• Morphological plurality ≠ semantic plurality:

(1) Dogs have tails.
   a. # “Generally, if something is a dog it has tails.”
   b. “Generally, if something is a dog it has a tail.”

(2) a. No aliens have ever walked the earth.
    b. All bankers wear suits.

(3) Carrying guns is illegal in Illinois.
    a. # “It’s OK to carry a gun as long as you don’t have more than one”
    b. “Carrying more than zero guns is illegal”

• No morphologically plural DPs in sentence 1, but continuation indicates semantic plurality for both DPs:

(4) Each boy drew a picture. They hung them on the wall.

• Common assumption: plurals are semantically unmarked, in the sense that they can have singular or plural reference indiscriminately (Krifka 1989; Zweig 2009, etc.). The implication of plurality in sentences such as

(5) I have children.

is a quantity implicature arising from the speaker’s failure to use the more informative

(6) I have exactly one child.

• Puzzle for such approaches: why is it that most (though not all) languages have singular as the morphologically unmarked category, and the plural is morphologically marked?
1 Sums

(7) a. Bob and Carl are wounded.
b. Bob is wounded and Carl is wounded.

(8) a. Bob and Carl are a couple.
b. Bob is a couple and Carl is a couple.

(9) a. Bob and Carl carried a piano.
b. Bob carried a piano and Carl carried a piano.

Ways of disambiguating:

(10) a. Bob and Carl both/each carried a piano/are wounded/#are a couple.
b. Bob and Carl carried a piano/?are wounded/#are a couple together.

N.B. why is Bob and Carl are couple together so weird? Is it just redundancy?

Logical representations?

(11) For (7a):
   a. $[\lambda P_{e,t}. P(bob) \land P(carl)](\text{wounded})$
      \hspace{2cm} (= \text{wounded}(bob) \land \text{wounded}(carl))
   b. wounded({bob, carl})
c. wounded(bob $\sqcup$ carl)

(12) For (8a):
   a. $[\lambda P_{e,t}. P(bob) \land P(carl)](\text{met}) = \text{met}(bob) \land \text{met}(carl)$
   b. met({bob, carl})
c. met(bob $\sqcup$ carl)

(13) For (9a):
   a. $[\lambda P_{e,t}. P(bob) \land P(carl)](\text{carried a piano})$
      \hspace{2cm} (= \exists y_e. \text{piano}(y) \land \text{carried}(y)(bob) \land \exists y_e. \text{piano}(y) \land \text{carried}(y)(carl))$
   b. $\exists y_e. \text{piano}(y) \land \text{carried}(y)({bob, carl})$
   c. $\exists y_e. \text{piano}(y) \land \text{carried}(y)(bob \sqcup carl)$

We need to explain why (11a) is a perfect rendition, (12a) is a terrible one, and (13a) is great but only under one reading.

Making properties functions over sets can be made to work, but it breaks what’s seen as an attractive correspondence between semantic types and set-theoretic hierarchy (properties = sets of individuals, GQs = sets of sets of individuals, etc.). Winter and Schwarzschild have defended the set-based approach in the (b) representations. A more common approach is to follow Link (1983) is using the (c) representations, by introducing a concept of plural...
individual or sum. Nouwen follows this route. To do this we need an ordered domain, a complete join semilattice (think back to the discussion of Szabolcsi (1997)). Recall the following definitions:

**Definition 1. (Partial order)** \( \leq \) is a partial order on set \( X \) iff

- \( \forall x \forall y : x \leq y \rightarrow x \in X \wedge y \in X \); and
- \( \leq \) is reflexive, transitive, and antisymmetric.

**Definition 2. (Partially ordered set)** \( (X, \leq) \) is a partially ordered set (poset) iff \( \leq \) is a partial order on \( X \).

**Definition 3. (Closure)** An ordered set \( (X, \leq) \) is closed under an operation \( * \) iff \( x * y \in X \) for all \( x,y \in X \).

**Definition 4. (Join semilattice)** A join semilattice is a partially ordered set which is closed under the join/sum operation \( \cup \). In other words, a join semilattice is a poset \( (X, \leq) \) s.t., for any two elements \( x, y \in X \), there is some \( z \in X \) s.t. \( x \cup y = z \).

**Definition 5. (Atom)** \( x \) is an atom in \( (X, \leq) \) if \( \neg \exists z \in X [z \leq x \wedge z \neq x] \) (nothing in \( X \) is a proper part of \( X \)).

**Definition 6. (Atoms)** \( Atoms \) is a function from sets to things in them that are atoms:

\[
Atoms(Y) := \{ y \mid y \in Y \wedge \neg \exists z [z \leq x \wedge z \neq x] \}.
\]

Link’s idea was to treat \( D_e \) not as a set of singular individuals in the intuitive sense, but as such a set of individuals closed under sum formation: e.g., if \( j, m, \) and \( s \) are all in \( D_e \), then so are \( j \cup m, j \cup s, m \cup s, \) and \( (j \cup m) \cup s \). Note that

- What we were thinking of previously as \( D_e \) is now \( Atoms(D_e) \).
- \( D_e \) is isomorphic to \( \phi(Atoms(D_e)) - \emptyset \).\(^1\)

The latter observation is important, since it suggests that there is formally much less difference between the set-based and plural-individual-based theories than it might seem at first glance.

The basic idea is that simple sentences with plural DPs (the boys) and conjoined individuals like (Bob and Carl) are translated as just \( P(X) \), where \( X \) is a possibly plural individual. This is meant to explain why the same plural DP is interpreted very differently, depending on whether the VP is meet, be wounded, carry a piano each, or carry a piano together. What it implies is that the action must occur in the verb phrase.

Interpreting singular and plural nouns:

\[
(14) \quad \text{a. boys picks out all the set containing the individual (atomic) boys (} x, y, z, \ldots \text{) also all of the sums that can be formed from them (} x \cup y, x \cup z, \ldots \text{).}
\]

\(^1\) ‘Isomorphic’ = ‘There is a structure-preserving 1-to-1 mapping between them’, with \( \leq \) corresponding to \( \leq \) and thus \( \cup \) corresponding to \( \cup \).
b. boy = \text{Atoms}(\text{boys}).

**Definition 7. (Star-operator)** The star-operator applies to a set and returns the poset which is that set closed under \( \sqcup \). That is, for any set \( X \), \( *X \) is the smallest set such that

- For all \( y \), if \( y \in X \) then \( y \in *X \)
- For all \( y,z \in *X \), \( y \sqcup z \in *X \).

Note that \( \text{boys} = *\text{boy} \), since \( \text{boy} \) picks out the atoms in \( \text{boys} \), and the star operation takes this set and undoes the effect of restricting to atoms. More generally, if \( X \) is atomic — i.e., if \( x \sqcup y \in X \) implies \( x \in X \cap y \in X \) — and \( X \) is closed under \( \sqcup \), then

\[
X = *(\text{Atoms}(X))
\]

We can also use the star-operator on predicates: e.g., if we suppose that a predicate like wounded contains only atoms, then \( *\text{wounded} \) will be a predicate which is true of all the wounded individuals and also all of any sum all of whose members are wounded.

2 Distributivity

Distributivity is a property of VPs:

**Definition 8. (Distributive)** VP is distributive iff \( \text{NP}_1 \) and \( \text{NP}_2 \) VP entails \( \text{NP}_1 \) VP and \( \text{NP}_2 \) VP.

- like carrots, be wounded, and carry a piano each are distributive.
- meet, be a couple, and carry a piano together are not distributive.

As Nouwen observes, both the distributive and non-distributive predicates in this list are cumulative:

**Definition 9. (Cumulative)** VP is cumulative iff \( \text{NP}_1 \) VP and \( \text{NP}_2 \) VP entails \( \text{NP}_1 \) and \( \text{NP}_2 \) VP.

\[
\begin{align*}
\text{(15) a.} & \quad \text{wounded is cumulative: Bill is wounded and Carl is wounded entail Bill and Carl are wounded} \\
\text{b.} & \quad \text{wrote songs together is cumulative: Bill and Carl wrote songs together and Sam and Sue wrote songs together entail (at least one reading of) Bill and Carl and Sam and Sue wrote songs together.}
\end{align*}
\]

This follows if combining with a plural argument like Bill and Carl forces a verb to pluralize using the \( * \)-operator, since

\[
P(a) \land P(b) \Rightarrow *P(a \sqcup b)
\]

First component of an account of distributive vs. non-distributive predicates, then:
• Predicates must combine with the ∗-operator first in order to take plural arguments (e.g., Bill and Carl or the boys).

Second component: explain the various kinds of predicates in terms of what kind of objects appear in their basic extension (the lexically specified extension, before applications of the ∗ operator).

• Distributive predicates have only atoms in their (basic) extension. When combined with a plural argument, they pluralize using the ∗-operation.
  
  – E.g., suppose \([\text{wounded}]^M = \{[\text{adam}]^M, [\text{bill}]^M, [\text{carl}]^M\} = \{a, b, c\}.
  
  – Then \(\ast[\text{wounded}]^M = \{a, b, c, a \cup b, a \cup c, a \cup b \cup c\}.
  
  – Bill and Carl are wounded is true because \(b \cup c \in \ast[\text{wounded}]^M\).
  
  – This is true even though \(b \cup c \notin [\text{wounded}]^M\) — the latter doesn’t contain any non-atoms.

• Non-distributive predicates have sums in their basic extension some of whose atoms are not in their (basic) extension.

  – E.g., suppose \([\text{carried a piano}]^M = \{b, c \cup a\}.
  
  – \(\ast[\text{carried a piano}]^M = \{b, c \cup a, b \cup c \cup a\} = \{b, a \cup c, a \cup b \cup c\}.
  
  – Carl and Adam carried a piano is true because \(c \cup a \in \ast[\text{carried a piano}]^M\).
  
  – Carl carried a piano is false because \(c \notin \ast[\text{wounded}]^M\).
  
  – Bill and Carl and Adam carried a piano is true because \(b \cup c \cup a \in \ast[\text{wounded}]^M\).
  
  – Importantly, this holds even though they weren’t all acting together in any piano-carrying event!
  
  – Note that there are several kinds of models that could make \(\text{Bill and Carl and Adam carried a piano}\) be true — including ones in which they each carried one, ones where they all collaborated, and ones where Carl and Adam collaborated and Adam and Bill collaborated, etc.
  
  – These are different “understandings” of the sentence, as Nouwen puts it, but not true ambiguities because the sentence is truth-conditionally unambiguous. (Compare: A man left is not ambiguous simply because various men could be the ones whose departure makes it true.)

• Anti-distributive predicates like be a couple are ones for which there are (as a matter of the meaning of the predicate) no atoms in the extension: Carl is a couple is crazy because being a couple is the sort of state which, as a matter of its meaning, requires multiple participants.
There are many alternatives to the proposal that Nouwen focuses on, including Boolean theories (Winter 2001). It’s sometimes been claimed that the Link-derived theories have difficulty with examples like *The shoes cost $75* on the grounds that the theory assigns them truth-conditions along the following lines:

(16) *The shoes cost $75* is true iff the maximal individual containing all of the shoes (in the relevant domain) is an element of the extension of *cost $75*.

But really, the objection isn’t that the theory fails to get the right truth-conditions; it’s simply that the theory assigns truth-conditions that fail to capture the very strong intuition that $75 is the cost of each pair of shoes. It’s not clear that this is a problem, though: it could well be that this additional (and context-sensitive) extra information comes in in a different way, through the interaction of semantic and world knowledge.

3 Cumulativity

(17) Two policemen inspected twelve boxes.
   a. Subject wide scope: ‘Each of the 2 policemen inspected 12 boxes.’
   b. Object wide scope: ‘12 boxes were doubly inspected.’
   c. Cumulative reading: ‘Two policemen participated in box-inspecting activities, and a total of 12 boxes were inspected.’

(18) Two authors wrote three novels.
   a. Subject wide scope: Each of the 2 authors wrote 3 novels.
   b. Object wide scope: 3 novels were (jointly) written by 2 authors.
   c. Cumulative reading: 3 novels were written, and 2 authors participated in some capacity. (e.g., author 1 and author 2 wrote novels 1 and 2, and author 2 wrote novel 3 alone)

Definition 10. (#) \#X := |Atoms(X)|  (# returns the number of atomic parts of X.)

(19) a. \([\text{two}]^M = \lambda P(e,t) \lambda Q(e,t) \exists x_e . \#x = 2 \land \star P(x) \land \star Q(x)\)
   b. \([\text{three}]^M = \lambda P(e,t) \lambda Q(e,t) \exists x_e . \#x = 3 \land \star P(x) \land \star Q(x)\)

(20) a. \([\text{two authors}]^M = \lambda P(e,t) \lambda Q(e,t) \exists x_e . \#x = 2 \land \star [\text{author}]^M(x) \land \star Q(x)\)
   b. \([\text{three authors}]^M = \lambda P(e,t) \lambda Q(e,t) \exists x_e . \#x = 3 \land \star [\text{author}]^M(x) \land \star Q(x)\)

(Recall that \([\text{authors}]^M = \star [\text{author}]^M\).) Familiar QR mechanisms plus the predicate-pluralization requirement give us

(21) \([\text{wrote three novels}]^M = \lambda z_e. \exists y_e . \#y = 3 \land \star [\text{novel}]^M(y) \land \star [\text{wrote}]^M(y)(z)\)

This is a predicate true of any individual z, atomic or non-atomic, such that there is a sum y consisting of three novels for which z *wrote y.

It has to be pluralized if it is to take a non-atomic subject, so we have for the subject wide scope reading (18a):
Two authors wrote three novels.

\[ \exists x : \#x = 2 \land \ast[\text{author}]^M(x) \land \ast[\text{wrote three novels}]^M(x) \]

In order for an individual \( x \) to verify the existential, it must be either

- A pair who jointly *wrote three novels
- the sum of exactly two atomic individuals each of whom is in the basic extension of *wrote three novels, so that their sum is too.

Neither of these is the cumulative reading (where, e.g., Bill wrote one novel and Carl wrote two).

In order to get the cumulative reading, Nouwen generalizes the *-operation to \( n \)-ary predicates. We do this by generalizing the \( \cup \) operation to tuples:

\( (x, \ldots, y) \cup (z, \ldots, w) := (x \cup z, \ldots, y \cup w) \)

This automatically makes * applicable to predicates like write (though technically we have to de-curry them first).

**Definition 11. (Star-operator)** For any set \( X \), \( \ast X \) is the smallest set such that

- For all (individuals or tuples) \( y \), if \( y \in X \) then \( y \in \ast X \)
- For all (individuals or tuples) \( y, z \in \ast X \), \( y \cup z \in \ast X \).

For example, if \( a \) ate \( y \), \( b \) ate \( z \), and nobody ate anything else, then

\[ [\ast \text{ate}]^M = \{(a, y), (b, z), (a \cup b, y \cup z)\}. \]

This allows us to express (18c) as

\[ \exists x \exists y : \#x = 2 \land \ast[\text{author}]^M(x) \land \#y = 3 \land \ast[\text{novel}]^M(y) \land \ast[\text{wrote}]^M(y)(x) \]

Success!

Or is there a problem?

In 2011, more than 100 authors wrote more than 200 novels.

Yes, if you interpret more than \( n \) as GQ theory would have it. We may need to turn to event semantics for a solution — or maybe the solution is to revise GQ theory in favor of a compositional analysis of more than \( n \) constructions.

**References**


