Prof. Chad Jones
Econ 202b
Spring 2007

Problem Set #9

Due Thursday, April 5, 2007

1. *Bellman's Equation in a model of Habit Persistence* (Sargent 1987). Consider the problem of choosing a consumption sequence $\{c_t\}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t (\log c_t + \gamma \log c_{t-1}), \quad 0 < \beta < 1, \gamma > 0$$

subject to

$$c_t + k_{t+1} \leq A k_t^{\alpha} \quad 0 < \alpha < 1, \quad A > 0$$

with $k_0 > 0$ and $c_{-1} > 0$ both given. Notice that the kernel of the agent's utility function depends on consumption today as well as consumption in the previous period (models of habit persistence in consumption generalize this dependence).

   (a) What are the state variables for a consumer at time t?

   (b) Formulate Bellman's functional equation for this problem.

   (c) Take the first order conditions to obtain two difference equations in c_t and k_t (and their lags).

2. *An RBC Model with Additive Technology Shocks.* (Blanchard-Fischer/Romer). Consider an economy consisting of a constant population of infinitely-lived individuals. The representative individual gets utility according to the expected value of

$$\sum_{t=0}^{\infty} (1+\rho)^{-t} u(C_t), \rho > 0.$$

The utility kernel is $u(C) = C - \theta C^2$, where $\theta > 0$. Assume that C is always in the range where $u'(C) > 0$. Output is linear in capital and a technology shock:

$$Y_t = AK_t + e_t,$$

where A is the (constant) marginal product of capital. Assume $A = \rho$ for simplicity. Capital accumulates with no depreciation:

$$K_{t+1} = K_t + Y_t - C_t.$$

Finally, the disturbance follows a first-order autoregressive process:

$$e_t = \phi e_{t-1} + \epsilon_t$$

and the ϵ_t's are mean-zero i.i.d. shocks that are normally distributed with variance σ_ϵ^2. Also, ϵ_t is realized before any choices are made at time t. Assume $-1 < \phi < 1$.

(a) Define a competitive equilibrium for an economy with this economic environment.

(b) Rather than solve for the competitive equilibrium allocation directly, we will make use of the welfare theorems to solve the planner's problem. What is Bellman's functional equation for the planner's problem?

(c) Find the Euler equation for consumption under uncertainty. That is, find the first-order condition that relates the marginal utility of consumption today to the conditional expection of marginal utility tomorrow.

(d) We will use the method of undetermined coefficients to solve this problem. Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess, what is K_{t+1} as a function of K_t and e_t?

(e) What values must the parameters α, β, and γ have for the first-order condition in part (a) to be satisfied for all values of K_t and e_t?

(f) What are the effects of a one-time shock to ϵ on the time paths of Y, K, and C?

(g) Now suppose $\phi = 1$. What are the effects of a one-time shock to ϵ on the time paths of Y, K, and C?