A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy*

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Abstract

This paper develops and estimates a macro-finance model that combines a canonical affine no-arbitrage finance specification of the term structure with standard macroeconomic aggregate relationships for output and inflation. From this new empirical formulation, we obtain several important results: (1) the latent term structure factors from finance no-arbitrage models appear to have important macroeconomic and monetary policy underpinnings, (2) there is no evidence of monetary policy inertia or a slow partial adjustment of the policy interest rate by the Federal Reserve, and (3) both forward-looking and backward-looking elements play important roles in macroeconomic dynamics.

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1. Introduction

Bonds of various maturities all trade simultaneously in a well-organized market that appears to preclude opportunities for financial arbitrage. Indeed, the assumption of no arbitrage is central to an enormous literature that is devoted to the empirical analysis of bond pricing and the yield curve. This research has found that almost all movements in the yield curve can be captured in a no-arbitrage framework in which yields are linear functions of a few unobservable or latent factors (e.g., Duffie and Kan 1996, Litterman and Scheinkman 1991, and Dai and Singleton 2000). However, while these affine no-arbitrage models are extremely popular and do provide useful statistical descriptions of the term structure, they offer little insight into the economic nature of the underlying latent factors or forces that drive movements in interest rates. To provide such insight, this paper combines a canonical affine no-arbitrage model of the term structure with a standard macroeconomic model.

The short-term interest rate is a critical point of intersection between the finance and macroeconomic perspectives. From a finance perspective, the short rate is a fundamental building block for rates of other maturities because long yields are risk-adjusted averages of expected future short rates. From a macro perspective, the short rate is a key policy instrument under the direct control of the central bank, which adjusts the rate in order to achieve the economic stabilization goals of monetary policy. Together, the two perspectives suggest that understanding the manner in which central banks move the short rate in response to fundamental macroeconomic shocks should explain movements in the short end of the yield curve; furthermore, with the consistency between long and short rates enforced by the no-arbitrage assumption, expected future macroeconomic variation should account for movements farther out on the yield curve as well.

In our combined macro-finance analysis, we find that the standard no-arbitrage term structure factors do have clear macroeconomic underpinnings, which provide insight into the behavior of the yield curve. Conversely, a joint macro-finance perspective can also illuminate various macroeconomic issues, since the addition of term structure information to a macroeconomic model can help sharpen inference. For example, in a macro-finance model, the term structure factors, which summarize expectations about future interest rates, in turn reflect expectations about the future dynamics of the economy. With forward-looking economic agents, these expectations should be important determinants of current and future macroeconomic variables. The relative importance of forward- versus backward-looking elements in the dynamics of the economy is an important unresolved issue in macroeconomics that the incorporation of term structure information may help resolve. Indeed, in our joint macro-finance estimates, the forward-looking elements play an important role in macroeconomic dynamics. Another hotly debated macro issue is whether central banks engage in interest rate smoothing or gradual partial adjustment in setting monetary policy (e.g., Rudebusch 2002b). With the inclusion of information from the term structure, we show that, contrary to much speculation in the literature, central banks do not conduct such inertial policy actions.

We begin our analysis in the next section by estimating an off-the-shelf affine no-arbitrage model of the term structure. As usual, this model is estimated using data on yields but not macroeconomic variables. We label this standard model the "yields-only" model to distinguish it from our later, more general "macro-finance" model that adds macroeconomic content. Our yields-only model introduces the affine, no-arbitrage term structure representation and provides a useful benchmark to evaluate the combined macro-finance model. One distinctive feature of our yields-only model is that it has only two latent factors instead of the three factors that are more commonly—though by no means exclusively—used. Our choice of just two factors reflects the fact that they appear quite sufficient to account for variation in the yield curve during our fairly short sample, which runs from 1988 to 2000. Our use of a short sample is motivated by our interest in relating the term structure factors to macroeconomic fundamentals. Although relationships among yields may have remained stable for much of the postwar period, as implicitly assumed by most term structure analyses, the preponderance of empirical evidence suggests that the relationships between interest rates and macroeconomic variables may have changed during the past 40 years, as the reaction function setting monetary policy has changed (e.g., Fuhrer 1996). Accordingly, while a yields-only model may appear stable during the entire postwar period, a macro-finance model likely will not; therefore, we limit our sample to a recent short interval of plausible stability in the monetary policy regime.

In Section 3, we provide some initial evidence on the relationship between the term structure factors and macroeconomic variables. Specifically, we are interested in reconciling the yields-only latent factor finance representation of the short rate with the usual macroeconomic monetary policy reaction function. In the former, the short rate is the sum of various latent factors, while in the latter, for example, in what is commonly known as the Taylor rule (Taylor 1993), the short rate is the sum of multiples of inflation and real resource utilization. Section 3 reconciles the finance and macro representations by suggesting an interpretation of one of the latent factors as a perceived inflation target and the other as a cyclical monetary policy response to the economy.

Section 4 builds on this interpretation and constructs a complete model that combines an affine no-arbitrage term structure with a small macroeconomic model that has rational expectations as well as inertial elements. The combined macro-finance model is estimated from the data by maximum likelihood methods and demonstrates a fit and dynamics comparable to the separate yields-only model and a stand-alone macroeconomic model. The contribution of Section 4 is to provide a unified framework containing both models that is estimated from the data. This new framework is able to interpret the latent factors of the yield curve in terms of macroeconomic variables. It also sheds light on the importance of inflation and output expectations in the economy and the extent of monetary policy inertia or partial adjustment. Section 5 concludes with suggestions for future applications of this model.

Several other recent papers also have explored macroeconomic influences on the yield curve, and it is perhaps useful to provide a brief comparison of our analysis to this research. Overall, the broad contour of our results is quite consistent with much of this recent research, which relates the general level of interest rates to an expected underlying inflation component and the slope or tilt of the yield curve to monetary policy actions. However, there are three distinctive features of our work. First, we use a structural macroeconomic specification of the kind that has been quite popular in recent macro research. A similar model—which is essentially a monthly version of the formulation in Rudebusch (2002a)—was employed in an analysis of German data by Hördahl, Tristani, and Vestin (2002). In contrast, many other papers have related macro variables to the yield curve using little or no macroeconomic structure, including, for example, Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2003), Wu (2001), Dewachter and Lyrio (2002), Kozicki and Tinsley (2001), Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2003), and Evans and Marshall (2001). Second, in conformity with the vast finance literature, we use an affine no-arbitrage structure in which the yield curve (and the price of risk) depends on a few latent factors. This arrangement allows a clear comparison of the term structure elements in our model to the parallel existing finance literature. In contrast, some recent research, such as Evans and Marshall (2001) does not impose the no-arbitrage finance restrictions, while other research, such as Ang and Piazzesi (2003) and Hördahl, Tristani, and Vestin (2002), impose the restrictions but model the term structure in terms of both observable macro factors and residual unobserved factors, which are not necessarily comparable to the unobserved factors in traditional finance models. Finally, as in Diebold, Rudebusch, and Aruoba (2003), our model also allows for a bi-directional feedback between the term structure factors and macro variables. In contrast, as in Ang and Piazzesi (2003), the macro sector is often modeled as completely exogenous to the yield curve.

2. A No-Arbitrage Yields-Only Model

We begin by estimating a standard finance model of the term structure, which is based on the assumption that there are no riskless arbitrage opportunities among bonds of various maturities. This model has no explicit macroeconomic content; however, it introduces the affine, no-arbitrage bond pricing specification and provides a yields-only baseline for comparison with the combined macro-finance model below.

The canonical finance term structure model contains three basic equations. The first is the transition equation for the state vector relevant for pricing bonds. We assume there are two latent factors L_t and S_t and that the state vector, $F_t = (L_t, S_t)'$, is a Gaussian $VAR(1)$ process:

$$F_t = \rho F_{t-1} + \Sigma \varepsilon_t, \tag{2.1}$$

where ε_t is *i.i.d.* $N(0, I_2)$, Σ is diagonal, and ρ is a 2×2 lower triangular matrix. The second equation defines the one-period short rate i_t to be a linear function of the latent variables with a constant δ_0:

$$i_t = \delta_0 + L_t + S_t = \delta_0 + \delta_1' F_t. \tag{2.2}$$

Without loss of generality, equation (2.2) implies unitary loadings of the two factors on the short rate because of the normalization of these unobservable factors. Finally, following Constantinides (1992), Dai and Singleton (2000, 2002), Duffee (2002), and others, the price of risk associated with the shocks ε_t is defined to be a linear function of the state of the economy[1]:

$$\Lambda_t = \begin{bmatrix} \Lambda_L \\ \Lambda_S \end{bmatrix}_t = \lambda_0 + \lambda_1 F_t. \tag{2.3}$$

The state transition equation (2.1), the short rate equation (2.2), and the price of risk (2.3) form a discrete-time "essentially affine" Gaussian two-factor term structure model (Duffee,

[1] As Duffee (2002) argues, compared to the specification in which the risk price is a multiple of the volatility of the underlying shocks, this alternative specification allows the compensation for interest rate risk to vary independently of such volatility. Such flexibility proves to be useful in forecasting the future bond yields both in and out of sample. For an intuitive discussion of the price of risk, see Fisher (2001).

2002). In such a structure, the logarithm of the price of a j-period nominal bond is a linear function of the factors

$$\ln(b_{j,t}) = \overline{A}_j + \overline{B}_j' F_t, \tag{2.4}$$

where, as shown in the appendix, the coefficients $\overline{A}_j$ and $\overline{B}_j$ are recursively defined by

$$\begin{aligned}
\overline{A}_1 &= -\delta_0; \qquad \overline{B}_1 = -\delta_1 && (2.5)\\
\overline{A}_{j+1} - \overline{A}_j &= \overline{B}_j'(-\Sigma\lambda_0) + \frac{1}{2}\overline{B}_j'\Sigma\Sigma'\overline{B}_j + \overline{A}_1 && (2.6)\\
\overline{B}_{j+1} &= \overline{B}_j'(\rho - \Sigma\lambda_1) + \overline{B}_1; \qquad j = 1, 2, ..., J. && (2.7)
\end{aligned}$$

Given this bond pricing, the continuously compounded yield to maturity $i_{j,t}$ of a j-period nominal zero-coupon bond is given by the linear function

$$i_{j,t} = -\ln(b_{j,t})/j = A_j + B_j' F_t, \tag{2.8}$$

where $A_j = -\overline{A}_j/j$ and $B_j = -\overline{B}_j/j$.

For a given set of observed yields, the likelihood function of this model can be calculated in closed form and the model can be estimated by maximum likelihood. We estimate this model using end-of-month data from January 1988 to December 2000 on five U.S. Treasury yields that have maturities of 1, 3, 12, 36, and 60 months. (The yields are expressed at an annual rate in percent.) Since there are two underlying latent factors but five observable yields, we follow the usual strategy and assume that the 3-, 12-, and 36-month yields are measured with *i.i.d.* error, as in Ang and Piazzesi (2003). The estimated size of such measurement error is one common metric to assess model fit.

We limit the estimation sample in order to increase the chance that it is drawn from a single stable period of monetary policy behavior. Over the entire postwar sample, the reaction of the Federal Reserve in adjusting the short rate in response to macroeconomic shocks appears to have changed.[2] In particular, the Fed's short rate response to changes in inflation during the 1970s has been found to be less vigorous than in the 1990s. Such a change across these two periods would likely alter the relationship between the term structure and macroeconomic variables. To avoid such instability, our fairly short sample period falls completely within Alan Greenspan's tenure as Fed Chairman, which is often treated as a consistent monetary policy regime.

[2] For example, see Fuhrer (1996), Judd and Rudebusch (1998), Clarida, Galí, and Gertler (2000), and Rudebusch (2003).

For our sample, just two factors appear sufficient to capture movements in the yield curve. This perhaps reflects the exclusion from our sample of the period of heightened interest rate volatility during the late 1970s and early 1980s. One indication of the superfluous nature of a third factor is provided by a principal component analysis. In our sample, the first principal component captures 93.1 percent of the variation in the five yields, and the first and second principal components together capture 99.3 percent of the variation. That is, just two components can account for essentially all of the movements in the yield curve.[3]

The parameter estimates of the yields-only model are reported in Table 1.[4] As is typically found in empirical estimates of such a term structure model, the latent factors differ somewhat in their time-series properties as shown by the estimated ρ. The factor L_t is very persistent, while S_t is less so. There is also a small but significant cross-correlation between these factors. The parameters in λ_1, which determine the time variation in the price of risk, appear significant as well. Finally, the model fits the 3-, 12-, and 36-month rates with measurement error standard deviations of 20, 35, and 16 basis points, respectively.

The factor loadings of the yields-only model are displayed in Figure 1. These loadings show the initial response of yields of various maturities to a one percentage point increase in each factor. A positive shock to L_t raises the yields of all maturities by almost an identical amount. This effect induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the L_t factor is often called a “level” factor, which is a term we will adopt. Likewise, a positive shock to S_t increases short-term yields by much more than the long-term yields, so the yield curve tilts and becomes less steeply upward sloped (or more steeply downward sloped); thus, this factor is termed the “slope” factor.

Table 2 reports the variance decomposition for 1-month, 12-month, and 5-year yields at different forecast horizons. The level factor L_t accounts for a substantial part of the variance at the long end of the yield curve at all horizons and at the short and middle ranges of the yield curve at medium to long horizons. At shorter horizons, the slope factor S_t accounts for much of the variance of the short and middle ranges of the yield curve.

Overall, the results in Tables 1 and 2 and Figure 1 reveal an empirical no-arbitrage model—even over our short sample—that is quite consistent with existing estimated models in the

[3] Bomfim (2003) also finds that a two-factor model fits a 1989-2001 term structure sample very well.

[4] Note that in the pricing formula (2.6), the constant λ_0 only enters the definition of $\bar{A}_j$; therefore, changes in λ_0 affect only the steady-state shape of the yield curve and not its variation over time. To reduce the number of parameters to be estimated, we impose the restriction that $\lambda_0 = 0$. Accordingly, we de-mean the bond yields and focus on the variations of yields from sample averages in the model estimation.

empirical finance literature on bond pricing.

3. Term Structure Factors and Monetary Policy

This section compares the finance and macro views of the short-term interest rate. It tries to reconcile these two views by relating the yields-only term structure factors obtained above to macroeconomic variables and monetary policy, in order to provide some motivation for the combined macroeconomic and term structure model that is rigorously estimated in Section 4.

As noted above, the model of choice in finance decomposes the short-term interest rate into the sum of unobserved factors:

$$i_t = \delta_0 + L_t + S_t. \tag{3.1}$$

These factors are then modeled as autoregressive time series that appear unrelated to macroeconomic variation.

In contrast, from a macro perspective, the short rate is determined by a monetary policy reaction function:

$$i_t = G(X_t) + u_t, \tag{3.2}$$

where X_t is a vector of observable macroeconomic variables and u_t is an unobserved shock (as in, for example, Rudebusch and Svensson 1999 and Taylor 1999). As an empirical matter, many different formulations of the reaction function $G(X_t)$ have been estimated by various researchers. In large part, this diversity reflects the complexity of the implementation of monetary policy. Central banks typically react with some flexibility to real-time data on a large set of informational indicators and variables. This reaction is difficult to model comprehensively with a simple linear regression using final revised data. (See, for example, discussion and references in Rudebusch 1998, 2002b.)

Still, a large number of recent empirical studies of central bank behavior have employed some variant of the Taylor (1993) rule to estimate a useful approximation to the monetary policy reaction function.[5] One version of the Taylor rule can be written as

$$i_t = r^* + \pi_t^* + g_\pi(\pi_t - \pi_t^*) + g_y y_t + u_t, \tag{3.3}$$

where r^* is the equilibrium real rate, π_t^* is the central bank's inflation target, π_t is the annual inflation rate, and y_t is a measure of the output gap or capacity utilization. In this Taylor

[5] In the U.S., these include Clarida, Galí, and Gertler (2000), Kozicki (1999), Judd and Rudebusch (1998), and Rudebusch (2002b).

rule, the short-term interest rate is set equal to its long-run level $(r^* + \pi_t^*)$ plus two cyclical adjustments to respond to deviations from the macroeconomic policy goals, specifically, the distance of inflation from an inflation target, $\pi_t - \pi_t^*$, and the distance of real output from its long-run potential, y_t.

Although the finance and macro representations of the short rate (3.1) and (3.3) appear dissimilar and are obtained in very different settings, we would argue that there is in fact a close connection between them.[6] A key element in making this connection is the identification of L_t, which captures movements in the general level of nominal interest rates, with the neutral level of the short rate; that is, we consider L_t to be a close approximation to $r^* + \pi_t^*$. To support this assumption, Figure 2 provides some suggestive evidence about the relationship between the yields-only level factor and inflation. It displays the factor L_t, annual inflation π_t (which is the de-meaned 12-month percent change in the price index for personal consumption expenditures), the one-year-ahead expectation of annual inflation (which is the de-meaned expectation from the Michigan survey of households—as in Rudebusch 2002a), and the 10-year-ahead inflation expectation. The last of these, which is measured as the spread between 10-year nominal and indexed Treasury debt, is only available starting in 1997 with the first issuance of indexed debt. In Figure 2, the estimated yields-only L_t appears to be closely linked to actual and expected inflation at both high and low frequencies. Over the entire sample, actual and expected inflation and the level factor all have slowly trended down about 2 percentage points. This decline is consistent with the view that over this period the Federal Reserve conducted an opportunistic disinflation, with a gradual ratcheting down of inflation and the inflation target over time (Bomfim and Rudebusch 2000). Alternatively, our analysis considers only private sector perceptions, and there may be differences between the true and perceived monetary policy inflation targets. Indeed, as shown in Orphanides and Williams (2003), when private agents have to learn about the monetary policy regime, long-run inflation expectations may drift quite far from even a constant true central bank inflation target.

The general identification of the overall level of interest rates with the perceived inflation goal of the central bank is a common theme in the recent macro-finance literature (notably, Kozicki and Tinsley 2001, Dewachter and Lyrio 2002, and Hördahl, Tristani, and Vestin 2002). Variation in r^* also could account for some of the variation in L_t; however, we assume that month-to-

[6] Ang and Piazzesi (2003) and Dewachter and Lyrio (2002) note a similar connection.

month variation in π_t^* dominates the L_t fluctuations during our 1988 to 2000 sample. This is quite plausible given the evidence from the market for indexed debt shown in Figure 2, in which movements in the two solid lines track each other very closely (after adjusting for the mean). Based on such evidence, Gürkaynak, Sack, and Swanson (2003) also argue that movements in long rates reflect fluctuations in inflation perceptions and not real rates. Similarly, a constant r^* is commonly assumed in the literature on Taylor rules.[7]

As a first approximation then, we identify movements in L_t with movements in the perceived inflation target of the central bank, π_t^*. However, finding a stable systematic link between an estimated level factor L_t and a small set of observables is difficult. In practice, as the perceived anchor for inflation, L_t is likely to be a complicated function of past inflation, expected future inflation, general macroeconomic conditions, and even Federal Reserve statements and other actions regarding policy goals. For tractability, we consider only a very simple filtering scheme where L_t is a weighted average of current inflation and the lagged level factor:

$$L_t = \rho_l L_{t-1} + (1 - \rho_l)\pi_t + \varepsilon_{L,t}. \tag{3.4}$$

We will embed this type of relationship in our complete macro-finance model estimated in the next section. However, even in a simple single-equation OLS regression using our very short sample of data, this formulation has some support using the yields-only level factor:

$$\begin{aligned} L_t &= \underset{(0.03)}{0.96} L_{t-1} + \underset{(0.03)}{0.04}\pi_t + \varepsilon_{L,t} \\ \bar{R}^2 &= .91, \qquad \sigma_{\varepsilon_L} = .27. \end{aligned} \tag{3.5}$$

Still, we view the specification (3.4) as only a useful but imperfect approximation.[8]

Given the identification of L_t with the inflation target, the remaining slope factor should capture the cyclical response of the central bank; that is, $S_t = g_\pi(\pi_t - L_t) + g_y y_t$. Again, in Section 4, we will rigorously estimate this relationship in a complete macro-finance model. However, the simple OLS regression of the yields-only slope factor on inflation and output gives remarkably promising results:

[7] Exceptions include Rudebusch (2001), Laubach and Williams (2003), and Trehan and Wu (2003); however, even in these exceptions, r^* varies quite slowly over time.

[8] Kozicki and Tinsley (2001) also support such an "adaptive learning" specification, while Hördahl, Tristani, and Vestin (2002) use an even simpler (near-) random walk process for underlying inflation. In our level regression, we cannot reject the restriction that the coefficients sum to one at the 10 percent level, and as will be clear below, this restriction provides nominal interest rates and inflation with dynamic homogeneity.

$$
\begin{aligned}
S_t &= 1.28(\pi_t - L_t) + 0.46 y_t + u_{S,t} \qquad (3.6)\\
&\quad\;\;\;(0.15) \qquad\qquad\;\; (0.06)\\
\bar{R}^2 &= .52, \qquad \sigma_{u_S} = .87,
\end{aligned}
$$

where y_t is de-meaned industrial capacity utilization, which we will typically refer to as output, although it is, strictly speaking, a measure of the output gap.[9] In this regression, the coefficients represent estimates of the policy response of the Fed. Specifically, the estimate of $g_\pi = 1.28$ reflects the inflation response: If inflation moves one percentage point above its target (L_t), the Fed raises S_t (or, roughly, the short rate relative to the long rate) by 128 basis points.[10] Similarly, given the estimated output response $g_y = .46$, if real utilization rises one percentage point, then the Fed raises S_t by 46 basis points. These estimated policy responses are very close to the values originally proposed by Taylor (1993) and to the Taylor rule estimates obtained in various empirical studies (for example, Kozicki 1999, Judd and Rudebusch 1998, and Rudebusch 2001). (Capacity utilization is about 1.4 times more cyclically variable than the output gap, so the equivalent g_y estimate in output gap terms is about .65.) For our purposes, however, what is most important about the Taylor rule regression (3.6) is that the associated fitted slope factor, $\hat{S}_t$ tracks the actual yields-only slope factor S_t quite well ($\bar{R}^2 = .52$). This correspondence is shown in Figure 3, which displays the yields-only slope factor S_t as a solid line and the fitted Taylor rule values $\hat{S}_t$ as a dashed line. The close connection between S_t and $\hat{S}_t$ suggests that the Taylor rule, which partitions the short rate into a neutral rate and a cyclical component, can be appropriately identified with the usual finance partition of the short rate into level and slope. However, in the next section, we will provide a structural macro-finance system estimation of this relationship that is much more rigorous and compelling.

Despite a fairly remarkable fit in Figure 3, some large persistent differences between S_t and $\hat{S}_t$ do remain, and these differences have been at the center of an important debate in macroeconomics. These serially correlated deviations—denoted $u_{S,t}$—have been given two different interpretations in the literature. The first of these is that the deviations reflect a slow partial adjustment by the Fed of the actual short-term interest rate to its desired value as given by the

[9] Given serial correlation in the regression errors, which is discussed below, robust standard errors for the coefficients are reported in parentheses. The restriction that the coefficients on π_t and L_t are equal with opposite sign cannot be rejected at the 5 percent significance level. While Taylor rule estimates are typically obtained using quarterly deviations of GDP from potential, for our monthly analysis, capacity utilization provides an anlogous measure of the output gap for the industrial sector.

[10] As typical for Taylor rule estimates in this sample, the inflation response coefficient is greater than one, so the Fed acts to damp increases in inflation by raising nominal and real interest rates—the so-called Taylor principle for economic stabilization.

policy rule. Such behavior is often called monetary policy inertia or interest rate smoothing, and it suggests a partial adjustment dynamic specification for the slope factor such as

$$S_t = (1-\rho_S)(g_\pi(\pi_t - L_t) + g_y y_t) + \rho_S S_{t-1} + \varepsilon_{S,t}. \tag{3.7}$$

This specification and its structural partial adjustment interpretation have been used by Woodford (1999), Clarida, Galí, and Gertler (2000), and many others.

In contrast, a second interpretation of the deviations between S_t and $\hat{S}_t$ is that they represent inadequacies in the Taylor rule in modeling all of the various influences on monetary policy. Under this view, the $u_{S,t}$ are the effect of policy responses to special circumstances and information that were not captured by the simple Taylor rule specification but were important to policymakers (as described in Rudebusch 2002b). Indeed, the persistent deviations between the actual and fitted slope factors in Figure 3 appear to correspond to several special episodes in which policy reacted to more determinants than just current readings on inflation and output. Most notably, the deviation in 1992 and 1993, when the actual slope factor (and associated short rates) was pushed much lower than the fitted slope based on inflation and output readings, is typically interpreted as a Federal Reserve response to a persistent "credit crunch" shock or disruption in the flow of credit.[11] This interpretation of the dynamics of the Taylor rule suggests a specification such as

$$S_t = g_\pi(\pi_t - L_t) + g_y y_t + u_{S,t}\ ; \qquad u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}. \tag{3.8}$$

In this specification, the AR(1) serially correlated shocks represent the Fed's reaction to persistent influences—beyond current inflation and output.[12]

Choosing between the partial adjustment and serially correlated shocks specifications depends crucially on separating the influences of contemporaneous and lagged regressors, which are typically difficult to untangle in a single equation context (e.g., Blinder 1986). As Rudebusch 2002b stresses, this problem is particularly acute for estimated monetary policy rules, because uncertainty in modeling the desired policy rate (given the endogeneity of regressors, the real-time nature of the information set, and the small samples available) makes the single-equation

[11] As Fed Chairman Alan Greenspan testified to Congress on June 22, 1994: "Households and businesses became much more reluctant to borrow and spend and lenders to extend credit—a phenomenon often referred to as the 'credit crunch.' In an endeavor to defuse these financial strains, we moved short-term rates lower in a long series of steps that ended in the late summer of 1992, and we held them at unusually low levels through the end of 1993—both absolutely and, importantly, relative to inflation."

[12] These rule deviations are not exogenous monetary policy shocks that represent actions independent of the economy; instead, they are endogenous responses to a variety of influences that cannot be captured by some easily observable variable such as output or inflation. For example, persistent deviations between the true and perceived inflation goals could show up as serially correlated residuals.

evidence on the rule's dynamic specification suspect.[13] Thus, a policy rule with slow partial adjustment and no serial correlation in the errors will be difficult to distinguish empirically on its own from a policy rule that has immediate policy adjustment but highly serially correlated shocks. However, information contained in the term structure can help distinguish between these two interpretations. In particular, Rudebusch (2002b) demonstrates that a slow partial adjustment of the short rate to new information by the Fed should imply the existence of predictable future variation in the short rate that is not present with serially correlated shocks. In fact, the general lack of predictive information in the yield curve about changes in the short rate suggests the absence of policy inertia. In the next section, for the first time, it will be possible to rigorously analyze this issue in a combined model that includes the macro variables as well as a no-arbitrage term structure. Our general model will allow for both types of policy rule dynamics—that is, partial adjustment and persistent shocks—and let the data judge between these interpretations.

4. A Complete Macro-Finance Model

Inspired by the above yields-only factor regression results, this section presents a combined macro-finance model in which the term structure factors are jointly estimated with macroeconomic relationships. These results provide a rigorous system estimation of the relationships described in Section 3. We first describe the equations of the model and then provide maximum likelihood estimates and analysis.

4.1. Model Structure

In the macro-finance model, the one-month short rate is defined to be the sum of two latent term structure factors

$$i_t = \delta_0 + L_t + S_t, \tag{4.1}$$

[13] Also, see English, Nelson, and Sack (2003) and Söderlind, Söderström, and Vredin (2003).

as in a typical affine no-arbitrage term structure representation. However, as suggested by the above yields-only factor regressions, the dynamics of these latent factors are given by

$$L_t = \rho_L L_{t-1} + (1-\rho_L)\pi_t + \varepsilon_{L,t} \tag{4.2}$$

$$S_t = \rho_S S_{t-1} + (1-\rho_S)[g_y y_t + g_\pi(\pi_t - L_t)] + u_{S,t} \tag{4.3}$$

$$u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}, \tag{4.4}$$

where π_t and y_t are inflation and output (specifically, capacity utilization) and L_t and S_t denote the unobserved macro-finance term structure factors. Although they retain the same notation, the estimated macro-finance L_t and S_t factors will of course differ somewhat from their yields-only counterparts. As shown below, however, the differences are small.

These equations provide macroeconomic underpinnings for the latent term structure factors. In equation (4.2), the factor, L_t, is interpreted as the underlying rate of inflation, that is, the inflation rate targeted by the central bank, as perceived by private agents. Agents are assumed to slowly modify their views about L_t as actual inflation changes. As we shall see from the empirical factor loadings below, L_t will be associated with the level of yields with maturities from 2 to 5 years, which is an important indication of the appropriate horizon to associate with the inflation expectations embodied in L_t. In equation (4.3), which mimics the classic Taylor rule, the slope factor S_t captures the central bank's dual mandate to stabilize the real economy and keep inflation close to its target level. Given the 2- to 5-year horizon of inflation expectations embodied in L_t, we believe this factor represents an interim or medium-term inflation target (as in Bomfim and Rudebusch 2000). Accordingly, in (4.3), the central bank is assumed to be attempting to close the gap between actual inflation and this interim inflation target. In addition, the dynamics of S_t allow for both partial adjustment and serially correlated shocks. If $\rho_u = 0$, the dynamics of S_t arise from monetary policy partial adjustment, as in equation (3.7). Conversely, if $\rho_S = 0$, the dynamics reflect the Fed's reaction to serially correlated information or events not captured by output and inflation, as in equation (3.8).

We close the above equations with a standard small macroeconomic model of inflation and output. Much of the appeal of this so-called New Keynesian specification is its theoretical foundation in a dynamic general equilibrium theory with temporary nominal rigidities; however, we focus on just the two key aggregate relationships for output and inflation.[14] One notable

[14] For explicit derivations and discussion, see Goodfriend and King (1997), Walsh (2003), Svensson (1999), Clarida, Galí, and Gertler (1999), Rudebusch (2002a), and Dennis (2003).

feature of our specification is its flexibility in being able to vary the amount of explicitly forward-looking versus backward-looking behavior in the determination of the macroeconomic variables. The relative contribution of these two elements is an important unresolved issue in the empirical macro literature.

A standard theoretical formulation for inflation is

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1 - \mu_\pi)\pi_{t-1} + \alpha_y y_t + \varepsilon_{\pi,t} \ , \tag{4.5}$$

where $E_t\pi_{t+1}$ is the expectation of period $t+1$ inflation conditional on a time t information set.[15] In this specification, the current (one-period) inflation rate is determined by rational expectations of future inflation, lagged inflation, and output. A key parameter is μ_π, which measures the relative importance of forward- versus backward-looking pricing behavior.[16] Since our model is estimated with monthly data, its empirical specification differs from (4.5). Given the institutional length of price contracts in the real world, the one-period leads and lags in theory are typically assumed to pertain to periods much longer than one month; indeed, empirical macroeconomic analyses invariably use data sampled at a quarterly or even annual frequency. For estimation with monthly data, we reformulate (4.5), with longer leads and lags as,[17]

$$\pi_t = \mu_\pi L_t + (1 - \mu_\pi)[\alpha_{\pi_1}\pi_{t-1} + \alpha_{\pi_2}\pi_{t-2}] + \alpha_y y_{t-1} + \varepsilon_{\pi,t}. \tag{4.6}$$

In this specification, inflation in the current month is set as a weighted average of the public's expectation of the medium-term inflation target, which we identify as L_t, and two lags of inflation. Also, there is a one-month lag on the output gap to reflect the usual adjustment costs and recognition lags.

The standard New Keynesian theory of aggregate demand can be represented by an intertemporal Euler equation of the form:

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} - \beta_r (i_t - E_t \pi_{t+1}) + \varepsilon_{y,t} \ . \tag{4.7}$$

Current output is determined by expected future output, $E_t y_{t+1}$, lagged output, and the ex ante real interest rate. The parameter μ_y measures the relative importance of expected future

[15] As above, data are de-meaned, so no constants are included in the macro equations.

[16] As a theoretical matter, the value of μ_π is not clearly determined. From well-known models of price-setting behavior, it is possible to derive an inflation equation with $\mu_\pi \approx 1$. However, many authors assume that with realistic costs of adjustment and overlapping price and wage contracts there will be some inertia in inflation, so μ_π will be less than one (Svensson 1999, Fuhrer and Moore 1995, and Fuhrer 1997).

[17] Again, for the empirical analysis, π_t is defined as the 12-month percent change in the personal consumption expenditures price index (P_t) in percent at an annual rate (i.e., $\pi_t \equiv 12(p_t - p_{t-1})$, where $p_t = 100 \ln P_t$).

output versus lagged output, where the latter term is crucial to account for real-world costs of adjustment and habit formation (e.g., Fuhrer 2000 and Fuhrer and Rudebusch 2003). For empirical implementation with monthly data, we estimate an equation of the form:

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y)[\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}] - \beta_r (i_{t-1} - L_{t-1}) + \varepsilon_{y,t} \ . \tag{4.8}$$

This equation has an additional lag of output, but the key difference is the specification of the ex ante real interest rate, which is given by $i_{t-1} - L_{t-1}$; that is, agents judge nominal rates against their view of the underlying future inflation not just next month's inflation rate.[18] Also, because our yields data are end-of-month observations, the $t - 1$ timing of the real rate is appropriate for the determination of time t output.

The factor L_t, which we interpret as medium-term inflation expectations, enters the macro-finance model in several contexts. It is the interim inflation target in the policy rule, the expectational anchor for price determination, and the benchmark for the evaluation of nominal interest rates in output determination. This triple role for L_t allows for substantial modeling simplification at the cost of some potential misspecification. Typically, policy rules involve longer-horizon inflation targets and inflation and output equations use shorter-horizon inflation expectations. We view our macro-finance specification as an economical compromise that, as shown below, provides a useful description of term structure and macroeconomic dynamics.

Finally, the specification of longer-term yields follows the standard no-arbitrage formulation described in Section 2 for the yields-only model. The state space of the combined macro-finance model can be expressed by equation (2.1) with the re-definition of the state vector F_t to include output and inflation. The dynamic structure of this transition equation is determined by the equations (4.2), (4.3), (4.4), (4.6), and (4.8). There are four structural shocks, $\varepsilon_{\pi,t}$, $\varepsilon_{y,t}$, $\varepsilon_{L,t}$, and $\varepsilon_{S,t}$, which are assumed to be independently and normally distributed. The short rate is determined by (4.1). For pricing longer-term bonds, the risk price associated with the structural shocks is assumed to be a linear function of just L_t and S_t and does not depend on the other state variables such as current or lagged π_t or y_t. Such a risk specification, which relies solely on the latent factors L_t and S_t to determine interest-rate risk compensations, matches the yields-only formulation in Section 2 and other empirical finance research and allows comparison with earlier work.[19] However, it should be noted that the macroeconomic shocks $\varepsilon_{\pi,t}$ and $\varepsilon_{y,t}$ are still able

[18] It would be interesting to augment i_t, as a determinant of output, with longer-maturity interest rates as well, but this is computationally difficult.

[19] Therefore, λ_1 continues to have just four non-zero entries, which greatly reduces the number of parameters

to affect the price of risk through their influence on π_t and y_t and therefore on the latent factors L_t and S_t. Given this structure, yields of any maturity are determined under the no-arbitrage assumption via equation (2.8). (See the appendix for details.)

4.2. Model Estimates

The above macro-finance model is estimated by maximum likelihood for the sample period from January 1988 to December 2000. (See the appendix for details.) The data on bond yields, inflation, and output (capacity utilization) are the same as defined above.

Before examining the parameter estimates of the model, it is useful to compare the time series of L_t and S_t extracted from the estimated macro-finance model with the ones extracted from the yields-only model. This is done in Figure 4 for L_t, and in Figure 5 for S_t. In both figures, the macro-finance model estimates of these factors (the solid lines) closely match the yields-only estimates (the dashed lines). Indeed, the two L_t factors have a correlation of .97, and the two S_t factors have a correlation of .98. This close correspondence suggests that our macro-finance factors L_t and S_t can indeed be treated (and termed) as level and slope factors and, more importantly, that our macro-finance interpretation of these factors has a direct bearing on the existing finance literature since we have obtained the very similar factors.

Table 3 reports the parameter estimates of the macro-finance model. First, consider the dynamics of the factors. The factor L_t is very persistent, with a ρ_L estimate of .989, which implies a small but significant weight on actual inflation. In contrast, the dynamics of S_t in the macro-finance model can be given a very different interpretation than in the yields-only model. Obviously, as evident in Figure 5, the estimates of S_t are persistent in both models; however, in the macro-finance model, this persistence does not come from partial adjustment since the ρ_S estimate is a minuscule .026. Instead, S_t responds with only a very short lag to output and inflation. The persistence in S_t reflects the fact that the Fed adjusts the short rate promptly to various determinants—output, inflation, and other influences in the residual u_t—that are themselves quite persistent (e.g., $\rho_u = .975$). Thus, our estimate of ρ_S decisively dismisses the interest rate smoothing or monetary policy inertia interpretation of the persistence in the short rate. The persistent deviations of slope from fitted slope shown in Figure 3 occur not because the Fed was slow to react to output and inflation but because the Fed responds to a variety of

to be estimated.

persistent determinants beyond current output and inflation.

The monetary policy interpretation of the slope factor is supported by the values of the estimated inflation and output response coefficients, g_π and g_y, which are 1.25 and 0.20, respectively. These estimates are similar to the usual single-equation estimates of the Taylor rule during this sample period (e.g., Rudebusch 2002b). Overall, the macro-finance model estimates confirm the interpretation suggested by the regressions in Section 3, although the system estimation of a complete model provides much tighter standard errors.

The estimated parameters describing the inflation dynamics also appear reasonable.[20] In particular, the estimated weight on explicit forward-looking expectations in determining inflation, μ_π, is 0.074. Since this estimate is based on monthly data, with time aggregation, it implies a weight of about 0.21 on the interim inflation objective at a quarterly frequency. This estimate appears consistent with many earlier estimates obtained using a variety of different methods and specifications. For example, using survey data on expectations, Rudebusch (2002a) obtains a broadly comparable μ_π estimate of 0.29, which is in the middle of the range of estimates in the literature. However, by using the yield curve to extract inflation expectations, our estimates bring new information to bear on this important macroeconomic question.

The estimated parameters describing the output dynamics also fall within reasonable ranges.[21] Specifically, the estimated value of $\mu_y = .009$, implies a negligible weight at a quarterly frequency on forward-looking output expectations in the determination of output behavior. This is very much in accord with the maximum likelihood estimation results reported by Fuhrer and Rudebusch (2003).

Finally, the risk price matrix (λ_1) appears significant, and the model fits the 3-month, 12-month, and 36-month yields with measurement error standard deviations that are quite comparable to the yields-only model.

4.3. Analysis of Dynamics

The dynamics of the estimated macro-finance model are quite interesting and intuitive. First, consider the instantaneous responses of the yield curve to a positive shock in L_t or S_t. These

[20] After taking into account time aggregation and the higher cyclical variability of capacity utilization compared with the output gap, the elasticity of inflation with respect to output ($\alpha_y = .014$) appears about half the size of estimates that use the entire postwar sample of quarterly data, for example, Rudebusch (2002a). The estimate does appear more in line with estimates obtained in recent shorter samples (Rudebusch 2001).

[21] The interest rate sensitivity of output ($\beta_r = .089$), after taking into account the time aggregation and the use of capacity utilization rather than the output gap, appears about twice the size of estimates that use the entire postwar sample of quarterly data, for example, Rudebusch (2002a).

responses are displayed in Figure 6.[22] As is clear from the structure of the factor dynamics above, a shock to L_t has two very different effects on the short rate i_t. First, it directly raises the short rate one-for-one according to equation (4.1). Second, from (4.3), an increase in L_t reduces S_t and pushes down the short rate by more than one-for-one—given the estimate of $g_\pi = 1.20$. The macroeconomic interpretation of this latter effect is that an increase in the perceived inflation target must be associated with an easing of monetary conditions so inflation can rise to its new target.[23] Given some persistence in inflation, easier monetary conditions (lower real rates) require an initial decline in the short-term nominal interest rate. This second effect dominates at the short end, so a positive shock to L_t initially lowers short-term yields. However, at intermediate- and long-term maturities, the first effect dominates, and the increase in L_t raises the yields one-for-one, as in the yields-only model. Therefore, the initial effect of an increase in L_t is not quite a parallel shift of the yield curve, but rather a tilt upward. The initial response of the yield curve to a positive shock in S_t is similar to the one shown in Figure 1 for the yields-only model. A positive shock to S_t (specifically to $\varepsilon_{S,t}$) increases short-term bond yields but has progressively less effect on bonds of greater maturity. Thus, the positive shock initially decreases the slope of the yield curve and produces a tilt downward.[24]

Similar to Figure 6, the solid lines in Figure 7 display the initial responses of the yield curve to inflation and output shocks in the estimated macro-finance model. Positive shocks to inflation and output in this model are followed by immediate increases in short-term interest rates, and for the inflation shock, these increases are more than one-for-one. These quick responses reflect the absence of monetary policy partial adjustment or inertia (the estimated $\rho_S = .026$). In contrast, the dashed lines in Figure 7 display the yield curve responses from a model that is identical to the estimated macro-finance model except that ρ_S is set equal to .9 and ρ_u equals 0. This hypothetical alternative model has substantial monetary policy inertia, and it displays markedly weaker responses to inflation and output shocks of yields that have maturities less than two years. The two quite different responses of the yield curve in these models illustrate the potential importance of the information from the term structure for discriminating between the two models. Given the system ML estimates, it is clear that the data prefer the macro-finance

[22] These instantaneous responses are analogous to those in Figure 1 for the yields-only model; but here the S_t "factor loadings" take into account the loadings on output and inflation.

[23] In a model without nominal rigidities or persistence, inflation would simply jump to the new target. Such a model, with $\mu_\pi = 1$, does not appear to fit the data.

[24] Figure 6 suggests that L_t and S_t might be better labled "Long-term" and "Short-term" factors because those are the locations of maximum influence; however, we will continue using the standard terminology.

model without policy inertia.

Now consider the dynamics of the macro-finance model more generally. Figures 8 and 9 display the impulse responses of the macroeconomic variables and bond yields to a one standard deviation increase in each of the four structural shocks in the model. Each response is measured as a percentage point deviation from the steady state. Figure 8 focuses on the macro shocks, and the first column shows the impulse responses to an inflation shock. Such a shock leads to an instant 25-basis point increase in the inflation rate, which is gradually reversed over the next two years. Inflation does not, however, return to its original level because the sustained period of higher inflation boosts perceptions of the underlying inflation target L_t. The initial jump in inflation also induces a tightening of monetary policy that raises the slope factor and short-term interest rates. Indeed, the 1-month rate first jumps about 30 basis points and then gradually falls. The 12-month and 5-year yields also increase in response to the inflation shock but by smaller amounts. The higher interest rates lead to a gradual decrease in output, which damps inflation.

The second column of Figure 8 displays the impulse responses to a positive output shock, which increases capacity utilization by .6 percentage point. The higher output gradually boosts inflation, and in response to higher output and inflation, the central bank increases the slope factor and interest rates. In contrast to the differential interest rate responses in the first column, all of the interest rates in the second column show fairly similar increases. The bond yields of all maturities are still approximately 5 basis points higher than their initial levels even 5 years after the shock, because the rise in inflation has passed through to the perceived inflation target L_t.

One particularly noteworthy feature of the responses in Figure 8 is how long-term interest rates respond to macroeconomic shocks. As stressed by Gürkaynak, Sack, and Swanson (2003), long rates do appear empirically to respond to news about macroeconomic variables; however, standard macroeconomic models generally cannot reproduce such movements because their variables revert to the steady state too quickly. By allowing for time variation in the inflation target, the macro-finance model can generate long-lasting macro effects and hence long rates that do respond to the macro shocks.

Figure 9 provides the responses of the variables to perceived changes in monetary policy. There are two types of such policy changes to consider (as in Haldane and Read 2000 and

Ellingsen and Söderström 2001): namely, changes in policy preferences and changes in macroeconomic policy determinants. In the macro-finance model, the first is a perceived shift in the inflation target or level factor.[25] The first column displays the impulse responses to such a level shock, which increases the inflation target by 34 basis points—essentially on a permanent basis. In order to push inflation up to this higher target, the monetary authority must ease rates, so the slope factor and the 1-month rate fall immediately after the level shock. The short rate then gradually rises to a long-run average that essentially matches the increase in the inflation target. The 12-month rate reaches the new long-run level more quickly, and the 5-year yield jumps up to that level immediately. The easing of monetary policy in real terms boosts output and inflation. Inflation converges to the new inflation target, but output returns to about its initial level.

The second column of Figure 9 displays the response to a slope shock, which is the second type of policy change: a perceived policy response to some development in the economy (other than current output and inflation). A one standard deviation slope shock raises the 1-month interest rate by 56 basis points, and raises the 12-month and 5-year yields by 42 and 5 basis points, respectively. In response to tighter monetary policy, the capacity utilization rate gradually declines, generating a decline in inflation as well. Falling inflation translates into perceptions of a declining inflation target, which eventually causes all interest rates to fall below their initial values.

Finally, a useful supplementary description of model dynamics can be obtained from the variance decomposition shown in Table 4. (However, we have limited confidence in the decompositions at the 60-month horizon because our sample only has two independent observations at that long horizon.) At the 12-month horizon, inflation is driven largely by shocks to inflation and the inflation target, and output is driven by shocks to output and the slope. The 1-month yield is driven by all four shocks, but predominantly by slope. The 12-month yield is driven by slope and level shocks and, to a lesser extent, by output and inflation shocks. Movements in the 5-year yield can be attributed to shocks to level.

[25] Such a shift could reflect the imperfect transparency of the underlying inflation goal in the U.S. or its imperfect credibility.

5. Conclusion

By constructing and estimating a combined macro-finance framework, this paper describes the economic underpinnings of the yield curve. In particular, it characterizes the relationships between the no-arbitrage latent term structure factors and various macroeconomic variables. The level factor is given an interpretation as the perceived medium-term central bank inflation target. The slope factor is related to cyclical variation in inflation and output gaps. In particular, the slope factor varies as the central bank moves the short end of the yield curve up and down in order to achieve its macroeconomic policy goals.

The estimated macro-finance model also provides several interesting empirical results. Notably, using a new methodology, the results confirm the conclusions of Rudebusch (2002b) that the amount of partial adjustment in the setting of monetary policy is negligible. Also, new information is drawn from the yield curve on the issue of the importance of expectations in the determination of output and inflation. These results confirm a statistically significant but limited role for expectations.

Still, there are several promising avenues for future research to improve the macro-finance linkages in this model. For example, the specification linking the level factor to inflation in our model is rudimentary and mechanical, since financial market participants in fact are undoubtedly conducting a subtle filtering of the available data to obtain underlying inflation objectives. Similarly, the link between the slope factor and output and inflation leaves much—notably, the large persistent residual $u_{s,t}$—to be explained rigorously. Presumably, an elaboration of the policy response to include real-time data and a forward-looking perspective would help. However, it should be noted that more complicated models quickly become computationally intractable for estimation.

A. Appendix on Bond Pricing and Macro-Finance Model Estimation

No-Arbitrage Bond Pricing

The state space of both models can be expressed as

$$F_t = \rho F_{t-1} + \Sigma\varepsilon_t. \tag{A.1}$$

In the yields-only model, the state vector F_t includes L_t and S_t, while in the macro-finance model, F_t includes output and inflation as described below. The above equation describes the evolution of the $n \times 1$ state vector F_t under the physical measure.

Suppose that under the equivalent martingale measure, the evolution of F_t follows

$$F_t = \kappa^Q + \rho^Q F_{t-1} + \Sigma\varepsilon_t, \tag{A.2}$$

where κ^Q is an $n \times 1$ vector and ρ^Q is an $n \times n$ transition matrix. The superscript Q denotes the parameters under the equivalent martingale measure.

Note that from the definition of the one-month interest rate (2.2), the logarithm of the price of a one-month bond can be expressed as

$$\begin{aligned} \ln(b_{1,t}) &= -\delta_0 - \delta_1' F_t \\ &= \overline{A}_1 + \overline{B}_1' F_t. \end{aligned} \tag{A.3}$$

Suppose that the logarithm of a j-month bond is

$$\ln(b_{j,t}) = \overline{A}_j + \overline{B}_j' F_t, \tag{A.4}$$

Thus the holding-period return on the j-month bond in period t is

$$\begin{aligned} hpr_{j,t} &= E_t^Q\left(\frac{b_{j-1,t+1}}{b_{j,t}}\right) - 1 \\ &= E_t^Q\{\exp(\overline{A}_{j-1} + \overline{B}_{j-1}' F_{t+1} - \overline{A}_j - \overline{B}_j' F_t)\} - 1 \\ &= \exp\{\overline{A}_{j-1} + \overline{B}_{j-1}' E_t^Q F_{t+1} - \overline{A}_j - \overline{B}_j' F_t + \frac{1}{2} Var_t^Q(\overline{B}_{j-1}' \varepsilon_{t+1})\} - 1 \\ &= \exp\{\overline{A}_{j-1} + \overline{B}_{j-1}'(\kappa^Q + \rho^Q F_t) - \overline{A}_j - \overline{B}_j' F_t + \frac{1}{2}\overline{B}_{j-1}'\Sigma\Sigma'\overline{B}_{j-1})\} - 1 \\ &= i_t = -\overline{A}_1 - \overline{B}_1' F_t. \end{aligned} \tag{A.5}$$

Comparing the coefficients yields

$$\begin{aligned} \overline{A}_1 + \overline{A}_{j-1} - \overline{A}_j + \overline{B}_{j-1}'\kappa^Q + \frac{1}{2}\overline{B}_{j-1}'\Sigma\Sigma'\overline{B}_{j-1} &= 0 \\ \overline{B}_1' + \overline{B}_{j-1}'\rho^Q - \overline{B}_j' &= 0. \end{aligned} \tag{A.6}$$

Next we provide links between the evolution of the state under the physical measure and the equivalent martingale measure, i.e., equations (A.1) and (A.2). Providing this connection is equivalent to specifying the dynamics of the price of risk. Given the risk price representation $\Lambda_t = \lambda_0 + \lambda_1 F_t$, the law of motion of the state vector F_t can be expressed as

$$\begin{aligned} F_t &= \kappa^Q + \rho^Q F_{t-1} + \Sigma\Lambda_t + \Sigma\varepsilon_t \\ &= (\kappa^Q + \Sigma\lambda_0) + (\rho^Q + \Sigma\lambda_1)F_{t-1} + \Sigma\varepsilon_t \\ &= \rho F_{t-1} + \Sigma\varepsilon_t. \end{aligned} \tag{A.7}$$

Therefore we have $\kappa^Q + \Sigma\lambda_0 = 0$ and $\rho = \rho^Q + \Sigma\lambda_1$. Substituting these into equation (A.6) gives

$$\begin{aligned} \overline{A}_j - \overline{A}_{j-1} &= \overline{B}'_{j-1}(-\Sigma\lambda_0) + \frac{1}{2}\overline{B}'_{j-1}\Sigma\Sigma'\overline{B}_{j-1} + \overline{A}_1 \\ \overline{B}'_j &= \overline{B}'_{j-1}(\rho - \Sigma\lambda_1) + \overline{B}'_1; \qquad j = 2, ... \end{aligned} \tag{A.8}$$

which match equations (2.6) and (2.7).

State-Space Representation of Macro-Finance Model

Substitute equation (4.1) into (4.8) and eliminate i_t:

$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y)[\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}] - \beta_r S_{t-1} + \varepsilon_{y,t}$$

Define $Y_t = [\ \pi_t \quad \pi_{t-1} \quad y_t \quad y_{t-1} \quad L_t \quad S_t \quad u_{s,t} \quad E_t y_{t+1}\ ]'$,

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & -\mu_\pi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\mu_y \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \rho_l - 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ (\rho_s - 1)\gamma_\pi & 0 & (\rho_s - 1)\gamma_y & 0 & (1-\rho_s)\gamma_\pi & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} (1-\mu_\pi)\alpha_{\pi_1} & (1-\mu_\pi)\alpha_{\pi_2} & \alpha_{y_1} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\mu_y)\beta_{y1} & (1-\mu_y)\beta_{y2} & 0 & -\beta_r & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_l & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \varepsilon_t = [\ \varepsilon_{\pi,t} \quad \varepsilon_{y,t} \quad \varepsilon_{l,t} \quad \varepsilon_{s,t}\ ]', \Pi = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\ ]',$$

and $\eta_t = (y_t - E_{t-1}y_t)$, which is the expectational error in forecasting y_t in period $t-1$. Then, the system can be written as

$$\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi\varepsilon_t + \Pi\eta_t. \tag{A.9}$$

We use Christopher Sims's algorithm (Sims 2001) to solve the system, and the solution is in the form of

$$Y_t = \Gamma Y_{t-1} + \Omega\varepsilon_t. \tag{A.10}$$

Moreover, the expectation of y_t in period $t-1$, $E_{t-1}y_t$, can be expressed by other variables in Y_t. Therefore, the state vector of the system becomes $F_t = [\ \pi_t \quad \pi_{t-1} \quad y_t \quad y_{t-1} \quad L_t \quad S_t \quad u_{s,t}\ ]'$, and the law of motion of the state

$$F_t = \rho F_{t-1} + \Sigma\varepsilon_t$$

can be obtained from the solution (A.10) where ρ is the 7×7 upper left corner of Γ, and Σ is the 7×4 upper part of Ω.

Log-Likelihood Function of Macro-Finance Model

We use data on π_t, y_t, and 1-, 3-, 12-, 36-, and 60-month bond yields. Since the underlying term structure model has two latent factors, three of the bond yields must be postulated to fit with measurement error. We assume that the 3-, 12-, and 36-month bond yields are measured with *i.i.d.* error. Therefore, given a specific vector of parameter values θ, both the latent factors L_t and S_t and the bond yield measurement errors can be obtained from the inversion of the bond pricing formula (2.8).

Define $R_t = [\ i_t \quad i_{60,t} \quad i_{3,t} \quad i_{12,t} \quad i_{36,t}\ ]'$ and $B = [\ B_1 \quad B_{60} \quad B_3 \quad B_{12} \quad B_{36}\ ]'$. Conditional on the first $t-1$ observations, the tth observation $z_t = (\pi_t, y_t, R_t)'$ is Gaussian:

$$z_t = \Gamma^z F_{t-1} + \Omega^z \xi_t \tag{A.11}$$

where

$$\Gamma^z = \begin{bmatrix} \rho_{1,.} \\ \rho_{3,.} \\ B\rho \end{bmatrix}, \Omega^z = \begin{bmatrix} \Sigma_{1,.} & 0 \\ \Sigma_{3,.} & 0 \\ B\Sigma & B^m \end{bmatrix}, B^m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sigma_3 & 0 & 0 \\ 0 & \sigma_{12} & 0 \\ 0 & 0 & \sigma_{36} \end{bmatrix}, \text{ and } \xi_t = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^m \end{bmatrix}.$$

The vectors $\rho_{1,.}$, $\rho_{3,.}$, $\Sigma_{1,.}$ and $\Sigma_{3,.}$ are the first and third rows of the matrices ρ and Σ, respectively, and ε_t^m is a 3×1 vector containing the measurement errors.

The logarithm of the conditional density of the tth observation can then be expressed as

$$\begin{aligned} llh_t &= \log f_{z_t|z_{t-1},...,z_1}(z_t|z_{t-1},...,z_1;\theta) \\ &= -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\det(\Omega^z\Omega^{z\prime})) - \frac{1}{2}(z_t - \Gamma^z F_{t-1})'(\Omega^z\Omega^{z\prime})^{-1}(z_t - \Gamma^z F_{t-1}) \end{aligned} \tag{A.12}$$

and the conditional likelihood function for the complete sample is

$$L_{z_t,z_{t-1},...,z_2|z_1}(z_t, z_{t-1}, ..., z_2|z_1;\theta) = \sum_{t=2}^{T} llh_t. \tag{A.13}$$

The log-likelihood is then maximized to obtain the ML estimates of the parameters, denoted $\widehat{\theta}_{MLE}$. Finally, the standard errors of $\widehat{\theta}_{MLE}$ are numerically computed based on the estimates of inverse of Hessian matrix at the convergence point.

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Table 1

Yields-Only Model Parameter Estimates

Factor dynamics (ρ)

	L_{t-1}		S_{t-1}	
L_t	0.997	(0.0014)	—	
S_t	0.021	(0.0013)	0.945	(0.0039)

Risk price (λ_1)

	L_t		S_t	
$\Lambda_{L,t}$	-0.0148	(0.0013)	0.0032	(0.0014)
$\Lambda_{S,t}$	-0.0028	(0.0014)	-0.0095	(0.0014)

Standard deviations (Σ)

σ_L	0.271	(0.0098)
σ_S	0.443	(0.0077)

Standard deviations of measurement error

3-month	0.201	(0.0055)
12-month	0.346	(0.0081)
36-month	0.159	(0.0078)

Note: Standard errors of the estimates are in parentheses.

Table 2

Yields-Only Model Variance Decomposition

Forecast Horizon	Level	Slope
	1-month yield	
1 month	28.7	71.3
12 months	44.3	55.7
60 months	76.8	23.2
	12-month yield	
1 month	35.4	41.4
12 months	58.2	36.4
60 months	84.6	13.8
	5-year yield	
1 month	88.6	11.4
12 months	93.1	6.9
60 months	97.8	2.2

Table 3

Macro-Finance Model Parameter Estimates

Factor dynamics					
ρ_L	0.989	(0.0068)	g_π	1.253	(0.0066)
ρ_S	0.026	(0.0111)	g_y	0.200	(0.0066)
ρ_u	0.975	(0.0062)			
Inflation dynamics					
μ_π	0.074	(0.0113)	$\alpha_{\pi 1}$	1.154	(0.0525)
α_y	0.014	(0.0074)	$\alpha_{\pi 2}$	-0.155	(0.0066)
Output dynamics					
μ_y	0.009	(0.0066)	β_{y1}	0.918	(0.0604)
β_r	0.089	(0.0067)	β_{y2}	0.078	(0.0066)

Risk price (λ_1)				
	L_t		S_t	
$\Lambda_{L,t}$	-0.0045	(0.0068)	0.0168	(0.0068)
$\Lambda_{S,t}$	-0.0223	(0.0064)	0.0083	(0.0067)

Standard deviations					
σ_L	0.342	(0.0089)	σ_π	0.238	(0.0110)
σ_S	0.559	(0.0313)	σ_y	0.603	(0.0128)
Standard deviations of measurement error					
3-month	0.288	(0.0162)			
12-month	0.334	(0.0194)			
36-month	0.127	(0.0094)			

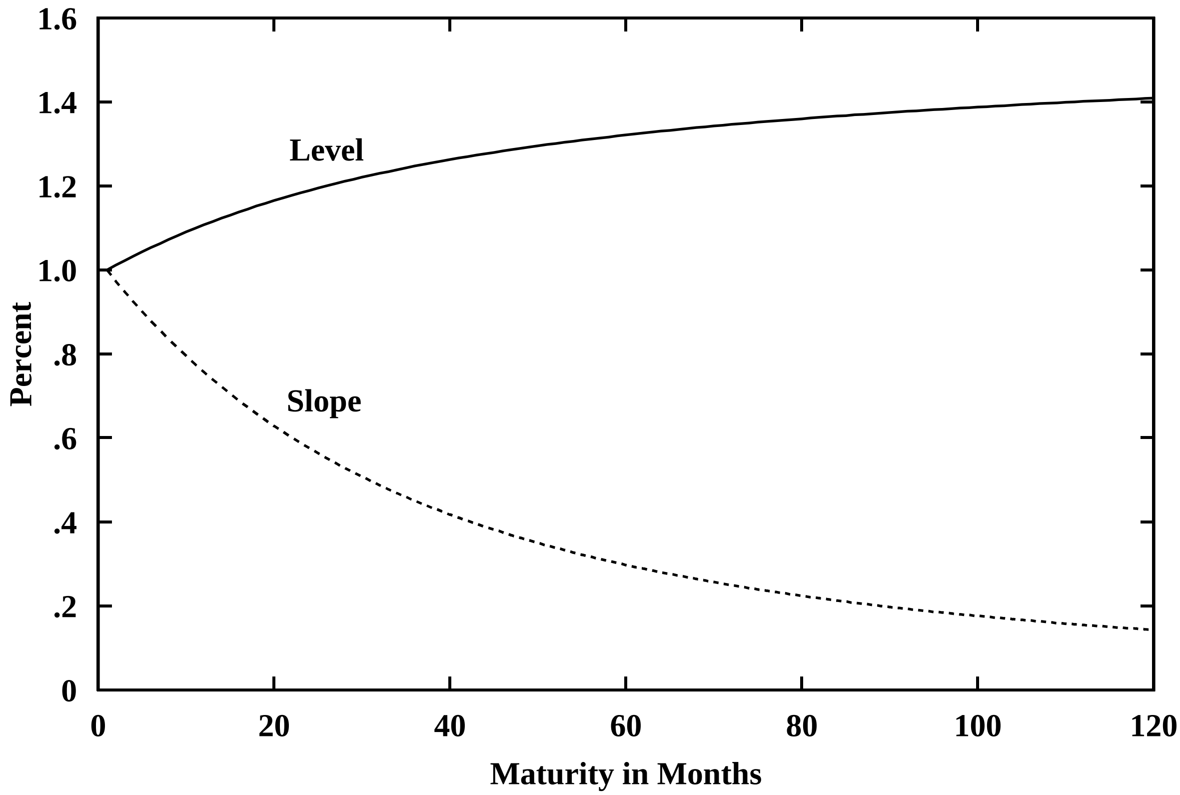
Note: Standard errors of the estimates are in parentheses.

Table 4

Macro-Finance Model Variance Decomposition

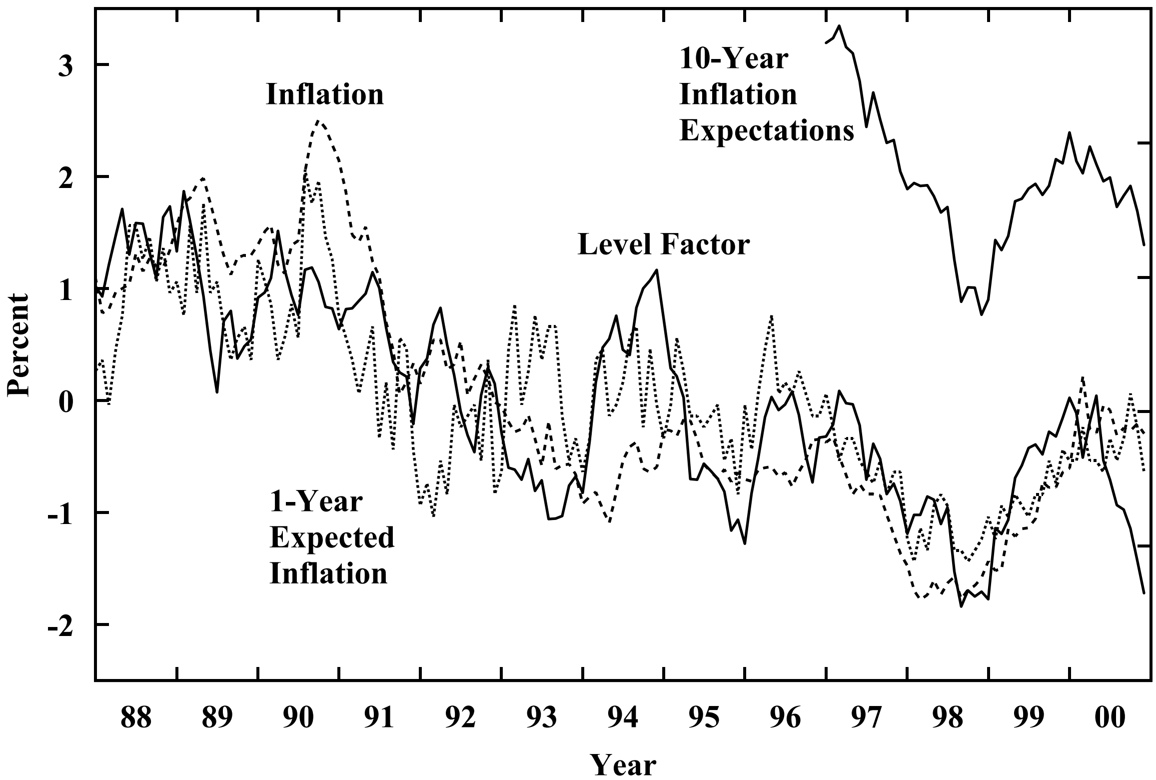
Forecast Horizon	Inflation	Output	Level	Slope
		Inflation		
1 month	97.3	0.1	2.7	0.0
12 months	52.1	2.8	44.7	0.4
60 months	6.8	2.6	82.1	8.6
		Output		
1 month	0.1	99.3	0.2	0.4
12 months	4.5	67.8	6.0	21.8
60 months	4.2	20.9	5.6	69.3
		1-month yield		
1 month	22.0	3.4	0.3	74.3
12 months	14.7	7.3	9.6	68.4
60 months	5.1	7.1	58.6	29.1
		12-month yield		
1 month	10.7	6.3	8.6	56.3
12 months	5.6	10.0	34.1	46.1
60 months	1.6	6.3	74.7	16.3
		5-year yield		
1 month	0.5	5.0	93.1	1.4
12 months	0.2	4.1	95.3	0.5
60 months	0.1	2.0	92.9	5.0

Figure 1: Factor Loadings of Yields-Only Model



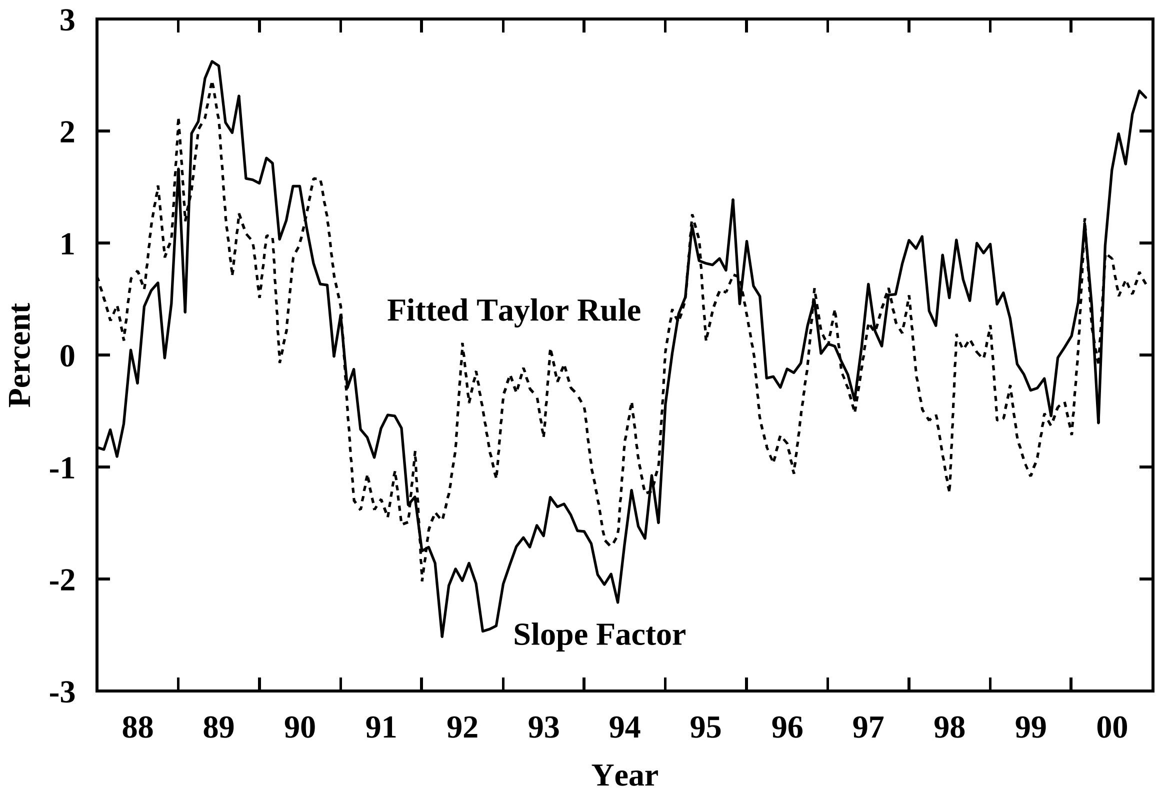
Note: These factor loadings show the impact response from a 1 percentage point increase in level or slope on the yield of a given maturity.

Figure 2: Yields-Only Level Factor and Inflation Indicators



Note: The estimated level factor from the yields-only model is shown, along with de-meaned annual inflation and one-year-ahead expected inflation from the Michigan Survey. From 1997 through 2000, 10-year-ahead expected inflation (not demeaned), which is the spread between 10-year nominal and indexed Treasury debt, is also shown.

Figure 3: Yields-Only Slope Factor and Fitted Taylor Rule



Note: The estimated slope factor from the yields-only model is shown, along with the fitted values from regressing the slope factor on inflation and output.

Figure 4: Level Factors from Yields-Only and Macro-Finance Models

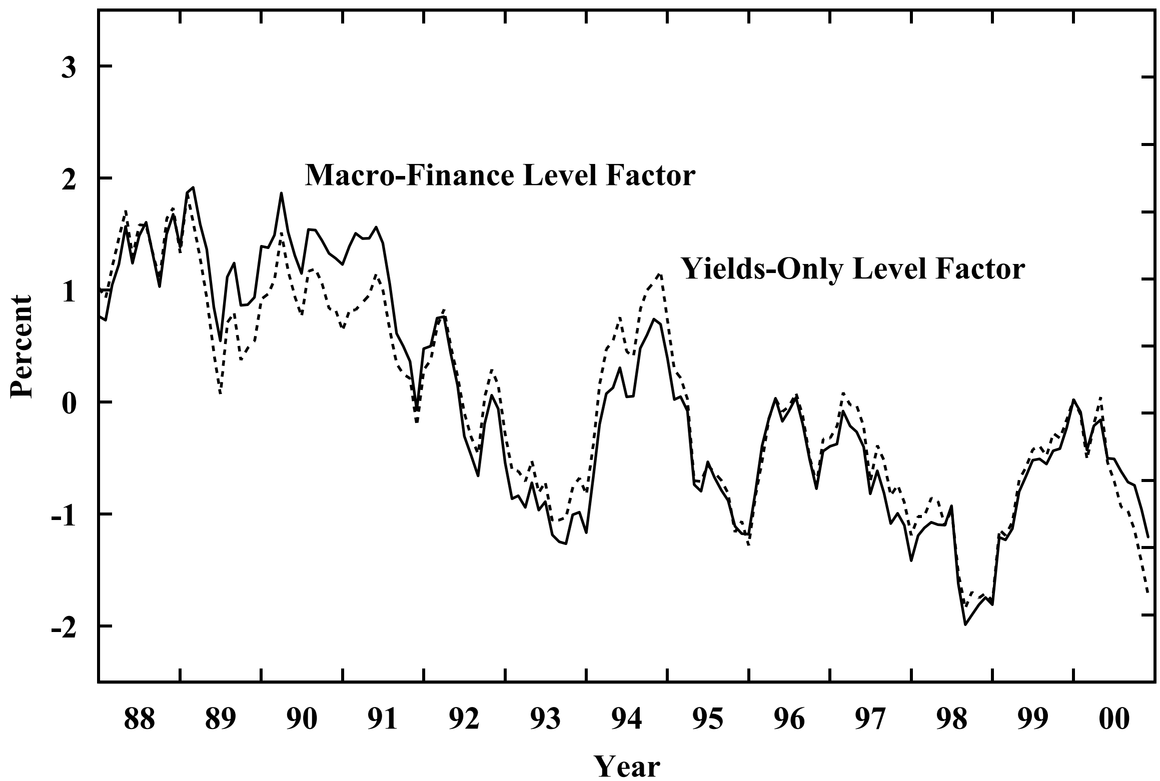


Figure 5: Slope Factors from Yields-Only and Macro-Finance Models

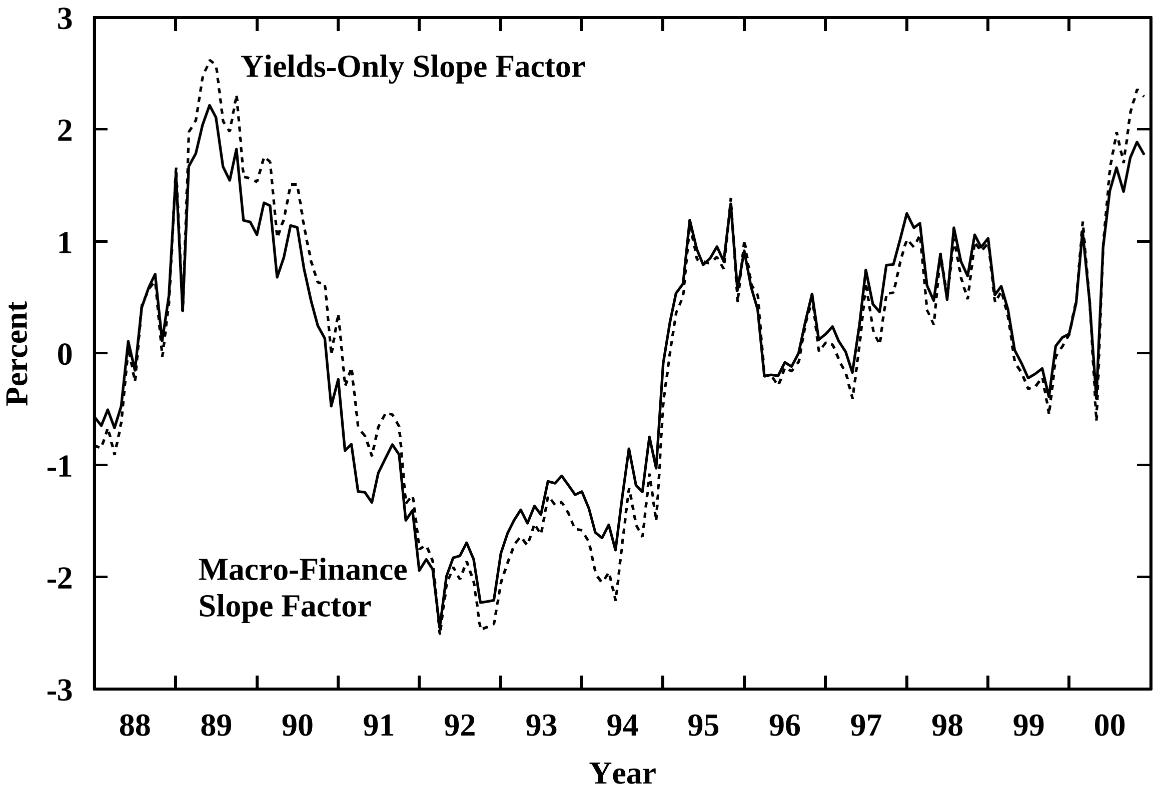
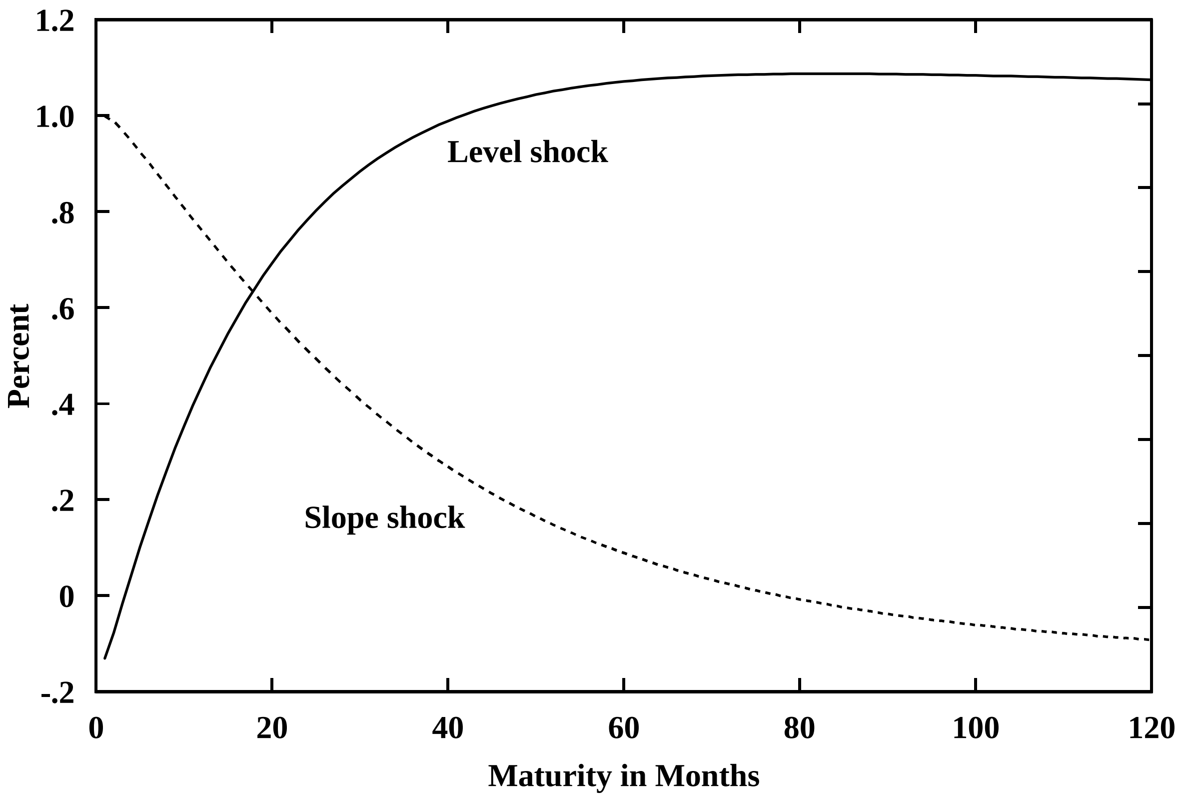
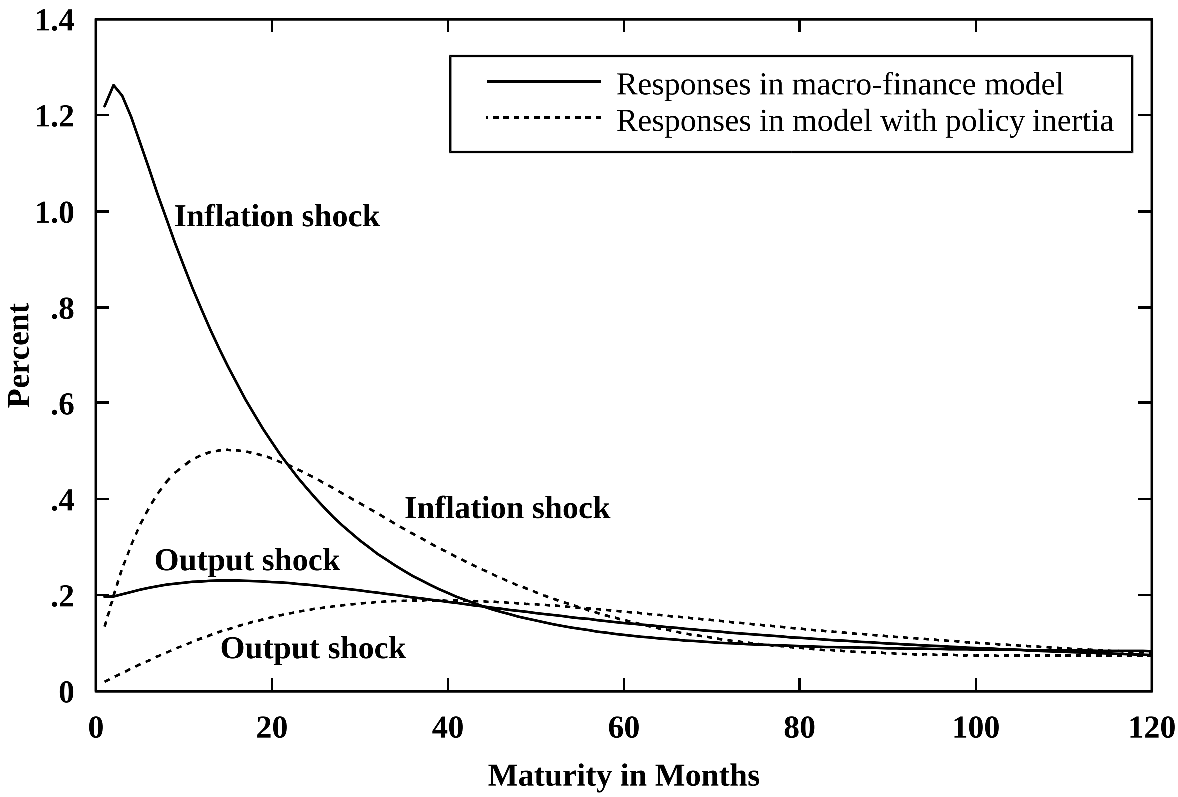


Figure 6: Initial Yield Curve Response to Level and Slope Shocks in Macro-Finance Model



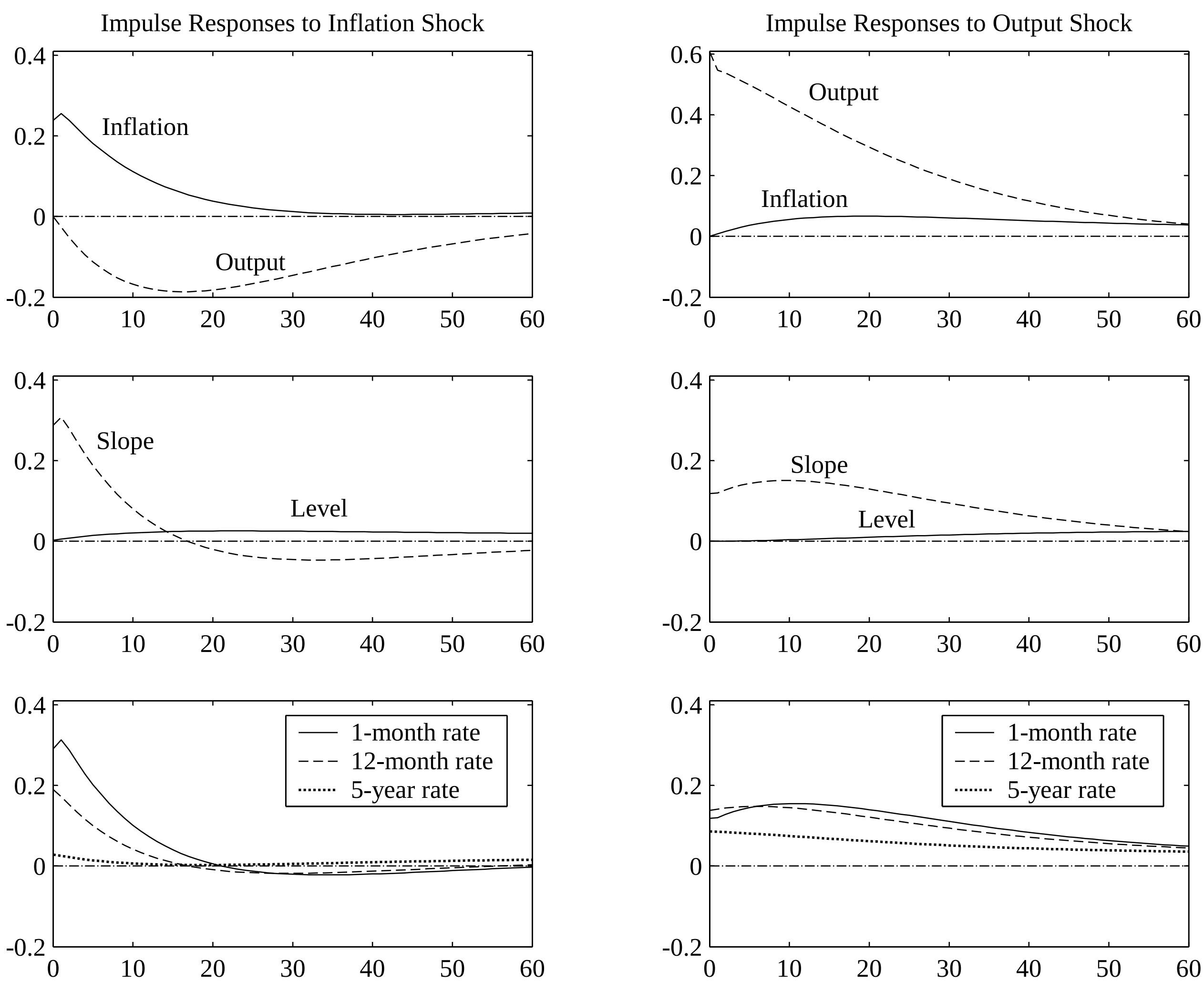
Note: These curves show the impact response from a 1 percentage point increase in level or slope on the yield of a given maturity.

Figure 7 : Initial Yield Curve Response to Output and Inflation Shocks



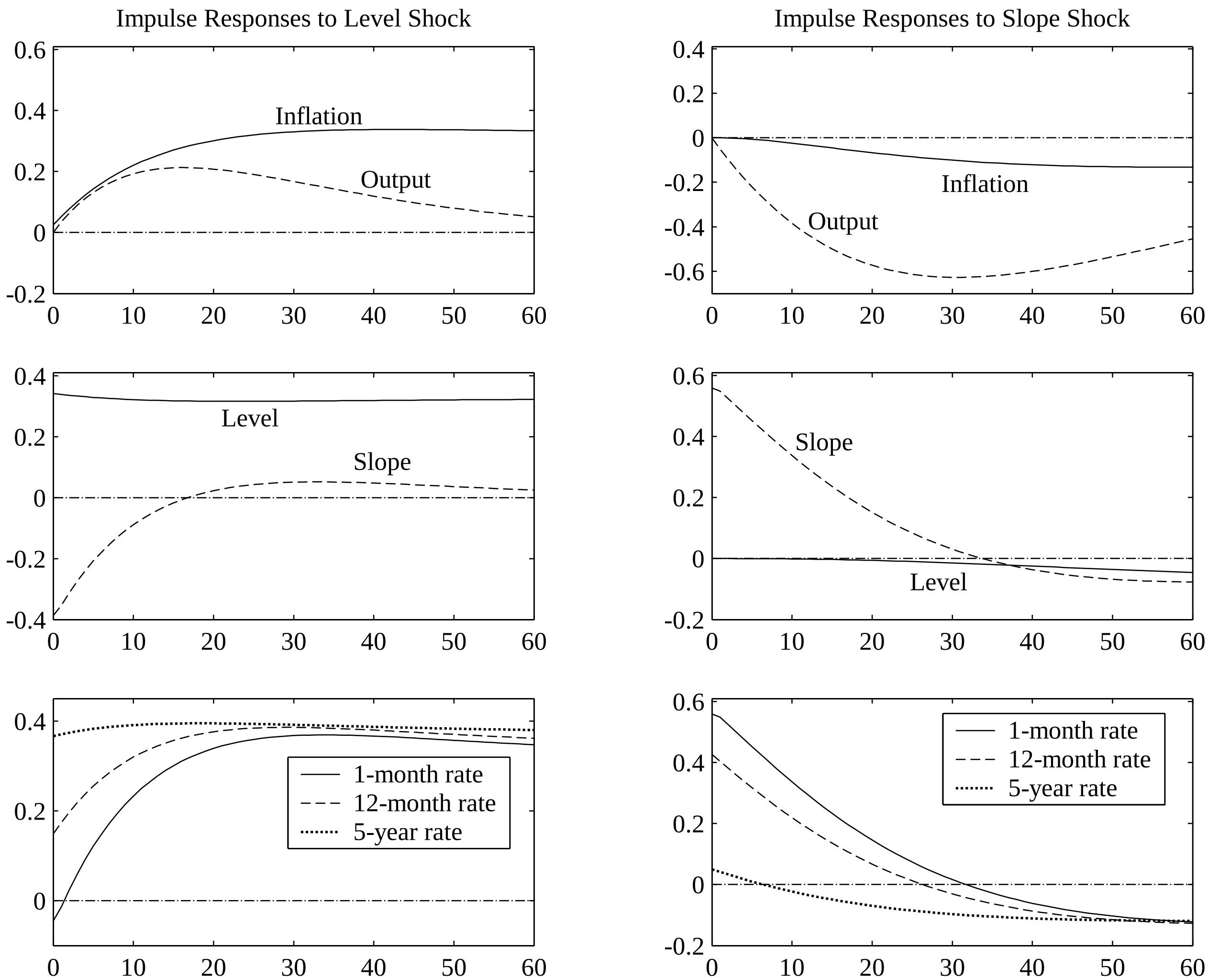
Note: The solid lines show the impact responses on the yield curve from a 1 percentage point increase in inflation or output in the estimated macro-finance model. The dashed lines give similar responses in a macro-finance model that assumes substantial monetary policy inertia ($\rho_S = 0.9$) and serially uncorrelated policy shocks ($\rho_u = 0$).

Figure 8: Impulse Responses to Macro Shocks in Macro-Finance Model



Note: All responses are percentage point deviations from baseline. The time scale is in months.

Figure 9: Impulse Responses to Policy Shocks in Macro-Finance Model



Note: All responses are percentage point deviations from baseline. The time scale is in months.