On the Lamb Vector Divergence as a Momentum Field Diagnostic Employed in Turbulent Channel Flow

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Overview

- Kinematics
- Dynamics
- Turbulent Channel Flow DNS

Hypothesis:
“The Lamb Vector Divergence and Transport Identifies Dynamically Important Motions.”

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{\omega} \times \mathbf{u} = -\nabla \left( \frac{p}{\rho} + \frac{u^2}{2} \right) - \nu \nabla \times \mathbf{\omega}
\]
Kinematics

- **Lamb Vector**
  \[ l = \omega \times u = (\nabla \times u) \times u \]

- **Lamb Vector Divergence**
  \[ \nabla \cdot l = u \cdot \nabla \times \omega - \omega \cdot \omega \]
  - Flexion Product
  - Enstrophy

- **Parity/Galilean Invariant**
\[ \nabla \cdot \mathbf{l} = \mathbf{u} \cdot \nabla \times \mathbf{\omega} - \mathbf{\omega} \cdot \mathbf{\omega} \]

- **Source/Sink Character**
  - \(-\mathbf{\omega} \cdot \mathbf{\omega} \leq 0\) Always (i.e. Sink)
  - \(\mathbf{u} \cdot \nabla \times \mathbf{\omega} \geq 0\) Typically (i.e. Source)

- **Positive \(\nabla \cdot \mathbf{l}\) Occurs Only From \(\mathbf{u} \cdot \nabla \times \mathbf{\omega}\)**
  - \(\mathbf{u} \cdot \nabla \times \mathbf{\omega}\) : High to Low Streamline Curvature.
  - Vortex “Unwinding” : Angular \(\rightarrow\) Linear Momentum
Dynamics

\[ \nabla \cdot \mathbf{l} = -\nabla^2 \Phi \quad \text{where} \quad \nabla \cdot \mathbf{u} = 0 \]

- Bernoulli Function is \( \Phi = \frac{p}{\rho} + \frac{u^2}{2} \)

- Harmonic Interpretation
  - If \( \nabla \cdot \mathbf{l} > 0 \), \( \Phi \) has a Local Maxima (Evacuates).
  - If \( \nabla \cdot \mathbf{l} < 0 \), \( \Phi \) has a Local Minima (Concentrates).

- Energy Curvature (Approaches Equilibrium)

- Distinct Capacity to Alter Momentum Field
Mean Lamb Vector Divergence: \[
\Lambda^+ = F^+ + E^+
\]

Mean Flexion Product: \[
F^+ = -U^+ \frac{d^2 U^+}{dy^+2}
\]

Mean Enstrophy: \[
E^+ = -\left(\frac{dU^+}{dy^+}\right)^2
\]

Mean Lamb Vector Divergence Components, \(Re\tau=590\).

Data courtesy of Moser, Kim, Mansour (1999).
Correlation Between Fluctuating $\nabla \cdot l$ and $\Phi$

Joint PDF of Fluctuating Lamb Vector Divergence and Bernoulli Function, $Re_t=180$.

Data courtesy of H.M. Blackburn.

$$y^+ = 2$$
$$y^+ = 12$$

$$\rho = -0.93$$
$$\rho = +0.70$$

- Viscous Layer: $\Phi$ and $\nabla \cdot l$ are Anti-correlated (Sink).
- Buffer Layer: $\Phi$ and $\nabla \cdot l$ are Correlated (Source).
Positive/Negative Character ("Unwinding")

- Angular ($\nabla \cdot \mathbf{l} < 0$) Converts to Linear ($\nabla \cdot \mathbf{l} > 0$) Momentum
Conclusions

- Lamb Vector Divergence:
  - Identifies Conversion of Angular to Linear Momentum
    - Interference of Dissipation and Enstrophy
  - Measures Energy Curvature (i.e. Bernoulli Function)
    - Superharmonic/Subharmonic Characterization
- $\nabla \cdot \mathbf{l}$ Identifies Regions Having a Distinct Capacity to Affect a Time Rate of Change of Momentum.
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Exact Lamb Vector Divergence Decomposition

Mean Lamb Vector Divergence:
\[ \Lambda^+ = \nabla \cdot l^+ \]
\[ = F^+ + E^+ + C^+ \]

Mean Flexion Product:
\[ F^+ = -U^+ \frac{d^2 U^+}{d y^+} \]

Mean Enstrophy:
\[ E^+ = - \left( \frac{d U^+}{d y^+} \right)^2 \]

Mean Correlation:
\[ C^+ = \frac{d}{d y^+} \left( \frac{u'_x \omega'_z - u'_z \omega'_x}{d y^+} \right)^+ \]

Viscous Layer  |  Buffer Layer  |  Outer Layer

Mean Lamb Vector Divergence Components, Reτ = 590.

Data courtesy of Moser, Kim, Mansour (1999).