Obligation, goodness, certainty, confidence, likelihood, knowledge, belief, and ability are examples of modal concepts — concepts whose conditions for application depend not only on how the real world is, but also on how the world ought to be, could be, might be given some information that is available to us, and so on. Ought, should, good, certain, likely, probable, think, know, believe, and be able are examples of modal words whose meaning is closely related to the modal concepts just mentioned. This chapter deals with two core questions. First, is there graded structure underlying (at least some) modal concepts – are they all-or-nothing, or can they come in degrees? Second, if there is evidence for graded structure in some modal concepts, how should semanticists model the meanings of the modal words whose meanings are intimately tied up with these concepts? The most influential tradition in logic and semantics conceives of modals on the analogy of quantifiers like some and all, which are clearly all-or-nothing concepts. There is also an old — and not necessarily incompatible — tradition which treats goodness, likelihood, and obligation on the analogy of graded concepts like height, weight, and temperature. In the recent semantics literature it has been increasingly common to focus on scalar aspects of the meanings of modal expressions, but there are still many interesting open questions about how far modal gradation extends, and how it relates to grammatical gradability.

1 Graded concepts and gradable expressions

According to an old and influential theory, concepts are defined in terms of necessary and sufficient conditions which determine exactly which things they apply to and which they do not. For instance,
we might analyze the concept SQUARE along the following lines: something is an instance of SQUARE if and only if it is a two-dimensional figure with four sides of equal length and four equal interior angles. This ‘Aristotelian’ theory has been very influential in studies of nominal concepts like FRUIT, CITY, and TABLE, but it has been largely abandoned in the wake of pioneering work by Eleanor Rosch and others (Rosch 1978). Rosch observed that even concepts like FRUIT do not seem to be all-or-nothing: experimental participants will rate some things (e.g., apples) to be better examples of fruit than others (e.g., crabapples and tomatoes). Such ‘prototype effects’ are unexpected under the Aristotelian theory, since — whatever the criteria associated with FRUIT are — there would seem to be only two possible answers to the question of whether something satisfies these criteria: ‘yes’ or ‘no’. Subsequent work has proposed some alternative explanations that might allow the Aristotelian theory to co-exist with prototype effects, but today most cognitive scientists have rejected the Aristotelian theory in favor of graded approaches to nominal concepts. (See Murphy 2002 for a good introduction to this and related topics.)

Despite these issues with a non-graded approach to nominal concepts, an analysis in terms of necessary and sufficient conditions remains quite plausible for logical concepts like SOME and ALL. Here, of course, we are dealing with a somewhat more complex question — the necessary and sufficient conditions for two concepts to be in a certain relation to each other [cross-ref: quantification chapter(s)].

(1) a. The SOME relation holds between two concepts A and B if and only if at least one instance of A is an instance of B.

b. The ALL relation holds between two concepts A and B if and only if each instance of A is also an instance of B.

The SOME relation would then hold between DOG and PET — i.e., Some dogs are pets would be true — if and only if at least one instance of DOG is also an instance of PET. There is obviously no gradation here: either at least one dog is a pet, and the relation holds, or no dogs are pets and it does not hold. Similar observations can be made about ALL and many other quantificational concepts.

However, this kind of treatment is not very plausible for concepts like TEMPERATURE or WEIGHT, which clearly admit of degrees. Monday could have a greater, lesser, or equal temperature to Tuesday — i.e., it could be hotter, colder, or exactly as hot. Mary could be a greater, lesser, or equal weight to Bill — i.e., she could be heavier, lighter, or exactly as heavy as he is. So, TEMPERATURE and WEIGHT must be concepts which do more than classify individuals into ‘yes’ and ‘no’ categories: they associate individuals with values which are ordered in a way that allows us to talk about things being associated with the concept to greater or lesser extents. These values are usually known as degrees, and a set of degrees with an ordering on them is known as a scale. For instance, the set of all possible degrees of weight — including tiny weights like 1 microgram and larger weights like 1 kilogram and 1 ton — forms a scale, once we add in the information that (e.g.) 1 microgram is less than 1 kilogram, which in turn is less than 1 ton.

Natural languages allow speakers to express concepts using words. When we are analyzing words we can ask questions not only about the structure of the concepts they express, but also about grammatical features of the linguistic means of expressing the concept. In other words, we can ask the question ‘Does it make sense to talk about degrees of concept X?’; if the answer is ‘yes’, we can then ask the further question ‘Does the language we are studying have the grammatical
and semantic resources to refer implicitly or explicitly to degrees of $X$?’. These are subtly, but importantly, different. To see this, consider the concept FRUIT and the word fruit. Psychological studies of concepts have given us strong reason to believe that FRUIT has graded structure. Some objects are clearly fruit, and some objects are clearly not fruit, but there are also objects which experimental subjects consistently classify as intermediate examples — neither clear cases nor clear non-cases. This is an answer to the question of conceptual gradation, then: FRUIT is not an all-or-nothing concept, but one that comes in degrees.

Nevertheless, the English word fruit is usually analyzed in linguistic semantics as a simple predicate of individuals: it denotes a function which takes an individual argument and returns either True or False, depending on whether or not the object is a fruit. This is an answer to the different question of grammatical gradability: fruit is a non-gradable predicate, even though the concept FRUIT makes available degrees that could in principle be made available for grammatical interaction with degree modifiers, comparative and superlative morphemes, and so forth. I do not know whether this is actually a good analysis of fruit; after all, English allows us to say things like This is more of a fruit than that and This is very much a fruit, and we need some semantic account of these sentences. But what is important here is that it is perfectly coherent for a theorist to endorse both the psychologists’ standard way of thinking about FRUIT and the linguists’ standard way of thinking about fruit. For example, we might suppose that fruit is a predicate which is true of anything whose degree of fruitiness exceeds a fixed, or perhaps context-sensitive, threshold. The claim that it is non-gradable is a claim about its grammar, and not about the structure of the underlying concept.

On this analysis, the meaning of the word fruit ‘talks about’ degrees, but does not make these degrees accessible for grammatical interaction with degree-binding operators. This is in contrast to expressions like heavy, which are readily gradable:

(2) a. This truck is heavier than that car.
   b. This truck is very heavy.

The concept WEIGHT clearly comes in degrees, and the expression heavy seems to expose these degrees grammatically for interaction with degree-binding operators like -er and very. One possible analysis of this interaction is sketched informally in (3), and (4) gives an analysis of the comparative sentence in (2a) as an example. (See Kennedy 1997, 2007 for a thorough development of a degree semantics along these lines.)

(3) WEIGHT is a concept which includes at least the following:
   a. a set of degrees $D_{weight}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{weight}$, whether or not $d \geq d'$; and
   c. a function $\mu_{weight}$ which assigns to each individual a unique $d \in D_{weight}$ representing their weight.$^1$

$^1$ We are glossing over the important issue of intensionality here. In a more complete theory, WEIGHT would assign a degree $d$ to pairs consisting of a world $w$ and an individual $x$, where $d$ represents how heavy $x$ would be if the facts were all as they are in $w$. This is necessary because, obviously, someone can understand the concept WEIGHT without knowing how heavy everything in the world is. A realistic theory would intensionalize all concepts and word meanings in this way, but I will simplify by ignoring this subtlety in the present chapter.
Technically, we need to assume that the ordering $\geq$ is at least reflexive and transitive. For concepts like weight it makes sense to assume also that it is connected: either $d \geq d'$ or $d' \geq d$ for any $d$ and $d'$ in $D_{weight}$. (Readers who do not recognize these technical terms need not linger over them for the moment: for our purposes it does little harm to think of ordered sets of degrees as intervals in the system of real numbers, such as $(0, \infty)$, $[0, \infty)$, or $(-\infty, \infty)$. Those who wish to go a bit deeper into order theory can consult chapter 3 of Partee, ter Meulen & Wall 1990.)

We can then analyze the comparative sentence (2a) along the following lines:

(4) For any $x$ and $y$, $x$ is heavier than $y$ is true if and only if $\mu_{weight}(x) > \mu_{weight}(y)$,

where `$d > d'$' abbreviates `$d \geq d'$', and it’s not the case that $d' \geq d'$. If we want our semantics to be compositional, we need to be able to assign a meaning to the comparative operator `-er`. So, we abstract away from the specific adjective that was used and give `-er` a meaning like (5) (which is still incomplete, but illustrative).

(5) `-er` combines with an adjective $A$ to return a relation between two individuals. The relation holds of $(x, y)$ if and only if the measure function associated with $A$ assigns a degree to the $x$ which exceed the degree that it assigns to $y$. In other words, for any $x$ and $y$, $x$ is $A$-er than $y$ is true if and only if $\mu_A(x) > \mu_A(y)$.

Note that style of analysis entails that a concept must have graded structure if there is a gradable expression which refers directly to degrees of the concept, e.g., by forming comparatives and superlatives or being modified by `very`. Otherwise there would be no degrees for these degree morphemes to act upon. However, we cannot determine conclusively whether a concept has graded structure simply by asking whether its linguistic exponents are gradable: as we saw in the case of `FRUIT` and `fruit`, it is perfectly possible for a concept to have graded structure without lexicalizing degrees.

In fact, there are many languages in which the words expressing graded concepts like `TEMPERATURE` and `WEIGHT` are not grammatically gradable. For instance, Bochnak (2014) shows that there is no evidence of grammatical gradability of any kind in the Native American language Washo. Nevertheless, Washo has a number of verbs which correspond to the simple (`positive`) forms of adjectives in English. These Washo verbs have the same kinds of meanings as English adjectives like `hot` and `heavy`: they are vague, and have context-dependent meanings which vary depending on (for example) whether we are talking about being heavy-for-a-baby, heavy-for-a-suitcase, or heavy-for-a-house. Washo speakers clearly possess the concept of one thing being heavier than another, and even use overt comparatives when speaking English, but it appears to be a grammatical fact about their language that they do not possess the means to talk explicitly about this kind of comparison. (See Beck, Oda & Sugisaki 2004; Beck, Krasikova, Fleischer, Gergel, Hofstetter, Savelsberg, Vanderelst & Villalta 2009; Bochnak 2013, 2014 for discussion of cross-linguistic variation in the grammar of scalar concepts.)

As we will see, it has been implicitly recognized in work on modality for some time that conceptual gradation is logically prior to — and does not entail — grammatical gradability. Graded concepts are routinely invoked in the literature, but grammatical gradability of modals has only recently begun to be discussed in detail.
2 Graded modality: Some history and examples

Following this line of thought, we can think of the subject of ‘graded modality’ as involving two related but different questions. On the one hand, we will ask which, if any, modal concepts are graded — whether there are any that are most insightfully analyzed as being like TEMPERATURE and WEIGHT rather than SOME, ALL, and SQUARE. On the other hand, we can ask whether there are modal expressions that are grammatically gradable. As we will see, there is good reason to answer both questions in the affirmative: graded structure is implicated in a wide range of modal concepts, and many — though probably not all — of the linguistic expressions which depend on these concepts are grammatically gradable.

2.1 Possible worlds and the ALL/SOME/NONE conception of modal meaning

In philosophy and formal semantics, the modal concepts of NECESSITY and POSSIBILITY are usually analyzed as ‘all’ and ‘some’ quantifiers over sets of ‘possible worlds’ — complete representations of the many possible ways that the world could be, with no remnant uncertainty.

(6) a. NECESSARY applies to a sentence \( \phi \) if and only if \( \phi \) is true in all possible worlds.
    b. POSSIBLE applies to a sentence \( \phi \) if and only if \( \phi \) is true in some (i.e., at least one) possible world.

This analysis has its roots in the work of Gottfried Wilhelm von Leibniz (1646-1716), who spoke of ‘possible worlds’ and whose analysis of necessary and contingent truth is equivalent to these more modern-looking statements (see Look 2013). Leibniz intended his version of (6) as an analysis of what is now called ‘absolute’ necessity and possibility. For instance, the ‘necessary truth’ of Two plus two is four could be analyzed as ‘In all possible worlds, two plus two is four’. Even God could not create a universe in which basic arithmetical facts are different. The ‘logical possibility’ that The Great Wall of China is on the moon could be analyzed as ‘In some possible world, the Great Wall of China is on the moon’. If God had wanted to, he could have transported the Great Wall to the moon.

As the latter example indicates, the absolute concept of possibility is much weaker than what the English word possible is generally used for. If I told a non-philosopher that it is a possibility that the Great Wall of China is on the moon, he would surely conclude that I am insane. Likewise, the treatment of NECESSARY is too strong for the English word ‘necessary’, which is frequently used to qualify statements which express (e.g.) very strong preferences rather than mathematical or conceptual truths. (7) is a naturally-occurring example:

(7) It is necessary that health workers have a basic understanding of the fundamental principles of maintaining a healthy body.

Clearly the author of (7) does not intend to express the claim that, in all possible worlds, health workers have a basic understanding of the fundamental principles of maintaining a healthy body. In fact, the natural interpretation of this sentence is that it is a norm or ideal which must be pursued, even if it is frequently violated in the real world. So, we have to weaken Leibniz’s interpretation of NECESSARY somehow if we want to capture the fact that (7) is not patently false.
The modern semantic literature on modal concepts — starting most notably with the work of Kripke (1959, 1963) and Hintikka (1962) — has built on the foundation of Leibniz’s ideas, but has extended them in an important way: the ‘all’ and ‘some’ in (6) do not really quantify over all possible worlds, but over a smaller set of worlds which represents some coherent domain of possibilities [cross-ref: other modality chapter(s)]. This modification makes the ‘all/some worlds’ approach to modality much more useful for the analysis of a wide range of concepts and linguistic expressions. For example, we might analyze Mary believes that it’s raining as in (8), where the domain of possible worlds is crucially restricted to the worlds which are compatible with what Mary believes.

(8) In all possible worlds that are compatible with Mary’s beliefs, it is raining.

Similarly, (7) could be analyzed as relativized to worlds where the norms of the healthcare profession are observed.

(9) In all possible worlds that conform to the norms of the healthcare profession, health workers have a basic understanding of the fundamental principles of maintaining a healthy body.

The intuitive falsity of It is possible that the Great Wall of China is on the moon can be understood along similar lines if possible is a SOME-quantifier, not over all possible worlds, but over those that are compatible with what some relevant person (say, the speaker) believes.

(10) In some possible world that is compatible with the speaker’s beliefs, the Great Wall of China is on the moon.

This interpretation of (10) does a better job of explaining why we would infer that someone who sincerely asserts this sentence has psychological problems.

In the wake of Kripke’s and Hintikka’s work in the early 1960’s, it has become a sort of default analytic choice in formal semantics to treat an enormous variety of modal expressions — of varying syntactic categories — as either SOME- or ALL-quantifiers over restricted domains of possible worlds: may, might, can, should, ought, have to, able, certain/certainly, possible/possibly, necessary/necessarily, believe, think, know, suspect, want, wish, and many more. Note that the choice to analyze a modal concept along these lines entails that the concept in question is not graded: SOME and ALL are all-or-nothing concepts, as we discussed in section 1. However, as we will see in section 3 below, there are ways to combine a ‘some/all worlds’ semantics for specific expressions with a graded analysis of modal concepts.

2.2 Graded modal concepts

The idea that some modal concepts have graded structure has a long history as well. For example, the philosopher John Locke announced in the Introduction to An Essay Concerning Human Understanding (1690) that one of the main purposes of the book was to inquire into “the grounds and degrees of belief, opinion, and assent” — taking for granted that these are things that come in degrees. Around the same time, Jacob Bernoulli’s Art of Making Conjectures (finished 1705, published 1714) proposed a scalar model for forming rational degrees of certainty in light of evidence. The focus on rational degrees of certainty here suggests that Bernoulli was concerned
primarily with KNOWLEDGE and JUSTIFICATION, rather than CERTAINTY as a matter of subjective psychology. (Interestingly, Bernoulli’s formal model eventually developed into modern probability theory, with some interesting twists and turns along the way: see Shafer 1996.)

Four centuries earlier, Saint Thomas Aquinas used the assumption that the modal concept of MORAL GOODNESS comes in degrees as a crucial premise in one of his logical deductions of the existence of God in *Summa Theologica* (written 1265-1274).²

Among beings there are some more and some less good, true, noble and the like. But ‘more’ and ‘less’ are predicated of different things, according as they resemble in their different ways something which is the maximum, as a thing is said to be hotter according as it more nearly resembles that which is hottest; so that there is something which is truest, something best, something noblest and, consequently, something which is uttermost being... Now the maximum in any genus is the cause of all in that genus; as fire, which is the maximum heat, is the cause of all hot things. Therefore there must also be something which is to all beings the cause of their being, goodness, and every other perfection; and this we call God.

Aquinas’ argument, while probably unconvincing to the modern reader, is linguistically interesting in several ways. It assumes that GOODNESS is a scalar concept, and it explicitly relies on an analogy between this modal concept and the non-modal scalar concept TEMPERATURE. What is more, Aquinas’ argument depends on the assumption that the scales resemble each other in that both are upper-bounded — both have a fixed maximum element.³

This is an issue about what linguists now call scale structure, a set of formal properties of degree scales that can be imposed by placing further restrictions on definitions like (3) above, and which have been shown to have semantic and pragmatic consequences. Specifically, the basic graded analysis of MORAL GOODNESS would look something like (11), and the optional upper-bounding condition would be (12). (TEMPERATURE is a slight variant, with ‘temperature’ replacing ‘good’ and ‘individual’ replacing ‘proposition’.)

(11) **MORAL GOODNESS** is a concept which includes at least the following:
   a. a set of degrees $D_{\text{good}}$;
   b. an ordering $\geq$ which determines, for each $d$ and $d' \in D_{\text{good}}$, whether or not $d \geq d'$; and
   c. a function $\mu_{\text{good}}$ which assigns to each proposition a unique $d \in D_{\text{good}}$ representing its goodness.

(12) In the concept of **MORAL GOODNESS**, there is a degree $d \in D_{\text{good}}$ such that $d \geq d'$ for all $d' \in D_{\text{good}}$.

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² The quote is taken from the First part, Question 2 of *The Summa Theologica of St. Thomas Aquinas*, Second and Revised Edition of 1920, translated by Fathers of the English Dominican Province. The full text is available online at http://www.newadvent.org/summa/.

³ The interpretation of the maximum as the ‘cause of all in that genus’ is related to Plato’s influential claim that all earthly objects reflect ideal ‘forms’ imperfectly and by degrees. Presumably all concepts admit of degrees in this worldview, even if they are only degrees of approximation to a Platonic Form.
The idea that goodness has graded structure as in (11) appears more or less explicitly in many other places in philosophy, notably utilitarianism, an influential moral theory which enjoins people to maximize the goodness of the consequences of their acts (Mill 1863). Obviously, it only makes sense to ‘maximize goodness’ if goodness is graded. Utilitarians do assume this, but they don’t need to assume that the scale of goodness has a fixed upper bound as Aquinas did, and as enforced by (12).

According to modern theories of the semantics and pragmatics of gradable adjectives, we should be able to use linguistic evidence to discern whether temperature and goodness do indeed have fixed maximum elements (or at least whether modern English speakers think that they do). The idea is that we can discern the structure of the degree scales associated with gradable adjectives by examining facts about the usage of expressions whose meanings make reference to these scales: for instance, warm, hot, good, and modified versions of these. Here is an example of how this might work. Linguistic work on scale structure has argued that words denoting graded concepts like height, fullness, and danger can differ in whether their scales have upper and/or lower bounds. For example, height clearly has no upper bound — in principle something could keep getting taller forever — but fullness does: there’s a principled limit to how much you can pack into a container before you couldn’t possibly get anything else in. This difference has been argued to influence a number of semantic and pragmatic properties of these adjectives, such as which degree modifiers they are able to combine with (Rotstein & Winter 2004; Kennedy & McNally 2005; Kennedy 2007).

The proposed test for the upper-boundedness of a graded concept’s degree scale involves whether an adjective which expresses that concept can be modified by ‘maximizing’ degree modifiers such as totally, perfectly, and completely. The test must be applied with a bit of finesse, since these degree modifiers may be polysemous — they may have other uses in addition to the degree-maximizing use that we are interested in. For example, Bill is totally tall is an acceptable English sentence, but it doesn’t have a maximum-degree reading: it can’t be understood to mean ‘Nothing could be taller than Bill’. When it can be used totally seems to have another meaning, for example, an emphatic reading that appears when we are trying to contradict someone (‘Bill is totally tall!’). This is not the reading that we are interested in, though. We want to use the maximizing-degree-modifiers test to discern whether a maximum-degree reading is possible, which would entail that the graded concept is associated with a fixed maximum degree. So the full statement of the test is this: ‘Can adjective A be modified by totally, perfectly, and/or completely with a “maximum-degree” meaning?’

With this in mind, let’s apply the test to a few adjectives:

(13) a. The glass is totally/perfectly/completely full.
    b. ?? Bill is totally/perfectly/completely tall.
    c. ?? This neighborhood is totally/perfectly/completely dangerous.

The symbol ‘??’ before a sentence indicates that the sentence is systematically odd or ‘infelicitous’, at least when we fix attention on the maximizing reading that we are interested in. The judgments are those of Kennedy & McNally (2005). If they are right, we have evidence that full lives on an upper-bounded scale, while tall and dangerous do not. This seems to accords well with intuition about these concepts.

Applying the test to the gradable adjective ‘good’, we get, for example,
(14) (??) It is totally/perfectly/completely good to lend money to friends.

As a sanity check, let’s try it out on hot too.

(15) (??) The weather is totally/perfectly/completely hot today.

(14) and (15) do not seem to admit of a ‘maximum-degree’ reading. If they did, (14) would entail on this reading that nothing could be better than lending money to friends, which it does not: there is no way to understand (14) on which it would conflict with the claim that curing cancer is even better. Similarly, the maximizing reading of (15) would entail that it could not possibly be hotter than it is today. Both of these interpretations seem to be unavailable. In the context of the theories of gradation just mentioned, we can take these data points as evidence against Aquinas’ claim that goodness and temperature are upper-bounded concepts.4

This is one example of a place where the structure of an interesting modal concept might be illuminated by an examination of the grammar and usage of modal language in light of linguistic theory.

3 Counterfactuals and deontic gradation

3.1 Lewis on counterfactuals

Soon after it was proposed in modern form by Kripke and Hintikka, Stalnaker (1968) and Lewis (1973) pointed out that certain important modal concepts seem to require more graded structure than ‘all’/‘some’-quantification over sets of worlds could provide. Here we will focus on Lewis’ book, which treats counterfactuals and certain deontic modals in a similar graded fashion. The basic problem motivating Lewis’ account is illustrated by the sentences in (16).

(16) a. If kangaroos had no tails, they would topple over.
    b. If kangaroos had no tails but used crutches, they would not topple over.

An initially plausible analysis of (16a) goes like this. Fix a set of possible worlds $R$ which are relevant to the interpretation of counterfactuals in the current conversation. Then (16a) is true just in case Kangaroos topple over is true in all of the possible worlds in $R$ in which Kangaroos have no tails is true. On this account, a counterfactual has the same basic meaning that necessary, believe, or must would receive in the Kripke/Hintikka-inspired semantics that we discussed above, with one important difference: the contextual restriction $R$ is provided by context, but the set of worlds quantified over is further restricted to the $R$-worlds in which Kangaroos have no tails is true.

The problem is that (16a) is intuitively consistent with (16b), but the analysis just sketched predicts otherwise. If all of the $R$-worlds in which kangaroos have no tails are worlds in which they topple over, then all of the $R$-worlds in which kangaroos have no tails but use crutches must also be worlds in which they topple over. In schematic form, the analysis predicts that the inference from If $\phi$, would $\chi$ to If $\phi$ and $\psi$, would $\chi$ should be valid for any sentences $\phi$, $\psi$, and $\chi$. This is apparently not a correct prediction.

4 Unfortunately the theoretical situation may be a bit less clear than this reasoning suggests, as Lassiter (2010) suggests: see section 4 below.
Lewis proposes that we should continue to interpret counterfactuals as ‘all’-quantifiers, and so as non-gradable expressions; but that we should relativize their interpretation to a graded concept of similarity to the actual world.\(^5\) The idea is that possible worlds can be ordered in terms of how similar they are to the actual world, and that a counterfactual is interpreted as an ‘all’-quantifier over the worlds that satisfy the antecedent and are otherwise as similar as possible to the actual world. Even without developing the formal apparatus, we can see how the use of this graded structure in the background helps. In the actual world, kangaroos have tails, and they do not use crutches. So, the possible worlds which are as similar as possible to the actual world, except that kangaroos have no tails, will be ones in which they still do not use crutches. In these worlds, they topple over, and so (16a) is true. However, when we want to interpret (16b) we have to look at worlds which are even less similar to the actual world, because in these worlds kangaroos have no tails and use crutches. In these worlds, they do not topple over, and so (16b) can be true at the same time that (16a) is false.

In Lewis’ formal development of this semantics, comparative similarity to the actual world is modeled using a qualitative ordering \(\leq_w\), where \(w_1 \leq_w w_2\) means that \(w_1\) is at least as similar to the actual world as \(w_2\) is. Lewis imposes several technical conditions on this ordering (including reflexivity, totality, and transitivity) which enforce features of the concept “at least as similar as” that he finds intuitive. While this kind of gradation appears somewhat different from the degree-based gradation that we discussed above, it is really very closely related: for any ordering satisfying the conditions on Lewis’ \(\leq_w\), we can find a degree-based representation that captures all of the same information about comparative similarity, and looks just like the ones we used for temperature and weight above. The basic idea is to set up a scale of similarity-to-the-actual-world-\(w\), and a function \(\mu_{\text{similar-to-}w}\) which assigns each world a degree on that scale. We then ensure that \(\mu_{\text{similar-to-}w}(w_1) \geq \mu_{\text{similar-to-}w}(w_2)\) if and only if \(w_1\) is at least as similar to \(w\) as \(w_2\) — that is, if and only if \(w_1 \leq_w w_2\). (We can actually find an infinite number of such representations for any such \(\leq_w\). See Klein 1991; Rullmann 1995; Lassiter 2016 for linguist-friendly presentations of this construction and discussion of the formal conditions that are needed to ensure that it works as intended.)

\[(17)\text{ similarity to the actual world} \text{ is a concept which includes at least the following:}\]
\[\text{a. a set of degrees } D_{\text{similar-to-}w};\]
\[\text{b. an ordering } \geq \text{ which determines, for each } d \text{ and } d' \in D_{\text{similar-to-}w}, \text{ whether or not } d \geq d';\]
\[\text{c. a function } \mu_{\text{similar-to-}w} \text{ which assigns to each world a unique } d \in D_{\text{similar-to-}w}.\]
\[\text{d. ‘Centering’: A requirement that nothing is more similar to the actual world } w \text{ than it is to itself: } \mu_{\text{similar-to-}w}(w) \geq d \text{ for all } d \in D_{\text{similar-to-}w}.\]

It’s interesting to note here that Lewis’ semantics for counterfactuals has a similar character to the theory of fruit and fruit that we discussed in section 1. For him, counterfactuals are ‘all’-
3.2 Deontic gradation

Lewis (1973: §5) suggests a way of modifying the semantics for counterfactuals and using it to interpret graded deontic concepts such as GOODNESS. That is, instead of reading \( w_1 \leq_w w_2 \) as \( w_1 \) is at least as similar to \( w \) as \( w_2 \), we will read it as \( w_1 \) is at least as good as \( w_2 \), relative to the moral standards that are relevant at \( w \). Equivalently, we can rewrite the degree-based representation in (17) but replace ‘similar to \( w \)’ with ‘good’, ending up with something that is at first glance quite similar to the MORAL GOODNESS scale that we defined in (11) above.

(18) COMPARATIVE GOODNESS (relative to \( w \)) is a concept which includes at least the following:
   a. a set of degrees \( D_{\text{good-at-w}} \);
   b. an ordering \( \geq \) which determines, for each \( d \) and \( d' \in D_{\text{good-at-w}} \), whether or not \( d \geq d' \);
   c. a function \( \mu_{\text{good-at-w}} \) which assigns to each world a unique \( d \in D_{\text{good-at-w}} \).

(The ‘at \( w \)’ qualification will mostly be left implicit in what follows, since we are not comparing possible moral codes.) Following Lewis, we leave out the Centering assumption (17d) in (18). This condition would not make sense when we are dealing with deontic concepts: the world that we live in, abounding in war and cruelty as it is, is probably not the possible world which best conforms to its own moral standards.

There is an important point of difference between MORAL GOODNESS as described in (11) and COMPARATIVE GOODNESS as in (18). Above we assumed that MORAL GOODNESS is a graded concept which assigns degrees of goodness to propositions — the things that declarative sentences denote. For example, \( \text{Bill talks to Mary at 7:35PM EST on August 28, 2014} \) denotes a proposition which we can think of a set of possible worlds. These worlds may differ in many ways, but they share the property that this sentence is true in all of them. \( \text{Bill talks to Sue at 7:35PM EST on August 28, 2014} \) denotes a different set of possible worlds. English expressions like better than appear to relate sentence-meanings of this type: e.g., we might say that \( \text{It is better that Bill talks to Mary than it is that he talks to Sue} \) (at 7:35PM EST on August 28, 2014). So, it makes intuitive sense to treat GOODNESS as a scalar concept which invokes a mapping from propositions to degrees on a scale.

Lewis’ concept of COMPARATIVE GOODNESS is different: it relates possible worlds, not propositions. In English, at least, we do not normally talk about the relative goodness of fully-specified possible worlds; probably the language does not even permit this. So, if we want to use COMPARATIVE GOODNESS to interpret deontic comparatives like \( A \) is better than \( B \), we have to find a way to use it to define a comparative goodness relation on propositions. A common way to do this — one which predates Lewis, and is still used in much work in formal semantics — is to rank two propositions \( \phi \) and \( \psi \) by considering the relative positions of the highest-ranked worlds in each.

(19) \( \phi \) is at least as good as \( \psi \) is true if and only if, for some \( w \in \phi \), \( \mu_{\text{good}}(w) \geq \mu_{\text{good}}(w') \) for all \( w' \in \psi \).
A characterization of \( \phi \) is better than \( \psi \) follows immediately.\(^6\)

(20) \( \phi \) is better than \( \psi \) is true if and only if there is some \( w \in \phi \) such that \( \mu_{\text{good}}(w) > \mu_{\text{good}}(w') \) for all \( w' \in \psi \).

In effect, what we are doing here is to ‘lift’ the graded concept COMPARATIVE GOODNESS (OF WORLDS) to a graded concept COMPARATIVE GOODNESS (OF PROPOSITIONS), and then to use the latter to interpret expressions like as good as and better than.

### 3.3 Deontic adjectives and gradability

Lewis (1973) also suggests interpreting the deontic adjectives obligatory and permissible as quantifiers over sets of worlds derived from the un-lifted COMPARATIVE GOODNESS (OF WORLDS) concept. Adapted for the degree-based version, his proposals are given informally in (21).

(21) a. It is obligatory that \( \phi \) is true at \( w \) if and only if all worlds in \( \text{BEST}(\mu_{\text{good-at-w}}) \) are worlds in which \( \phi \) is true.

b. It is permissible that \( \phi \) is true at \( w \) if and only if at least one world in \( \text{BEST}(\mu_{\text{good-at-w}}) \) is a world in which \( \phi \) is true.

\( \text{BEST} \) is essentially an ‘arg max’ operator: \( \text{BEST}(\mu_{\text{good-at-w}}) \) returns the set containing all worlds \( w' \) whose degree of (world-)goodness is at least as high as that of any other world.

This proposal predicts that the adjectives permissible and obligatory should be non-gradable. Even though their meanings are partly determined by a graded concept of comparative (world-)goodness, once we have fixed this ordering there is a determinate fact of the matter about whether all/some of the best worlds make some proposition true, with no room for degrees. We can test whether this prediction is correct by looking at corpus evidence, using three tests that were discussed in the last section: the availability of degree modification, comparisons (comparatives and equatives), and superlatives. The examples in (22)-(25) were found in the Corpus of Contemporary American English (COCA, Davies 2008-), except as noted.

(22) Comparative/equative/superlative + obligatory:

a. In 1984, he was the only candidate to utter the word ‘gay’; now, it is as \underline{obligatory} in Democratic rhetoric as references to Jefferson.

b. Their attendance at periodic commemorations of the case and of their parents’ execution became less \underline{obligatory}.

c. The most \underline{obligatory} task is to manage the list and set it according to the items in order to gain good results. (web)

\(^6\) The ‘highest-ranked’ gloss and (20) are only appropriate for theories like Lewis’, where the ordering is connected. Kratzer’s (1991) theory is formally equivalent to Lewis’, except that incomparabilities are possible: it could be that \( w \) is not ranked as high as \( w' \), and \( w' \) is not ranked as high as \( w \). This means that we frequently cannot identify a maximum element among some set of degrees. However, if we know that all of the degrees in some set are comparable, the ‘maximum’ gloss is often appropriate.

There is another, orthogonal technical problem which affects our ability to identify maxima, known in the literature as the ‘limit assumption’. Lewis (1973) and others have devoted much energy to discussing whether this assumption is appropriate in the analysis of counterfactuals, but it does not seem necessary to consider it in detail here.
(23) Comparative/equative/superlative operator + permissible:
   a. The ethical analysis of fictional models ... is not only as permissible as the ethical analysis of real-life models and friends, but, for all those who really care about the quality of their life, just as necessary.
   b. It is a peculiarity of the Arab-Islamic political culture that a ruler’s authoritarianism is more permissible than his identification with Western powers.
   c. Humorous X-do-Y exaggerations are the most permissible, because people like being entertained. (Jones 2010: 35)

(Note that (23a) also provides an example of the adjective necessary occurring with a comparison operator. This suggests that it may be gradable as well.)

Similarly, it is not difficult to find examples of these adjectives occurring with degree modifiers.

(24) Degree modifier + obligatory:
   a. It’s almost obligatory these days for a young-adult novel to include e-mail and instant messaging as part of its plot machinery.
   b. It is not - as I understand it - TOTALLY obligatory to be stark naked there all the time, but if you feel so inclined, you can be.

(25) Degree modifier + permissible:
   a. All I was trying to do was to protect hearth, home, and business, which in simpler times would have been a perfectly permissible and legally defensible option.
   b. Before agreeing to go to confession, I had told Mother I wasn’t ready yet to recite this important prayer in English, but she assured me it was quite permissible to say any prayer in any language, since God was omniscient.

While these adjectives directly modify proposition-denoting expressions in a number of examples — (24a), (24b), (25b), and, implicitly, (22a) and (25a), quite a few of the COCA examples involve modification of event nominals like attendance, authoritarianism, and analysis of real-life models and friends. It is easy to associate these nominals with propositions that we can apply our semantics to: that some relevant person attends something, that someone behaves in an authoritarian way, etc. Still, some account needs to be given about the relative infrequency in COCA of sentence-modifying obligatory and permissible with overt markers of gradability. I suspect that this feature can be attributed to the overall low frequency of the construction. It is permissible (for x) to/that ... occurs only 41 times in COCA, and It is obligatory (for x) to/that ... only four times in this 450-million-word corpus.

If permissible and obligatory are indeed gradable, as these examples suggest, they cannot be identified with simple ‘some’/‘all’ quantifiers over sets of worlds. An alternative is to suppose that they are in fact scalar adjectives. Permissible could be related to the ‘minimum’ (‘partial’) adjectives (26), whose meaning in the unmodified form can often be paraphrased using ‘some’. Similarly, necessary could be related to the ‘maximum’ (‘total’) adjectives like dry (27), which in their unmodified form can often be paraphrased using the universal ‘all’ or the universal negative
‘no’. Alternatively, 

necessary might be related to the ‘extreme’ adjectives discussed in §3.5 below. 7

Despite being roughly paraphrasable by ‘some’ and ‘all’/‘no’, minimum and maximum adjectives lexicalize degrees and are fully gradable, as the (b) and (c) examples indicate.

(26) Minimum adjective wet
  a. This towel is wet.
     = It has a non-zero degree of wetness.
     ≈ It has some amount of water on it.
  b. This towel is wetter than that one. (comparison)
  c. This towel is slightly wet. (degree modification)

(27) Maximum adjective dry
  a. The towel is dry.
     = It has a maximal degree of dryness.
     ≈ It has no amount of water on it.
  b. This towel is drier than that one. (comparison)
  c. This towel is completely/perfectly/absolutely dry. (degree modification)

It seems plausible, then, permissible is a minimum adjective meaning ‘having a non-zero degree of permissibility’, and obligatory is a maximum adjective meaning something like ‘having a maximal degree of obligation’.

3.4 Inferential properties of deontic scales

Lewis’ graded-but-not-gradable semantics for deontic modals has a second controversial feature which is also maintained in many current theories of deontic modality. Lewis’ account predicts that φ is better than ψ is true when there is just one φ-world that are better than all worlds in ψ — even all other φ-worlds are strictly worse than all ψ-worlds. Similarly, the analysis predicts that It is obligatory that Mary leave is true if all Mary leaves in all of the best possible worlds, regardless of how few or far-fetched these worlds are. However, a number of theorists have offered empirical arguments which suggest that it is a mistake to look only at ‘highest-ranked’ worlds when asking about how good a proposition is, or whether it is obligatory. This is a feature shared by many systems of deontic logic, some predating Lewis, and also by Kratzer’s (1981; 1991) semantics, which has been very influential in linguistics.

For example, Jackson (1991) imagines a doctor who must choose whether to recommend to a patient a safe but mediocre treatment or a risky new treatment. His best information suggests that the risky treatment will probably kill the patient, but there is a small chance of a total cure. The safe treatment will definitely work, but will leave the patient with some lasting pain and discomfort. It seems clear that the best worlds are worlds in which the patient is totally cured; but this will happen

7 See Cruse (1986); Yoon (1996); Rotstein & Winter (2004); Kennedy & McNally (2005); Kennedy (2007) for discussion of minimum and maximum adjectives.
only if the doctor recommends the risky treatment. These worlds are strictly better than worlds
in which the patient has lasting pain and discomfort, which in turn are better than the worlds in
which the patient dies as a result of the risky treatment. So, if we look only to the best worlds in
determining the goodness of propositions, and whether each is obligatory, we seem to be committed
to the following judgments:

(28)  a. It is better for the doctor to prescribe the risky treatment than it is for him to prescribe
the safe but mediocre treatment.
     b. It is obligatory that the doctor prescribe the risky treatment.

I have a strong intuition that both of these sentences are clearly false in the situation described: it
would be morally reprehensible to do what they recommend, since this would probably kill the
patient. If so, the proposed way of lifting COMPARATIVE GOODNESS (OF PROPOSITIONS) to
COMPARATIVE GOODNESS (OF WORLDS) in (19) and (20) is in trouble. Somehow, we need to
take into account all of the possible outcomes of each action and their probabilities, rather than
just looking at the best possible outcomes of each. (See Jackson 1985, 1991; Goble 1996; Lassiter
2014, 2016 for discussion of this and related puzzles involving information-sensitivity, including
the much-discussed ‘Miners’ Puzzle’. The ‘best-worlds’ lifting in (19) and (20) is defended by von
Fintel (2012) and Dowell (2013).)

Goble (1996); Cariani (2009); Lassiter (2014, 2016) discuss versions of this problem involving
(variously) good or ought, and propose to resolve them by using a more complex lifting. On
their proposal, the scalar concept COMPARATIVE GOODNESS (OF PROPOSITIONS) is created from
the COMPARATIVE GOODNESS (OF WORLDS) concept by assigning to each proposition a degree
of goodness which is a weighted average of the goodness of the worlds in it, where each world
receives a weight equal to the probability that it will be true if the proposition occurs. On this
construction, when we consider whether (28a) is true, we must take into account all of the possible
outcomes. Even though the best result of the risky treatment is very good, this fact receives very
little weight in the final result because it is much more likely that the risky treatment will result in a
terrible outcome, death. The safe but mediocre treatment is thus better, even though its best possible
outcome is not as good as the best possible outcome of the risky treatment. (See Lassiter 2014, 2016
for a formal development of this proposal, along with some surprising connections with part-whole
structure of scales, such as the interaction of non-modal scales with plural-denoting expressions.)

Similar considerations are relevant to Kratzer’s 1981; 1991 analysis of the modal auxiliary
must, and its extension to have to, ought, and should by von Fintel & Iatridou (2008) [cross-ref:
relevant modality chapter(s)]. In relevant respects these proposals are equivalent to Lewis’ proposal
for obligatory: they denote universal quantifiers over sets of worlds which are chosen by looking
at the highest-ranked worlds according to a scalar concept of COMPARATIVE GOODNESS. The
objection to looking only at highest-ranked worlds just noted could be applied here as well: The
doctor must/has to/ought to prescribe the risky treatment are intuitively false in the Medicine
scenario, even while the best outcomes are ones in which The doctor prescribes the risky treatment
comes out as true. See however Katz, Portner & Rubinstein 2012; Silk 2012; von Fintel 2012 for
efforts to explain these facts within the broad framework of Kratzer’s theory.
3.5 Gradability and deontic auxiliaries

In addition, there is a certain amount of gradability among the verbal exponents of deontic modality. For instance, *ought* and *should* appear to participate in comparative constructions and occur with degree modifiers (Lassiter 2011, 2016).

(29) a. [O]nce the damage is done, Constance ought to help George — or, at least, she ought to help him more than she ought to help anyone else similarly situated. (Driver 1997: 853)
   b. A war between Great Britain and the U.S. ought very much to be deprecated.

(30) a. I don’t think he [UFC fighter Phil Davis] should be compared to Rosholt as much as he should be to Houston Alexander.
   b. If you desperately need to change an old post then PM one of the moderators ... This should very much be considered the exception though. The normal edit window should usually be enough.

The constructions in (29) and (30) are grammatically somewhat different from examples of comparison and degree modification constructions that we have seen with adjectives. However, they are typical of comparison and degree modification structures with gradable verbs such as *like*. Here are some examples found on the web.

(31) a. I think I like to read more than I like to write...
   b. Someone they like very much to work with may not be someone they very much like to socialize with, and vice versa.

It is not clear how we could make sense of the examples in (29) and (30) if *ought* and *should* denote ‘all’-quantifiers over sets of worlds—except, perhaps, by reference to an unanalyzed notion of ‘coercion’. Lassiter (2011) proposes to explain the gradability of *ought* and *should* by treating them as degree expressions closely related to *good* in its moral sense. According to this proposal, unmodified *ought* is analogous to unmodified *good*, *tall*, or *heavy*, in that all have meanings which invoke a context-dependent threshold for obligation, goodness, height, or weight. *The doctor ought to prescribe the risky treatment* is roughly equivalent to *It is (morally) good for the doctor to prescribe the risky treatment*. As long as the MORAL GOODNESS (OF PROPOSITIONS) concept is spelled out in the information-sensitive way sketched above, this sentence is correctly predicted to be false for the same reason that (28a) is: since the probable outcome of the risky treatment is death, it would be very bad for the doctor to prescribe it, and so this is not what the doctor ought to do.

However, this proposal encounters a number of empirical problems. For example, it predicts that *ought* $\phi$ is incompatible with $\psi$ *is better than* $\phi$, for any $\psi$. This means ruling out the possibility of supererogatory actions, where someone acts in a way that is even better than what they strictly *ought* to do (Chisholm 1963). See Hansson 2001; von Fintel 2012 for this and other problems, and see Lassiter 2016: §8 for discussion and a proposal to model *ought* and *good* as gradable items which refer to related but distinct information-sensitive scales.

*Must* and *have to*, on the other hand, are usually thought to be non-gradable. Consider, for example, the intuitive contrast between *should* and *must* in acceptability with degree modifiers and in comparatives.

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Several analyses suggest themselves. One option is that must and have to are indeed ‘all’-quantifiers of some kind. Another is that they are some sort of degree expressions, but that they have syntactic restrictions on their ability to combine with degree modifiers and form comparatives. This is at least a plausible suggestion, given that English auxiliaries are syntactically very complicated, and rather different from main verbs such as like. A third possibility is that must and have to really are scalar expressions, but their gradability is limited for principled semantic or pragmatic reasons. Portner & Rubinstein (2015) argue that the distinction between must/have to and ought/should is analogous to the distinction between relative adjectives like big and extreme adjectives like huge, as discussed by Cruse (1986); Paradis (2001); Morzycki (2012). The empirical analogies are striking: for example, must and have to readily combine with the degree modifier positively, while should and ought are odd with this modifier. Positively is also natural with extreme adjectives, but odd but not relative adjectives.

The analogy may not be perfect: for example, Morzycki (2012) identifies downright, flat-out, and full-on as degree modifiers that prefer extreme adjectives, but these seem rather less natural with must and have to. Still, the resemblance is striking. A second connection is that must/have to resemble extreme adjectives in being more natural in equative constructions than in comparatives, and that the oddness of the comparative is alleviated by the addition of even. (The examples in (36) and (38) are adapted from Portner & Rubinstein (2015: §2.3.2).)

These empirical connections suggest that it may be possible to treat even deontic must and have to as gradable expressions, albeit ones with certain principled peculiarities. However, further empirical and theoretical work is needed before we can embrace this conclusion with confidence.
4 Epistemic gradation and gradability

As we have seen, modal gradation appears to be widespread, and it has been recognized implicitly in analyses of modal meaning for a long time. The observation that some modal expressions of natural languages are grammatically gradable is more recent, and the idea that this gradability should be given a compositional semantic treatment in the same terms as adjectival and verbal gradability is younger still. Lewis’s (1973) discussion of deontic comparatives, for example, does not draw out the analogies with non-modal comparatives or other phenomena related to gradability. Hamblin (1959) comes a bit closer, giving a degree semantics for the epistemic adjective probably and using it to interpret both the comparative (φ is more probable than ψ) and positive (It is probable that φ) forms. But he too limits the discussion to this specific empirical domain.

Another important question which is not addressed by Hamblin or Lewis is whether gradability in the modal domain extends also to degree modification. The fact that it does (of which we already saw some evidence in the last section) was to my knowledge first explicitly noted by Kratzer (1981: 46-51). Kratzer’s focus here is the epistemic vocabulary of German: the adjective wahrscheinlich ‘probable’ and epistemic interpretations of the noun Möglichkeit ‘possibility’ and auxiliary können ‘can’. Her examples are reproduced in (39).

(39) a. Es kann gut sein, dass der Gauzner-Michl der Mörder war.
   ‘There is a good possibility that G.-M. was the murderer.’
   It can well be that the G.-M. the murderer was
b. Es besteht aber immer noch eine geringe Möglichkeit, dass der Kastenjakl der Mörder war.
   ‘There is, however, still a slight possibility that the Kastenjakl the murderer was
   There is however always still a slight possibility that the Kastenjakl the Mörder war.
c. Der Gauzner-Michl kann eher der Mörder sein als der Kastenjakl.
   ‘G.-M. is more likely to be the murderer than K.’
   The Gauzner-Michl can rather the murderer be than the Kastenjakl.
d. Es ist wahrscheinlich, dass der Gauzner-Michl der Mörder war.
   ‘It is probable that the Gauzner-Michl the murderer was
   It is probable that G.-M. was the murderer.’

Kratzer suggests that the epistemic modal kann ‘can’ is modifiable by gut ‘well’, and can form comparatives using ehem, which she glosses ‘rather’. Also, the epistemic noun Möglichkeit ‘possibility’ can combine with the adjectival modifier gering ‘slight’. The epistemic adjective möglich ‘possible’ can also be modified by gut, as in example (40) (a headline from the May 23, 2013 edition of the newspaper Berner Zeitung).

(40) Es ist gut möglich, dass das Volk heute anders entscheiden würde.
   ‘It is quite possible that the people today otherwise decide would
   It is good possible that the people today otherwise decide would
As Kratzer discusses, modeling the distinction between kann and kann gut — or between möglich and gut möglich, or possible and quite possible — requires making more distinctions between
grades of modality than is possible if the only tools we have at our disposal are ‘all’ and ‘some’-quantification over a fixed set of worlds. Kratzer proposes to use an ‘ordering source’ — a set of propositions — to draw distinctions among the relevantly possible worlds, ordering them according to whether one world satisfies all of the ordering-source propositions that another does (and possibly more). She suggests a number of ways to use this ordering to encode grades of possibility that have appropriate logical relations to each other: kann gut entails kann, and so forth.

As mentioned above, Kratzer’s (1981) proposal for epistemic modals is formally equivalent to Lewis’ semantics for deontic modals, except that she drops Lewis’ assumption of connectedness. In addition to drawing attention to the phenomenon of graded modality, Kratzer’s (1991) most important contribution to our understanding of this phenomenon was to propose lexical entries for a number of modal words and complex modal expressions, roughly as in (41). (These are simplified and modified somewhat from Kratzer’s original proposals, but are very much in their spirit.)

\[(41) \quad a. \quad \text{Es kann gut sein, dass } \phi \text{ is true if and only if, of the worlds that are maximal in the relevant world-ordering, at least one is a world in which } \phi \text{ holds.} \\
\quad b. \quad \text{Es besteht eine geringe Möglichkeit, dass } \phi \text{ is true if and only if } \phi \text{ is in the domain of the world-ordering (i.e., if it is ‘epistemically accessible’ — among the possible worlds being ordered).}\]

These proposals could naturally be extended to the English translations of these expressions.

### 4.1 Compositionality and likelihood

Kratzer’s influential analysis of complex graded modal constructions has an important lacuna: it is not compositional. Entire constructions are assigned meanings, and there is no attempt to derive the meanings of the constructions from the meanings of their parts. But it is obviously desirable to give a compositional analysis, where the meaning of geringe Möglichkeit ‘slight possibility’ would be derived from the meanings of gering and Möglichkeit using standard compositional machinery (cf. Yalcin 2007; Portner 2009). This process should ideally happen in the same way that gering is combined with Selbstachtung to form geringe Selbstachtung ‘low self-esteem’. Similarly, we should analyze quite and more as having the same meaning in expressions like quite possible and more probable that they do in quite hungry and more important.

One way to pursue this goal is discussed by Villalta (2008); Portner (2009): we can use Kratzer’s rule of Comparative Possibility to induce modal degrees from a world-ordering. Comparative Possibility is the same ‘lifting’ rule that we saw when discussing Lewis’ semantics in the last section: the degree assigned to a proposition is determined by the highest position of any world in the proposition.

\[(42) \quad \text{Given a world-ordering, assign epistemic degrees to propositions in any way you like as long as the following constraint is observed: } \mu_{epi}(\phi) \geq \mu_{epi}(\psi) \text{ if and only if, for every world } w_1 \text{ in } \psi, \text{ there is a world } w_2 \text{ in } \phi \text{ which is ranked at least as high as } w_1 \text{ in the world-ordering.} \]

(Note: This rule has the consequence that the set of degrees will not be totally ordered if the world-ordering is not, and so we cannot think of degrees as real numbers anymore.)
This gives us the beginnings of a compositional account of complex epistemic constructions. For example, we might treat *slight possibility* as analogous to *slight pain*, as sketched informally here.

(43) a. *x* is a pain if and only if the degree of pain associated with *x* is non-zero.
   b. *φ* is possible if and only if $\mu_{epi}(\phi) > \min(\mu_{epi})$.

(44) a. *x* is a slight pain if *x* is a pain and the degree of pain associated with *x* is not great.
   b. *φ* is a slight possibility if *φ* is a possibility and $\mu_{epi}(\phi)$ is not great.

Epistemic comparisons formed using *kann eher*, or English *more likely*, could be interpreted by analogy to non-modal comparatives like ‘*a* is heavier than *b*’. (See, however, Herburger & Rubinstein 2014 for objections to this treatment of the German construction).

(45) a. *φ* is as likely as *ψ* is true if and only if $\mu_{epi}(\phi) \geq \mu_{epi}(\psi)$.
   b. *φ* is more likely than *ψ* is true if and only if $\mu_{epi}(\phi) > \mu_{epi}(\psi)$.

We can also interpret modified forms of *likely*, as in (46).

(46) a. *Bill* is twice as heavy as *Sam* is true if and only if $\mu_{weight}(Bill) \geq 2 \times \mu_{weight}(Sam)$.
   b. *Rain* is twice as likely as *snow* is true if and only if $\mu_{epi}(rain) \geq 2 \times \mu_{epi}(snow)$.

The introduction of degrees is a good start on the compositional problem. However, as Lassiter (2010); Yalcin (2010) point out, it leads to another problem. Suppose that I am about to toss a single fair die. Since the die is fair, all numbers in \{1, 2, 3, 4, 5, 6\} are equally likely to turn up. So the following sentences are intuitively true:

(47) a. The die is exactly as likely to land on 1 as it is to land on 2. ($\mu_{epi}(1) = \mu_{epi}(2)$)
   b. The die is exactly as likely to land on 1 as it is to land on 4. ($\mu_{epi}(1) = \mu_{epi}(3)$)
   c. The die is exactly as likely to land on 1 as it is to land on 6. ($\mu_{epi}(1) = \mu_{epi}(3)$)

The problem is that, according to the proposal just sketched, if (47a) and (47b) are both true then (48) must be true as well:

(48) So, the die is exactly as likely to land on 1 as it to land on an even number. ($\mu_{epi}(1) = \mu_{epi}(2 \text{ or } 4 \text{ or } 6)$)

But this conclusion is clearly false: it’s clearly more likely that a fair die will land on an even number than it is that it will land on 1.

This is a problem for the way of constructing degrees in (42). Intuitively the reason that the semantics has this property is that degrees are assigned to propositions in a way that is only sensitive to the positions of the highest-ranked worlds in the propositions. When we form a disjunction such as *φ* or *ψ*, the positions of worlds in the disjunction are unaffected, and so the highest-ranked worlds in the disjunction will just be the highest-ranked *φ*-worlds or the highest-ranked *ψ*-worlds, whichever is greater. In the case at hand, the highest-ranked 2-worlds and 3-worlds are equally ranked, and so the disjunction gets the same degree as both disjuncts. This is wrong: as (47) illustrates, disjunctions are frequently *more likely* than any of their disjuncts, because likelihoods add up in a particular way.
Both Lassiter (2010) and Yalcin (2010) propose dealing with this issue by treating the likelihood scale as one of probabilities, following Yalcin (2005, 2007); Swanson (2006). Probabilities are numbers between 0 and 1, inclusive, and a probability measure is a mapping from propositions to probabilities. This mapping must satisfy some technical conditions, most crucially: a tautology has probability 1, a contradiction has probability zero, and the probability of any two mutually exclusive propositions is the sum of their individual probabilities (additivity). As an example, since rain and no rain are mutually exclusive, $P(\text{rain or no rain})$ is the sum of $P(\text{rain})$ and $P(\text{no rain})$. Furthermore, rain or no rain is a tautology, so its probability is 1. So, we have $P(\text{rain}) = 1 - P(\text{no rain})$, or, more generally, $P(\phi) = 1 - P(\not\phi)$.

This proposal correctly predicts that (47) should be invalid, because probabilities of mutually exclusive propositions add up: when $P(\phi), P(\psi), \text{and } P(\chi)$ are all equal, $P(\psi \text{ or } \chi)$ will be greater than $P(\phi)$ except in certain special cases. The proposal also predicts correctly that (49) should be intuitively true in the die-throwing example:

(49) It is exactly three times as likely that the die will land on an even number as it is that the die will land on 1.

Constructions like this — including, for example, Bill is exactly twice as heavy as Sam — seem to have the interpretation in (50).

(50) $x$ is exactly three times as $A$ as $y$ is true if and only if $\mu_A(x) = 3 \times \mu_A(y)$.

Clearly $P(\text{throw 1}) = P(\text{throw 2}) = P(\text{throw 4}) = P(\text{throw 6})$, and these outcomes are all mutually exclusive. So, $P(\text{throw even}) = P(\text{throw 2 or 4 or 6}) = P(\text{throw 2}) + P(\text{throw 4}) + P(\text{throw 6}) = 3 \times P(\text{throw 1})$. (49) thus comes out true, as desired.

In addition, the proposal that epistemic degrees are probabilities has the consequence that the epistemic scale has a maximum and a minimum element. Recall from the discussion of boundedness in section 2 that a maximum degree is one that cannot be exceeded: if $x$ is maximally full, then it is not possible that it could be more full. Similarly, if $\mu_{epi}$ is a probability measure then there is some proposition such that nothing can receive a greater degree that it does — namely, a tautology. As far as likely is concerned, this seems to be a good prediction: surely nothing could be more likely than the proposition that I will either be in Paris tomorrow at noon, or I will not. The prediction that likely’s scale has a minimum element is similarly plausible: surely nothing could be less likely than the proposition that I will both be in Paris tomorrow at noon, and I will not.

As we saw in section 2 (discussing Aquinas’ argument for the existence of God), another test for the existence of a maximal degree on a scale involves whether an adjective can be modified by totally, perfectly, and completely with a ‘maximum-degree’ meaning. Here things get interesting: as Portner (2009) points out, likely differs from full in that it does not satisfy this test.

(51) a. The glass is totally/perfectly/completely full.

b. (??) It is totally/perfectly/completely likely that it will rain tomorrow.

It doesn’t seem to be possible to interpret (51b) as meaning ‘Rain is as likely as a tautology’, ‘Rain is absolutely certain/beyond doubt’, or anything of the sort. Similarly, Kennedy & McNally

8 Specifically, when either $\psi$ and not $\chi$ or $\chi$ and not $\psi$ has probability 0, including when $\psi$ and $\chi$ are logically equivalent.
(2005); Kennedy (2007) use the possibility of modification by slightly as a diagnostic for lowerboundedness. If this is right, then the probabilistic semantics seems to predict that (52b) should be acceptable with a ‘just greater than zero’ meaning. This prediction appears to be incorrect.

(52)  

a. The towel is slightly/barely wet.

b. (??) It is slightly/barely likely that it will rain tomorrow.

These data points are very puzzling. Should we conclude that probability is the wrong choice for likely’s scale, as Portner (2009) argues? That likely lives on a scale which is exactly like the probability scale except that it excludes the endpoints of 0 and 1, so that the range of $\mu_{epi}$ is $(0, 1)$, as Klecha (2012, 2014) suggests? Or that there are principled restrictions on the applicability of Kennedy & McNally’s (2005) degree-modification tests to certain kinds of adjectives, as Lassiter (2010) argues?

Each of these proposals accounts for (51), but all have drawbacks. For example, Portner’s (2009) seems to require positing a number of distinct epistemic scales, and then adding additional machinery in order to account for the logical connections between likely, certain, and possible. We will return to this point momentarily. Lassiter’s (2010) interpretation requires complicating our understanding of standard degree modification tests. This move, even if it can be motivated empirically, certainly muddies the waters in our understanding of how modification relates to the presence or absence of a endpoints on an adjective’s scale. Klecha’s (2012; 2014) proposal has the consequence that it should be nonsensical to talk about the likelihood of a tautology or a contradiction in examples like (53) (based on Lassiter 2016: §4.)

(53)  

a. It’s very unlikely that you’ll win, but it’s more likely than it is that $2 + 2 = 3$ (since that is impossible).

b. It’s very likely that a child, both of whose parents have two copies of a certain recessive allele, will have two copies of the same allele. In fact, it’s absolutely certain.

But these examples, far from being contradictory or unintelligible, seem to be readily interpretable.

The theoretical upshot of this debate about the structural aspects of the likelihood scale is still unclear. However, probabilistic treatments of likely and probable are increasingly common: see for example Cariani 2015; Moss 2015; Swanson 2015 for many interesting theoretical variations and connections. In addition, Holliday & Icard (2013) investigate the logic of comparative likelihood in detail, showing that it is possible to capture the crucial inferential properties of comparative likelihood using a scale that is slightly weaker logically than the probability scale. There is still much work to be done in integrating these insights into a compositional semantics for a larger fragment of English. See Lassiter 2015 for some efforts in this direction, and an argument that the full probabilistic treatment may be necessary to account for constructions involving including ratio modifiers like three times as likely.

4.2 Certainty, degree modification, and monotonicity

A related problem involves the relationship between likely and certain. Certain comes in several forms: it can be used impersonally or with an animate subject, and its complement can be propositional or a wh-question.
(54)  a. Bill is certain that it will rain.
    b. It is certain that it will rain.
    c. Bill is certain who will win the tournament.
    d. It is certain who will win the tournament.

Focusing on the impersonal proposition-embedding (54b), we can ask what the connection is between (54b) and similar examples with *likely*.

(55)  a. It is certain that it will rain.
    b. It is likely that it will rain.

Earlier I spoke of ‘µepi’, taking for granted that there is a single epistemic scale. This is in the spirit of Lewis (1973) and Kratzer (1981), who attempted to define a variety of modal expressions of varying strength in terms of a single graded modal concept. But it’s not the case that all epistemic concepts live on a common scale: for instance, *clear*, *evident*, and *obvious* have more semantic texture than the simple epistemic concepts that we have focused on so far (cf. section 4.4). For these, some additional dimension of meaning will surely be needed in addition to or instead of probability. Even for the basic epistemic concepts, it’s not obvious that all live on a single scale, and in fact Portner (2009) points out that the puzzle about degree modification that we discussed in the last section could be interpreted as an argument for distinguishing the scales of likelihood and certainty. Specifically, while *likely* failed the *totally* test, *certain* seems to pass it.

(56)  It is totally certain that it will rain. (It couldn’t be more certain.)

If the acceptability of *totally* with a ‘maximizing’ meaning is a reliable test for scalar upper-boundedness, this suggests that *likely* and *certain* cannot be on the same scale: *certain*’s scale has a maximum element which *likely*’s lacks.

Nevertheless, likelihood and certainty are clearly not unrelated. For example, (55a) entails (55b), and a typical assertion of (55b) would implicate the negation of (55a) [cross-ref: implicature chapter(s)]. They even appear to order propositions in the same way. For example, if Stanford and Alabama are both playing football this weekend (not against each other, of course!), it would be very strange to assert (57a) and deny (57b).

(57)  a. It is more certain that Alabama will win on Saturday than it is that Stanford will.
    b. It is more likely that Alabama will win on Saturday than it is that Stanford will.

Lassiter & Goodman (2015) give experimental evidence for this **monotonicity** property using a reasoning experiment involving judging the strength of arguments. When experimental participants were more likely to judge a conclusion C ‘certain’ than they were to judge a different conclusion C’ ‘certain’, they were also more likely to judge conclusion C ‘likely’ (or indeed ‘probable’, ‘plausible’, ‘possible’, or ‘necessary’) than C’. This suggests that these scales, while perhaps not identical, are strongly — maybe even perfectly — correlated in the pairwise comparisons that they deliver between individuals.

The latter property is again reminiscent of adjective pairs like *beautiful* and its ‘extreme’ counterpart *gorgeous*. Just as odd as asserting (57a) while denying (57b) would be to assert (58a) while denying (58b):
(58)  a. Paris is more gorgeous than London.
     b. Paris is more beautiful than London.

*Beautiful* and *gorgeous* seem to be on the same scale, at least in the sense that (58a) entails (58b). *Lassiter (2010)* argues similarly that *likely* and *certain* are both on a probability scale, and attempts to explain away the degree modification differences in terms of differences among lexically specified adjective classes. *Lassiter (2011, 2016)* suggests as an alternative that *certain’s* scale may be one of *log probability*, which is a monotonic transformation of the probability scale. Because it is a monotonic transformation, \( \log(P(\phi)) \geq \log(P(\psi)) \) if and only if \( P(\phi) \geq P(\psi) \), and both scales are upper-bounded at the same point (probability 1, which is log probability 0). Either hypothesis would explain the data in (56) and (57) and the clear intuition that nothing can be more likely — or certain — than a tautology. However, certain facts involving proportional degree modifiers pointed out by *Klecha (2012)* seem to favor the log probability hypothesis. See also *Klecha (2014); Lassiter (2016)* for further discussion.

4.3 Possibility, modification, and kinds of comparison

A related issue is the status of *possible*. Unmodified *possible* seems to be the dual of unmodified *certain*, meaning that (59a) is true if and only if (59b) is, and likewise with (60a) and (60b).

(59)  a. It’s certain that it’s raining.
     b. It’s not possible that it’s not raining.

(60)  a. It’s possible that it’s raining.
     b. It’s not certain that it’s not raining.

If *certain* lives on a (log?) probability scale and indicates a maximal degree of probability, we can explain (59) and (60) by supposing that *possible* indicates a non-zero degree of probability (*Yalcin 2005; Lassiter 2010*). Then, recalling the equation \( P(\phi) = 1 - P(\text{not } \phi) \), we have the equivalences:

(61)  a. \( P(\phi) = 1 \) (\( \phi \) is certain) if and only if \( P(\text{not } \phi) = 0 \) (\( \text{not } \phi \) is not possible).
     b. \( P(\phi) > 0 \) (\( \phi \) is possible) if and only if \( P(\text{not } \phi) < 1 \) (\( \text{not } \phi \) is not certain).

(There is an additional question about whether the meaning of *possible*, *wet*, and minimum adjectives generally should be stated in terms of non-zero deviation from the minimum, or a large enough deviation. See *Rotstein & Winter 2004; Kennedy 2007; Lassiter & Goodman 2013* for relevant discussion.)

Given this, it would be natural to suppose that *possible* is a minimum adjective, like *wet* and perhaps *permissible* (*Lassiter 2010, 2011*). Minimum adjectives are gradable, and in the positive form they have a `non-zero degree’ meaning which is frequently well-paraphrased using ‘some’. The latter property can explain the intuitive plausibility of the traditional analysis of *possible* as a SOME quantifier over possible worlds, as it did with *permissible*.

(62)  Minimum adjective *wet*
     a. This towel is *wet*.
It has a non-zero degree of wetness.
≈ It has some amount of water on it.

b. This towel is wetter than that one.
(c omparative)
c. This is the wettest towel we have.
(s uperlative)
d. This towel is slightly wet.
(degree modification)

(63) possible

a. It’s possible that it will rain.
= Rain has a non-zero degree of probability.
≈ There is some possibility in which it rains.

b. In fact, it is more possible that tomorrow is the zombie apocalypse than people magically floating away into the clouds.
(comparative; web)
c. “But I did not become what I am and survive without cataloguing the facts or by ignoring the most possible explanation for these facts, however improbable,” he continued.
(superlative; Harbaugh 2003: 189)
d. In all seriousness yes, it is slightly possible that the NSA could have a hand in the Linux code, but unlikely.
(degree modification; web)

The analysis of possible as a minimum adjective faces some challenges. Klecha (2012) argues that possible is not a gradable adjective at all, based on two main observations. First, he notes that more/too/very possible are intuitively less natural than (e.g.) more/too/very likely. Second, he reports corpus work showing that comparatives and certain degree modifiers are less frequent in COCA (Davies 2008-) with possible than with likely, tall, or several other uncontroversially gradable adjectives. (However, Klecha does not report how possible compares to bent, wet and other uncontroversial minimum adjectives, which would be a more appropriate baseline.) Klecha proposes that possible denotes an existential quantifier over epistemically accessible worlds, but that it can sometimes be ‘coerced’ into a gradable interpretation—which would, presumably, behave like a minimum adjective in relevant respects. (See also Herburger & Rubinstein 2014 for additional relevant considerations from German.)

The invocation of coercion here brings with it some additional interesting connections with adjective interpretation more broadly. In this domain (unlike, for example, Pustejovsky’s (1995) Generative Lexicon theory), ‘coercion’ is not a theoretical concept with a clear formal interpretation, but a descriptive label for certain situations in which adjectives that are intuitively non-gradable are used in ways that force an interpreter to search for an associated gradable property in an ad hoc way. Pregnant is a standard example.

(64) Mary is more pregnant than Sue.

a. ‘Mary has been pregnant for a longer time than Sue.’
b. ‘Evidence of Mary’s pregnancy is more clearly visible than evidence of Sue’s.’
c. ‘Mary is behaving in a way that is more stereotypically associated with pregnant people than the way that Sue is behaving.’

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It is not clear whether the semantic relationship between *possible* and *more possible* is open-ended in the way that the relationship between *pregnant* and the possible interpretations of *more pregnant* is. In addition, it would be desirable to have a clear, falsifiable theory of coercion, with limitations on when and how this operation can apply. Hopefully future work will be able to clarify the empirical situation and provide principled grounds for distinguishing ‘coerced’ from ‘basic’ scalar meanings.

4.4 Clarity

A number of authors have recently discussed the gradable adjective *clear*, as exemplified in (65) (Barker & Taranto 2003; Taranto 2006; Bronnikov 2008; Barker 2009; Wolf & Cohen 2011; Crone 2015).

(65) It is clear that Bill is not feeling well.

Clarity statements are interesting for a number of reasons. For example, Barker & Taranto (2003) identify the ‘paradox of asserting clarity’: an assertion of (65) could be taken to be self-defeating, on the grounds that — if it is indeed clear to all involved — it is a mystery why the speaker would bother saying so. Second, *clear* is clearly a gradable adjective, and specifically a maximum adjective.

(66) a. It is totally clear that Charter does not want to talk to their current customers.
   b. [I]t is clearer that Lewis’ view is antirealistic with respect to propositions than with respect to properties, and clearer with respect to properties than with respect to possible worlds. (Plantinga 2003: 222)

Various proposals exist to explain these facts. All agree that publicly available evidence is a key component: (65) is not true if the speaker has private evidence which is not available to the listener, for example. Barker’s (2009) explanation of the Paradox involves the idea that clarity assertions help to negotiate the standard of evidence relevant to the conversation: if (65) is asserted and accepted, then everyone agrees that the evidence in favor of *Bill is not feeling well* is strong enough to justify taking it for granted from here on. The gradability, in turn, is explained by the fact that justification is a graded concept: a conclusion can be more or less justified by some evidence. Plausibly, justification is an upper-bounded concept in the sense that there is some degree of justification such that nothing could possibly be more justified; Descartes seemed to think that his *I think therefore I am* reached this level. If so, this would account for the fact that, as a maximum adjective, *clear* needs to be on an upper-bounded scale.

An alternative account due to Wolf & Cohen (2011) suggests instead that clarity statements are have to do with the probabilistic belief states that ideal reasoners would have if they had access to the evidence available to the conversational participants. This, too, provides an upper-bounded scale since the probability scale is upper-bounded. If this is right, we might conclude that *clear* and *certain* are identical except that *certain* relates to relevant individuals’ *actual* belief states, while *clear* relates to the belief states that they *would* have if they were ideal reasoners in some sense.
5 Additional varieties of graded modality

5.1 Ability modals

Ability seems to be a graded concept, and *able* appears to be gradable. Here are a few examples.

(67) a. [T]he programs effectively represent an effort to render other countries more willing and more able to defeat imminent threats to the United States that may emerge from those states.

b. In many application fields experts possess a large amount of implicit knowledge, since they are often more able to do than to explain what they do.

(68) a. The observer was herself struggling with the recognition of the unfamiliarity of the complex multicultural mix of the nursery, very much able to identify with this. (Rustin & Bradley 2008: 18)

b. We were very much able to fight under these conditions, as our enemy would learn, to his own detriment. (Russell 2011: 88)

There is a well-developed literature on semantically related root modals like *can* (e.g., Brennan 1993; Hacquard 2006). However, the gradability of *able* and its implications for the analysis of root modality in general has not been studied in depth to date.

5.2 Desire verbs

*Want*, *wish*, *desirable*, and certain other expressions of desire are gradable.

(69) a. Kodak’s warranty is an iron clad 1 year. They want more to repair the camera than to buy a new one.

b. I am an American and I want very much to travel to Cuba.

(70) a. Obviously, it is far more desirable for a building to sustain a limited amount of deformation than for it to suffer a complete breakage failure.

b. Because heavy water is a very expensive commodity, it is very desirable to repair a pipe or other piece of equipment that develops a heavy water leak.

(71) a. Do you wish more that unicorns or mermaids existed?

b. Sometimes I wish very much that I wasn’t so socially awkward and virtually incapable of sustaining fannish friendships.

Villalta (2008) discusses gradability of *want* and *wish* among numerous other embedding verbs, proposing that they encode degrees of desire in some form. The specific proposal is equivalent to the Lewis/Kratzer Comparative Possibility relation, where an ordering of goodness/desirability on worlds is lifted to a corresponding ordering on propositions. This proposal is subject to many of the same criticisms that were mentioned for Lewis’ account of goodness described in section 3.4 above: for instance, it does not appear to make desirability of propositions sensitive to probabilities, and
many examples suggest that it is (Levinson 2003; Lassiter 2011). In contrast, it may be possible to resolve these issues by modeling want’s scale as expected utility, as Levinson (2003) proposes. Information-sensitivity is an automatic consequence of this account, for the same reasons that it follows from the expected-value semantics for deontic concepts sketched above.

6 Conclusion

Much remains to be done at the intersection of theories of gradation and theories of modality, and this work is of potentially great value for linguistics, philosophy, and cognitive science. In this chapter I have intentionally isolated for discussion a number of phenomena at the intersection of modality and gradation that are not currently well-understood, or which have been discussed very little in the literature so far and are ripe for careful empirical study. In addition, the study of graded modality is a theoretically exciting area for researchers with interdisciplinary interests, since it has the potential to inform and be informed by a broad range of disciplines: philosophy of language and mind, epistemology, probability-, decision-, and game theory, the psychological study of reasoning and concepts, and artificial intelligence topics such as knowledge representation, inference, and planning, to name just a few. When another survey like this is written in ten or twenty years, I expect that the theoretical and empirical landscape will look rather different, and many of the ideas floated here will turn out to be incorrect or too simplistic (though I hope that some will last).

References


Herburger, Elena & Aynat Rubinstein. 2014. Is ‘more possible’ more possible in german?


Portner, Paul & Aynat Rubinstein. 2015. Extreme and non-extreme deontic modals. To appear in
M. Chrisman and N. Charlow (eds.), *Deontic Modals*. Oxford University Press.

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