1 GQ theory

Inspirational quote from Barwise & Cooper 1981: 159, a foundational paper on applications of GQ theory to natural language:

The quantifiers of standard first-order logic (as presented in elementary logic textbooks) are inadequate to treat the quantified sentences of natural languages in at least two respects. First, there are sentences which cannot be symbolized in a logic which is restricted to the first-order quantifiers $\forall$ and $\exists$. Second, the syntactic structure of quantified sentences in predicate calculus is completely different from the syntactic structure of quantified sentences in natural language. The work on generalized quantifiers referred to above has led to new insights on the nature of quantifiers, insights which permit logical syntax to correspond more closely to natural language syntax. These insights, we argue, may also make a significant contribution to linguistic theory.

Some examples:

(1) a. Every dog barks.
   b. Mary watched at least three movies.
   c. More than half of Americans failed to vote.
   d. Most Americans failed to vote.

(1a) and (1b) could be explicated using first-order logic, as in

$$\forall x. \text{dog}(x) \rightarrow \text{barks}(x)$$

and

$$\exists x \exists y \exists z. x \neq y \neq z \land \text{watched}(m)(x) \land \text{watched}(m)(y) \land \text{watched}(m)(z)$$

But (1c) and (1d) could not, unless we were to enrich FOL to include numbers and functions from subsets of the domain to numbers (Barwise & Cooper 1981: 161).

Original motivations for GQs in natural language semantics (Lewis 1970; Montague 1973):

- Many quantifiers can’t be associated with individuals: some/many/most/no girls
- uniform semantic type for all NPs
• Coordination of names and GQs (*Mary and most boys*)

Some terminological matters:

• GQs are **semantic**, not syntactic objects. *Every dog* is not a generalized quantifier — it’s an expression that denotes a generalized quantifier.

• *Every, some, more than half* are not GQs, nor do they denote GQs. These expressions are **determiners** (a syntactic category) which combine with noun phrases like *dog* and *boy in fourth grade* to form phrases which denote GQs.

The basic idea: GQs denote properties of properties, i.e., sets of sets of individuals.

*Every man* denotes the set of properties that every man has. The property of walking is in this set iff every man walks. ... $\lambda P \forall x [\text{man}(x) \to P(x)]$ is the (characteristic function of the) set of properties every man has. Other ways of writing the same thing are $\lambda P [\text{MAN} \subseteq P]$ or $\{P : \text{MAN} \subseteq P\}$.

*At least two men* denotes the set of properties that at least two men have, written as $\lambda P \exists x \exists y [x \neq y \land \text{man}(x) \land \text{man}(y)]$. Other ways of writing the same thing are: $\lambda P [|\text{MAN} \cap P| \geq 2]$ or $\{P : |\text{MAN} \cap P| \geq 2\}$. ...

Since GQs are sets (of sets of individuals), they have elements. For instance, *Every man walks* is true iff the set of walkers is an element of $[\text{every man}]$, the GQ denoted by *every man*. (from Szabolcsi 1997)

### 1.1 Determiners

It follows from the above that quantificational determiners like *every* and *at least two* denote curried relations between sets, i.e., functions from sets (noun phrase denotations) to GQ denotations.

• *Every boy*:
  
  – $[\text{every boy}] = \lambda Q. \forall x [\text{boy}(x) \to Q(x)]$ (official)
  
  – $[\text{every boy}] = \lambda Q. \text{boy} \subseteq Q$ (helpful but abusive of notation)

• Backing out the meaning of *every*:
  
  – $[\text{every}] = \lambda P \lambda Q. \forall x [P(x) \to Q(x)]$
  
  – $[\text{every}] = \lambda P \lambda Q. P \subseteq Q$

For other GQs, the key modification will be that, instead of only trues, you’re looking for at least $n$, between $n$ and $m$, more than half, etc. **True** values in this list.

  – *more than two boys*:
more than two boys = λQ.∃x∃y∃z[x ≠ y ≠ z ∧ boy(x) ∧ boy(y) ∧ boy(z) ∧ Q(x) ∧ Q(y) ∧ Q(z)]

– Backing out the meaning of more than two:

* [more than two] =

λPλQ.∃x∃y∃z[x ≠ y ≠ z ∧ P(x) ∧ P(y) ∧ P(z) ∧ Q(x) ∧ Q(y) ∧ Q(z)]

* [more than two] = λPλQ.|P ∩ Q| > 2

Obviously more than two isn’t a lexical item, though! What would we have to do to decompose it in a more realistic way? Here’s an (incomplete) beginning:

• [two] = 2

• [more than] = λnλPλQ.|{x | P(x) ∧ Q(x)}| > n

But it would be even better to have a cross-categorial semantics for more for this and other uses (more beautiful, etc.). (Extended meditations on this issue: Hackl 2001; Nouwen 2010.)

1.2 Properties of quantificational determiners


Example: Every/no/some/at least 2 are conservative because Every boy sneezed is equivalent to Every boy is a boy who sneezed, etc.

Possible linguistic universal: All quantificational determiners attested in natural languages are conservative. (It’s not obvious why this should be so!)


• some/at least 2 are intersective because some/at least 2 boys sneezed are equivalent to some/at least 2 boys who sneezed sneezed.

• Every/no are not intersective because Every/no boy sneezed is not equivalent to Every/no boy who sneezed sneezed.

1.3 Monotonicity

Monotonicity is not an arbitrary property of determiners: if you’ve defined the denotation of the GQ correctly, its monotonicity properties will fall out from the way that the sets that satisfy the GQs are arranged in the powerset algebra (℘(D_e), ⊆).
Definition 3. (Upward monotonic) $GQ$ is upward monotonic iff, whenever $P \in GQ$, all supersets of $P$ are also in $GQ$.

Definition 4. (Downward monotonic) $GQ$ is downward monotonic iff, whenever $P \in GQ$, all subsets of $P$ are also in $GQ$.

Definition 5. (Non-monotonic) $GQ$ is non-monotonic iff $GQ$ is neither upward monotonic nor downward monotonic.

Graphical correspondents of (upward-/downward-/non-monotonicity) in the BA?

- If $GQ$ is upward monotonic, then
  
  - $GQ(\text{run}) = GQ(\text{move})$, because $\{x \mid \text{run}(x)\} \subseteq \{x \mid \text{move}(x)\}$.
  
  - $GQ(\text{run}) \neq GQ(\text{run-fast})$, because $\{x \mid \text{run}(x)\} \nsubseteq \{x \mid \text{run-fast}(x)\}$.

- If $GQ$ is downward monotonic, then
  
  - $GQ(\text{run}) = GQ(\text{run-fast})$, because $\{x \mid \text{run-fast}(x)\} \subseteq \{x \mid \text{run}(x)\}$.
  
  - $GQ(\text{run}) \neq GQ(\text{move})$, because $\{x \mid \text{move}(x)\} \nsubseteq \{x \mid \text{run}(x)\}$.

- If $GQ$ is non-monotonic, then $GQ(\text{run}) \neq GQ(\text{run-fast})$ and $GQ(\text{run}) \neq GQ(\text{move})$.

(2) Exercise: Are the following GQs upward-, downward-, or non-monotonic? Check your answers by considering inferences to super- and subsets: [every man], [no man], [some man], [at least one man], [more than one man], [fewer than two men], [exactly two men]
Exercise: Fix $D_e = \{a, b, c, d\}$, letting $a, b, c$ be men and let $d$ be a dog (as in p.13 of the reading). Check your answers to exercise (2) by finding the denotation of each GQ in the exercise within the powerset algebra below.

References


