Top Income Inequality in the United States and France

Income share of top 0.1 percent

United States

France

Year

Overview

- Atkinson / Piketty / Saez stylized facts on top income inequality
  - Rising sharply in US since 1980
  - More stable in France and Japan
  - Why?

- Fractal nature of top income inequality

- Standard tax explanation fails
Related literature

- **Rent Seeking:** Piketty, Saez, and Stantcheva (2011) and Rothschild and Scheuer (2011)
- **Globalization:** Haskel, Lawrence, Leamer, and Slaughter (2012)
- **Finance:** Philippon and Reshef (2009) and Bell and Van Reenen (2010)
- **Not just finance:** Bakija, Cole, and Heim (2010) and Kaplan and Rauh (2010)
- **Use Pareto to get growth:** Kortum (1997), Lucas and Moll (2013), Perla and Tonetti (2013).
Outline

• Facts

• Model

• Numerical examples
Top Income Inequality around the World

Income share of top 0.1 percent

Year


0% 2% 4% 6% 8%

Argentina
France
Germany
India
U.S.
Norway
Japan
China

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Top Income Inequality around the World

Top 1% share, 2004−06

Top 1% share, 1980−82

45−degree line

United States
South Africa
Singapore
Norway
Ireland
Portugal
Australia
New Zealand
Italy
Japan
Spain
Sweden
Indonesia
Mauritius
Denmark

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Pareto Distributions

\[ \Pr [Y > y] = \left( \frac{y}{y_0} \right)^{-\xi} \]

- Let \( \tilde{S}(p) \) = share of income going to the top \( p \) percentiles, and \( \eta \equiv 1/\xi \) be a measure of Power law inequality:

\[ \tilde{S}(p) = \left( \frac{100}{p} \right)^{\eta-1} \]

- If \( \eta = 1/2 \), then share to Top 1% is \( 100^{-1/2} \approx .10 \)

- If \( \eta = 3/4 \), then share to Top 1% is \( 100^{-1/4} \approx .32 \)

- Let \( S(a) \) = share of 10\( a \)'s income going to top \( a \):

\[ S(a) = 10^{\eta-1} \]
Fractal Inequality Shares in the United States

From 27% in 1975 to 40+% in 2010
The Power-Law Inequality Exponent $\eta$, United States

$\eta$ rises from .42 in 1975 to .65 in 2010
The Rise in Top Income Inequality by Occupation

The ratio for the top 0.1% group as a whole = 2.6

Source: Bakija, Cole, and Heim (2010)
The Model
Basic Mechanism: Random growth w/ death $\Rightarrow$ Pareto
Entrepreneur’s Problem

\[
\max_{\{c_t, e_t\}} \mathbb{E} \int_0^\infty e^{-(\rho+\delta+\bar{\delta})t} \left[ \log c_t + \beta \log \ell_t \right] dt
\]

subject to

\[
c_t = \psi_t x_t
\]

\[
e_t + \ell_t + \tau = 1
\]

\[
dx_t = \mu(e_t)x_t dt + \sigma x_t dB_t
\]

\[
\mu(e) = \phi e
\]

\(x\) = idiosyncratic productivity of a variety

\(\delta\) = endogenous creative destruction

\(\bar{\delta}\) = exogenous destruction

\(\psi\) = determined in GE
Solution for Entrepreneur’s Problem

• The Bellman equation for the entrepreneur:

\[
\rho V(x) = \max_e \log \psi + \log x + \beta \log(\Omega - e) + \frac{\mathbb{E}[dV(x)]}{dt} \\
+ (\delta + \bar{\delta})(V^w - V(x))
\]

where \( \Omega \equiv 1 - \tau \).

• Equilibrium effort:

\[
e^* = \Omega - \frac{1}{\phi} \cdot \beta (\rho + \delta + \bar{\delta})
\]
Stationary Distribution of Entrepreneur’s Income

- Unit measure of entrepreneurs / varieties
- Displaced in two ways
  - Exogenous destruction ($\bar{\delta}$): new entrepreneur $\rightarrow x_0$.
  - Endogenous creative destruction ($\delta$): inherit existing productivity $x$.
- Distribution $f(x, t)$ satisfies Kolmogorov forward equation:
  \[
  \frac{\partial f(x, t)}{\partial t} = -\frac{\partial}{\partial x} [\mu(e^*) x f(x, t)] + \frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left[ \sigma^2 x^2 f(x, t) \right] - \bar{\delta} f(x, t)
  \]
- Stationary distribution $\lim_{t \rightarrow \infty} f(x, t) = f(x)$ solves
  \[
  \frac{\partial f(x, t)}{\partial t} = 0
  \]
• Guess that \( f(\cdot) \) takes the Pareto form \( f(x) = C x^{-\xi - 1} \) \( \Rightarrow \)

\[
\xi^* = -\frac{\tilde{\mu}^*}{\sigma^2} + \sqrt{\left(\frac{\tilde{\mu}^*}{\sigma^2}\right)^2 + \frac{2 \bar{\delta}}{\sigma^2}}
\]

\[
\tilde{\mu}^* \equiv \mu(e^*) - \frac{1}{2} \sigma^2 = \phi \Omega - \beta (\rho + \delta^* + \bar{\delta}) - \frac{1}{2} \sigma^2
\]

• Power-law inequality is therefore given by

\[
\eta^* = 1/\xi^*
\]
Comparative Statics (given $\delta^*$)

$$\eta^* = 1/\xi^*, \quad \xi^* = -\frac{\mu^*}{\sigma^2} + \sqrt{\left(\frac{\mu^*}{\sigma^2}\right)^2 + \frac{2\delta}{\sigma^2}}$$

$$\tilde{\mu}^* = \phi \Omega - \beta (\rho + \delta^* + \bar{\delta}) - \frac{1}{2}\sigma^2$$

- Power-law inequality $\eta^*$ increases if
  - $\uparrow \phi$: better technology for converting effort into $x$
  - $\downarrow \delta$ or $\bar{\delta}$: less destruction
  - $\downarrow \tau$: More entrepreneurial time ($\uparrow \Omega$)
  - $\downarrow \beta$: Lower utility weight on leisure
Growth and Creative Destruction

Final output

\[ Y = \left( \int_0^1 Y_i^{\theta} di \right)^{1/\theta} \]

Production of variety \( i \)

\[ Y_i = \gamma^n x_i^\alpha L_i \]

Resource constraint

\[ L_t + R_t + 1 = \bar{N}, \quad L_t \equiv \int_0^1 L_{it} di \]

Flow rate of innovation

\[ \dot{n}_t = \lambda R_t \]

Creative destruction

\[ \delta_t = \dot{n}_t \]
Suppose \( \frac{R}{\bar{L}} = \bar{s} \) where \( \bar{L} \equiv \bar{N} - 1 \).

Define \( X \equiv \int_{0}^{1} x_i di = \frac{x_0}{1-\eta} \). Markup is \( 1/\theta \).

Aggregate PF
\[
Y_t = \gamma^{n_t} X^\alpha L
\]

Wage for \( L \)
\[
w_t = \theta \gamma^{n_t} X^\alpha
\]

Profits for variety \( i \)
\[
\pi_{it} = (1 - \theta) \gamma^{n_t} X^\alpha \left( \frac{x_i}{X} \right)
\]

Definition of \( \psi_t \)
\[
\psi_t = (1 - p)(1 - \theta) \gamma^{n_t} X^{\alpha - 1}
\]

Note that \( \eta \) has a level effect on output and wages.
Growth and Inequality in the $\bar{s}$ case

- Creative destruction and growth
  \[ \delta^* = \lambda R = \lambda \bar{s} \bar{L} \]
  \[ g_y^* = \dot{n} \log \gamma = \lambda \bar{s} \bar{L} \log \gamma \]

- Does rising top inequality always reflect positive changes?
  - No! $\uparrow \bar{s}$ raises growth but reduces inequality (more creative destruction).
Endogenizing $s = \frac{R}{\bar{L}}$

- Researcher can sell a new idea at price $P_t$
  - Assume $x$ not observed until after idea used
  - So $P$ does not depend on $x$.
  - Entrepreneurs pay a constant fraction of their profits, $p$, to rent an idea
- People are indifferent ex ante to being researcher, worker, or entrepreneur
  - Worker and researcher: $w_t = \lambda P_t (1 - \tau_R)$
  - Worker and entrepreneur: $\mathbb{E}[V(x, p)] = V^w$
• Price of an idea ($P_t = \text{pdv of rental payments } \psi_t x_t p$)

\[
P_t = \frac{\psi_t p}{r + \delta + \bar{\delta} - g - \mu} \cdot \frac{x_0}{1 - \eta}
\]

• Value function for worker (no leisure)

\[
V^w = \frac{1}{\rho} \left( \log w_t + \frac{g}{\rho} \right)
\]

• Value function for entrepreneur

\[
(\rho + \delta + \bar{\delta}) EV(x, p) = \log x_0 + \eta + \beta \log(\Omega - e^*) + \log \psi_t(p) + \frac{\tilde{\mu}}{\rho + \delta + \bar{\delta}} + (\delta + \bar{\delta}) V^w
\]
Equilibrium solution

Drift of log x
\[ \tilde{\mu}^* = \phi \Omega - \beta (\rho + \delta^* + \bar{\delta}) - \frac{1}{2} \sigma^2 \]

Pareto inequality
\[ \eta^* = \frac{1}{\xi^*}, \quad \xi^* = -\frac{\tilde{\mu}^*}{\sigma^2} + \sqrt{\left(\frac{\tilde{\mu}^*}{\sigma^2}\right)^2 + \frac{2}{\sigma^2}} \]

Research allocation
\[ \frac{s^*}{1-s^*} = \frac{1-\theta}{\theta} \cdot \frac{\delta^* p^* (1-\tau_R)}{\rho + \delta^* + \bar{\delta} - \mu} \]

Creative destruction
\[ \delta^* = \lambda s^* \bar{L} \]

Growth
\[ g^* = \delta^* \log \gamma \]

Rental price of idea
\[ p^* = 1 - \frac{\theta}{1-\theta} \cdot \frac{\exp(-\eta)}{1-\eta} \cdot \frac{\exp\left\{\frac{g}{\rho} - \frac{\tilde{\mu}}{\rho + \delta^* + \bar{\delta}}\right\}}{(1-s^*)L(\Omega-c^*)^\beta} \]
Understanding Real World Inequality?

<table>
<thead>
<tr>
<th></th>
<th>1980</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>.44</td>
<td>.65</td>
</tr>
<tr>
<td>France</td>
<td>.40</td>
<td>.47</td>
</tr>
</tbody>
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• Pareto inequality is rising throughout the world
  ○ Technological change: $\phi \uparrow$ – World wide web, etc. make it easier for an entrepreneur to grow her market

• Does rising top inequality always reflect positive changes?
  ○ No! $\bar{s} \uparrow$ raises growth but reduces inequality (more creative destruction).
Why does $\eta$ for France and Japan rise less?

• Technological change needs to be offset by something
  ◦ Rise in $\bar{s}$ and creative destruction?
  ◦ Rise in misallocation: $\uparrow \bar{\delta}$
  ◦ Globalization: rise in $\delta^*$ because of opening up to competition from U.S., etc.
    – flat inequality could result from a “good” policy

• Policies that encourage entrepreneurs to exert less effort
  ◦ Rise in utility weight on leisure in France
  ◦ Lost decade in Japan?
Numerical examples

\[ \rho = 0.01, \quad \bar{L} = 1, \quad \theta = \frac{2}{3}, \quad \Omega = 1 \]
\[ \phi = 0.13, \quad \beta = 0.5, \quad \gamma = 1.1, \quad \lambda = 2.1 \]
\[ \sigma^2 = 0.1, \quad \bar{s} = 0.10, \quad \bar{\delta} = 0.10 \]
Varying the $x$-technology parameter $\phi$

Power law inequality

Growth rate (percent)

x–technology, $\phi$

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How the model is working

• $\uparrow \phi$ raises top inequality, but leaves the growth rate of the economy unchanged.
  
  ◦ Surprising in that we have a “linear differential equation” for $x$.

• Key: the distribution of $x$ is stationary!

• Higher $\phi$ has a positive level effect through higher inequality, raising everyone’s wage.
  
  ◦ But growth is only determined through research, not through $x$...
Varying the Research Share, $\bar{s}$

![Graph showing the relationship between research share and inequality](image-url)
Varying Exogenous Destruction, $\tilde{\delta}$

Power law inequality

Growth rate (percent)

Exogenous destruction, $\delta$ bar
Varying the Utility Weight on Leisure, $\beta$

![Graph showing the relationship between utility weight on leisure and growth rate](graph.png)

- **Power law inequality**: The graph plots the growth rate (percent) against the utility weight on leisure, $\beta$, for different values of $\beta$.
- **Utility weight on leisure, $\beta$**: The x-axis represents the utility weight on leisure, ranging from 0.2 to 0.7.
- **Growth rate (percent)**: The y-axis represents the growth rate in percent, ranging from 0 to 4.

The graph illustrates how varying the utility weight on leisure affects the growth rate, highlighting the relationship between these two economic indicators.
The Role of Luck, $\sigma$

![Graph showing the relationship between Stdev of shocks, $\sigma$, Power law inequality, and Growth rate (percent).]

- **Power law inequality**
  - Stdev of shocks, $\sigma$
  - Growth rate (percent)
  - A Schumpeterian Model of Top Income Inequality – p.33/50
Numerical examples: endogenous $s$

\[ \rho = .01, \quad \bar{L} = 36, \quad \theta = 2/3, \quad \Omega = 1 \]
\[ \phi = .13, \quad \beta = .5, \quad \gamma = 1.1, \quad \lambda = .06 \]
\[ \sigma^2 = .1, \quad \tilde{\delta} = .15 \]
Varying the x-technology parameter $\phi$

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Why does $\uparrow \phi$ reduce growth?

- $\uparrow \phi \Rightarrow \uparrow e^* \Rightarrow \uparrow \mu^*$

- Two effects
  - **Direct effect**: raises drift, so entrepreneurship worth more $\Rightarrow$ more research and faster growth
  - **Indirect effect**: raises Pareto inequality ($\eta$), so entrepreneurship is riskier $\Rightarrow$ worth less $\Rightarrow$ less research and slower growth

- Pareto distribution $\Rightarrow$ inequality channel is big so the 2nd effect dominates.
Varying Exogenous Destruction, $\bar{\delta}$

Graph showing the relationship between power law inequality and growth rate (percent) with respect to exogenous destruction, $\delta$.
Varying the Utility Weight on Leisure, $\beta$
Varying the Tax on Research, $\tau_R$
Varying the Size of the Market, $\bar{L}$
The Role of Luck, $\sigma$

[Graph showing the relationship between Power law inequality and Growth rate (percent) with respect to Stdev of shocks, $\sigma$.]
Matching Top Inequality in the US and France

US / France: $\phi$ goes from .106 to .172

France: Another parameter changes as well...
US: Varying the $x$-technology parameter $\phi$

- Power law inequality
- Growth rate (percent)

$\phi$ in US rises from 0.106 to 0.172
France: A rise in exogenous destruction, \( \bar{\delta} \)

\(\bar{\delta}\) in France rises from 0.175 to 0.223
France: A rising utility weight on leisure, $\beta$

$\beta$ in France rises from 0.350 to 0.460
France: A rising subsidy to research, $\tau_R$

$\tau_R$ in France falls from -0.400 to -1.000

Power law inequality

Growth rate (percent)

US, $\eta$

France, $\eta$
France: Rising size of the Market, $\bar{L}$

\[ \bar{L} \text{ in France rises from 15.000 to 30.000 (with } \bar{\delta} = 0.22) \]

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France: A fall in idiosyncratic risk, $\sigma$

$\sigma$ in France falls from 0.280 to 0.120
Conclusions

- Pareto income inequality
  - Entrepreneurs exert effort to raise productivity
  - Creative destruction moderates inequality

- Worldwide rise in top income inequality
  - $\phi \uparrow$: Technological change makes it easier for effort to raise productivity/market share (world wide web)

- France and Japan
  - More misallocation ($\tilde{\delta}$)?
  - Policies that have discouraged entrepreneurs?
  - Opening up to creative destruction from rest of world?
To Do

- Optimal allocation
  - Research externalities suggest $\delta^*$ too low $\Rightarrow \eta^*$ too high??

- Use Chetty et al IRS panel data to estimate the stochastic process for top incomes? (Guvenen, others?)
  - Random walk with periodic “death”? 
  - Changing stochastic process over time?
  - How important is luck?
  - Doing this for France or other countries as well would be really valuable...