$$K_{t+1} = K_t + Y_t - C_t.$$

(a) Define a competitive equilibrium for an economy with this economic environment.

(b) Rather than solve for the competitive equilibrium allocation directly, we will make use of the welfare theorems to solve the planner's problem. What is Bellman's functional equation for the planner's problem?

(c) Find the Euler equation for consumption.

(d) Guess that consumption takes the for $C_t = \alpha + \beta K_t + \gamma\eta_t$. Given this guess, what is K_{t+1} as a function of K_t and η_t?

(e) What values must the parameters α, β, and γ have for the first-order condition in part (a) to be satisfied for all values of K_t and η_t?

(f) What are the effects of a one-time shock to η on the time paths of Y, K, and C?

(g) "Temporary consumption binges are paid for by lower consumption forever." Discuss this statement.