Prof. Chad Jones
Econ 202b
Spring 2007

Problem Set #4

Due Thursday, February 15, 2007

1. *The Optimal Allocation in a Neoclassical Growth Model.* Consider the following economic environment:

$$U_0 = \int_0^\infty u(c_t)e^{-\rho t}\ dt,$$

$$u(c) \equiv \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}$$

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

$$\dot{K}_t = Y_t - C_t - \delta K_t, \quad K_0 > 0,$$

$$L_t = 1, \quad c \equiv C/L.$$

where the notation is the same as in class, and $\alpha \in (0,1)$, and δ, ρ, and σ are all positive parameters, with ρ sufficiently large to keep utility finite. Notice that $c = C/L$ as usual.

   (a) Define the optimal allocation of resources in this economy. That is, define what an allocation *is*, and then define what an optimal allocation is.

   (b) Write down the Hamiltonian for this problem. What is an economic interpretation of the Hamiltonian?

   (c) Obtain the first order conditions that characterize the solution. What economic interpretation can you give to these?

   (d) Solve for the optimal allocation in steady state (i.e. when the allocation is constant over time). What is the optimal saving rate in steady state?