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Econ 202b
Spring 2007

Problem Set #5

Due Tuesday February 27, 2007

1. *Intertemporal and Dynamic Budget Constraints.* Consider the following intertemporal budget constraint:

$$\int_t^\infty C_s e^{-\bar{r}_s(s-t)} ds = V_t + \int_t^\infty (Y_s - T_s) e^{-\bar{r}_s(s-t)} ds$$

where C is consumption, Y is labor income, T is a lump sum tax, V is financial wealth, and $\bar{r_s} = 1/(s-t)\int_t^s r_u du$. Derive the dynamic budget constraint for this consumer. Now go back the other way: show how the DBC can be integrated to yield the IBC, assuming the transversality condition holds.

2. *Optimal Allocations in the Neoclassical Growth Model.* Suppose you are appointed the social planner of an economy with the following preferences and technology:

$$U_0 = \int_0^\infty u(c_t) e^{-(\rho-n)t}\ dt,$$

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha},$$

$$\dot{K}_t = Y_t - C_t - \delta K_t, \quad K_0 > 0, \quad \delta > 0$$

$$\dot{L}_t / L_t = n, \quad L_0 > 0, \quad n > 0$$

$$\dot{A}_t / A_t = g, \quad A_0 > 0, \quad g > 0$$

where the notation is the same as in class and α lies between zero and one. Notice that $c \equiv C/L$ as usual. Assume $u(.)$ is CRRA with an intertemporal elasticity of substitution given by $\sigma > 0$.

   (a) The problem is to find the utility-maximizing allocation of resources. Set up this problem in a reasonable fashion and write down the Hamiltonian.
   (b) Obtain the first order conditions that characterize the solution.
   (c) Analyze the solution in a phase diagram.
   (d) Suppose the economy begins in steady state. Using phase diagrams, show how the economy responds to

i. A one-time reduction in capital (e.g. because of a war).
ii. An increase in n (e.g. because of increased migration).
iii. An increase in g.

3. *Distortionary Taxation in the Neoclassical Growth Model.* Consider the basic setup of the NGM discussed in class, augmented to include distortionary taxation of capital. The representative household solves

$$\max_{\{c_t\}} \int_0^\infty u(c_t)e^{-(\rho-n)t}\ dt$$

subject to

$$\dot{v}_t = w_t + (1-\tau)r_t v_t - nv_t - c_t + f_t$$

and to the no Ponzi game condition

$$\lim_{t\to\infty} v_t e^{-(\bar{r}_t - n)t} \geq 0$$

where $u(c)$ has a constant intertemporal elasticity of substitution given by σ, $\rho > n > 0$, $\tau > 0$, and f_t is a lump-sum rebate to households of the revenue collected from the capital tax, and $\bar{r}_t = 1/t \int_0^t r_s ds$. (Each household is "small" relative to the size of the government.) Assume output is produced according to the production function $y = k^\alpha A^{1-\alpha}$ where A is exogenously growing at rate $g > 0$, and assume that capital does not depreciate in production.

(a) Define the competitive equilibrium for this economy with taxes. (Include every endogenous variable from the model in your definition, and make sure to count equations and unknowns!)

(b) Solve the model to find the Euler equation for consumption and the steady state ratio of capital per capita to the level of technology. Be sure to indicate how the asset accumulation equation (the $\dot{v}$ equation) implies the standard accumulation equation for capital.

(c) Show the dynamics of the system in a phase diagram.

(d) Suppose, starting from steady state, there is a permanent, unanticipated increase in the tax rate to τ'. Analyze the change in a phase diagram. Be sure to show how consumption evolves over time.

(e) Suppose instead that the change in the tax rate is *anticipated*, i.e. it is announced one year in advanced. Analyze this change in a phase diagram. How and why is the consumption path different?