Nested conditionals and genericity in the de Finetti semantics

Daniel Lassiter  
Stanford University

Jean Baratgin  
CHArt, Université de Paris VIII


Abstract The trivalent, truth-functional theory of conditionals proposed by de Finetti in 1936 and developed in a scattered literature since has enjoyed a recent revival in philosophy, psychology, and linguistics. However, several theorists have argued that this approach is fatally flawed in that it cannot correctly account for nested conditionals and compounds of conditionals. Focusing on nested conditionals, we observe that the problem cases uniformly involve generic predicates, and that the inference patterns claimed to be problematic are very plausible when we ensure that only non-generic (episodic and stative) predicates are used. In addition, the trivalent theory makes correct predictions about the original, generic counter-examples when combined with an off-the-shelf theory of genericity. The ability of the trivalent semantics to account for this complex interaction with genericity thus appears as a strong argument in its favor.

1 The trivalent semantics for indicative conditionals


Stalnaker’s thesis: \( P(\text{If } A, C) = P(C | A) \).

Lewis’s (1976) proof is widely thought to show that this equation cannot hold. However, Lewis’ proof tacitly assumes bivalence, as do a wide range of other triviality proofs for Stalnaker’s thesis (Lassiter 2019).

In fact, a trivalent, truth-functional semantics that supports Stalnaker’s thesis while avoiding triviality results was provided more than 3 decades before this thesis was proposed, in de Finetti 1936 (English translation in de Finetti 1995). According to de Finetti, the indicative conditional \( \text{If } A, C \) is true if \( A \land C \), and false if \( A \land \lnot C \), and otherwise undefined (here noted ‘#’). Most crucially, the indicative conditional is always undefined when its antecedent is false.\footnote{For de Finetti, the third truth-value represents an epistemic relation of “doubt” between an agent and a sentence (de Finetti 1995: 182). Other interpretations are possible: for example, Hailperin (1996) interprets the # value as ‘don’t care’. For our purposes, it is not the meaning of the # value that is critical but its formal role in the semantics of conditionals, and so we will not take a stand on this interpretive question.}
Table 1  de Finetti’s trivalent truth-table for the indicative conditional.

<table>
<thead>
<tr>
<th>If A, C</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 0 #</td>
</tr>
<tr>
<td>A</td>
<td>0   # # #</td>
</tr>
<tr>
<td>#</td>
<td># # #</td>
</tr>
</tbody>
</table>

This semantics supports Stalnaker’s thesis quite directly, if we equate the probability of a sentence with the probability that it is true, divided by the probability that it is defined. When A and C are atomic propositions, this comes down to

\[
P(\text{‘If A, C’}) = \frac{P(\text{‘If A, C’ is true})}{P(\text{‘If A, C’ is defined})} = \frac{P(A \land C)}{P(A)} = P(C | A).
\]

See *inter alia* Milne 1997; Cantwell 2006, 2008; Mura 2009; Rothschild 2014; Lassiter 2019 for discussion. The basic idea of the trivalent semantics for indicative conditionals was rediscovered and/or developed by a number of philosophers in the intervening decades, with interesting variations (e.g., Reichenbach 1944: §32; Quine 1960: §46; Cooper 1968; Belnap 1970; Ducrot 1972; Mackie 1973; Adams 1975; Farrell 1979; McDermott 1996; see also Hailperin 1996 for further historical context and formal developments).

Of the various philosophers who have considered a trivalent treatment of indicative conditionals along these lines, several—notably Adams, Mackie, and Ducrot—were particularly interested in expounding the connection between conditionals and supposition that is suggested by the Ramsey test. Indeed, recent psychological studies have supported the trivalent theory (e.g., Politzer, Over & Baratgin 2010; Baratgin, Over & Politzer 2013; Baratgin, Politzer, Over & Takahashi 2018; Politzer, Jamet & Baratgin 2020), and it has been taken up in psychological work as the main representative of a ‘suppositional’ theory of conditionals, supplanting the ‘no truth-value’ theory popularized in philosophical work under the influence of Adams, Edgington, and Bennett. In addition, several more recent formal studies have expanded on de Finetti’s observation that his theory enforces the equation between probabilities of conditionals and conditional probabilities, showing that it circumvents a variety of triviality results (Paneni & Scozzafava 2003; Mura 2009; Rothschild 2014; Lassiter 2019).

Despite these encouraging signs, several theorists have recently argued that the trivalent theory is fatally flawed because it cannot account for complex conditionals—compounds and nestings. In her survey article on indicative conditionals, Edgington (2014: §4.4) rejects de Finetti’s semantics out of hand, noting that it makes problematic predictions about conjunctions of conditionals under one treatment of conjunction and gesturing at ‘equally unappetizing consequences’ under others. Bradley (2002: §8) gives de Finetti’s semantics short shrift for largely the same reasons as Edgington. Douven (2016) gives a detailed critique of the de Finetti proposal based on its predictions regarding nested conditionals, concluding (p.265):
De Finetti’s semantics for conditionals has much to recommend it. However, this paper argues that it is materially inadequate because it gets the truth conditions and probabilities of nested conditionals badly wrong.

This critique goes to the heart of the issue, since it relies only on the core trivalent semantics without any auxiliary assumptions about other operators. Our aim here is to show that de Finetti’s semantics actually makes correct predictions about both right- and left-nested conditionals. The key observation is that the theory is explicitly aimed at single-case probabilities—i.e., probabilities of singular, episodic events. In contrast, the apparent counter-examples discussed below are all generic in form, and so implicitly quantificational. When they are modified to ensure that single events are under discussion, the implausibility of de Finetti’s predictions disappears. The de Finetti semantics also correctly predicts the invalidity of the core examples when combined with an off-the-shelf treatment of genericity and some recent linguistic insights about the use of conditionals to restrict operators. We thus vindicate de Finetti’s restriction on the scope of his account, along with Adams’s (1965: 302) warning that conditionals with antecedents referring to ‘repeatable’ and ‘unrepeatable’ events require very different semantic treatments.

2 Nested conditionals in the de Finetti semantics

The de Finetti semantics is truth-functional and allows arbitrary left- and right-nesting of conditionals. For right-nested conditionals, the predictions are welcome: the semantics validates the so-called ‘Import-Export’ rule, according to which right-nested conditionals with the form of (1a) are equivalent to simple conditionals with conjunctive antecedents like (1b).

(1)  a. If $A$, then (if $B$ then $C$)
    b. If $(A$ and $B)$, then $C$

The validity of this pattern is supported by many instances, for example:

(2)  a. If it’s foggy, then if your brother is here then we won’t go to the beach.
    b. If it’s foggy and your brother is here then we won’t go to the beach.

Fitelson (2015) shows that, in any bivalent semantics, the validity of Import-Export is incompatible with Stalnaker’s thesis, the equation between probabilities of conditionals and conditional probabilities. While sophisticated maneuvering may allow us to evade this result without giving up bivalence (Khoo & Mandelkern 2019), the fact that de Finetti’s semantics validates both Import-Export and Stalnaker’s thesis without further ado appears as a strong argument in its favor (see Lassiter 2019: §9).

However, Douven (2016) argues that de Finetti’s predictions around left-nestings—conditionals in the antecedent of other conditionals—are less welcome: the conditionals in (3) are predicted to be equivalent.

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2 De Finetti’s motivation for this restriction is related to his opposition to the frequentist interpretation of probability and his epistemic interpretation of the # value (see, for instance, de Finetti 1995: 187-188). Our conclusion here is much more limited, and involves only the interpretation of conditionals in natural language. Our conclusions are fully compatible with de Finetti’s philosophical position, but do not rely on it.
a. If \((B \text{ if } A)\), then \(C\)
b. If \((A \text{ and } B)\), then \(C\)

In the de Finetti semantics, the nested conditional \(B \text{ if } A\) in (3a) is true only if \(A \wedge B\). Since the nesting conditional is defined only if the antecedent is true, the entire conditional in (3a) is only defined if \(A \wedge B\), and in that case is true iff \(C\). But these are also the definedness- and truth-conditions of the conditional in (3b). So, de Finetti predicts that conditionals with the forms in (3a) and (3b) should be logically equivalent.

Examining the particular cases that Douven cites, this prediction about left-nested conditionals seems to be obviously wrong.

a. If this material becomes soft if it gets hot, it is not suited for our purposes.
b. If this material gets hot and becomes soft, it is not suited for our purposes.

These sentences are plainly not equivalent. (4b) leaves open the possibility that the material in question is suited for our purposes simply because it does not, in fact, become hot. However, (4a) indicates that the material is unsuitable if it would be soft when hot, regardless of whether it ever actually does become hot.

Douven also discusses counter-examples involving left- and right-nested conditionals.

a. If your mother gets angry if you come home with a B, then she’ll get furious if you come home with a C.
b. If you come home with a B and your mother gets angry and you come home with a C, then your mother will get furious.

These are not intuitively equivalent. The situation with (6) is even worse:

a. If the glass breaks if it’s dropped on the floor, then it breaks if it’s dropped on a pillow.
b. If the glass is dropped on the floor and it breaks and it is dropped on a pillow, then the glass breaks.

As Douven notes, the probability of (6a) is intuitively low, but the probability of (6b) is clearly 1. Yet de Finetti’s theory also predicts that these sentences are logically equivalent: (6a) is defined only if the glass is dropped on the floor, breaks, and is dropped on a pillow, which is the equivalent to the antecedent of (6b). Since logically equivalent sentences cannot fail to have the same probabilities, Douven concludes that de Finetti’s proposal is interesting, but incorrect.

3 De Finetti’s predictions are plausible (perhaps even correct) for left-nested episodic conditionals

As noted above, de Finetti’s semantics is intended to apply only to singular events (de Finetti 1979). Yet the (a) examples in (4), (5), and (6) all involve generalizations about dispositional properties of persons or physical objects. The subject of (4a) involves a certain material’s propensity to respond to heat by becoming soft. (5a) deals with the addressee’s mother’s temperament. The intuitive interpretation of (6a) is about the hardness of the glass and its tendency to break under various circumstances—regardless of whether it ever actually does.
As we will see in a moment, our best linguistic theories of genericity posit additional operators in the logical forms of generic conditionals that disrupt the supposed equivalence of the examples in (4)-(6). However, no such operators occur in conditionals that refer to unique, temporally-bounded events or states—the type that de Finetti’s proposal was intended to apply to. If we modify the examples to ensure that they are episodic, de Finetti’s predictions appear much less implausible.

One simple way to force an episodic interpretation is to focus on past events framed by specific time adverbials.

(7) a. If this material became soft at 3:05PM if it got hot at 3:04PM, our workers were not able to use it at 3:10PM.
   b. If this material got hot at 3:04PM and became soft at 3:05PM, our workers were not able to use it at 3:10PM.

Unlike the generic instance on this argument in (4), it is very plausible that the examples in (7) are truth-conditionally equivalent. For instance, one could verify or falsify (7a) by showing that the material did get hot at 3:04PM and become soft at 3:05PM, and that the workers were/were not able to use the material at 3:10PM. Note, however, that these are the same conditions under which one could verify or falsify (7b)—just as predicted by the de Finetti semantics.

One example obviously does not corroborate a general claim of logical equivalence. For good measure, here is another:

(8) a. If the candidate got angry this morning if he did not sleep well last night, his demeanor is not right for this job.
   b. If the candidate did not sleep well last night and got angry this morning, his demeanor is not right for this job.

If there is any difference in meaning between the examples in (8), it is very subtle indeed.

What about the grading and glass examples (5) and (6)? An episodic variant of (5) is:

(9) a. If your mother got angry if you came home with a B last Thursday, then she’ll get furious if you come home with a C tomorrow.
   b. If you came home with a B last Thursday and your mother got angry and you come home with a C tomorrow, then your mother will get furious.

There is no obvious problem with de Finetti’s prediction that these episodic conditionals are equivalent.

As the reader can verify, an episodic version of (6) sounds bizarre; but this could be due to the fact that its truth would require a particular glass to be broken twice. If we modify the example to make reference to repeatable events, the sentences do appear to be equivalent, as the de Finetti semantics predicts.

(10) a. If the car was dented at 3PM if it bumped into a tree at that time, then it was dented at 3:05PM if it bumped into another car at that time.
    b. If the car bumped into a tree at 3PM and was dented at that time and bumped into another car at 3:05PM, then it was dented at 3:05PM.
In addition, unlike the generic examples in (6), it is difficult to imagine assigning different probabilities to the two sentences.

We conclude that the equivalence between left-nested conditionals and conditionals with conjunctive antecedents predicted by de Finetti is plausible in its intended domain, with predicates referring to unique events.

4 Left-nested generic conditionals

In order to understand why left-nested generic conditionals differ from their episodic counterparts, let’s look to theories of genericity.

In linguistic work on genericity examples (4), (5), and (6) would be treated as generic sentences, with an interpretation that involves an interaction between the conditional antecedent and an implicit generic operator \( \text{GEN} \) (e.g., Krifka, Pelletier, Carlson, ter Meulen, Chierchia & Link 1995; Pelletier & Asher 1997; Cohen 1999a,b). The precise semantics of \( \text{GEN} \) are a matter of much debate. However, nearly all theories agree that (a) \( \text{GEN} \) has an intensional meaning, and (b) it is roughly equivalent to an adverb of quantification over events or situations along the lines of \( \text{generally} \). One rough-and-ready way to test for generic interpretation is to insert ‘generally’ or ‘normally’ and observe whether the interpretation changes dramatically.

(11)  
\begin{align*}
a. & \text{This material becomes soft if it gets hot.} \\
b. & \text{This material generally/normally becomes soft if it gets hot.}
\end{align*}

According to this diagnostic, the antecedent of the original left-nested example (4a) is indeed generic: (11a) has roughly the same interpretation with or without ‘generally/normally’. In contrast, the antecedent of our modified example (7a) has a very different meaning with this addition: it is interpretable, but only as indicating quantification over instances (say, repeated heating events that took place at the same time each day).

(12)  
\begin{align*}
a. & \text{This material became soft at 3:05PM if it got hot at 3:04PM.} \\
b. & \text{This material generally/normally became soft at 3:05PM if it got hot at 3:04PM.}
\end{align*}

The fact that the interpretation is quite different with ‘generally/normally’ indicates that (12a) is most naturally interpreted as non-generic, as claimed above.

The left-nested conditionals in (7) are subjected to this test in (13) and (14). According to the test just described, the antecedent of (13a) (= (7a)) has a \( \text{GEN} \) operator in its logical form.

(13)  
\begin{align*}
a. & \text{If this material becomes soft if it gets hot, it is not suited for our purposes.} \\
b. & \text{If this material (generally/normally) becomes soft if it gets hot, it is not suited for our purposes.}
\end{align*}

However, the example with a conjunctive antecedent does not: the sentences in (14) are not remotely equivalent.

(14)  
\begin{align*}
a. & \text{If this material gets hot and becomes soft, it is not suited for our purposes.} \\
b. & \text{If this material (generally/normally) gets hot and becomes soft, it is not suited for our purposes.}
\end{align*}
(14a) is readily interpreted as about what happens if the material (actually, in the real world) happens to get hot and become soft. In contrast, the antecedent of (14) is about what usually or normally happens. ‘This material generally gets hot and becomes soft’ could be false even if the material does happen to do both—just as ‘Bill is generally happy’ can be true in a situation where he is currently sad.

This means that the logical forms of the left-nested generic conditionals that we have discussed differ from their purported conjunctive equivalents in a critical way: the left-nested examples have a generic operator in the embedded antecedents that has no counterpart in the conjunctive examples. As a result, de Finetti’s semantics would not predict that the sentences are logically equivalent.

(15) a. (4a) \(\rightarrow\) If \[GEN (B if A)\], then \(C\)
    b. (4b) \(\rightarrow\) If \((A and B)\), then \(C\)

Applying the same test to the left-and-right-nested conditionals in (5) and (6) gives similar results: in that case, the respective logical forms of the (a) and (b) examples are as in (16).

(16) a. (6a) \(\rightarrow\) If \[GEN (B if A)\], then \[GEN (D if C)\]
    b. (6b) \(\rightarrow\) If \((A and B and C)\), then \(D\)

This difference in logical form defuses the objection that de Finetti’s semantics incorrectly predicts the equivalence of the (a) and (b) examples in (4) and (6): this holds only if we assign these sentences an incorrect logical form based on a superficial analysis. And when we compare episodic sentences without extraneous operators, the data are as predicted by the trivalent semantics.

5 Domain restricting uses of if

Another parallel between \(GEN\) and adverbs of quantification is that both have domains that can be restricted by conditional antecedents. For adverbs of quantification, this was noted by Lewis (1975): for instance, ‘generally/normally’ in (17) quantifies only over instances in which the material gets hot. Situations in which the material does not get hot, so that the antecedent is false, are simply ignored by the quantificational adverb.

(17) This material generally/normally becomes soft if it gets hot.

It is common in linguistics to take Lewis’ observations to motivate a treatment of conditionals quite generally as devices of domain restriction (Kratzer 1991). However, Huitink (2008: §5), following insights in Belnap 1970, points out that a trivalent semantics can handle domain-restricting uses of conditionals quite straightforwardly. The idea is to define the quantificational operator so that it ignores \# values, corresponding to cases that falsify the conditional antecedent. So, the semantics would be something like the following:

(18) generally(\(P\)) is true iff most situations in which \(P\) is defined are ones in which it is true.

(Note that this is formally very similar to the trivalent definition of probability given in §1, which was crucial in enforcing Stalnaker’s thesis.) If \(P\) happens to be a conditional \(If A, C\), the restriction to situations in which \(P\) is defined will then have the effect of excluding \(\neg A\)-situations from consideration. So, for example, (17) has the logical form generally(this material becomes soft if
it gets hot). The sentence makes the bivalent assertion that the majority of the cases in which the material gets hot (so that the conditional embedded by generally is defined) are ones in which it becomes soft.

A conditional antecedent can also function to restrict the domain of GEN (e.g., Krifka et al. 1995). Intuitively, (19) makes a generalization about what happens in those cases in which the material gets hot, without saying anything about what happens in cases where it does not get hot.

(19) This material becomes soft if it gets hot.

A trivalent semantics can capture the use of conditionals to restrict generics in the same way that Huitink treats adverbs of quantification: simply ensure that the GEN operator ignores # values.

(20) GEN(\(P\)) is true iff most situations in which \(P\) is defined are ones in which it is true.

(This treatment of GEN is very simplistic, but the basic idea is readily adapted to more sophisticated theories of genericity.)

For the treatment of left-nested conditionals, these details are important because they illuminate precisely why Douven’s arguments from left-nested generic conditionals fail, and why the matched episodic conditionals behave differently. Sentences with the logical form If [\(GEN(B if A)\)], \(C\) do not have a ‘conditional proposition’—a three-valued semantic object—in the antecedent at all. Their antecedent is a bivalent, quantificational semantic object which is true iff most \(A\)-situations (in some suitable domain) are \(B\)-situations.

\[
\text{‘If } [GEN(B \text{ if } A)], \text{ then } C' \sim \begin{cases} 
\# & \text{if not most } A\text{-situations are } B\text{-situations} \\
1 & \text{if most } A\text{-situations are } B\text{-situations, and } C \text{ holds} \\
0 & \text{if most } A\text{-situations are } B\text{-situations, and } \neg C \text{ holds}
\end{cases}
\]

So, (7a)/(13a) should be undefined if the material does not (normally) become soft in situations in which it is heated; otherwise, it should be true just in case the material is not suited for our purposes. This is indeed a close paraphrase of the intuitive interpretation.

De Finetti’s semantics, when joined with an appropriate treatment of the use of if to restrict generics, does not predict that left-nested generic conditionals should be equivalent to any particular complex conditionals involving singular events. In particular, it does not predict the equivalence of left-nested generic conditionals with conjunctive conditionals of any stripe: conjunctions do not have an analogous domain-restricting function.

6 Conclusion

Modern linguistic analysis vindicates de Finetti’s insistence on restricting the basic semantics of conditionals to singular events, for different (though not incompatible) reasons to his own. Generalizations have a more complicated semantics than their surface form indicates, and they interact with conditionals in a complex, non-truth-functional way. We have shown that some apparent fatal counter-examples to de Finetti’s trivalent semantics evaporate under careful analysis. While further challenges certainly remain to be addressed, the trivalent semantics certainly deserves continued attention from philosophers, linguists, and psychologists.
References

Cohen, A. 1999b. *Think generic! the meaning and use of generic sentences.* CSLI.