Prof. Chad Jones
Econ 202b
Spring 2007

Problem Set #7

Due Tuesday, March 13, 2007

1. *Problem 2.17 in Romer's text.*

2. *Capital Externalities in a Neoclassical Growth Model (Frankel 1962 AER).* Consider the usual Ramsey-Cass-Koopmans model as discussed in class (i.e. with population growth at rate n and exogenous technological progress at rate g). Assume instantaneous utility is CRRA. Let final output be produced by a continuum of firms of mass one, with each firm using the same production function

$$Y = \kappa^{\beta} K^{\alpha} (AL)^{1-\alpha}.$$

where $\kappa = \int_0^1 K_i di$. Assume that there are positive externalities to capital accumulation. That is, firms take the aggregate capital stock, κ, as given at each point in time. Notice that because the measure of firms is one, $\kappa = K = K_i$. Assume that $\beta > 0$ but that the externality is not too strong, so that $\alpha + \beta < 1$. The rest of the model is identical to the neoclassical growth model.

   (a) Describe the environment for this economy (technology and preferences).

   (b) Define the competitive equilibrium for this economy.

   (c) What is the per capita growth rate of the economy along the balanced growth path? If $g = 0$ is there per capita growth? Why?

   (d) Solve for the investment rate $s = I/Y$ along the balanced growth path in the competitive equilibrium.

   (e) What is the socially optimal investment rate along the balanced growth path, and why are these rates different?

3. *Achieving the Social Optimum in an AK Model.* Consider the competitive equilibrium of the basic AK model, augmented to included taxes

(subsidies). Here are the key places where the framework differs from the model considered in class: The representative agent solves

$$\max \int_0^\infty u(c_t)e^{-(\rho-n)t}dt$$

subject to

$$\dot{v}_t = (r_t - n)v_t + (1 - \tau_w)w_t - (1 + \tau_c)c_t$$

where $u(\cdot)$ is a CRRA utility function, $v_0 > 0$ is given, and a NPG condition, taking prices and tax rates as given.

The representative firm solves

$$\max \tilde{A}_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t(1 + \tau_r)K_t$$

where the firm takes $\tilde{A}_t$, τ_r and prices as given. For the aggregate economy, $\tilde{A}_t = Ak_t^{1-\alpha}$, where $k_t \equiv K_t/L_t$ and $A > 0$ is a constant. Finally, $v_t = k_t$ closes the model.

(a) Solve for the growth rates of c, y, and k along the balanced growth path.

(b) Which taxes can affect the long-run growth rate and which cannot? Explain.

(c) Discuss a combination of taxes and subsidies (negative taxes) that the government could employ that will maintain a balanced budget and simultaneously implement the socially optimal allocation of resources.