

# How to Set a Deadline for Auctioning a House?

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## **Abstract**

We investigate the optimal choice of an auction deadline by a seller who commits to this deadline prior to the arrival of any buyers. In our model buyers have evolving outside options, and their bidding behaviors change over time. We find that if the seller runs an optimal auction, then she should choose a longer deadline. However, if the seller runs a second-price auction, then a shorter deadline could potentially help her. Moreover, the seller can extract information about buyers' outside options by selling them contracts similar to European call options. Finally, the optimal dynamic mechanism is equivalent to setting a longer deadline and running an auction in the last day.

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# 1 Introduction

Economists used to model house selling as a bargaining problem between a seller and a buyer. Recent literature (e.g. Mayer (1998), Albrecht et al. (2016), Arefeva (2016), and Han and Strange (2014)) began to notice that over 30 % of house sales in the U.S. involve multiple buyers, and they model house selling as an auction instead of bargaining. However, housing auctions differ significantly from the traditional optimal auction models. Housing auctions are dynamic; they often last for weeks. During the auction new buyers might arrive, and existing buyers might lose interest if they find a great outside option (i.e. another house appears on the market). Moreover the seller not only has to design the auction rule, but also specifies the end date of the auction – the deadline for submitting bids. In this paper we study the optimal deadline that a seller should set for auctioning a house.

We study the optimal choice of an auction deadline using a two-period model. Prior to the arrival of any buyers, the seller commits to a date to run an auction. Shorter deadline means the seller runs an auction in period one, and longer deadline means the seller runs an auction in period two. Buyers arrive in period one and draw their value for the house, and their outside options for period one are normalized to zero. In period two new outside options become available, and buyers update their value for the house, which is equal to the value they draw minus their outside option. Thus if a buyer gets a great outside option, his value for the house decreases. We assume that no new buyers arrive in period two because we have implicitly modeled arrivals and departures through the evolving outside options. In period one buyers only know the distribution of their future outside options, and in period two they observe the actual realizations. Arrival is equivalent to a buyer expecting a great outside option, but ends up with a disappointing one. Departure is equivalent to the buyer finding a great outside option in period two and is no longer interested in bidding for this house.

The seller decides which period she wants to run an auction. We consider two types of auction formats: the optimal auction and the second-price auction. For each auction format, the seller takes the auction rule as given and selects a date to the auction. The seller's optimal choice of an auction deadline boils down to the trade-off between arrivals and departures. Running an auction in period one prevents bidders from searching for outside options, which reduces departure. Running an auction in period two allows the bidders to learn their outside options, and they might lose interest in this house if they find great outside options. However running an auction in period two also has potential

benefits: if a bidder gets a bad outside option, then his value for this house increases, which is analogous to a high valued buyer arriving in period two. Intuitively, period one prevents departures, but period two creates arrivals, and the seller needs to figure out which effect dominates the other.

Our first result is that for the optimal auction the seller always runs the auction in period two. In an optimal auction, the seller first calculates each bidder's marginal revenue, which is equal to the bidder's value for the house minus his information rent. The seller allocates the good to the bidder with the highest marginal revenue, and the seller's profit is equal to the maximum of the marginal revenues. The marginal revenues change from period one to period two, and the seller compares the maximal marginal revenue from each periods. The reason the seller always runs the auction is period two is due to the convexity of the max function: in period two there is a shock to the buyers' values, and the expected maximal marginal revenue is greater than the maximal of the marginal revenue from period one. This convexity argument is a useful tool in auction theory; for example, Bulow and Klemperer (1996) used this argument to show that a second-price auction with  $N + 1$  bidders generates more profit than an optimal auction with  $N$  bidders.

We also analyze the optimal deadline for a second-price auction. For a second-price auction with two bidders, we get the exact opposite result of the optimal auction case: the seller always runs the auction in period one. The logic is that the seller's revenue is the minimum of the two bidders' values, and since min is a concave function, the minimal expected bid from the second period is smaller. Simon Board (2009) also discovered this example in the context of revealing information in auctions. Note that for the optimal auction convexity of the max function suggests a longer deadline, but for the second-price auction with two bidders concavity of the min function implies a shorter deadline. However this two-bidder result is a knife-edge case both in our setting and in Board (2009). If there are more than two bidders, we find that the optimal deadline depends on the departure rate. The seller runs the auction in period one if the departure rate is high and in period two if the departure rate is low. For example, if the seller expects many other houses will appear on the market tomorrow, then she wants to run the auction today to lock in the existing bidders. The seller sets a shorter deadline if she expects fierce competition in the future.

Although we set up a model for optimal deadline of running an auction, the main driving force in our model is the information structure of the outside options, so we can alternatively interpret our model in terms of information disclosure in auctions. A shorter

deadline prevents bidders from acquiring information about their outside options, and a longer deadline allows the bidders to learn this information. Consequently our results on auction timing has natural analogs in the literature on revealing information in auctions. For example, for optimal auctions Milgrom and Weber (1992) and Eso and Szentes (2007) both argue for full information disclosure, which is analogous to a longer deadline in our setting. However our approach differs from the Linkage Principle in Milgrom and Weber (1982): in their model the increase in revenue is due to the decrease in information rent, but in our model the information rent could increase under a longer deadline. In fact we show in Example 3.3 that efficiency, information rent, and revenue could all increase. For the second-price auction Board (2009) studies no information disclosure for two bidders and full information disclosure for a sufficiently large number of bidders (under some regularity conditions). Bergemann and Pesendorfer (2007) argue for partial information disclosure in auctions, which could serve as a middle ground if we weaken the seller's commitment power in our model. We elaborate on the connections between our work and the information disclosure models in Section 4.1.

In Section 4 we discuss two extensions of our model. First we study the optimal dynamic mechanism. Our baseline model assumes that the seller commits to a specific date to run an auction, but in general the seller could use any dynamic mechanism. For example, she could set a high reserve price in period one, and if the house doesn't sell, she lowers her reserve price in period two. Or she could charge bidders a participation fee in each period, as a screening for serious bidders. The seller could also ask bidders to pay a deposit in period one and then let them search for outside options. It turns out that these tactics are not helpful, because the buyers would strategically respond to the seller's schemes. We show that the optimal dynamic mechanism is to do nothing in period one and run an optimal auction in period two. We also discuss an extension where the outside options are the buyers' private information. In this case the seller cannot calculate the marginal revenue from each bidder in period two, so she cannot run an optimal auction as before. However the seller can achieve the same profit as the optimal auction using the handicap auction introduced by Eso and Szentes (2007). The handicap auction first asks bidders to purchase from a menu of contracts similar to European call options and then screens the bidders based on the contracts they purchased.

Our paper is related to the literature on comparison of the selling mechanisms for houses. Quan (2002) and Chow Hafalir Yavas (2015) show that the optimal auction mechanism produces higher expected revenue than the sequential search by examining

the model with private values<sup>1</sup>. In this paper we show that the optimal auction with a longer deadline is a dynamic optimal mechanism for selling the property in the model with private as well as correlated values. Mayer (1995) argues that the auction produces lower prices relative to the negotiated sales because the negotiated sale allows to wait for a buyer with a high value. We show that the seller can optimally wait to auction the property which delivers higher price as compared to a quick auction sale as in Mayer (1995). Merlo, Ortalo-Magné, Rust (2014) consider the home seller’s problem, and show that the seller should set initial list price and over time adjust this price until the house is sold or withdrawn from the market. In this paper we add the strategic behavior of buyers and show that the dynamic optimal mechanism for the selling the house is to set a long deadline for auctioning a house.

Our work contributes to the study of designing deadlines. Empirical literature find ambiguous results on the effect of auction duration on revenue. Tanaka (2014) reports that a study by Redfin Realtors shows that houses that have deadlines not only sell faster, but also sell at higher prices. Similarly Larsen et al. (2016) find that for auto auctions the good auctioneers sell faster and generate more revenue. On the other hand, Einav et al. (2015) study online auctions on Ebay, and they find no difference in revenue between a one-day auction and a one-week auction. A large literature in bargaining studies the “eleventh hour” deadline effect (e.g. Fuch and Skrzypacz (2010, 2013)), and a large literature on optimal pricing studies the optimal selling strategy before a deadline (e.g. Board and Skrzypacz (2015), Lazear (1986), Riley and Zeckhauser (1983)). However the literature on bargaining and optimal pricing usually take deadlines as exogenous instead of the seller’s design. A recent paper by Chaves and Ichihashi (2016) also investigates the optimal timing of auctions, but they focus on the accumulation of bidders instead of a pre-determined deadline.

## 2 The Model

We consider a two-period model of housing selling. A risk-neutral seller has two periods to sell her house. In the first period  $N$  potential buyers arrive, and in the second period no new buyers arrive. Assume  $N \geq 2$  for the purpose of studying auctions, but most of our results still hold for a single buyer (seller just chooses a posted price). After buyers arrive in the first period, they independently draw their value  $v_i \sim F_i[\underline{v}_i, \bar{v}_i]$ ; note that the

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<sup>1</sup>Chow Hafalir Yavas (2015) argue that the revenue is higher in the auction of a homogenous properties during the hot markets, and when it attracts buyers with high values.

value distributions could be asymmetric. We focus on private-value auctions and abstract away from the common-value component. As in standard auction models, we assume that  $F_i$  has full support, and that  $v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$  is non-decreasing.

In the first period buyers have no outside option; outside options are normalized to zero for all buyers. In the second period the outside option for bidder  $i$  is a random variable with mean  $\lambda_i$ . In the first period buyers only know that the expected value of their future outside option is equal to  $\lambda_i$ . Then in the second period buyers observe their actual outside option  $\hat{\lambda}_i = \lambda_i + \epsilon_i$ , where  $\epsilon_i$  has mean 0. Assume that  $\epsilon_i$  is common knowledge in the second period. Moreover assume that  $\mathbb{E}[\epsilon_i|v_1, \dots, v_N] = 0$  for all  $v$ , but the  $\epsilon$ 's could be correlated with each other, as long as their expected values conditional on  $v$  is equal to 0. We interpret this change in outside option as follows: buyers know that in the next period other houses might appear on the market, but they do not know exactly how good these houses are.

Though we assume that no new buyers arrive in the second period, we could interpret arrivals and departures through the change in buyers' outside options. Indeed in the first period buyer  $i$ 's value for the house is equal to  $v_i - \lambda_i$ , but in the second period his value becomes  $v_i - \lambda_i - \epsilon_i$ . Arrival means the buyer expects a high outside option ( $\lambda_i$  is large), but ends up with a terrible outside option in the second period ( $\epsilon_i$  is negative). Departure means a buyer gets a great outside option in the second period ( $\epsilon$  is positive and large) and therefore is no longer interested in bidding for this house.

The seller commits to a period to run an auction. She either runs an auction in period 1 or period 2. We interpret period 1 as a shorter deadline and period 2 as a longer deadline. Running the auction in period 1 is equivalent to treating buyers' values as  $v_i - \lambda_i$ , whereas a period 2 auction treats buyers' values as  $v_i - \lambda_i - \epsilon_i$ . For example, suppose the seller chooses a period to run a (static) optimal auction. If she runs the auction in period 1, she treats buyer  $i$ 's marginal revenue as

$$MR_{1i}(v_i) = v_i - \lambda_i - \frac{1 - F_i(v_i)}{f_i(v_i)},$$

and if she runs the auction in period 2, she treats buyer  $i$ ' marginal revenue as

$$MR_{2i}(v_i, \epsilon_i) = v_i - (\lambda_i + \epsilon_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} = MR_{1i}(v_i) - \epsilon_i.$$

In an optimal auction the seller allocates the good to the bidder with the highest marginal revenue, so allocation could be different in period 1 and period 2. The bidder with the

highest marginal revenue in period 1 might have a low marginal revenue in period 2 if  $\epsilon_i$  is positive and large.

In Section 3 we analyze the seller’s optimal timing for two auction formats: the optimal auction and the second-price auction. In both cases the allocation is different in the two periods. The trade-off between these two periods is between “arrivals” and “departures”: running the auction in period 1 prevents buyers from searching for outside options, but if a buyer gets a bad outside option in period 2 (i.e.  $\epsilon_i$  is negative), he would bid more on the house. We show that for the optimal auction the seller always chooses period 2, but for the second-price auction the seller might choose period 1.

We make two qualifications about our model. First we assume that the seller commits to one period to run an auction. In general the seller could be using any dynamic mechanism. For example, the seller could set a high reserve price in period 1, and lower the reserve price in period 2 if the house didn’t sell. We show in Section 4.2 that in fact the optimal dynamic mechanism is to run an optimal auction in period 2. We also assume that  $\epsilon$  is common knowledge; that is, the seller can observe the buyers’ outside options. One might object to this assumption because a buyer’s outside option depends on his taste, which could be private information. We show in Section 4.3 that the seller can achieve the same profit even if she cannot observe  $\epsilon$ .

### 3 Optimal Timing

In this section we assume the seller commits to a period to run an auction. We discuss the optimal timing for two auction formats: the optimal auction and the second-price auction. For each auction format, we compare the seller’s revenue from running the auction in period 1 versus running the auction in period 2. We also discuss the change in efficiency and information rent over the two periods.

#### 3.1 Optimal auction

In an optimal auction the seller first calculates the marginal revenue of each bidder, which is equal to the bidder’s value minus his information rent. If the marginal revenue of every bidder is negative, then the seller retains the good. Otherwise she allocates the good to the bidder with the highest marginal revenue. More precisely, in period 1 bidder  $i$ ’s

marginal revenue is equal to

$$MR_{1i}(v_i) = v_i - \lambda_i - \frac{1 - F_i(v_i)}{f_i(v_i)},$$

and in period 2 bidder  $i$ ' marginal revenue is equal to

$$MR_{2i}(v_i, \epsilon_i) = v_i - (\lambda_i + \epsilon_i) - \frac{1 - F_i(v_i)}{f_i(v_i)} = MR_{1i}(v_i) - \epsilon_i.$$

If the seller runs an optimal auction in period 1, she allocates the good to the bidder with for whom  $MR_{1i}(v_i)$  is the highest (if it is positive). If the seller runs an optimal auction in period 2, then she allocates the good to the bidder for whom  $MR_{2i}(v_i, \epsilon_i)$  is the highest (conditional on the MR being positive). For either period the seller's revenue is equal to the expected maximum of the marginal revenue and zero.

Our first result is that the seller should always wait until period 2 to run the auction.

**Theorem 3.1.** *If the seller runs an optimal auction, she should wait until period 2.*

*Proof.* The seller's revenue in the first period is equal to

$$R_1 = \mathbb{E}_v \max\{MR_{11}(v_1), \dots, MR_{1N}(v_N), 0\},$$

and the seller's revenue in the second period is equal to

$$\begin{aligned} R_2 &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{21}(v_1, \epsilon_1), \dots, MR_{2N}(v_N, \epsilon_N), 0\} \\ &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{11}(v_1) - \epsilon_1, \dots, MR_{1N}(v_N) - \epsilon_N, 0\}. \end{aligned}$$

Since  $\mathbb{E}[\epsilon|v] = 0$  and  $\max$  is convex, Jensen's inequality implies that  $R_2 \geq R_1$ . Hence the seller runs the auction in the second period.  $\square$

*Remark.* Bulow and Klemperer (1996) used the same convexity argument to show that a second-price auction with  $N + 1$  bidders generates more profit than an optimal auction with  $N$  bidders.

In Theorem 3.1 the seller is not committed to sell the good. Now suppose that the seller commits to selling the good; for example, she is moving to a new city and must sell her house. The convexity argument for waiting remains valid.

**Proposition 3.2.** *If the seller runs an optimal auction, but is committed to sell the good, then she should still wait until period 2.*

*Proof.* The seller’s revenue in the first period is now equal to

$$R_1 = \mathbb{E}_v \max\{MR_{11}(v_1), \dots, MR_{1N}(v_N)\},$$

(note we dropped the 0 from the max), and the seller’s revenue in the second period becomes

$$\begin{aligned} R_2 &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{21}(v_1, \epsilon_1), \dots, MR_{2N}(v_N, \epsilon_N)\} \\ &= \mathbb{E}_\epsilon \mathbb{E}_v \max\{MR_{11}(v_1) - \epsilon_1, \dots, MR_{1N}(v_N) - \epsilon_N\}. \end{aligned}$$

Since  $\mathbb{E}[\epsilon|v] = 0$  and max is convex, we again obtain that  $REV_2 \geq REV_1$ . Hence the seller runs the auction in the second period.  $\square$

We could interpret waiting as revealing information in auctions. Waiting until period 2 allows bidder  $i$  to acquire information about his outside option and thus learn his “true value”  $v_i - \widehat{\lambda}_i$ . Running an auction in period 1, on the other hand, prevents the bidders from learning their outside options. Hence Theorem 3.1 and Proposition 3.2 are analogous to full information disclosure. We discuss the connection of our model to the relevant literature on information disclosure in Section 4.1.

We have thus far demonstrated that revenue increases if the seller waits until period 2 to run an optimal auction. Could we obtain similar results for efficiency and information rent? Unfortunately we cannot derive an analog of Theorem 3.1 because the (marginal) efficiency and information rent are neither convex nor concave (see Section A in the appendix). Information disclosure models (e.g. Milgrom and Weber (1982)) often suggest that revenue increases because information rent decreases. In our model, however, information rent could increase in period 2, in which case efficiency increases even more. We next illustrate this point through an example.

*Example 3.3.* We give an example in which efficiency, information rent, and revenue all increase in the second period. Suppose all bidders draw their value from  $U[0, 1]$  and  $\lambda_i = \lambda$  for all  $i$ . In the second period each bidder’s outside option is either 0 or 1, with probability  $\lambda$  of getting 1. In other words with probability  $\lambda$  a bidder finds a great outside option and leaves the market.

In period 1 the optimal auction is a second-price auction with reserve price  $\frac{1+\lambda}{2}$ .

Efficiency, information rent, and revenue are as follows:

$$\begin{aligned}
E_1(\lambda) &= N\lambda + N \int_{\frac{1+\lambda}{2}}^1 v^{N-1}(v-\lambda)dv = (N-1)\lambda + \frac{N}{N+1} \left(1 - \frac{(1+\lambda)^{N+1}}{2^{N+1}}\right) + \lambda \frac{(1+\lambda)^N}{2^N} \\
I_1(\lambda) &= N\lambda + N \int_{\frac{1+\lambda}{2}}^1 v^{N-1}(1-v)dv = N\lambda + \frac{1}{N+1} - \frac{(1+\lambda)^N}{2^N} + \frac{N(1+\lambda)^{N+1}}{(N+1)2^{N+1}} \\
R_1(\lambda) &= E_1(\lambda) - I_1(\lambda) = \frac{N-1}{N+1} - \lambda + \frac{(1+\lambda)^{N+1}}{(N+1)2^N}
\end{aligned}$$

In period 2 some bidders leave the auction. The probability that exactly  $n$  bidders remain is equal to  $\binom{N}{n}\lambda^{N-n}(1-\lambda)^n$ . For these remaining  $n$  bidders, each one has outside option 0, so in the second period the efficiency, information rent, and revenue are as follows:

$$\begin{aligned}
E_2(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ N - n + \frac{n}{n+1} \left(1 - \frac{1}{2^{n+1}}\right) \right] \\
I_2(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ N - n + \left( \frac{1}{n+1} - \frac{1}{2^n} + \frac{n}{(n+1)2^{n+1}} \right) \right] \\
R_2(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ \frac{n-1}{n+1} + \frac{1}{(n+1)2^n} \right]
\end{aligned}$$

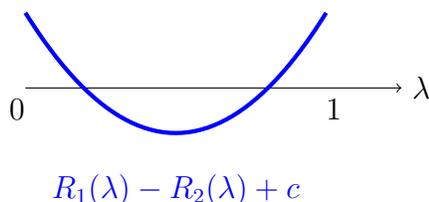
We now compare the efficiency, information rent, and revenue between the two periods. To simplify calculation, we note that the optimal auction in period 1 is equivalent to the following auction: bidders have values in  $[-\lambda, 1-\lambda]$  and outside option 0, and the seller runs a second-price auction with reserve price  $\frac{1-\lambda}{2}$ . We can rewrite efficiency, information rent, and revenue as below:

$$\begin{aligned}
E_1(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ N - n + (1-\lambda) \cdot \frac{n}{n+1} \left(1 - \frac{1}{2^{n+1}}\right) \right] \\
I_1(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ N - n + (1-\lambda) \cdot \left( \frac{1}{n+1} - \frac{1}{2^n} + \frac{n}{(n+1)2^{n+1}} \right) \right] \\
R_1(\lambda) &= \sum_{n=0}^N \binom{N}{n} \lambda^{N-n}(1-\lambda)^n \cdot \left[ (1-\lambda) \cdot \left( \frac{n-1}{n+1} + \frac{1}{(n+1)2^n} \right) \right]
\end{aligned}$$

Now it's easy to see that  $E_1(\lambda) < E_2(\lambda)$ ,  $I_1(\lambda) < I_2(\lambda)$ , and  $R_1(\lambda) < R_2(\lambda)$ . Indeed for each quantity the coefficients of the summands are the same, and the only differences are the expression in the brackets. In period 2 the  $(1-\lambda)$  term in the bracket becomes 1.

Therefore efficiency, information rent, and revenue all increase from the first period to the second period. Since revenue is equal to efficiency minus the information rent, we deduce that efficiency increases by an amount larger than the increase in information rent.

We end this section with a note on waiting cost. We have so far ignore waiting cost or discounting in order to present Theorem 3.1 in the most clean manner. In reality however the seller has to incur a large waiting cost; she has to pay a fee to her realtor and endure psychological stress. Suppose the seller has to incur a cost of  $c$  if she waits until period 2. Then in Example 3.3 she would sell in period 1 if  $R_1(\lambda) - (R_2(\lambda) - c) \geq 0$ . This scenario happens if  $\lambda$  is close to 0 or close to 1. The curve  $R_1(\lambda) - (R_2(\lambda) - c)$  is u-shaped as shown below:



If  $c = 0$ , then Theorem 3.1 implies that the blue curve is always below the  $x$ -axis, and the seller always waits until period 2. If  $c > 0$ , then the blue curve would be above the  $x$ -axis when  $\lambda$  is close to 0 or 1. If  $\lambda$  is close to 0, then buyers have a low exit rate, and they shade their bids very slightly in period 1. If  $\lambda$  is close to 1, many buyers will leave the auction (i.e. other houses will appear); the seller will face high competition tomorrow, so she sells today.

## 3.2 Second-price auction

An optimal auction requires the seller to know the bidders' outside options  $\hat{\lambda}$  and calculate the information rent for each bidder. In reality these information may not be readily available to the seller, so a more realistic approach would be a detail-free mechanism like the second-price auction.

In this section we assume the seller chooses a period to run a second-price auction (with no reserve price). In contrast to Theorem 3.1 in a second-price auction the seller might want to run in the auction in the first period. We first consider a simple example with two bidders, adopted from Simon Board's paper [5], in which the auction always takes place in period 1. Then we analyze an second-price auction with  $N$  bidders, in which the seller runs the auction in period 1 if she faces high competition in period 2.

Our first observation is that if there are two bidders, then we get the exact opposite result of Theorem 3.1: the seller always runs a second-price auction in period 1. Simon Board (2009) also discovered this example in the context of revealing information in second-price auctions.

**Proposition 3.4.** *(From Board (2009)) In a second-price auction with two bidders, seller should always run the auction in period 1.*

*Proof.* The expected profit from first period is equal to

$$REV_1 = \mathbb{E}_v \min\{v_1 - \lambda_1, v_2 - \lambda_2\},$$

and the expected profit from second period is equal to

$$REV_2 = \mathbb{E}_\epsilon \mathbb{E}_v \min\{v_1 - \lambda_1 - \epsilon_1, v_2 - \lambda_2 - \epsilon_2\}.$$

Since min is concave, Jensen's Inequality implies that  $REV_1 \geq REV_2$ , so the seller should run the auction in the first period.  $\square$

At first glance Proposition 3.4 seems to contradict Proposition 3.2. Indeed, if the value distributions are symmetric, and the seller is committed to selling the good, then the optimal auction from Proposition 3.2 becomes a second-price auction. To highlight this apparent contradiction, consider the symmetric case when  $F_1 = F_2$ . Then the optimal auction is a second-price auction, and the Revenue Equivalence Theorem implies that

$$\mathbb{E}_v \min\{v_1 - \lambda_1, v_2 - \lambda_2\} = \mathbb{E}_v \max\{MR_{11}(v_1), MR_{12}(v_2)\}.$$

Hence in period 1 the revenue from Proposition 3.4 and Proposition 3.2 are exactly the same. Why should the seller run a second-price auction in period 1, but wait until period 2 to run an optimal auction? The difference occurs in period 2. The period 2 auctions in Proposition 3.2 are different from a second-price auction. In period 2, outside options make bidders' distributions asymmetric; even if all  $F_i$  are identical, the  $\epsilon_i$  are different. Under asymmetric distributions, in a second-price auction the seller may not allocate the good to the bidder with the highest marginal revenue. As a result a second-price auction yields less profit than an optimal auction in period 2, and the seller wants to run a second-price auction in period 1.

We now consider the case with more than two bidders. Then the seller might run the auction in either period 1 or period 2, depending on the distribution of buyers' outside

options. Similar to the intuition for an optimal auction with waiting cost, if the seller expects many buyers to find great outside option in period 2, then she will run the auction in period 1. Moreover information rent could increase in the second period. We illustrate these facts through an example.

*Example 3.5.* We illustrate how the seller's optimal timing depends on the buyers' departure rate. Suppose all buyers draw their value from  $U[0, 1]$ . All buyers have an expected future outside option equal to  $\lambda_i = \lambda$  for all  $i$ . Moreover in the second period outside options are either 0 or 1. Unlike in Example 3.3, we now assume that buyers' outside options are correlated. In particular, exactly a fraction  $\lambda$  of the buyers (selected at random) find an outside option equal 1, so in the second period  $\lambda N$  buyers leave the auction. The remaining  $(1 - \lambda)N$  buyers have outside option 0. In this example  $\lambda$  represents the buyers' departure rate: large  $\lambda$  corresponds to a high departure rate, and low  $\lambda$  corresponds to a low departure rate.

In the first period the efficiency, information rent, and revenue are as follows:

$$\begin{aligned} E_1(\lambda) &= N \int_{\lambda}^1 v^{N-1}(v - \lambda)dv = \frac{N}{N+1} + \frac{\lambda^{N+1}}{N+1} - \lambda \\ I_1(\lambda) &= N \int_{\lambda}^1 v^{N-1}(1 - v)dv = \frac{1}{N+1} - \lambda^N + \frac{N}{N+1}\lambda^{N+1} \\ R_1(\lambda) &= E_1(\lambda) - I_1(\lambda) = \frac{N-1}{N+1}(1 - \lambda^{N+1}) - \lambda + \lambda^N \end{aligned}$$

In the second period the efficiency, information rent, and revenue become

$$\begin{aligned} E_2(\lambda) &= \frac{(1 - \lambda)N}{(1 - \lambda)N + 1} \\ I_2(\lambda) &= \frac{1}{(1 - \lambda)N + 1} \\ R_2(\lambda) &= \frac{(1 - \lambda)N - 1}{(1 - \lambda)N + 1} \end{aligned}$$

We can easily check that  $E_1(\lambda) - E_2(\lambda) \leq 0$  and  $I_1(\lambda) - I_2(\lambda) \leq 0$  for all  $\lambda \in [0, 1]$ , which means efficiency and information rent both increase in the second period. For the change in revenue we have  $R_1(\lambda) - R_2(\lambda)$  is convex in  $\lambda$ . Moreover we know that  $R_1(0) - R_2(0) = 0$  and  $R_1(1) - R_2(1) > 0$ , so there exists a  $\lambda^*$  such that  $R_1(\lambda) - R_2(\lambda) < 0$  for all  $\lambda < \lambda^*$ , and  $R_1(\lambda) - R_2(\lambda) > 0$  for all  $\lambda > \lambda^*$ . Therefore revenue increases for small values of  $\lambda$ , but decreases for large values of  $\lambda$ . Figure 1 illustrates the change in efficiency, information rent, and revenue.

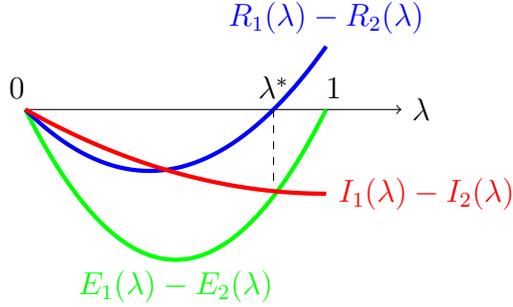


Figure 1: Changes in efficiency, information rent, and revenue

Figure 1 shows that both efficiency and information rent increase in the second period. On the other hand revenue could either increase or decrease. If  $\lambda < \lambda^*$ , then revenue increases, so the seller waits until period 2. If  $\lambda > \lambda^*$ , then revenue decreases, and the seller runs the auction in period 1. Intuitively, if the departure rate is high, the seller is facing a lot of competition in the future. Many houses will appear on the market, and existing bidders will leave, so the seller prefers to run the auction sooner. Hence when  $\lambda$  is close 1, the departure rate is high, and the seller should run the auction in period 1; otherwise she should wait until period 2.

## 4 Discussion

### 4.1 Information disclosure

We set up our model in terms of optimal timing: period 1 is a shorter deadline, and period 2 is a longer deadline. However our model concerns little about the time structure and focuses more on the information structure. Between the two periods, the only change is the bidders' outside options, so we can reinterpret our model as an information disclosure problem. The seller decides whether to allow the bidders to acquire more information about their outside options. A shorter deadline corresponds to no information disclosure, and a longer deadline corresponds to full information disclosure.

We can reformulate our results in Section 3 in the language of information disclosure. Theorem 3.1 and Proposition 3.2 state that in an optimal auction the seller should fully reveal all the information, while Proposition 3.4 says that in a second-price auction with two bidders the seller should reveal no information. Example 3.5 on the other hand suggests that in general a seller might reveal no information in a second-price auction if

the signals have a large variance.

We now discuss how our results connects to the relevant literature on revealing information in auctions.

#### **4.1.1 Milgrom and Weber (1982)**

Milgrom and Weber (1982) is one of the seminal papers on revealing information in auctions. They proposed a Linkage Principle, which says the auctioneer should always reveal all her information to the bidders. Our setting differs from the Linkage Principle in two ways. First outside options make distributions asymmetric, and second the allocation changes from the first period to the second period. Moreover in Milgrom and Weber (1982), revealing information decreases the information rent, but in our case the information rent could go up (and efficiency goes up even more).

Consider a simple example. Suppose there are two bidders. In period 1, the high bidder submits 100, and the low bidder submits 50. Milgrom and Weber (1982) would say that in period 2, the high bidder might submit 80 and the low bidder 70. Waiting until period 2 brings the bids closer and thereby raises the second price. In our setting, however, in period 2 the high bidder might find a great outside option and bid lower than the low bidder, so the allocation could change.

#### **4.1.2 Board (2009)**

Board (2009) studies revealing information in second-price auctions. He showed that for two bidders the seller reveals no information (same as Proposition 3.4), but for a sufficiently large number of bidders the seller should always reveal information (under some regularity conditions). We also find that the two-bidder case presents a knife-edge result in which the seller always runs the auction in period 1. In general the seller waits until period 2 unless  $\lambda$  is sufficiently high, which means the outside options have a large variance.

The result of “never waits” for two bidders stands in contrasts with the result of “always waits” for the optimal auction (even if there are two bidders). Although for symmetric bidders the optimal auction is equivalent to a second-price auction, in period 2 the bidders get different outside options and therefore become asymmetric. As aforementioned in Section 3.2 this asymmetry in period 2 differentiates the “never waits” result for second-price auction with two bidders with the “always waits” result for the optimal auction.

### 4.1.3 Bergemann and Pesendorfer (2007); Eso and Szentes (2007)

In Bergemann and Pesendorfer (2007) the bidders cannot observe their own values, and the seller reveals signals for bidders to learn their value. They showed that the optimal signal structure is a partition of  $[\underline{v}_i, \bar{v}_i]$  for each bidder  $i$ , and bidder  $i$  can observe which interval of the partition his value falls in, but cannot observe his exact value. In particular, if there is only one bidder, the seller reveals no information: the partition is just the whole interval. In our language, Bergemann and Pesendorfer (2007) suggest that if there is only one bidder, the seller would use a posted price  $v - \lambda$  in period 1. In contrast Theorem 3.1 says the seller waits until period 2 and proposes  $\max\{v - \lambda - \epsilon, 0\}$  instead of  $v - \lambda$ .

Eso and Szentes (2007) study a situation similar to our setting. The bidders first draw their raw value  $v_i$ , but their actual value also depends on another parameter  $\epsilon$ , which the seller could choose to release. In their model revealing  $\epsilon$  is the optimal strategy for the seller. In our setting the outside option  $\epsilon$  is common knowledge, whereas in their setting the signal  $\epsilon$  is unobservable to the seller. In the case when outside options are the bidders' private information, the seller could run a handicap auction proposed by Eso and Szentes (2007), which we will further discuss in Section 4.3.

We now present an example with one bidder to highlight the connections between Theorem 3.1, Bergemann and Pesendorfer (2007), and Eso and Szentes (2007).

*Example 4.1.* There is one bidder. His value  $v$  is drawn from  $U[-1, 1]$ . In period 1 neither the seller nor the buyer knows  $v$ . In period 2 both the seller and the buyer observe  $v$ . There is no outside option in either periods. (This set-up is equivalent to saying the buyer has value 0 and an outside option from  $U[-1, 1]$ .) What's the maximal profit the seller can extract?

Theorem 3.1 states the seller should do nothing in period 1 and post a price  $v$  in period 2. Indeed, in period 1, the seller can only post price 0, so her expected profit is 0. In period 2 the seller posts price  $v$  and earns an expected profit of  $\int_0^1 \frac{1}{2}v \cdot dv = \frac{1}{4}$ .

Bergemann and Pesendorfer (2007) would say the seller gets 0. The seller immediately sells in period 1, so the best she can do is to post a price equal to the expected value of  $v$ , which is 0.

Eso and Szentes (2007) would propose the following mechanism. In period 1 the seller sells a European call option at price  $\frac{1}{4}$ , and in period 2 the buyer can purchase the object at a strike price 0. The bidder is willing to buy this call option because his expected payoff in period 2 is equal to  $\int_0^1 \frac{1}{2}v \cdot dv = \frac{1}{4}$ . In period 1 he is willing to pay up to  $\frac{1}{4}$  for this call option with strike price 0. Notice the price in period 2 is still 0, but unlike in Bergemann

and Pesendorfer (2007), the seller now takes advantage of the bidder's uncertainty in period 1. Notice that for Eso and Szentes (2007) the seller achieve the same profit as Theorem 3.1, but their mechanism does not require the seller to know buyers' values in period 2. In Section 4.3 we show how their mechanism works for multiple bidders.

## 4.2 Optimal dynamic mechanism

Theorem 3.1 assumes the seller chooses a specific period to run an auction. More generally, the seller could use any dynamic mechanism. For example, she could set a high reserve price in period 1, and if no one submits a bid, she lowers the reserve price to period 2. It turns out that such a tactic is not helpful, because the bidders would strategically wait. The optimal dynamic mechanism is to do nothing in period 1 and run an optimal auction in period 2.

We define a dynamic mechanism as follows. There are  $N$  bidders. In period 1, bidder  $i$  privately observes his value  $v_i$  and chooses whether to report  $v_i$ . If he does not report his value in period 1, then he must report his value in period 2.

As before, outside option  $\widehat{\lambda}_i$  is realized in period 2. In period 1 bidder  $i$  only knows that that  $\widehat{\lambda}_i$  has mean  $\lambda_i$ , and in period 2 he observes  $\widehat{\lambda}_i = \lambda_i + \epsilon_i$  and reports  $\epsilon_i$ . The individual rationality (IR) constraint must be satisfied for both periods. The IR constraints imply that the mechanism must guarantee bidder  $i$  at least  $\lambda_i$  in period 1 (if he makes a report) and at least  $\lambda_i + \epsilon_i$  in period 2.

The seller can allocate the good and make transfers in both period 1 and period 2. Suppose bidders  $i_1, \dots, i_k$  report their values in period 1. A mechanism consists of the following four functions for each bidder  $i$ :

- $X_{1i}(v_{i_1}, \dots, v_{i_k})$ : allocation in period 1 based on reported values
- $T_{1i}(v_{i_1}, \dots, v_{i_k})$ : transfer in period 1 based on reported values
- $X_{2i}(v_1, \dots, v_N; \epsilon_1, \dots, \epsilon_N)$ : allocation in period 2
- $T_{2i}(v_1, \dots, v_N; \epsilon_1, \dots, \epsilon_N)$ : transfer in period 2.

The mechanism must satisfy IR and IC whenever the bidders make a report. In particular, if a bidder makes a report in period 1, then his IC constraint must take into account of his period 1 payoff as well as his expected payoff in period 2.

**Theorem 4.2.** *The optimal dynamic mechanism is to make no allocation in period 1 and run an optimal auction in period 2.*

We defer the proof to the appendix. Here is the intuition. In equilibrium all bidders report their value in period 1; otherwise we can assume they report in period 1, but the seller ignored this information. We can also assume that the seller makes transfers at the end of period 2 because both the seller and the bidders are risk-neutral. Buyers announce their types in period 1, and all transfers are made in period 2, so the only dynamic nature of this problem is that the seller could potentially allocate the good in period 1. We basically have to prove that the seller always allocates the good in period 2. If there is only one bidder, then Eso and Szentes (2015) imply that the dynamic nature of this problem is irrelevant. In our setting their “irrelevance result” extends to multiple bidders. The seller allocates the good and makes transfers all in period 2.

### 4.3 Seller cannot observe $\epsilon$ .

So far we have assumed that the seller can observe the bidders’ outside options:  $\epsilon$  is common knowledge in period 2. In reality outside options could be the bidders’ private information, so the second period auction generates less profit than our model predicts, and therefore waiting may not be optimal as Theorem 3.1 suggests.

In this section we show that even if the seller doesn’t know  $\epsilon$ , she can achieve the same profit as the optimal auction in Theorem 3.1, as long as  $(1 - F_i)/f_i$  is decreasing for all  $i$ . The seller can use a handicap auction introduced by Eso and Szentes (2007).

If there is only one bidder, the handicap auction is equivalent to a European call auction. For an intuitive explanation of how this European call auction could extract all the surplus, see Example 4.1. In general the handicap auction goes in three steps:

1. In period 1, bidder  $i$  reports  $v_i$  and pays  $c_i(v_i)$ .
2. In period 2, the seller runs a second-price auction with no research price.
3. Winner of the period 2 auction pays an additional premium equal to  $\frac{1-F_i(v_i)}{f_i(v_i)}$ .

In period 1 the seller has to design a payment rule  $c_i$ . In the case of one bidder,  $c_i$  is the price of the call option. In period 2 the allocation is the same as before; the mechanism allocates the good to the highest marginal revenue bidder as in Theorem 3.1). Indeed, in period 2 bidder  $i$ ’s value is  $v_i - \widehat{\lambda}_i$ , but he has to pay an additional premium of  $\frac{1-F_i(v_i)}{f_i(v_i)}$  in case he wins. As a result bidder  $i$ ’s adjusted value is  $v_i - \widehat{\lambda}_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ , which is equal to his marginal revenue. He will not bid more than his marginal revenue. If he wins the auction, his payoff is equal to  $v_i - \widehat{\lambda}_i - \frac{1-F_i(v_i)}{f_i(v_i)}$  minus the second highest bid. If he loses

the auction, he gets 0. Therefore the handicap auction allocates the good to the bidder with the highest marginal revenue.

We are left to solve for an incentive compatible  $c_i$  and check the bidders have the same payoffs as before:

$$\pi_i(v_i) = \max_{v'_i} \mathbb{E}_{\hat{\lambda}} \mathbb{E}_{v_{-i}} \max\{v_i - \hat{\lambda}_i - \frac{1 - F_i(v'_i)}{f_i(v'_i)} - \text{2nd-price}, 0\} - c_i(v'_i).$$

The 2nd-price does not depend on  $v_i$  or  $v'_i$ , so the single crossing condition is equivalent to  $(1 - F_i)/f_i$  is decreasing. Envelope Theorem implies that

$$\pi_i(v_i) = \mathbb{E}_{\hat{\lambda}} \mathbb{E}_{v_{-i}} \int_0^{v_i} \mathbf{1} \left( x - \hat{\lambda}_i - \frac{1 - F_i(x)}{f_i(x)} - \text{2nd-price} \geq 0 \right) dx,$$

which is the same as in the optimal auction in period 2. Indeed, in the optimal auction, bidder  $i$ 's payoff is also given by the integral envelope formula above. Hence the seller can achieve the same profit even if she doesn't now the outside options.

Note: we have

$$c_i(v_i) = \mathbb{E}_{\hat{\lambda}} \mathbb{E}_{v_{-i}} \max\{v_i - \hat{\lambda}_i - \frac{1 - F_i(v_i)}{f_i(v_i)} - \text{2nd-price}, 0\} - \mathbb{E}_{\hat{\lambda}} \mathbb{E}_{v_{-i}} \int_0^{v_i} \mathbf{1} \left( x - \hat{\lambda}_i - \frac{1 - F_i(x)}{f_i(x)} - \text{2nd-price} \geq 0 \right) dx,$$

where 2nd-price is equal to  $\max\{v_{-i} - \hat{\lambda}_{-i} - \frac{1 - F_{-i}(v_{-i})}{f_{-i}(v_{-i})}, 0\}$ . These calculations follow from Proposition 2 in Eso and Szentes (2007). Essentially the seller uses  $c_i$  to screen the buyers' valuations in period 1. Since in period 1 the buyers do not know  $\hat{\lambda}$ , they cannot extract any information rent from  $\hat{\lambda}$ . Therefore the seller can achieve the same profit as in Theorem 3.1 even if she cannot observe the buyer's outside options.

## 5 Conclusion

We analyzed the optimal choice of an auction deadline through a two-period model. We found that for the optimal auctions the seller always sets a longer deadline, but for the second-price auction the seller might choose a shorter deadline if she expects high departure rate (i.e. a fierce competition) in the future. Moreover we showed that we can without loss of generality assume the seller commits a date to run the auction; the optimal dynamic mechanism is to set a longer deadline and run the optimal auction in

the last period. Our results have many analogs in the literature on information disclosure in auctions, which suggests there is potentially a connection between optimal timing and optimal information structure.

# A Efficiency and Information Rent

In Theorem 3.1 we used a convexity argument to prove that revenue increases in the second period. Can we apply the same argument to study efficiency and information rent? Unfortunately the answer is no. The highest marginal revenue is a convex function, but same property fails for efficiency and information rent. They are neither convex nor concave, so we can't conclude either increase or decrease.

Consider a simple example with only two bidders. Figure 2 illustrates the marginal efficiency (ME), marginal information rent (MI), and the marginal revenue (MR) for an optimal auction. The horizontal axis is the first bidder's outside option  $\hat{\lambda}_1$ , and the vertical axis the second bidder's outside option  $\hat{\lambda}_2$ . We fix  $(v_1, v_2)$  and calculate the ME, MI, and MR for each  $(\hat{\lambda}_1, \hat{\lambda}_2)$ . The solid lines partition the first quadrant into three regions: bidder 1 gets the good; bidder 2 gets the good, and neither gets the good. We see that MR is convex, but ME and MI are neither convex nor concave. As a result we cannot obtain the analogs of Theorem 3.1 for efficiency and information rent.

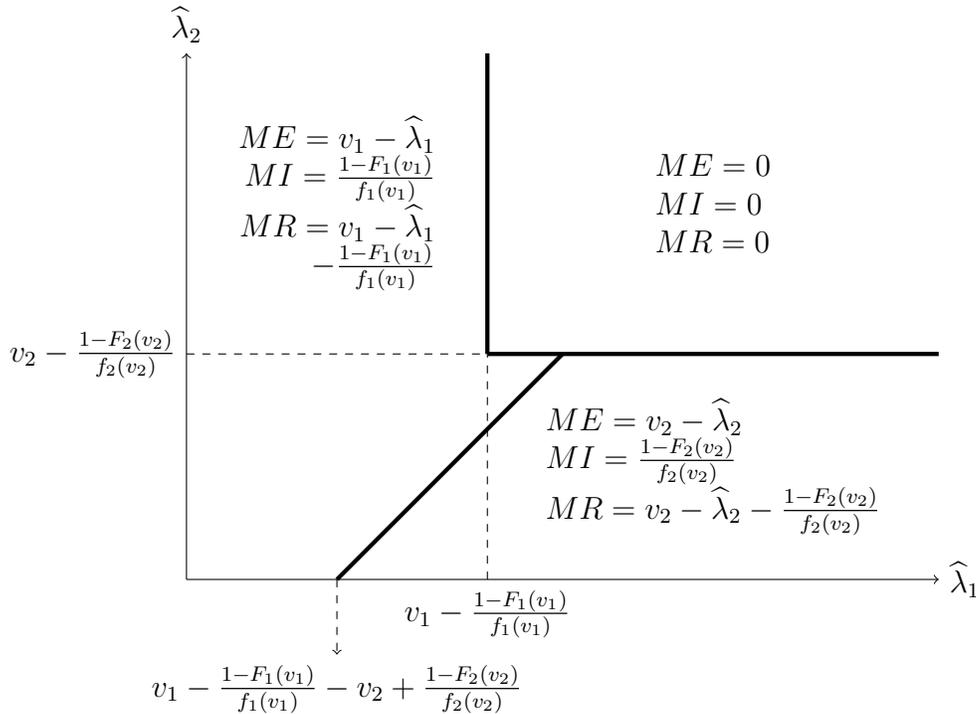


Figure 2: Optimal auction: (marginal) efficiency, information rent, and revenue

## B Proof of Theorem 4.2

In equilibrium, all bidders report their values in period 1. Suppose bidder  $i$  reports in period 2. In equilibrium she must report her true value in period 2 for all realizations of  $\epsilon$ , so we can assume she reports in period 1, but the seller didn't use that information in period 1.

Now we can simplify the mechanism as follows. Let  $v = (v_1, \dots, v_N)$  and  $\epsilon = (\epsilon_1, \dots, \epsilon_N)$ . A mechanism consists of four functions for each bidder  $i$ :

$$X_{1i}(v), T_{1i}(v), X_{2i}(v, \epsilon), T_{2i}(v, \epsilon).$$

Since there is only one object, the allocation rule must satisfy

$$\sum_{i=1}^N X_{1i}(v) + \sum_{i=1}^N X_{2i}(v, \epsilon) \leq 1 \quad \forall \epsilon. \quad (\text{B.1})$$

Let  $P_{1i}(v_i) = \int_{v_{-i}} X_{1i}(v_i, v_{-i}) \cdot dv_{-i}$  denote bidder  $i$ 's chance of winning the object in period 1. Let  $T_{1i}(v_i) = \int_{v_{-i}} T_{1i}(v_i, v_{-i}) \cdot dv_{-i}$  denote bidder  $i$ 's expected transfer in period 1. Let  $P_{2i}(v_i, \epsilon) = \int_{v_{-i}} X_{2i}(v_i, v_{-i}; \epsilon) \cdot dv_{-i}$  denote bidder  $i$ 's chance of winning the object in period 2. Let  $T_{2i}(v_i, \epsilon) = \int_{v_{-i}} T_{2i}(v_i, v_{-i}; \epsilon) \cdot dv_{-i}$  denote bidder  $i$ 's expected transfer in period 2.

The incentive constraint is as follows:

$$S(v_i) = \max_{v'_i} [P_{1i}(v'_i) \cdot v_i - T_{1i}(v'_i)] + \mathbb{E}_\epsilon [P_{2i}(v'_i, \epsilon) \cdot v_i - T_{2i}(v'_i, \epsilon) + (1 - P_{1i}(v'_i) - P_{2i}(v'_i, \epsilon)) \cdot (\lambda_i + \epsilon_i)].$$

IR must hold for each period:

$$\begin{aligned} P_{1i}(v_i) \cdot v_i - T_{1i}(v_i) &\geq \lambda_i \\ P_{2i}(v_i, \epsilon) \cdot v_i - T_{2i}(v_i, \epsilon) &\geq \lambda_i + \epsilon_i \quad \forall \epsilon. \end{aligned}$$

The envelope formula implies that

$$\begin{aligned} S_i(v_i) &= S_i(\underline{v}_i) + \int_{\underline{v}_i}^v P_{1i}(x) \cdot dx + \mathbb{E}_\epsilon \int_{\underline{v}_i}^v P_{2i}(x, \epsilon) \cdot dx \\ &= \lambda_i + \int_{\underline{v}_i}^v P_{1i}(x) \cdot dx + \mathbb{E}_\epsilon \int_{\underline{v}_i}^v P_{2i}(x, \epsilon) \cdot dx. \end{aligned}$$

The seller's profit from type  $v_i$  is equal to

$$\begin{aligned}
\pi_i(v_i) &= T_{1i}(v_i) + \mathbb{E}_\epsilon T_{2i}(v_i, \epsilon) \\
&= P_{1i}(v_i) \cdot (v_i - \lambda_i) + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon) \cdot (v_i - \lambda_i - \epsilon_i)] - (S_i(v_i) - \lambda_i) \\
&= P_{1i}(v_i) \cdot (v_i - \lambda_i) + \mathbb{E}_\epsilon [P_{2i}(v_i, \epsilon) \cdot (v_i - \lambda_i - \epsilon_i)] - \int_{\underline{v}_i}^v P_{1i}(x) \cdot dx - \mathbb{E}_\epsilon \int_{\underline{v}_i}^v P_{2i}(x, \epsilon) \cdot dx \\
&= \left[ P_{1i}(v_i) \cdot (v_i - \lambda_i) - \int_{\underline{v}_i}^v P_{1i}(x) \cdot dx \right] + \mathbb{E}_\epsilon \left[ P_{2i}(v_i, \epsilon) \cdot (v_i - \lambda_i - \epsilon_i) - \int_{\underline{v}_i}^v P_{2i}(x, \epsilon) \cdot dx \right].
\end{aligned}$$

Hence the seller's expected profit from bidder  $i$  is equal to

$$\int_{\underline{v}_i}^{\bar{v}_i} \pi_i(v_i) \cdot dv_i = \int_{\underline{v}_i}^{\bar{v}_i} [MR_{1i}(v_i) \cdot P_{1i}(v_i) + \mathbb{E}_\epsilon MR_{2i}(v_i, \epsilon_i) \cdot P_{2i}(v_i, \epsilon)] \cdot dv_i.$$

For each  $v$  the seller chooses  $P_{1i}(v_i)$  and  $P_{2i}(v_i, \epsilon)$  to maximize

$$\sum_{i=1}^N MR_{1i}(v_i) \cdot P_{1i}(v_i) + \mathbb{E}_\epsilon \sum_{i=1}^N MR_{2i}(v_i, \epsilon_i) \cdot P_{2i}(v_i, \epsilon).$$

From (B.1) we know that for any  $\epsilon$  we have

$$\sum_{i=1}^N P_{1i}(v_i) + P_{2i}(v_i, \epsilon) \leq 1.$$

If  $\max_i MR_{1i}(v_i) \leq 0$  for all  $i$ , then the seller should not allocate the object in period 1. If  $\max_i MR_{1i}(v_i) > 0$  for some  $i$ , then without loss of generality assume bidder 1 has the highest MR, and let  $P_1$  denote  $P_{11}(v_1)$ . Then in period 2, the seller should allocate the object to the highest MR (if it's positive) with probability  $1 - P_1$ . The seller's total profit is equal to

$$P_1 \cdot \max\{MR_{11}(v_1), \dots, MR_{1N}(v_N), 0\} + (1 - P_1) \cdot \mathbb{E}_\epsilon \max\{MR_{21}(v_1, \epsilon_1), \dots, MR_{2N}(v_N, \epsilon_N), 0\}.$$

By Theorem 3.1 we know that the seller should set  $P_1 = 0$ . The seller allocates the object to the highest MR in period 2 (if it's positive). Therefore the optimal mechanism is to do nothing in period 1 and run an optimal auction in period 2.

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