CS255: Cryptography and Computer Security **Winter 2021**

Assignment #2

Due: Wednesday, Feb. 17, 2021, by Gradescope (each answer on a separate page).

Problem 1. RawCBC attacks. In class we discussed the ECBC (encrypted CBC) MAC for messages in $\mathcal{X}^{\leq L}$ where $\mathcal{X} = \{0,1\}^n$. Recall that RawCBC is the same as ECBC, but without the very last encryption step. We showed that RawCBC is an insecure MAC for variable length messages. Here we show a more devastating attack on RawCBC. Let m_1 and m_2 be two multi-block messages. Show that by asking the signer for the MAC tag on m_1 and for the MAC tag on one additional multi-block message m_2' of the same length as m_2, the attacker can obtain the MAC tag on $m = m_1 \parallel m_2$, the concatenation of m_1 and m_2.

Problem 2. Multicast MACs. Suppose user A wants to broadcast a message to n recipients $B_1, \ldots, B_n$. Privacy is not important but integrity is. In other words, each of $B_1, \ldots, B_n$ should be assured that the message he is receiving were sent by A. User A decides to use a MAC.

a. Suppose user A and $B_1, \ldots, B_n$ all share a secret key k. User A computes the MAC tag for every message she sends using k. Every user B_i verifies the tag using k. Using at most two sentences explain why this scheme is insecure, namely, show that user B_1 is not assured that messages he is receiving are from A.

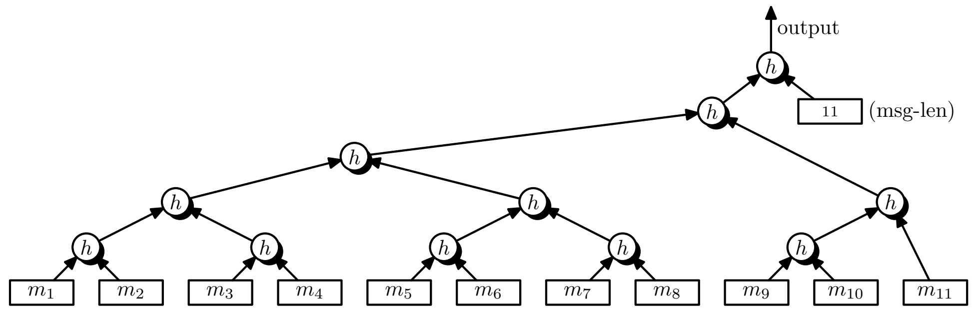
b. Suppose user A has a set $S = \{k_1, \ldots, k_\ell\}$ of ℓ secret keys. Each user B_i has some subset $S_i \subseteq S$ of the keys. When A transmits a message she appends ℓ MAC tags to it by MACing the message with each of her ℓ keys. When user B_i receives a message he accepts it as valid only if all tags corresponding to keys in S_i are valid. Let us assume that the users $B_1, \ldots, B_n$ do not collude with each other. What property should the sets $S_1, \ldots, S_n$ satisfy so that the attack from part (a) does not apply?

c. Show that when $n = 10$ (i.e. ten recipients) it suffices to take $\ell = 5$ in part (b). Describe the sets $S_1, \ldots, S_{10} \subseteq \{k_1, \ldots, k_5\}$ you would use.

d. Show that the scheme from part (c) is completely insecure if two users are allowed to collude.

Problem 3. Parallel Merkle-Damgård. Recall that the Merkle-Damgård construction gives a *sequential* method for extending the domain of a CRHF. The tree construction in the figure below is a parallelizable approach: all the hash functions h within a single level can be computed in parallel. Prove that the resulting hash function defined over $(\mathcal{X}^{\leq L},\ \mathcal{X})$ is collision resistant, assuming h is collision resistant. Here h is a compression function $h : \mathcal{X}^2 \to \mathcal{X}$, and we assume the message length can be encoded as an element of $\mathcal{X}$.

More precisely, the hash function is defined as follows:



input: $m_1 \dots m_s \in \mathcal{X}^s$ for some $1 \le s \le L$
output: $y \in \mathcal{X}$

let $t \in \mathbb{Z}$ be the smallest power of two such that $t \ge s$ (i.e., $t := 2^{\lceil \log_2 s \rceil}$)
for $i = s+1$ to t: $m_i \leftarrow \perp$
for $i = t+1$ to $2t-1$:
    $\ell \leftarrow 2(i-t)-1, \quad r \leftarrow \ell + 1$ // *indices of left and right children*
    if $m_\ell = \perp$ and $m_r = \perp$: $m_i \leftarrow \perp$ // *if node has no children, set node to null*
    else if $m_r = \perp$: $m_i \leftarrow m_\ell$ // *if one child, propagate child as is*
    else $m_i \leftarrow h(m_\ell, m_r)$ // *if two children, hash with h*
output $y \leftarrow h\big(m_{2t-1},\ s\big)$ // *hash final output and message length*

Problem 4. In the lecture we saw that Davies-Meyer is used to convert an ideal block cipher into a collision resistant compression function. Let $E(k, m)$ be a block cipher where the message space is the same as the key space (e.g. 128-bit AES). Show that the following methods do not work:

$$f_1(x, y) = E(y, x) \oplus y \quad \text{and} \quad f_2(x, y) = E(x,\ x \oplus y)$$

That is, show an efficient algorithm for constructing collisions for f_1 and f_2. Recall that the block cipher E and the corresponding decryption algorithm D are both known to you.

Problem 5. In lecture we saw that an attacker who intercepts a randomized counter mode encryption of the message "To:bob@gmail.com", can change the ciphertext to be an encryption of the message "To:mel@gmail.com". In this exercise we show that the same holds for randomized CBC mode encryption.

Suppose you intercept the following hex-encoded ciphertext:

85e2654a8b52038c659360ecd8638532 b365828d548b3f742504e7203be41548

You know that the ciphertext is a randomized CBC encryption using AES of the plaintext "To:bob@gmail.com", where the plaintext is encoded as ASCII bytes. The first 16-byte block is the IV and the second 16-byte block carries the message. Modify the ciphertext above so that it decrypts to the message "To:mel@gmail.com". Your answer should be the two block modified ciphertext.

Problem 6. Authenticated encryption. Let (E, D) be an encryption system that provides authenticated encryption. Here E does not take a nonce as input and therefore must be a randomized encryption algorithm. Which of the following systems provide authenticated encryption? For those that do, give a short proof. For those that do not, present an attack that either breaks CPA security or ciphertext integrity.

a. $E_1(k, m) = \big[c \leftarrow E(k, m),\ \text{output}\ \ (c, c)\ \big]$ and $D_1(k,\ (c_1, c_2)\ ) = D(k, c_1)$

b. $E_2(k, m) = \big[c \leftarrow E(k, m),\ \text{output}\ \ (c, c)\ \big]$ and $D_2(k,\ (c_1, c_2)\ ) = \begin{cases} D(k, c_1) & \text{if } c_1 = c_2 \\ \textsf{fail} & \text{otherwise} \end{cases}$

c. $E_3(k, m) = \big(\ E(k, m),\ E(k, m)\ \big)$ and $D_3(k,\ (c_1, c_2)\ ) = \begin{cases} D(k, c_1) & \text{if } D(k, c_1) = D(k, c_2) \\ \textsf{fail} & \text{otherwise} \end{cases}$

To clarify: $E(k, m)$ is randomized so that running it twice on the same input will result in different outputs with high probability.

d. $E_4(k, m) = \big(\ E(k, m),\ H(m)\ \big)$ and $D_4(k,\ (c_1, c_2)\ ) = \begin{cases} D(k, c_1) & \text{if } H(D(k, c_1)) = c_2 \\ \textsf{fail} & \text{otherwise} \end{cases}$

where H is a collision resistant hash function.

Problem 7. Let F be a secure PRF defined over $(\mathcal{K}, \mathcal{X}, \mathcal{Y})$ where $\mathcal{Y} := \{0, 1\}^n$. Let $(E_{\text{ctr}}, D_{\text{ctr}})$ be the cipher derived from F using randomized counter mode. Let $H : \mathcal{Y}^{\leq L} \to \mathcal{Y}$ be a collision resistant hash function. Consider the following attempt at building an AE-secure cipher defined over $(\mathcal{K},\ \mathcal{Y}^{\leq L}, \mathcal{Y}^{\leq L+2})$:

$$E'(k, m) := E_{\text{ctr}}\big(k,\ (H(m), m)\big)\ ; \qquad D'(k, c) := \left\{ \begin{array}{l} (t, m) \leftarrow D_{\text{ctr}}(k, c) \\ \text{if } t = H(m) \text{ output } m, \text{ else } \textsf{reject} \end{array} \right\}$$

Note that when encrypting a single block message $m \in \mathcal{Y}$, the output is three blocks: the random IV, a ciphertext block corresponding to $H(m)$, and a ciphertext block corresponding to m. Show that (E', D') is not AE-secure by showing that it does not have ciphertext integrity. Your attack should make a single encryption query.

At some point in the past, this type of construction was used to protect secret keys in the Android KeyStore. Your attack resulted in a compromise of the key store.

Problem 8. Exponentiation algorithms. Let $\mathbb{G}$ be a finite cyclic group of order p with generator g. In class we discussed the repeated squaring algorithm for computing $g^x \in \mathbb{G}$ for $0 \leq x < p$. The algorithm needed at most $2 \log_2 p$ multiplications in $\mathbb{G}$.

In this question we develop a faster exponentiation algorithm. For some small constant w, called the window size, the algorithm begins by building a table T of size 2^w defined as follows:

$$\text{set}\ \ T[k] := g^k\ \ \text{for } k = 0, \ldots, 2^w - 1\ . \tag{1}$$

a. Show that once the table T is computed, we can compute g^x using only $(1+1/w)(\log_2 p)$ multiplications in $\mathbb{G}$. Your algorithm shows that when the base of the exponentiation g is fixed forever, the table T can be pre-computed once and for all. Then exponentiation

is faster than with repeated squaring.
Hint: Start by writing the exponent x base 2^w so that:

$$x = x_0 + x_1 2^w + x_2 (2^w)^2 + \ldots + x_{d-1} (2^w)^{d-1} \quad \text{where } 0 \le x_i < 2^w \text{ for all } i = 0, \ldots, d-1.$$

Here there are d digits in the representation of x base 2^w. Start the exponentiation algorithm with x_{d-1} and work your way down, squaring the accumulator w times at every iteration.

b. Suppose every exponentiation is done relative to a different base, so that a new table T must be re-computed for every exponentiation. What is the worse case number of multiplications as a function of w and $\log_2 p$?

c. Continuing with Part (b), compute the optimal window size w when $\log_2 p = 256$, namely the w that minimizes the overall worst-case running time. What is the worst-case running time with this w? (counting only multiplications in $\mathbb{G}$)

Problem 9. Feedback. As in homework 1, we would love to hear your feedback on how the course is going so far.

a. How long did you spend on this assignment?

b. Do you have any feedback on the course material? How can the teaching team support you better?