Selection Detection and Two-Sample-Testing: Generalized Greenwood Statistics and their Applications

Đan Daniel Erdmann-Pham, Jonathan Terhorst & Yun S. Song

University of California, Berkeley

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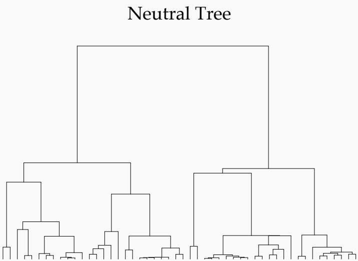
SPA 2019

Two Problems

Population Genetics: Detecting Selective Pressure



- At each depth, leaf set sizes are approximately equidistributed
- Leaf set sizes are highly unbalanced close to the root

- Given a tree, how can we tell whether it was generated under selection or not?
- Data allows computation of sum of squares of leaf set sizes

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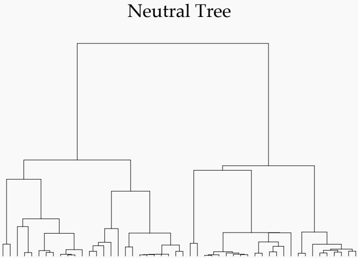

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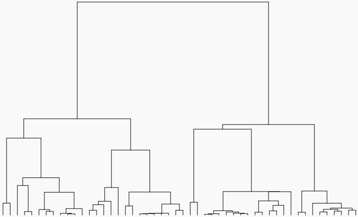
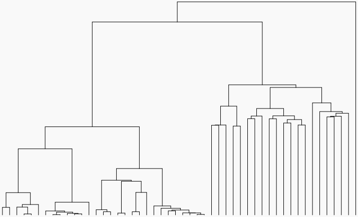


Tree with Selection



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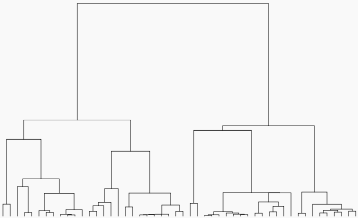

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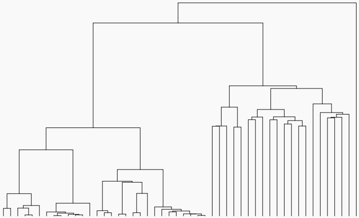

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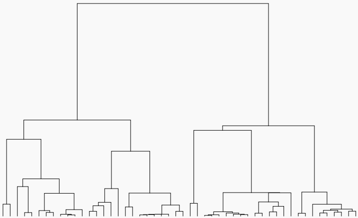

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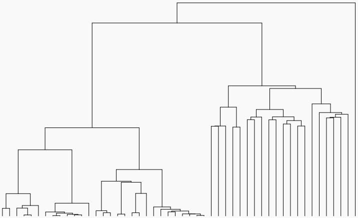

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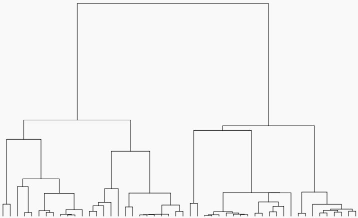

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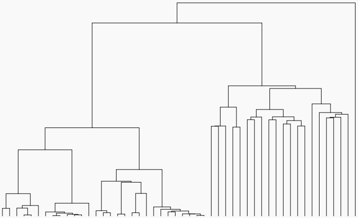

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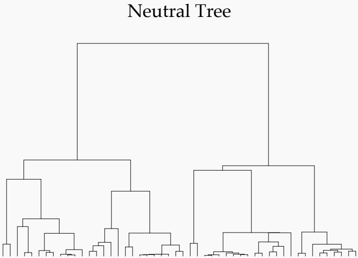

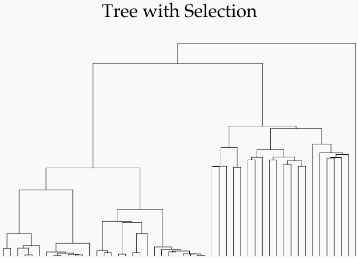

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Two-Sample Tests: Comparing $\{X_k\}_{k\in[n]}$ and $\{Y_k\}_{k\in[m]}$

How to test the hypothesis whether $\{X_k\}$ and $\{Y_k\}$ are identically distributed?

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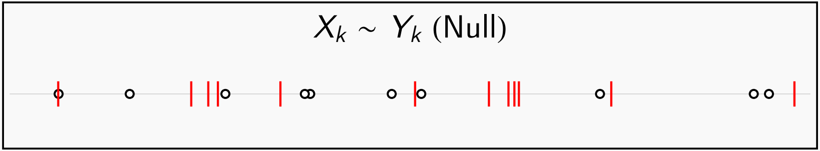

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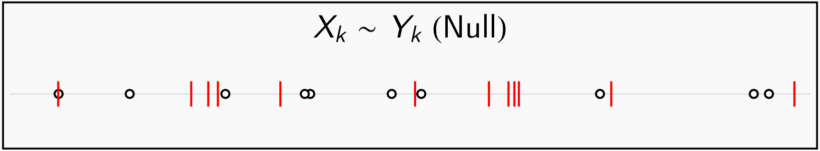

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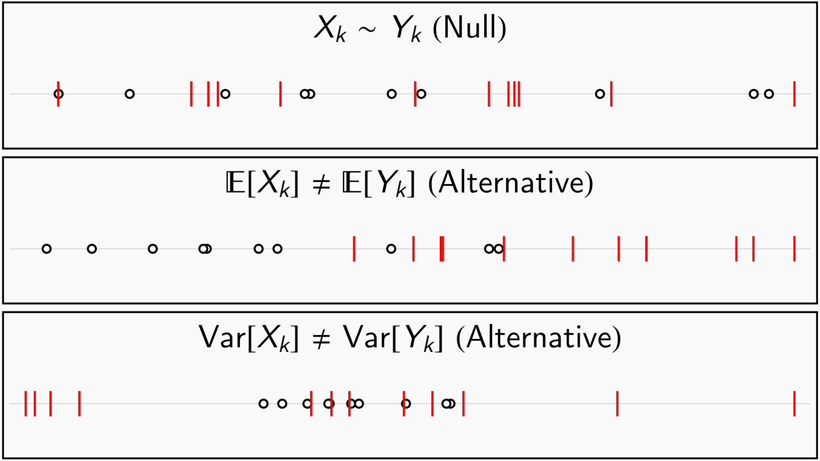

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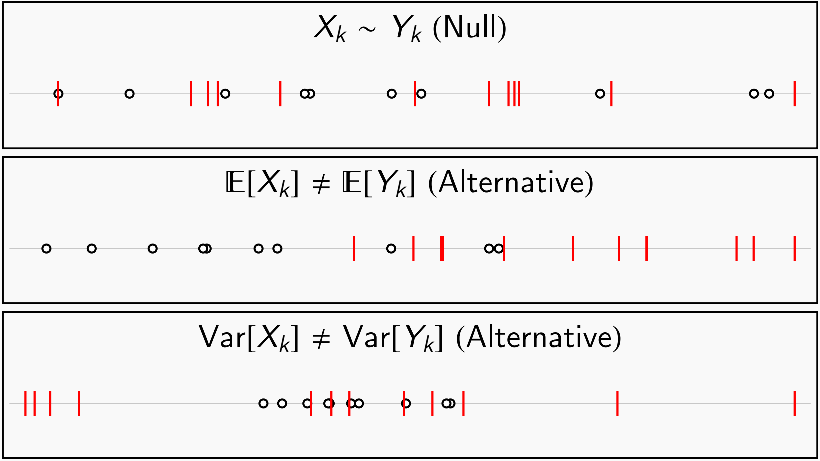

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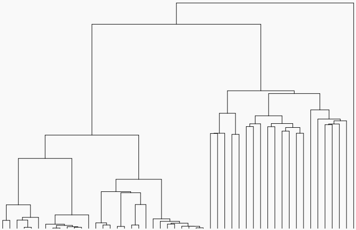
Sampling uniformly from the k-dimensional simplex Δ^{k-1}

Balls and bins



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Limit as $n \to \infty$ for fixed k

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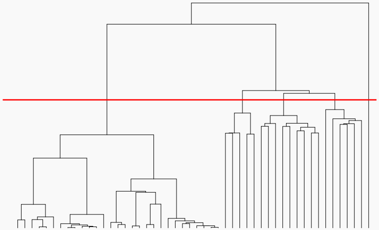

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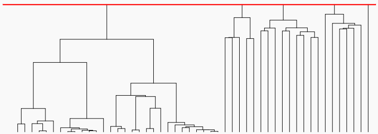

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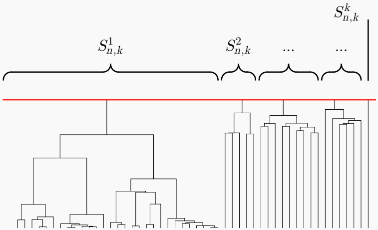

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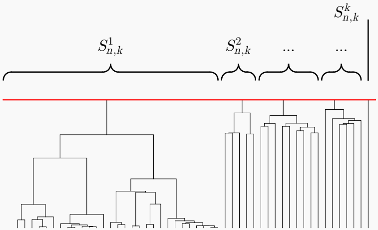

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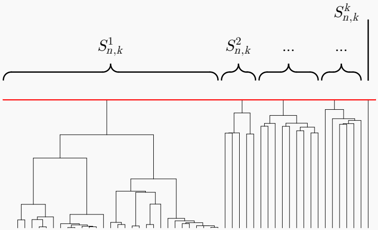

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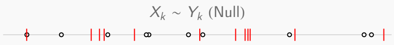

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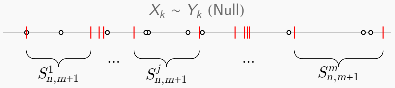

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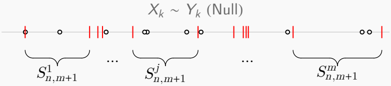

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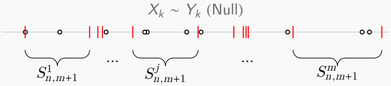

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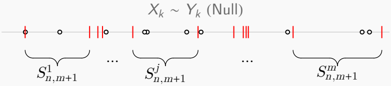

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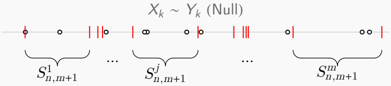

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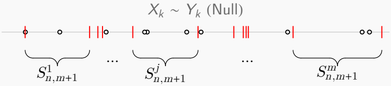

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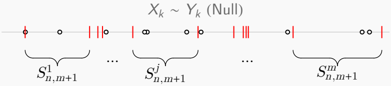

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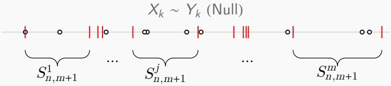

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Results

Observation (Recursion)

Let $G(x) = \sum_{m=0}^{\infty} \mathrm{Li}_{-2m}(x)/m!$, then

$$\mathbb{E}\|S_{n,k}\|_2^{2m} = \frac{m!}{\binom{n-1}{k-1}} \left[x^n\right] \left(S_m(x)\right)^{\star(k)} .$$

Corollaries (Discrete)

1. ε-approximation in $O\left(\frac{n}{\varepsilon}\log\left(\frac{n}{\varepsilon}\right) + \frac{n}{\varepsilon}\log k\right)$ time
2. Conservative hypothesis tests
3. Alternative Scaling limits: CLT, LLN, large deviations

Corollaries (Continuous)

1. Continuum approximation: $\|F_{n,k} - F_k\|_\infty \in O\left(n^{-1}\right)$
2. Monotonicity: $F_{n,k} - F_k \geq 0$
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1. ε-approximation in $O\left(\frac{n}{\varepsilon} \log\left(\frac{n}{\varepsilon}\right) + \frac{n}{\varepsilon} \log k\right)$ time
2. Conservative hypothesis tests
3. Alternative Scaling limits: CLT, LLN, large deviations

Corollaries (Continuous)

1. Continuum approximation: $\|F_{n,k} - F_k\|_\infty \in O\left(n^{-1}\right)$
2. Monotonicity: $F_{n,k} - F_k \geq 0$
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Results

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Let $G(x) = \sum_{m=0}^{\infty} \mathrm{Li}_{-2m}(x)/m!$, then

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Application to Two-Sample Testing

Comparing Non-Parametric Two-Sample Tests

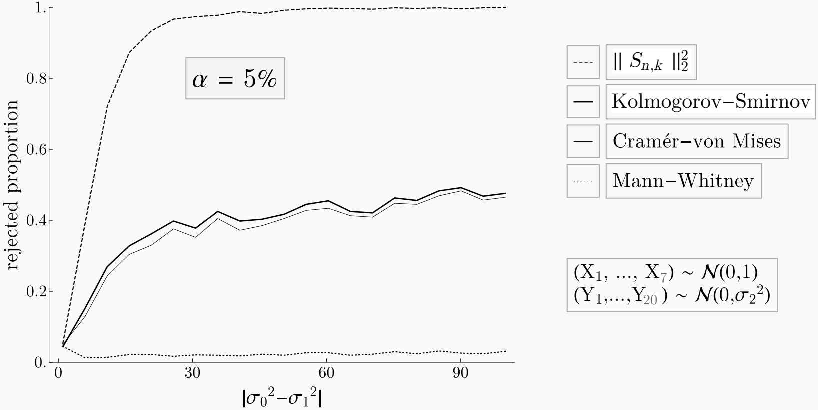

Figure: Hypothesis testing based on $\|S_{n,k}\|_2^2$ is more sensitive to variance changes than common other two-sample tests.

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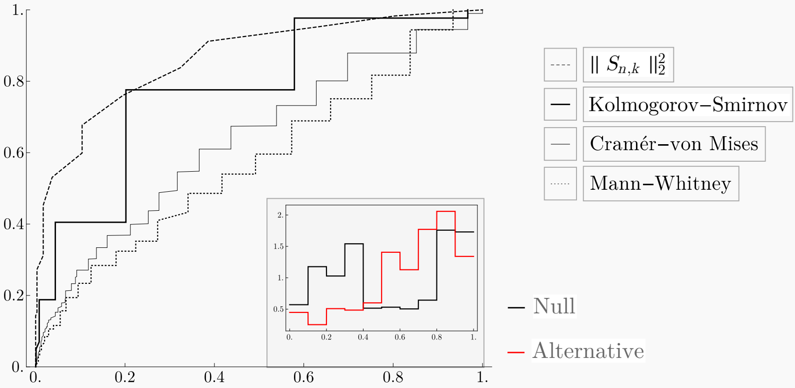

Figure: Hypothesis testing based on $\|S_{n,k}\|_2^2$ is more sensitive to compound mean and variance changes than common other two-sample tests, for randomly generated null and alternative of common support.

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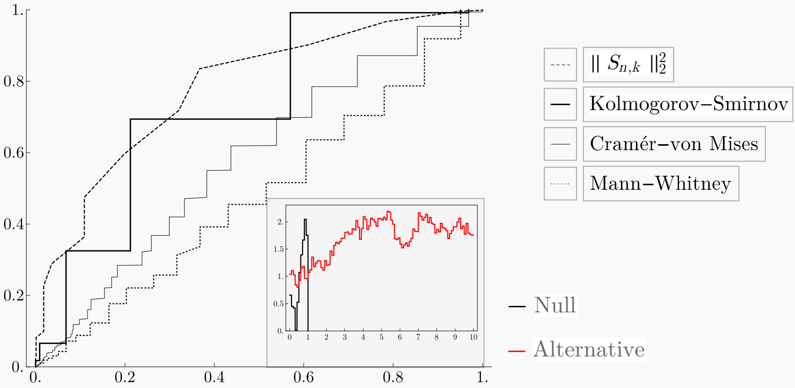

Figure: Hypothesis testing based on $\|S_{n,k}\|_2^2$ is more sensitive to compound mean and variance changes than common other two-sample tests, for randomly generated null and alternative of distinct support.

New Perspectives on old Questions

What happened?

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2. Understood discretized problem through generating functions of moments
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1. Apply hypothesis test to real data
2. Quantify more precisely the power against given classes of alternatives

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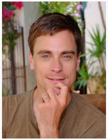

Jonathan Fischer

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