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Econ 202b
Spring 2007

# Problem Set #8

Due Thursday, March 22, 2007

1. *An "AK" Model with Physical and Human Capital.* Consider the following aggregate production function:

$$Y_t = K_t^{\alpha}(h_t L_t)^{1-\alpha}$$

where $h_t \equiv H_t/L_t$ is human capital per person, $Y_t$ is output, and $K_t$ is physical capital. Accumulation of the capital inputs is given by

$$\dot{K}_t = s_K Y_t - \delta K_t$$

$$\dot{H}_t = s_H Y_t - \delta H_t$$

where $s_K$ and $s_H$ are constant and exogenously given. The labor force, $L_t$, grows at the exogenous rate $n > 0$.

   (a) What is the per capita growth rate in the economy along the balanced growth path?

   (b) Show that along the balanced growth path the production function can be written as $Y = AK$ where $A$ is constant. What is the value of $A$?

   (c) Define a competitive equilibrium for this economy that endogenizes the allocations $s_{Kt}$ and $s_{Ht}$ and uses standard preferences. Are there any externalities needed? Will the decentralized equilibrium be socially optimal? [HINT: Do not solve the model explicitly to answer this part.]

2. *Achieving the Social Optimum in Romer (1990).* Consider a simplified version of Romer (1990) with taxes/subsides. The environment is given by

$$Y_t = L_{yt}^{1-\alpha} \int_0^{A_t} x_{it}^{\alpha}\, di$$

$$\int_0^{A_t} x_{it}\, di = K_t, \quad \dot{K}_t = Y_t - C_t, \quad K_0 > 0$$

$$\dot{A}_t/A_t = \nu L_{At}, \quad \nu > 0, \quad A_0 > 0$$

$$L_{At} + L_{yt} = L$$

where $0 < \alpha < 1$ and $L$ is fixed. Preferences are given by the standard CRRA utility function with a discount rate $\rho$ and an intertemporal elasticity of substitution $\sigma$.

Consider an imperfectly competitive equilibrium with taxes and subsidies. In particular, the available tax instruments are:

- Capital goods firms have to pay a (constant) proportional tax $\tau_K$ (or receive a subsidy if $\tau_K < 0$) for every unit of raw capital that they use. As in class, one unit of raw capital, combined with a design, can be costlessly transformed into a capital good. Thus, the marginal cost of production is $r_t(1 + \tau_K)$.
- Wages earned in the R&D sector are taxed at rate $\tau_A$ (or subsidized if $\tau_A < 0$), and this tax is paid by research firms. Therefore, the free entry / zero-profit condition for this sector is

$$P_{At}\dot{A}_t = (1 + \tau_A)w_t L_{At}.$$

With this setup, answer the following questions.

(a) Define an imperfectly competitive equilibrium for this economy, following the template we used in class. Assume that all taxes or subsidies are rebated/financed with a lump sum transfer to households so that the government's budget balances in every period. Be sure to count equations and count unknowns.

(b) Solve the Final Good sector's profit maximization problem. What is the Final Good sector's demand for labor and capital goods as a function of the prices $w_t$ and $p_{it}$?

(c) Solve the profit maximization problem for a capital good's firm that owns a patent and show that all capital goods sell for the same price. What is the instantaneous profit earned by a capital good's firm as a function of aggregate output and the stock of designs?

(d) A researcher who creates a new design sells the patent to the capital goods sector at price $P_{At}$. Assuming free entry into the bidding process, what is $P_{At}$ along a balanced growth path?

(e) Solve for $s_A \equiv L_A/L$ along the balanced growth path. What is the steady state growth rate of per capita income?

(f) Solve for $s_K \equiv \dot{K}/Y$ along the balanced growth path.

(g) Solve for the socially optimal allocation of resources in this problem and the socially optimal growth rate.

(h) Assuming lump-sum financing of subsidies, can an R&D subsidy by itself be used to achieve the social optimum in steady state? Why or why not?

(i) Along a balanced growth path, find the combination of taxes and subsidies that achieves the social optimum. Discuss the relevant distortions.

3. *R&D and Productivity Growth (Jones 2002, AER)*. Consider the Romer model, augmented by Jones (1995 JPE). In particular, focus on the production function for new ideas:

$$\dot{A} = \delta L_A^{\lambda} A^{\phi}.$$

Empirically, the following facts are a reasonable characterization of the U.S. economy in the post-war era:

- Scientists and Engineers engaged in R&D have grown at an average rate of 4% per year.
- Labor force growth has been about 1% per year.
- TFP Growth (Hick's neutral) has averaged 1% per year.
- Labor's share is about 2/3.

Assume that these growth rates have been constant over time in the post-war era.

(a) Suppose $\lambda = 1$. Use these facts to calculate an implied estimate of $\phi$. Interpret.

(b) Suppose $\lambda = 1/2$. Use these facts to calculate an implied estimate of $\phi$. Interpret.

(c) Is the U.S. economy in the post-war period in steady state? What is the long-run steady state growth rate of per capita income implied by these numbers? Discuss.