Prof. Chad Jones
Econ 202b
Spring 2007

Problem Set #6

Due Tuesday, March 6, 2007 in class

1. *The Behavior of the Saving Rate.* Consider the differential equations of the neoclassical growth model:

$$\dot{\tilde{c}}/\tilde{c} = \sigma(f'(\tilde{k}) - \delta - \rho) - g$$

$$\dot{\tilde{k}} = f(\tilde{k}) - \tilde{c} - (n + g + \delta)\tilde{k}.$$

   Assume that $\tilde{y} = f(\tilde{k}) = \tilde{k}^{\alpha}$, with $0 < \alpha < 1$.

   (a) Define $z = c/y$ so that $s = 1 - z$ and write down a differential equation for the growth rate of z.

   (b) Construct a phase diagram in $(\tilde{k}, z)$ space. What is the slope of the $\dot{\tilde{k}} = 0$ schedule?

   (c) Why is the slope of the $\dot{z} = 0$ schedule ambiguous? Under what conditions is the locus horizontal? What is the value of z for which this condition holds? What is the associated saving rate? Under this condition, what is the behavior of the saving rate along the transition from a low value of $\tilde{k}_0$?

   (d) Under what conditions is the $\dot{z} = 0$ schedule downward sloping? In this case, what is the behavior of the saving rate along the transition path from a low value of $\tilde{k}_0$?

   (e) Under what conditions is the $\dot{z} = 0$ schedule upward sloping? In this case, what is the behavior of the saving rate along the transition path from a low value of $\tilde{k}_0$?

   (f) Provide some intuition for these results.

2. *Convergence Speeds in the NGM.* Recall the key differential equations of the neoclassical growth model:

$$\dot{\tilde{c}}/\tilde{c} = \sigma(f'(\tilde{k}) - \delta - \rho) - g$$

$$\dot{\tilde{k}}/\tilde{k} = \frac{f(\tilde{k})}{\tilde{k}} - \frac{\tilde{c}}{\tilde{k}} - (n + g + \delta).$$

Assume the production function is Cobb-Douglas with capital share $0 < \alpha < 1$.

(a) Derive the log-linearized system of differential equations, i.e. the two equations in terms of $(\log \tilde{c} - \log \tilde{c}^*)$ and $(\log \tilde{k} - \log \tilde{k}^*)$.

(b) Draw the phase diagram for the log-linearized model. What is the equation for the stable arm (i.e. $\log \tilde{c} - \log \tilde{c}^*$ as a linear function of $\log \tilde{k} - \log \tilde{k}^*$)? What is the slope of the unstable arm?

(c) Derive an equation describing the behavior of $\log \tilde{y}_t$ as a function of $\log \tilde{y}_0$ and $\log \tilde{y}^*$ and time. What is the speed of convergence in this model? In what sense is this a "speed of convergence"?

(d) Discuss qualitatively how this speed of convergence relates to the speed of convergence in the Solow model, i.e. when the saving rate is constant.

(e) Consider two economies, say the U.S. and Japan. Discuss the behavior of $x_t \equiv (\log \tilde{y}_t^{Japan} - \log \tilde{y}_t^{U.S.})$ over time and relate this to the speed of convergence. Interpret.

3. *Problem 2.16 in Romer's text.* Two additional parts to this question (insert them at the start of parts (a) and (b)):

2.16 (a)(0) Define the competitive equilibrium for this Diamond economy with taxes and transfers.

2.16 (b)(0) Explain how the definition of competitive equilibrium from part (a) changes for this new economy.