Misallocation, Economic Growth, and Input-Output Economics

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Important theme of recent growth literature:

- Enhanced appreciation of the role that misallocation plays in explaining cross country income differences

Examples:

- Restuccia and Rogerson (2008), Banerjee and Duflo (2005), Hsieh and Klenow (2009)
Three Points

- **Misallocation**: Overview of misallocation

- **Theory**: The input-output structure of the economy can amplify effects of misallocation

- **Empirics**: Quantifying the input-output multiplier

  Asks more questions than it answers...
I. Misallocation
1. Misallocation and TFP: A Simple Example

Production: \[ X_{steel} = L_{steel}, \quad X_{latte} = L_{latte} \]

Resource constraint: \[ L_{steel} + L_{latte} = \bar{L} \]

GDP (aggregation): \[ Y = X_{steel}^{1/2}X_{latte}^{1/2} \]

\[ x \equiv L_{steel}/\bar{L} \] denotes the allocation (markets, distortions, central planner, etc).

Then GDP and TFP are

\[ Y = A(x)\bar{L} \]

\[ A(x) = \sqrt{x(1-x)} \]
Misallocation Reduces TFP

Total factor productivity, $A(x)$

Fraction of labor making steel, $x$
An Alternative View of Misallocation

Total factor productivity, $A(x)$

Fraction of labor making steel, $x$
Advantages of “Alternative View”

- Intermediate degrees of misallocation can have large effects
- In a poor country, small improvements in the allocation may have small effects: growth miracles are hard.

What models deliver this “alternative view”?

- O-ring complementarity of Kremer (1993)?
- Others?

Simple example misleads on one key point

- Misallocation may not only be across sectors
- *Within* sector?
- *Within* firms and plants?
2. Misallocation of Ideas?

**Romer (1990) variety framework**

- Romer (1994) suggests effects can be large
- But not so when goods are highly substitutable
- Broda and Weinstein (2006): Gains from new varieties imported into the U.S. between 1972 and 2001 only 2.6% of GDP.

**Is a different approach needed?**

- Quality ladders, a la Aghion-Howitt?
3. Key Questions

**What is the nature of misallocation?**

- Within sector? Between sectors? Within firms?
- Ideas?

**Why is there misallocation?**

- Active literature on political economy and growth
- Acemoglu, Johnson, and Robinson (2005)
- “Alternative view” of misallocation may help...

**How does misallocation lead to 50-fold income differences?**

- Amplification question.
- Significant in business cycle models; much more needed in growth!
II. Input-Output Economics: Overview
Capital multiplier: more $K \rightarrow$ more $Y \rightarrow$ more $K$, etc.

- Multiplier is $1 + \alpha + \alpha^2 + \ldots = \frac{1}{1-\alpha} = 3/2$ if $\alpha = 1/3$.
- Mankiw, Romer, and Weil (1992): This is too small...
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Broaden capital: Need $\alpha = 2/3 \Rightarrow$ multiplier = 3

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A Brief History of the Growth Literature

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**Intermediate goods are another possibility!**

- Very similar to capital, only depreciate fully in production
- Easily measured, share of gross output is large
A Simple Example

- Gross output and intermediate goods

\[ Q_t = \bar{A} \left( K_t^\alpha L_t^{1-\alpha} \right)^{1-\sigma} X_t^\sigma \]

\[ X_{t+1} = \bar{x} Q_t \]

- GDP is \( Y_t \equiv (1 - \bar{x}) Q_t \). In steady state:

\[ Y = \text{TFP} \cdot K^{\alpha} L^{1-\alpha} \]

\[ \text{TFP} \equiv \left( \bar{A} \bar{x}^\sigma (1 - \bar{x})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]
With capital accumulation...

- A constant fraction $\bar{s}$ of GDP is invested:

$$K_{t+1} = \bar{s}Y_t + (1 - \delta)K_t$$

- GDP per worker in steady state is

$$y^* \equiv \frac{Y}{L} = \left(\bar{A}\bar{x}\sigma (1 - \bar{x})^{1-\sigma} \left(\frac{\bar{s}}{\delta}\right)^{\alpha(1-\sigma)}\right)^{\frac{1}{(1-\alpha)(1-\sigma)}}$$
The effects of misallocation and differences in $\bar{A}$ are multiplied:

- A 1% increase in $\bar{A}$ raises output by more than 1% because of the multiplier $\frac{1}{(1-\alpha)(1-\sigma)}$.
- With no intermediate goods, just the standard $\frac{1}{1-\alpha} = 1 + \alpha + \alpha^2 + ...$
- With intermediate goods, an additional effect from the induced production of intermediates, so multiplier is larger.
- Can be written as $\frac{1}{1-\beta}$, where $\beta \equiv \sigma + \alpha(1 - \sigma)$ is the total factor share of produced goods.

Quantitatively significant

- Standard values: $\alpha = 1/3$, $\sigma = 1/2$
- Share of produced goods: $\beta = \sigma + \alpha(1 - \sigma) = 2/3$
- Total multiplier: $\frac{1}{(1-\alpha)(1-\sigma)} = 3$
Input-Output Economics: Model

Is the multiplier effect diluted by a realistic I-O structure?
Economic Environment: \( N \) sectors

\[
Q_i = A \cdot A_i \left( K_i^{\alpha_i} H_i^{1-\alpha_i} \right)^{1-\sigma_i-\lambda_i} \\
\begin{array}{l}
d_i^{\sigma_1} d_i^{\sigma_2} \cdot \ldots \cdot d_i^{\sigma_N} \\
\quad \text{domestic IG}
\end{array} \\
\begin{array}{l}
m_i^{\lambda_1} m_i^{\lambda_2} \cdot \ldots \cdot m_i^{\lambda_N} \\
\quad \text{imported IG}
\end{array}
\]

Resource constraint (j):
\[
c_j + \sum_{i=1}^{N} d_{ij} = Q_j
\]

Aggregation:
\[
Y = c_1^{\beta_1} \cdot \ldots \cdot c_N^{\beta_N}
\]

Resource constraint:
\[
C + X = Y
\]

Physical capital:
\[
\sum_{i=1}^{N} K_i = K \text{ given}
\]

Human capital:
\[
\sum_{i=1}^{N} H_i = H \text{ given}
\]

Balanced trade:
\[
\overline{PX} = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{p}_j m_{ij}
\]
Equilibrium with Misallocation

**Allocation of Resources:** A standard competitive equilibrium, where some heterogeneous fraction $\tau_i$ of firm $i$’s output is expropriated.

- Could be a tax.

- Could also be theft, regulations, special relationships, etc.

- A more general model could allow input-specific distortions at the firm level as well.

- To keep presentation short, I omit a formal definition of equilibrium (see paper).
Proposition 1 (Solution for $Y$ and $C$)

In the competitive equilibrium with misallocation, the solution for total production of the aggregate final good is

\[ Y = A\tilde{\mu} K^{\tilde{\alpha}} H^{1-\tilde{\alpha}} \epsilon \]

where

\[ \mu' \equiv \frac{\beta'(I-B)^{-1}}{1-\beta'(I-B)^{-1}} \lambda \quad (N \times 1 \text{ vector of multipliers}) \]

\[ \tilde{\mu} \equiv \mu' \mathbf{1} \]

\[ \log \epsilon \equiv \omega + \mu' \tilde{A}, \quad \text{where} \quad \tilde{A}_i \equiv A_i (1 - \tau_i). \]
Understanding the Key Multiplier, $\tilde{\mu}$

$$
\mu' \equiv \frac{\beta'(I - B)^{-1}}{1 - \beta'(I - B)^{-1}\lambda}
$$

The matrix $L \equiv (I - B)^{-1}$ is known as the Leontief inverse.

- $I$ is the $N \times N$ identity matrix
- $B$ is the $N \times N$ input-output matrix, with typical element $\sigma_{ij}$
- Let $\ell_{ij}$ denote the typical element of $L$
- Then a 1% increase in $A_j$ raises output in sector $i$ by $\ell_{ij}$%

Then $\beta'(I - B)^{-1}$ just adds up these effects across all sectors

- Weight by value-added
- Typical element reveals the effect of sector $j$ on GDP.

Finally $\tilde{\mu} \equiv \mu'1$

- This reveals the effect on GDP if economy-wide productivity rises by 1%.
Proposition 2 (Multiplier in a special case)

- Assume each sector has the same total exponent on intermediate goods (though composition can vary):

  \[ \sigma_i \equiv \sum_{j=1}^{N} \sigma_{ij} = \hat{\sigma} \text{ and } \lambda_i \equiv \sum_{j=1}^{N} \lambda_{ij} = \hat{\lambda} \text{ for all } i. \]

- Define \( \bar{\sigma} \equiv \hat{\sigma} + \hat{\lambda} < 1 \) to be the total intermediate goods share.

- Then,

\[ \frac{\partial \log Y}{\partial \log A} = \mu' 1 = \frac{\beta'(I-B)^{-1} 1}{1 - \beta'(I-B)^{-1} \lambda} = \frac{1}{1 - \bar{\sigma}}. \]
Proposition 3 (Symmetry and Distortions)

- Suppose all parameters are identical across sectors:
  \[ \sigma_{ij} = \hat{\sigma}/N, \lambda_{ij} = \hat{\lambda}/N, \beta_i = 1/N, \text{ and } \tau_i = \bar{\tau} \]

- Define \( \bar{\sigma} \equiv \hat{\sigma} + \hat{\lambda} < 1 \) to be the total intermediate goods share.

- Then,
  \[
  \log C = \text{Constant} + \frac{\bar{\sigma}}{1-\bar{\sigma}} \log (1 - \bar{\tau}) + \log (1 - \bar{\sigma}(1 - \bar{\tau}))
  \]
Consumption vs. $\bar{\tau}$ with $\bar{\sigma} = 1/2$
Proposition 4 (Symmetry with Random Distortions)

• Suppose all parameters are identical across sectors:

\[ \sigma_{ij} = \hat{\sigma}/N, \lambda_{ij} = \hat{\lambda}/N, \text{ and } \beta_i = 1/N \]

• Define \( \bar{\sigma} \equiv \hat{\sigma} + \hat{\lambda} < 1 \) to be the total intermediate goods share.

• Assume \( \log(1 - \tau_i) \sim N(\theta, \nu^2) \) and let \( 1 - \bar{\tau} \equiv e^{\theta + \frac{1}{2}\nu^2} \) reflect the average distortion.

• Then,

\[
\text{plim} \ \log C = \frac{\bar{\sigma}}{1-\bar{\sigma}} \cdot (1 - \bar{\tau}) + \log (1 - \bar{\sigma}(1 - \bar{\tau})) - \frac{1}{1-\bar{\sigma}} \cdot \frac{1}{2} \cdot \nu^2
\]
Summary

Key Result: The input-output structure of the economy multiplies the effects of distortions.

- Closely related to the Diamond-Mirrlees result about not taxing intermediate goods.

- It would be nice to derive a result for log-normal distortions in the general input-output model, but I have not been able to do so thusfar.

- The multiplier $\tilde{\mu}$ surely plays a key role.
III. Input-Output Economics: Data

The empirical version of the point that $\tilde{\mu} \approx \frac{1}{1-\bar{\sigma}}$
The U.S. Input-Output Matrix, 480 Industries

The good being used

Industry using the input

Wholesale trade (381)
Trucking (385)
Management of Companies (431)
Iron & Steel Mills (201)
Real Estate (411)
Paperboard products (125)

Ag/Mi/Con | ------------------ Manufacturing ------------------ | ---- Services ----
The U.S. Input-Output Matrix, 48 Industries

The Good Being Used

Wholesale/retail trade (31)
Land/pipeline transport (33)
Radio/telecomm/semi−conductors (19)
Office/accounting/computing mach. (17)
F.I.R.E. (38−39)

Other business activities (43)
Japan’s Input-Output Matrix, 48 Industries
China’s Input-Output Matrix, 48 Industries

The Good Being Used

Electricity (26)
Metals (13−15)
Conclusions

Input-Output Data

- The simple $1/1 - \sigma$ formula works remarkably well.
- Input-output matrices are surprisingly similar across countries.

Input-Output Models

- The input-output structure of an economy has the potential to substantially amplify the effect of distortions.
- If 1/2 of output gets stolen at each stage of production, then the effect on final GDP is much larger: 1/2 of the steel is lost, 1/2 of the cars are lost, and 1/2 of the pizzas are lost — so the steel is essentially stolen three times!

Misallocation

- Intermediate goods are misallocated, just like capital and labor.
- Would be valuable to redo the Hsieh-Klenow (2009) exercise taking this into account.