Estimating and Simulating a SIRD Model of COVID-19

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(Preliminary and incomplete)

Outline

- Basic model
  - Social distancing via a time-varying β
- Estimation and simulation
  - Different countries, U.S. states, and New York City
  - Robustness to parameters
  - "Forecasts" from each of the last 7 days
- Re-opening and herd immunity
  - How much can we relax social distancing?

Basic Model

Notation

- Number of people who are (stocks):

$$\begin{aligned} S &= \text{Susceptible} \\ I &= \text{Infectious} \\ R &= \text{Recovered} \\ D &= \text{Dead} \end{aligned}$$

- Constant population size is N

$$S_t + I_t + R_t + D_t = N$$

SIRD Model: Overview

- Susceptible get infected at rate $\beta I_t/N$

  New infections = $\beta I_t/N \cdot S_t$

- Infections resolve at Poisson rate γ, so the average number of days until resolution is $1/\gamma$ so $\gamma = .2 \Rightarrow 5$ days.

- Resolution happens in one of two ways:
  - Death: fraction δ
  - Recovery: fraction $1 - \delta$

SIRD Model: Laws of Motion

$$\Delta S_{t+1} = \underbrace{-\beta S_t I_t / N}_{\text{new infections}}$$

$$\Delta I_{t+1} = \underbrace{\beta S_t I_t / N}_{\text{new infections}} - \underbrace{\gamma I_t}_{\text{resolving infections}}$$

$$\Delta R_{t+1} = \underbrace{(1-\delta)\gamma I_t}_{\text{recover}}$$

$$\Delta D_{t+1} = \underbrace{\delta \gamma I_t}_{\text{die}}$$

$$R_0 = D_0 = 0$$

Recyled notation (terrible) R_0: Initial infection rate

- Initial reproduction number $R_0 \equiv \beta/\gamma$

R_0	=	β	$\times$	$1/\gamma$
# of infections from one sick person		# of lengthy contacts per day		# of days contacts are infectious

- R_0 = expected number of infections via the first sick person
  - $R_0 > 1 \Rightarrow$ disease initially grows
  - $R_0 < 1 \Rightarrow$ disease dies out: infectious generate less than 1 new infection
- If $1/\gamma = 5$, then easy to have $R_0 >> 1$

Basic Properties of Differential System (Hethcote 2000)

- Let $s_t \equiv S_t/N$ = fraction susceptible
- If $R_0 s_t > 1$, the disease spreads, otherwise declines
- Initial exponential growth rate is $\beta - \gamma$
- As $t \to \infty$, the total fraction of people ever infected, e^*, solves (assuming $s_0 \approx 1$)

$$e^* = -\frac{1}{R_0} \log(1 - e^*)$$

Long run is pinned down by R_0 (and death rate),
γ affects timing

Social Distancing

- What about a time-varying infection rate β_t?
  - Disease characteristics – fixed, homogeneous
  - Regional factors (NYC vs Montana) – fixed, heterogeneous
  - Social distancing – varies over time and space
- Reasons why β_t may change over time
  - Policy changes on social distancing
  - Individuals voluntarily change behavior to protect themselves and others
  - Superspreaders get infected quickly but then recover and "burn out" early

Model of Social Distancing

- Assume two key parameters β_0 and β^*
- Economy decays exponentially from β_0 to β^* at rate λ:

$$\beta_t = \beta_0 e^{-\lambda t} + \beta^*(1 - e^{-\lambda t})$$

$\Rightarrow$ *can think about initial* $R_0 = \beta_0/\gamma$
$R_0(t) = \beta_t/\gamma$ *and final* $R_0^* = \beta^*/\gamma$

- Interpretation:
  - λ governs the rate of convergence to R_0^*

Estimates and Simulations

Estimation: Countries and States

- Parameters that are fixed and homogeneous
  - $\gamma = 0.2$: average duration is 5 days (or $\gamma = 0.1$)
  - $\delta = 0.003$

    Apr 1: 15% of mothers giving birth in NYC infected

    With $\delta = .004$, model says only 13.6% ever infected on Apr 1
- Parameters that vary across countries/states
  - β_0 and β^*
  - λ: speed at which you move to β^*
  - I_0: initial number of infections (gets timing right)
- Objective function:
  - Equally weighted SSR for Cumulative deaths (logs) and Daily deaths (logs)

Estimation based entirely on death data

- Excess death issue
  - New York City added 3000+ deaths on April 15 $\approx$ 45% more
  - *The Economist* $\Rightarrow$ increases based on vital records

  $\Rightarrow$ We adjust all NYC deaths before April 15 by this 45% and non-NYC deaths upwards by 33%

- We use 5-day moving averages (centered)
  - Otherwise, very serious "weekend effects" in which deaths are underreported
  - Even zero sometimes, followed by a large spike

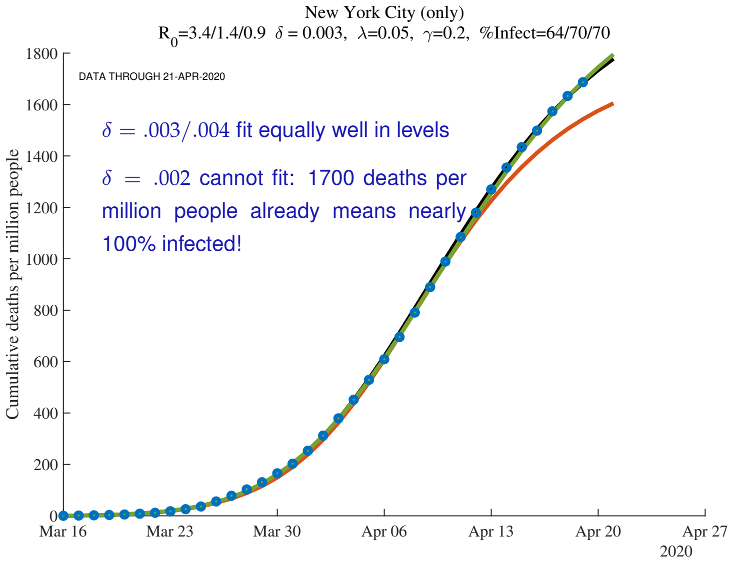
Guide to Graphs

- 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black=current
  - Red = oldest, Orange = second oldest, Yellow =third oldest...
  - Violet (purple) = one day earlier
- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = .003$)
  - Red = lowest parameter value (e.g. $\delta = .002$)
  - Green = highest parameter value (e.g. $\delta = .004$)

Guide to Graphs (continued)

- R_0 in subtitle: R_0 / $R_0(today)$ / R_0^*
  - Initial / Today / Final
- "%Infect"
  - Today / t+30 / Final
  - This is the percent ever infected
  - (so fraction δ will eventually be deaths)

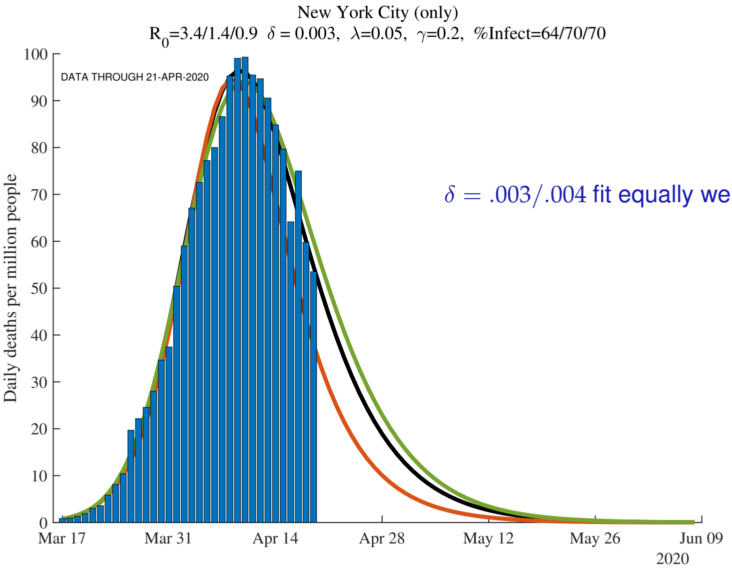
New York City: Cumulative Deaths per Million ($\delta = .003/.002/.004$)



$\delta = .003/.004$ fit equally well in levels

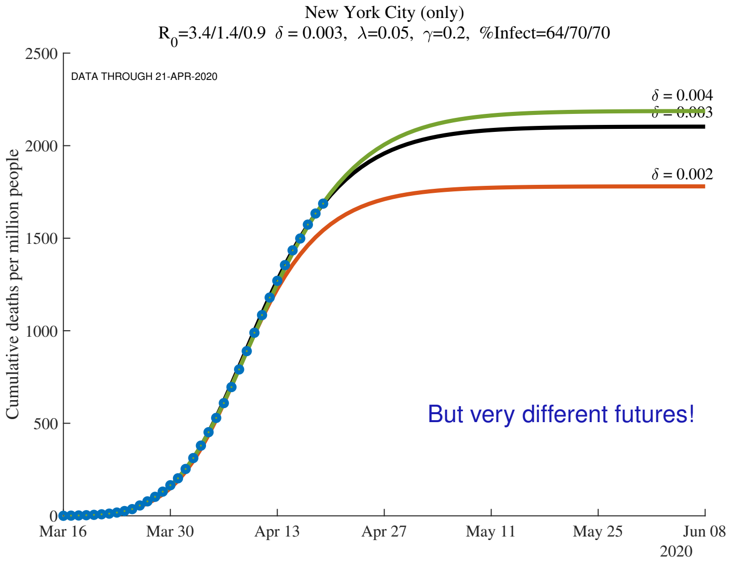
$\delta = .002$ cannot fit: 1700 deaths per million people already means nearly 100% infected!

New York City: Daily Deaths per Million People ($\delta = .003/.002/.004$)

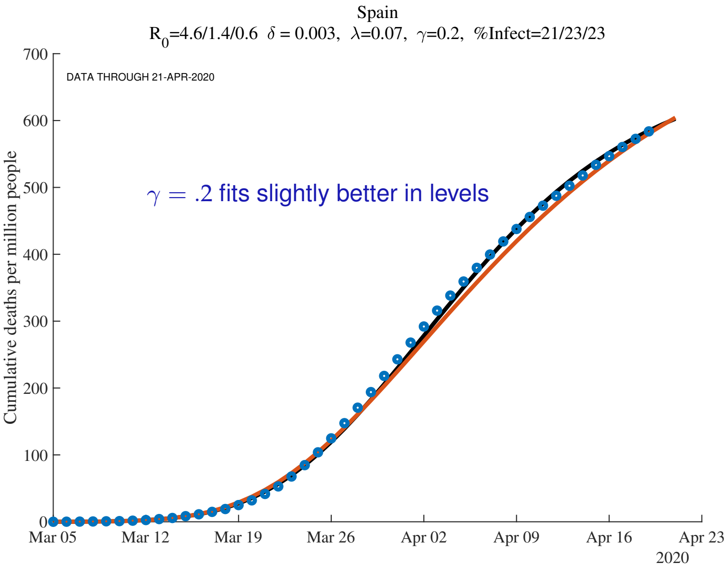


$\delta = .003/.004$ fit equally well

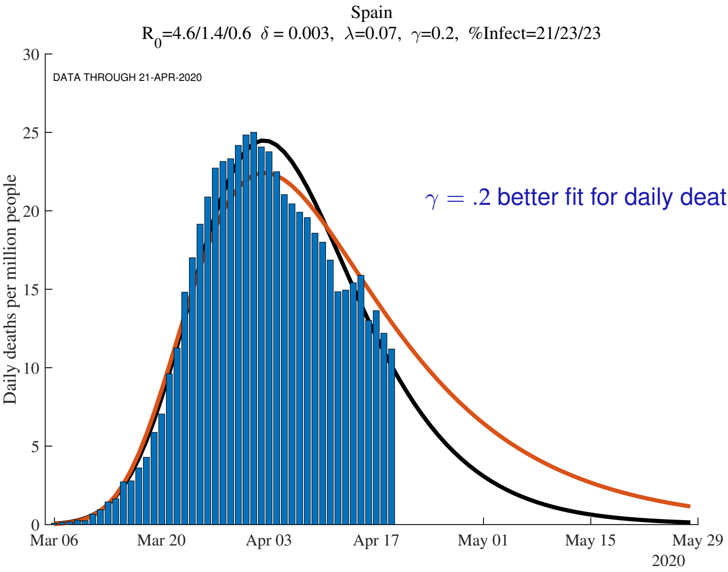
New York City: Cumulative Deaths per Million ($\delta = .003/.002/.004$)



Spain: Cumulative Deaths per Million People ($\gamma = .2/.1$)

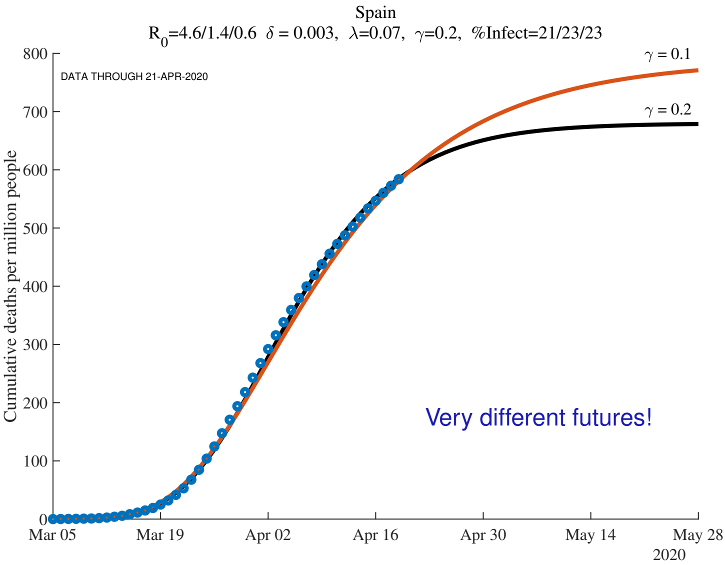


Spain: Daily Deaths per Million People ($\gamma = .2/.1$)

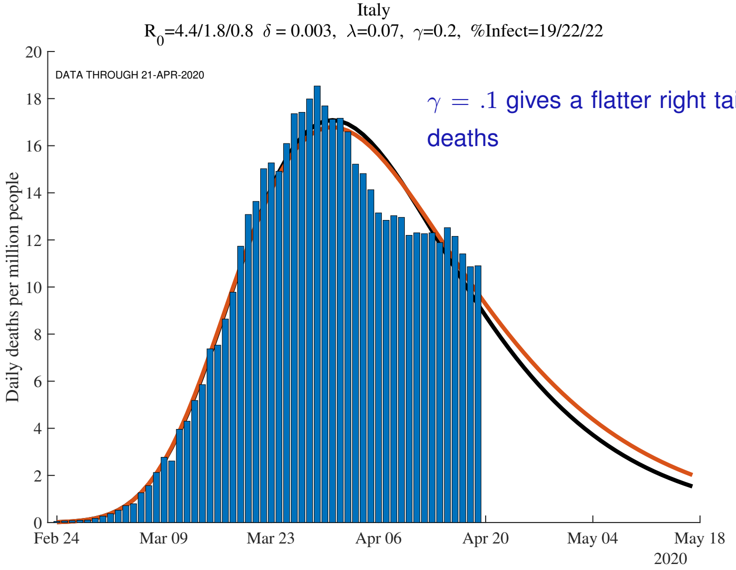


$\gamma = .2$ better fit for daily deaths

Spain: Cumulative Deaths per Million (Future, $\gamma = .2/.1$)



Italy: Daily Deaths per Million People ($\gamma = .2/.1$)



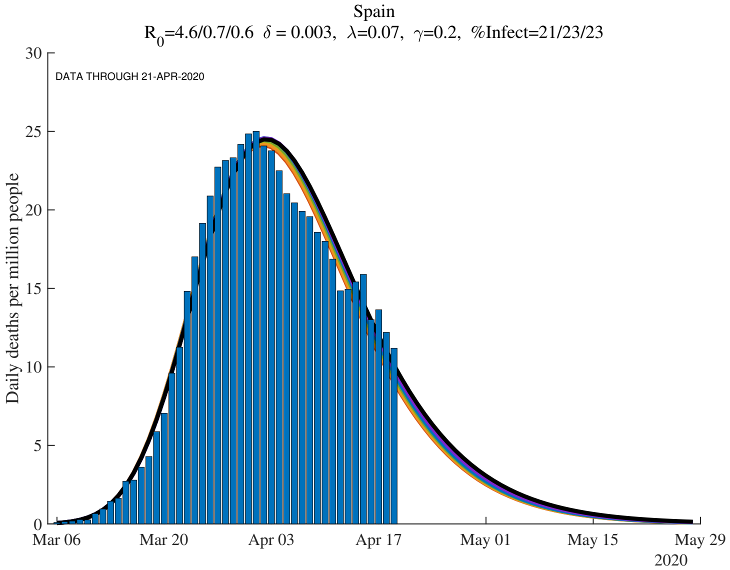
$\gamma = .1$ gives a flatter right tail of deaths

Repeated "Forecasts" from the past 7 days of data

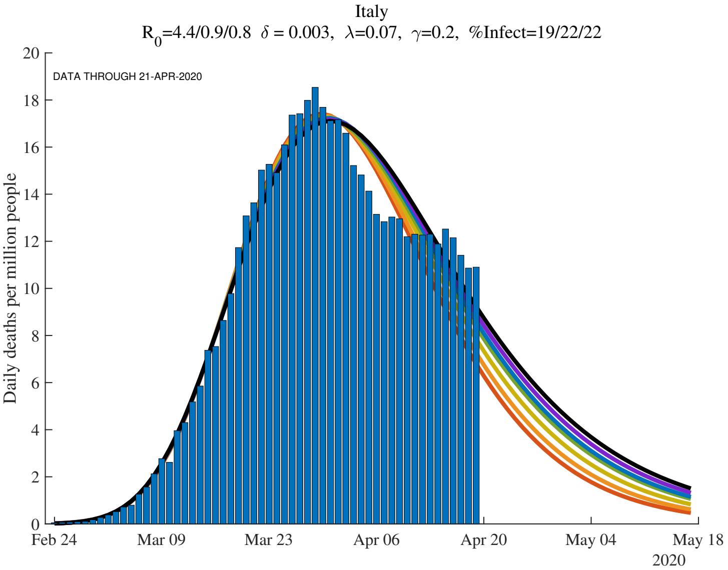
– After peak, forecasts settle down.

– Before that, very noisy!

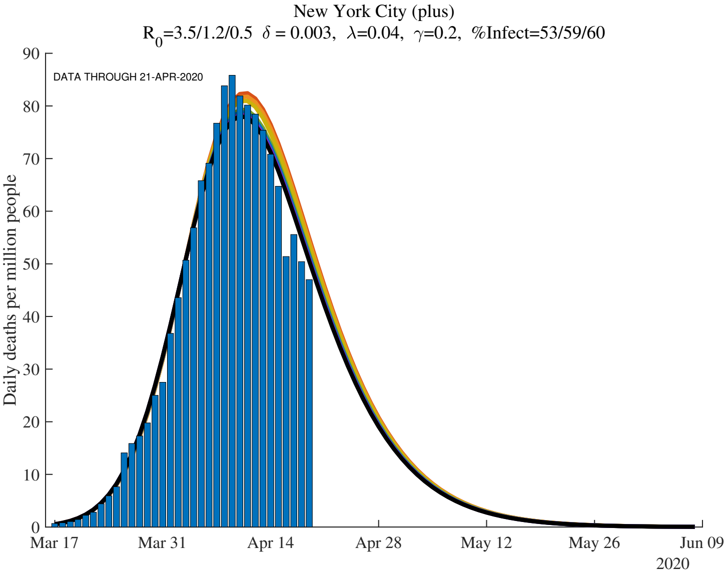
Spain (7 days): Daily Deaths per Million People



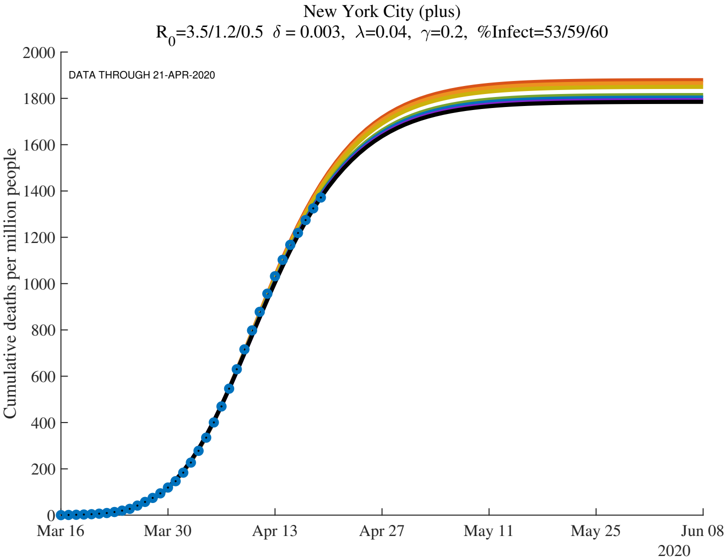
Italy (7 days): Daily Deaths per Million People



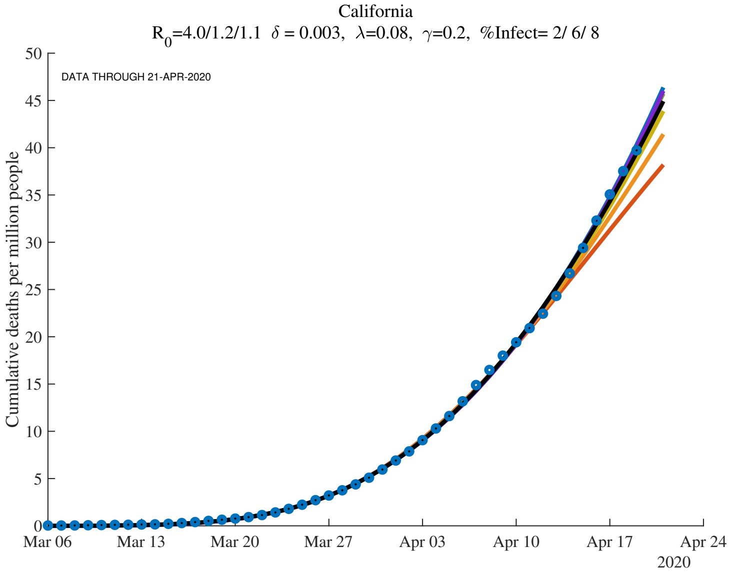
New York City (7 days): Daily Deaths per Million People



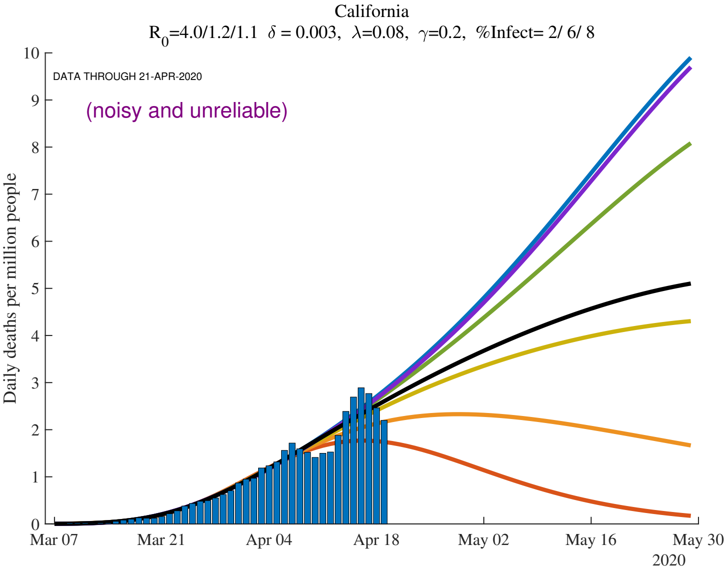
New York City (7 days): Cumulative Deaths per Million (Future)



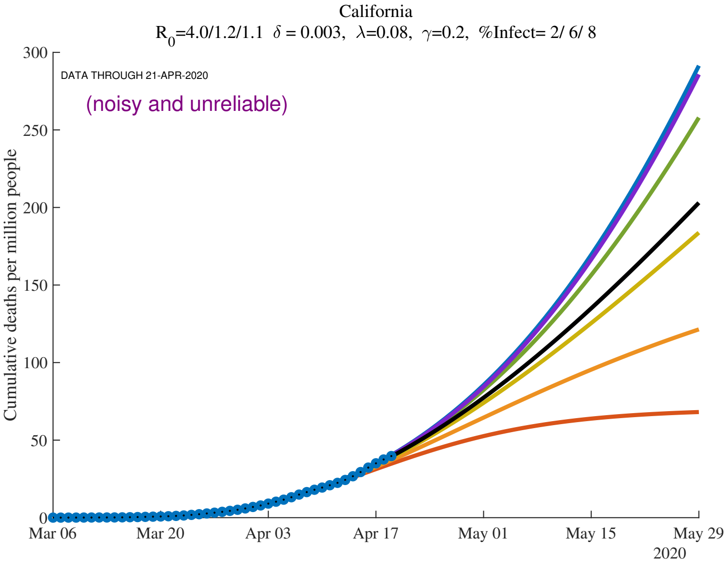
California (7 days): Cumulative Deaths per Million



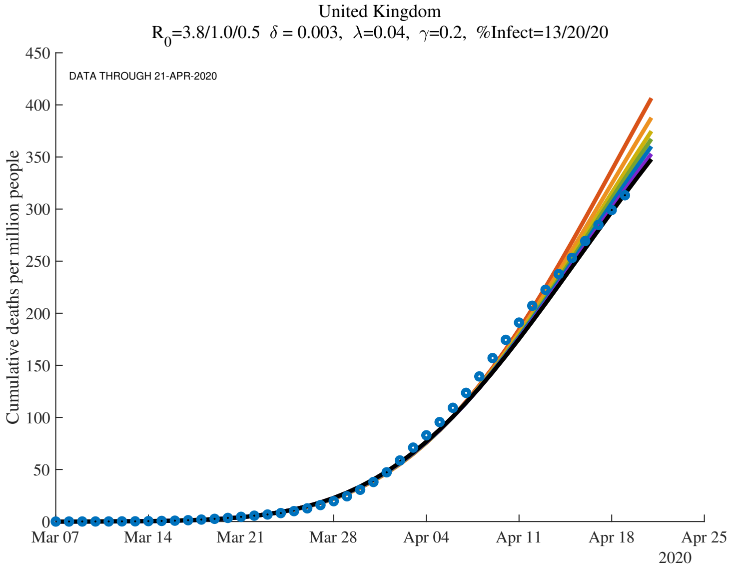
California (7 days): Daily Deaths per Million People



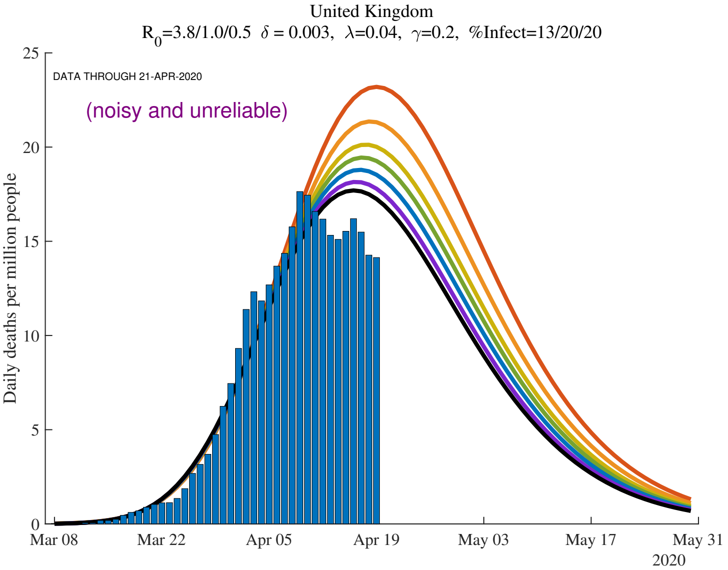
California (7 days): Cumulative Deaths per Million (Future)



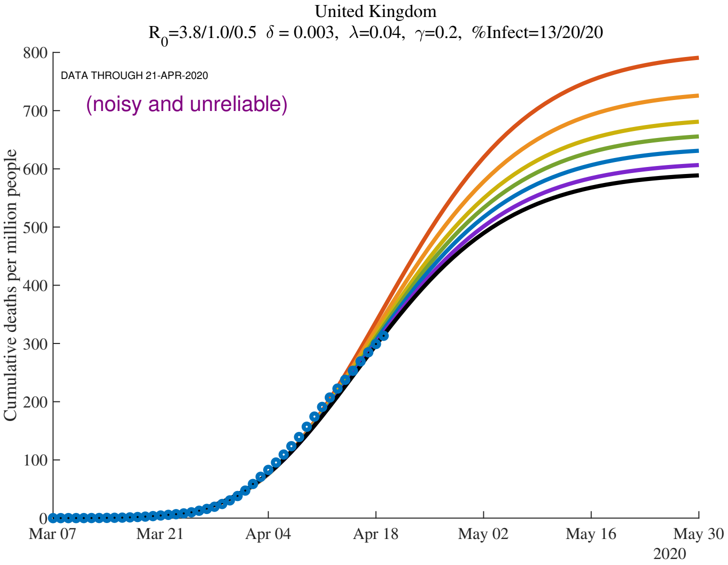
U.K. (7 days): Cumulative Deaths per Million



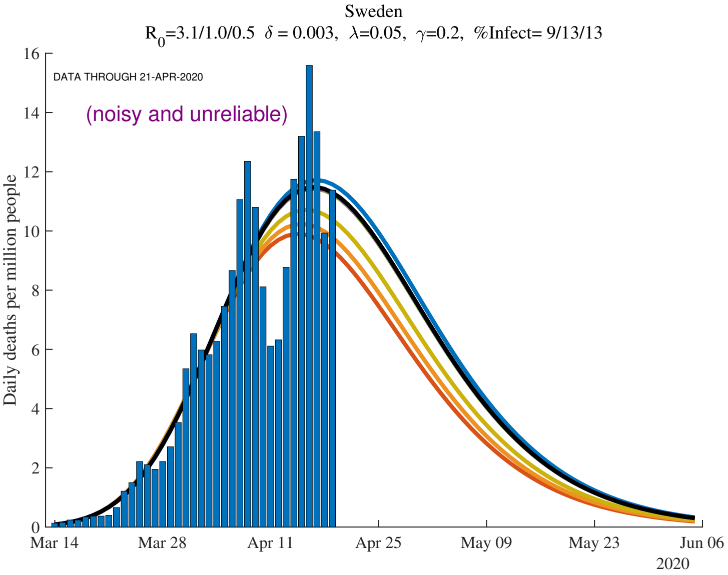
U.K. (7 days): Daily Deaths per Million People



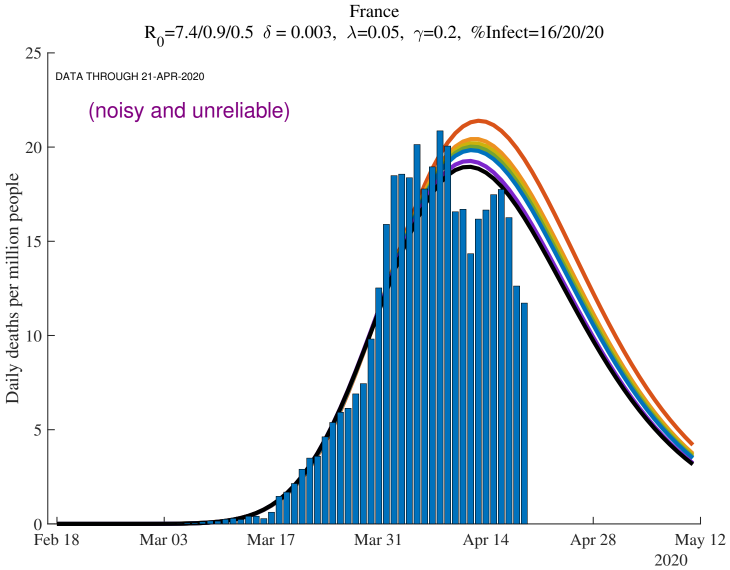
U.K. (7 days): Cumulative Deaths per Million (Future)



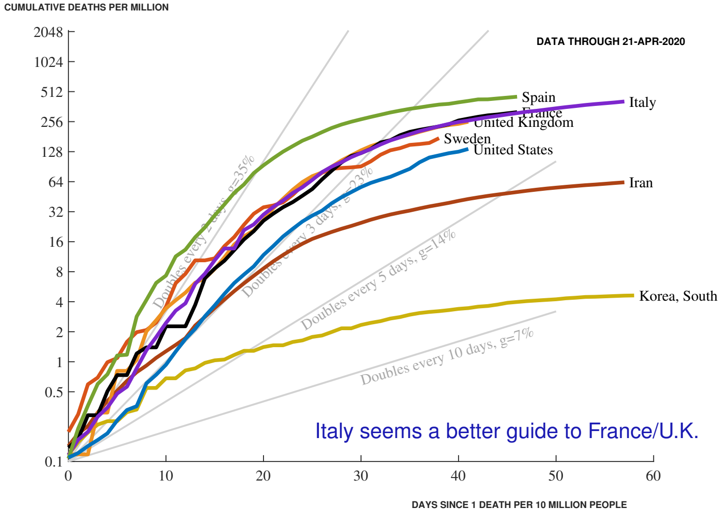
Sweden (7 days): Daily Deaths per Million People



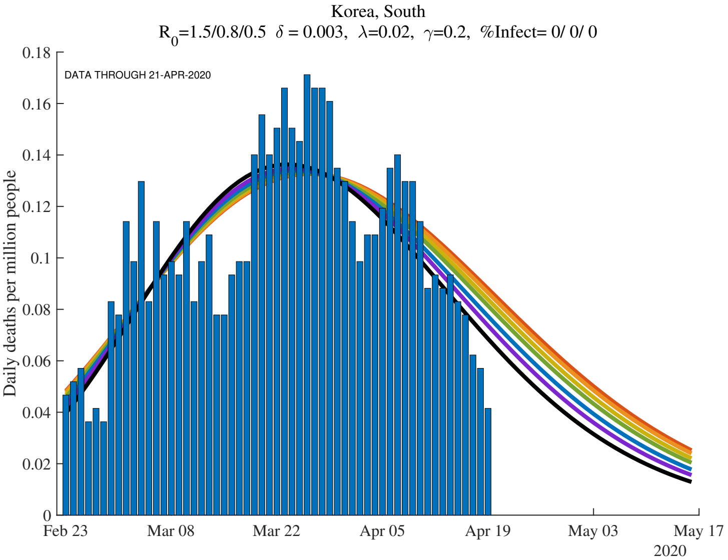
France (7 days): Daily Deaths per Million People



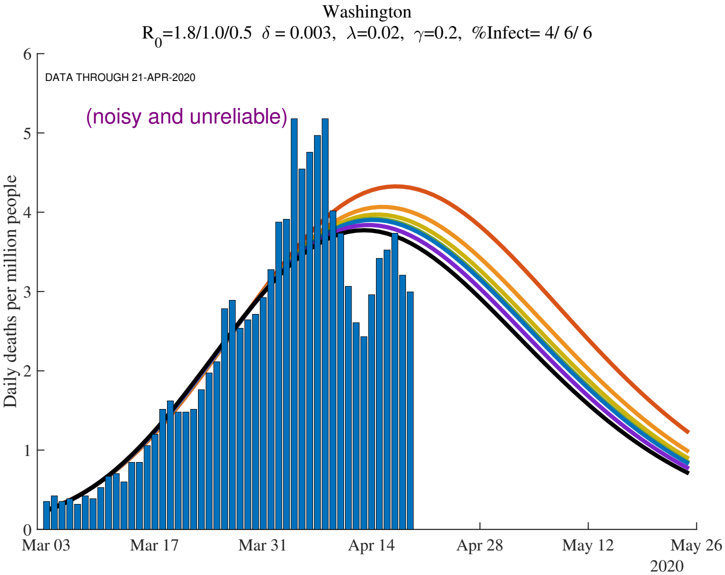
Cumulative Deaths per Million, Log Scale



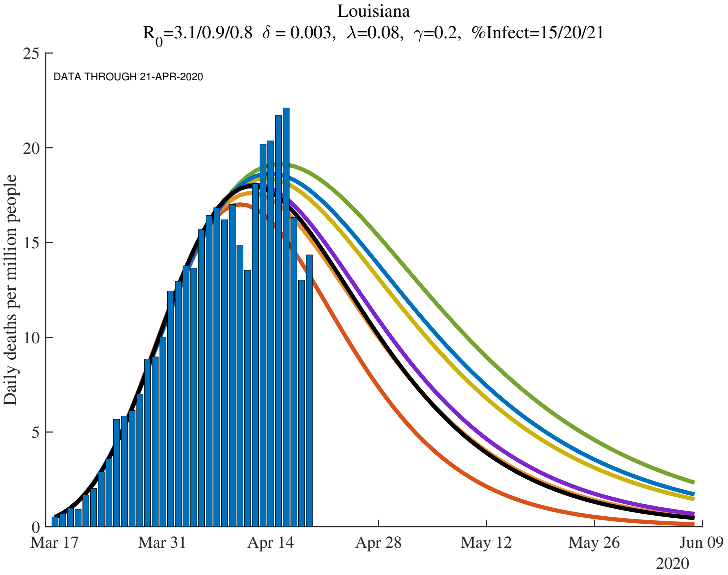
S. Korea (7 days): Daily Deaths per Million People



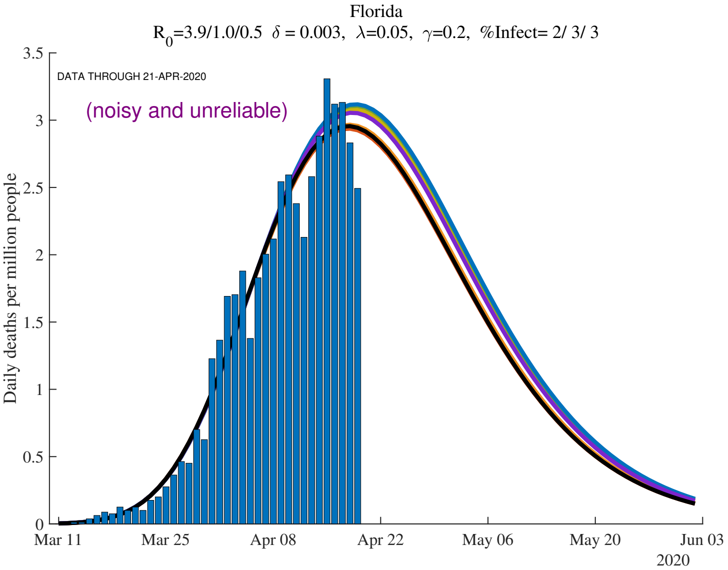
Washington (7 days): Daily Deaths per Million People



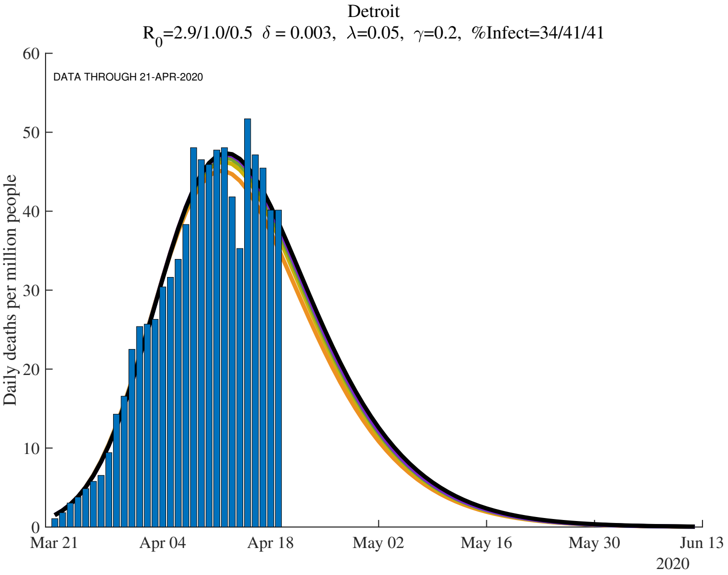
Louisiana (7 days): Daily Deaths per Million People



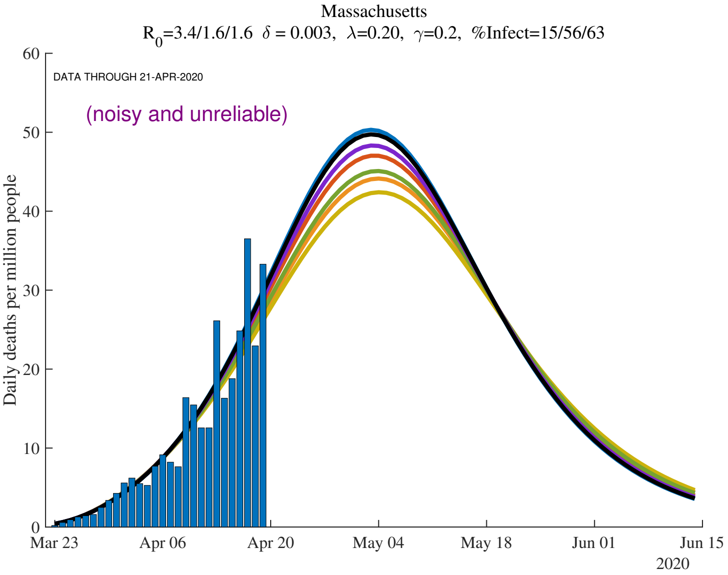
Florida (7 days): Daily Deaths per Million People



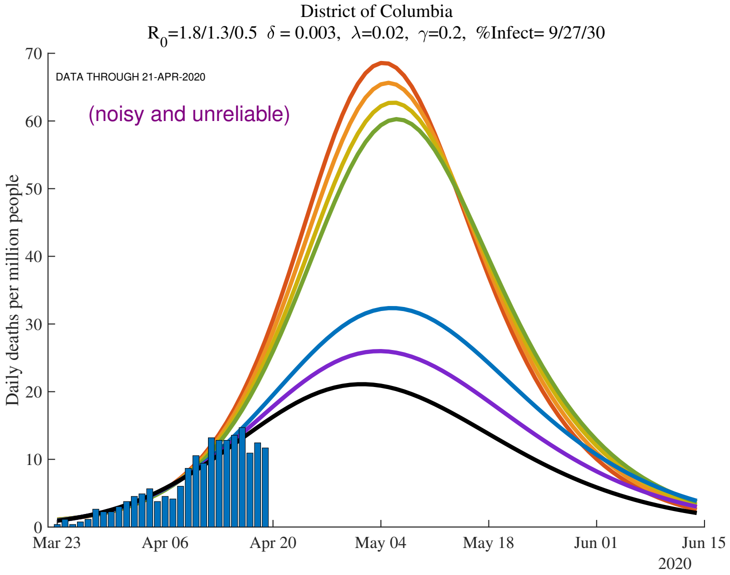
Detriot (Wayne County, 7 days): Daily Deaths per Million People



Massachusetts (7 days): Daily Deaths per Million People



District of Columbia (7 days): Daily Deaths per Million People



Reopening and Herd Immunity

Percent Ever Infected would be very informative

	— Percent Ever Infected (today) —		
	$\delta = .002$	$\delta = .003$	$\delta = .004$
New York City (plus)	78	53	41
Detroit	51	34	26
Spain	32	21	16
Italy	28	19	14
Michigan	25	16	12
France	24	16	12
Massachusetts	23	15	12
United Kingdom	20	13	10
Sweden	13	9	6
District of Columbia	13	9	7
New York excluding NYC	8	5	4
Denmark	5	3	2
Germany	4	3	2
Florida	3	2	2
California	3	2	1
Tennessee	2	1	1

Herd Immunity

- How far can we relax social distancing?
- Let $s(t) = S(t)/N$ = the fraction still susceptible
  - The disease will die out as long as

$$R_0(t)s(t) < 1$$

  - That is, if the "new" R_0 is smaller than $1/s(t)$
  - Today's infected people infect fewer than 1 person on average
- We can relax social distancing to raise $R_0(t)$ to $1/s(t)$

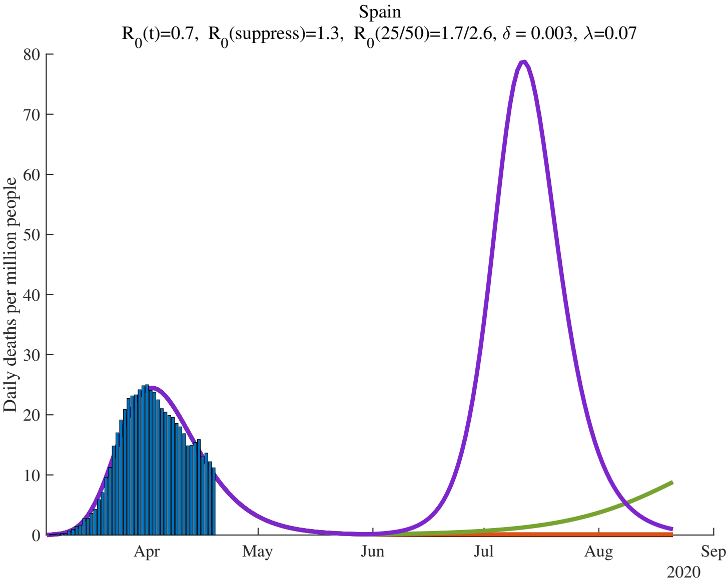
Herd Immunity and Opening the Economy?

	R_0	$R_0(t)$	Percent Susceptible t+30	R_0(t+30) with no outbreak	Percent way back to normal
New York City	3.5	1.2	40.5	2.5	55.6
Detroit	2.9	1.0	58.7	1.7	34.8
Spain	4.6	0.7	77.4	1.3	15.1
Italy	4.4	0.9	78.4	1.3	10.9
Michigan	3.4	1.0	76.3	1.3	13.3
France	7.4	0.9	79.8	1.3	5.1
Massachusetts	3.4	1.6	43.8	2.3	38.2
United Kingdom	3.8	1.0	80.5	1.2	8.4
Sweden	3.1	1.0	87.0	1.1	8.0
District of Columbia	1.8	1.3	73.4	1.4	4.2
New York excluding NYC	3.5	0.9	92.9	1.1	8.3
Washington	1.8	1.0	93.9	1.1	13.1
Denmark	2.7	0.7	96.4	1.0	16.1
Germany	4.6	0.9	95.3	1.0	3.7
Florida	3.9	1.0	96.6	1.0	2.9
California	4.0	1.2	94.1	1.1	-4.3

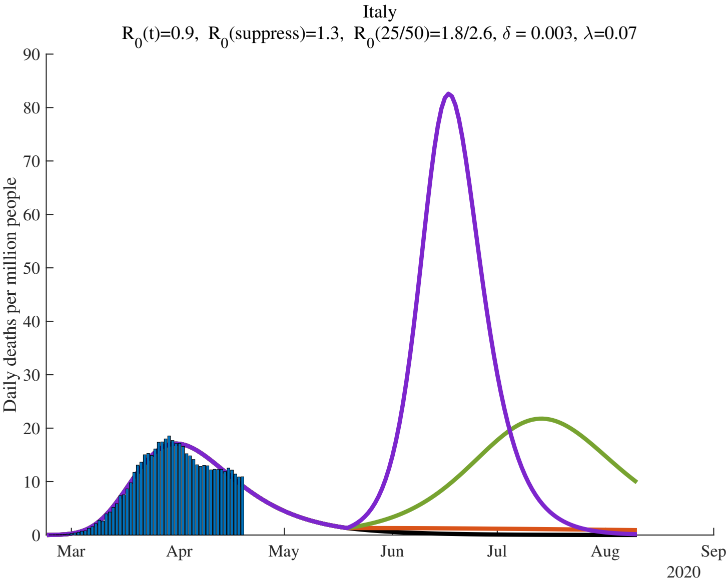
Simulations of Re-Opening

- Begin with the basic estimates shown already
- Different policies are then adopted starting around May 20
  - Black: assumes R_0(today) remains in place forever
  - Red: assumes R_0(suppress)$= 1/s(today)$
  - Green: we move 25% of the way from R_0(today) back to initial R_0 = "normal"
  - Purple: we move 50% of the way from R_0(today) back to initial R_0 = "normal"
- We assume these R_0 values stay in place forever
  - In practice, over course, β would likely start to fall as mortality rises

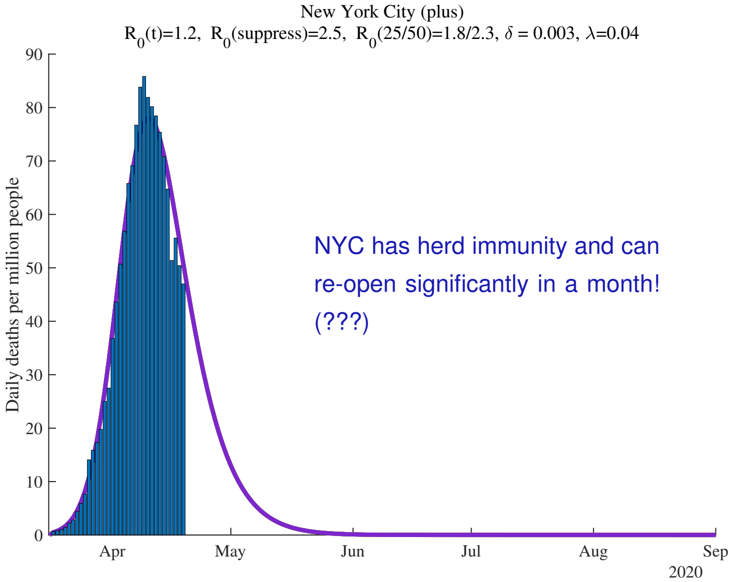
Spain: Re-Opening



Italy: Re-Opening

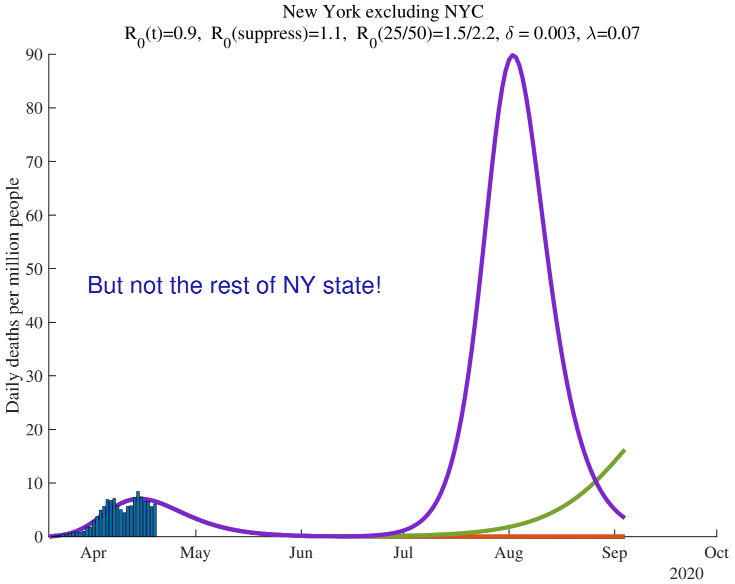


New York City: Re-Opening

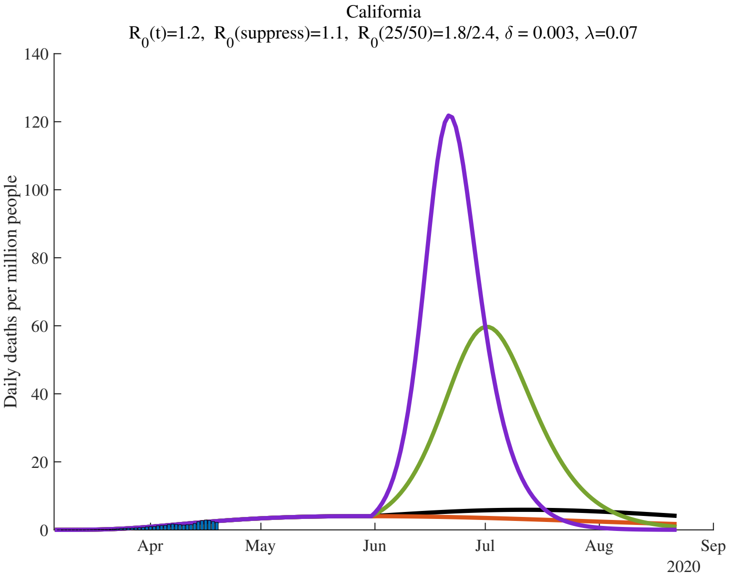


NYC has herd immunity and can re-open significantly in a month! (???)

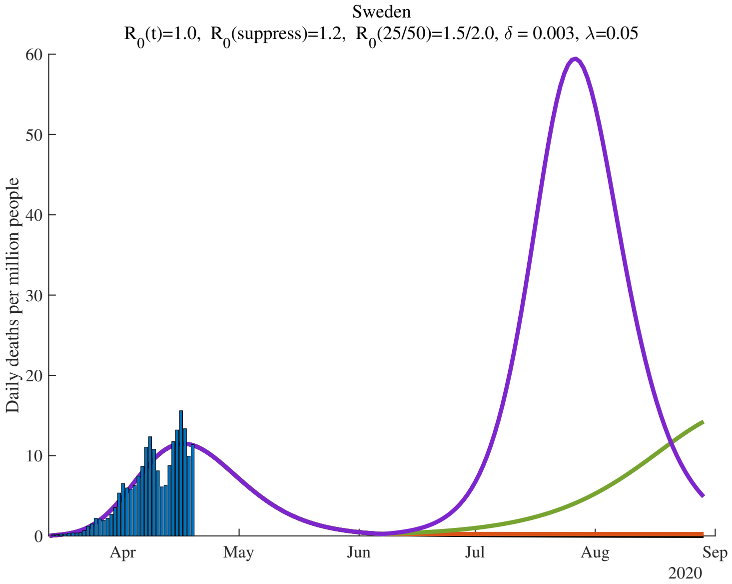
New York excluding NYC: Re-Opening



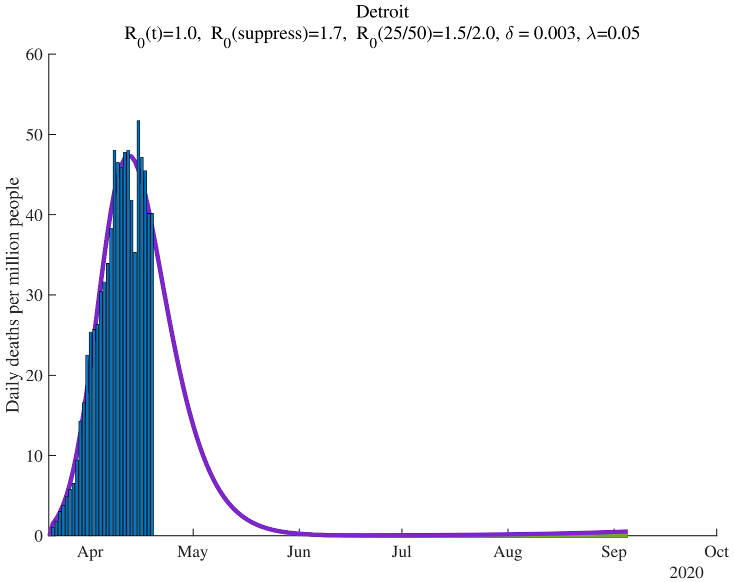
California: Re-Opening



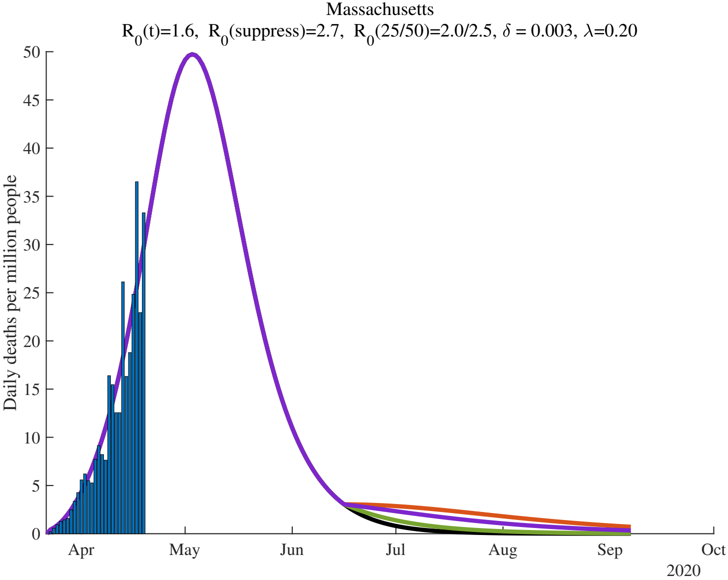
Sweden: Re-Opening



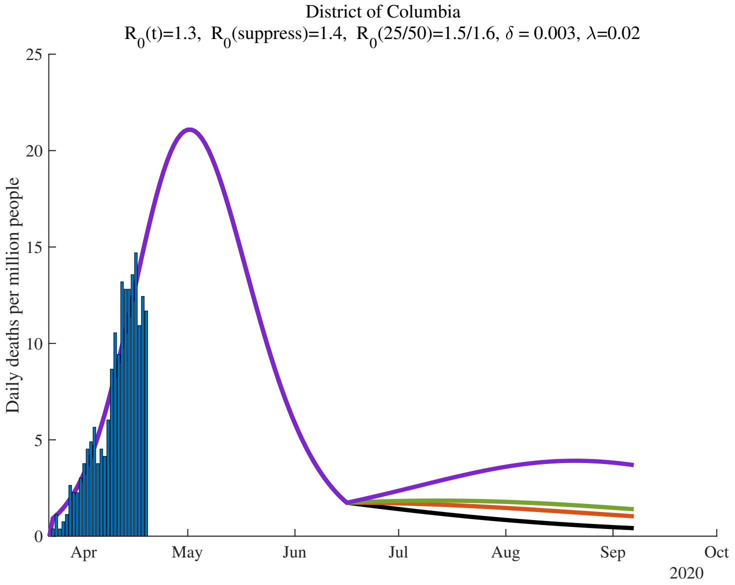
Detroit: Re-Opening



Massachusetts: Re-Opening



Washington, DC: Re-Opening



Conclusions

Speculations based on model, we are not epidemiologists

- Time-varying β (or R_0) needed to capture social distancing, by individuals or via policy

- New York City already has around 1700 deaths per million people
  - So $\delta > .002$ is required by model
  - 100% are not already infected and resolved

(continued...)

Conclusions (continued)

- Random sampling in NYC would be very informative about δ and percent ever infected (for herd immunity effects)
    - NEJM study of 215 mothers giving birth in NYC found a 15 percent infection rate from Mar 22 to Apr 4
    - Suggests a high current ever-infected rate (infections doubling every 3-4 days)
    - Model suggests $\delta < .004$ to match this
    - California serology testing also suggests $\delta \approx .003$ (but problems?)
- "One size fits all" will not work for re-opening
    - NYC could potentially re-open soon w/ basic precautions
    - Rest of NY state could not