Contract Complexity and Performance

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Contract Complexity as a Design Factor

- Today, approximately half of the revenue generated in the U.S. manufacturing industry is spent on procurement (U.S. Department of Commerce 2006).
- Since products and their production processes have become more complex, firms are now buying more complex items and services.
- Hence it is critical for firms to streamline these intricate procurement processes to maintain a competitive edge in the market.
- Many factors influence the design of procurement contracts such as price, availability, cost, and delivery schedule.
- In this tutorial, we provide a brief overview of some of the commonly used procurement contracts and discuss *contract complexity* as a design factor.
Two Commonly Used Procurement Contracts

Wholesale Price Contracts:

- Common in the supply chain management literature motivated by the observation that “many supply-chain transactions are governed by simple [price-only] contracts, defined only by a per-unit wholesale price” (Lariviere and Porteus 2001).

- Employed for a wide variety of products and services, including paper, alarm systems, pharmaceuticals, software, components for airplanes, electronics design, assembly and components, and healthcare services.

- Simple: they require only the specification of a single parameter – the wholesale price.

Quantity Discount Contracts:

- Praised by academic literature as they have been shown theoretically to increase sales, reduce costs, increase channel efficiency, allow for self-selected price discrimination, and eliminate inefficiencies due to information asymmetry.

- Observed regularly in practice.

- However, there is evidence that they do not perform consistently in practice (Altintas, Erhun, and Tayur 2008).
Outline

1 Introduction

2 Optimal Contracts Under Complete Information

3 A Little Bit of Background

4 Optimal Contracts Under Asymmetric Cost Information

5 A Behavioral Analysis of the Efficiency of Simple Contracts Under Asymmetric Demand Information

6 Dynamic Procurement

7 Concluding Remarks
Consider a simple stylized two-stage supply chain with one supplier and one buyer where the buyer (she) buys goods from the supplier (he) and sells them in an end consumer market.

The supplier’s unit cost of production is $c$.

The supplier quotes unit wholesale price $w$ to the buyer.

The buyer chooses order quantity $q$.

The buyer faces a market where the price $P$ is inversely related to the quantity sold.

Let us assume a linear demand curve $P = a - bq$ where $a$ is the market potential and $b$ is the price sensitivity.

Assume that all of this information is common knowledge.

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Decentralized Supply Chain

- Both the supplier and the retailer maximize their own profits
  - Supplier’s profit: $\Pi_S = (w - c)q$
  - Buyer’s profit: $\Pi_B = (P - w)q = (a - bq - w)q$

- A dynamic game of complete information
  - First, the supplier chooses the unit wholesale price $w$.
  - After observing $w$, the buyer chooses the order quantity $q$.

- Solve this game using backwards induction:

$$\frac{\partial \Pi_B}{\partial q} = a - 2bq - w = 0 \Rightarrow q(w) = \frac{a - w}{2b}.$$ 

- Given $q(w) = (a - w)/(2b)$, the supplier maximizes $\Pi_S = (w - c)q = (w - c)(a - w)/(2b)$.

$$\frac{\partial \Pi_S}{\partial w} = a - w - w + c = 0 \Rightarrow w = \frac{a + c}{2}.$$
A single decision-maker who is concerned with maximizing the entire chain's profits $\Pi = (a - bq - c)q$.

$$\frac{\partial \Pi}{\partial q} = a - 2bq - c = 0 \Rightarrow q(c) = \frac{a - c}{2b}.$$
## Centralized vs. Decentralized Supply Chains

<table>
<thead>
<tr>
<th></th>
<th>Decentralized supply chain</th>
<th>Centralized supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price ((w))</td>
<td>(w = (a + c)/2)</td>
<td>(w)</td>
</tr>
<tr>
<td>Quantity ((q))</td>
<td>(q = (a - c)/(4b))</td>
<td>(q^* = (a - c)/(2b))</td>
</tr>
<tr>
<td>Market price ((P))</td>
<td>(P = (3a + c)/4)</td>
<td>(P^* = (a + c)/2)</td>
</tr>
<tr>
<td>Supplier’s profit ((\Pi_S))</td>
<td>(\Pi_S = (a - c)^2/(8b))</td>
<td>(\Pi_S^* = (w - c)q)</td>
</tr>
<tr>
<td>Buyer’s profit ((\Pi_B))</td>
<td>(\Pi_B = (a - c)^2/(16b))</td>
<td>(\Pi_B^* = (P^* - w)q)</td>
</tr>
<tr>
<td>Total profits ((\Pi))</td>
<td>(\Pi = 3(a - c)^2/(16b))</td>
<td>(\Pi^* = (a - c)^2/(4b))</td>
</tr>
</tbody>
</table>

- In the decentralized supply chain
  - Sales quantity is lower
  - Market price is higher
  - Total profit is lower

  compared to the centralized supply chain.

- This inefficiency is due to **double marginalization** (Spengler 1950).
Supply Chain Coordination

- Global optimization: Identify what is best for the entire system in terms of system-wide costs/profits
  - Who will optimize? (Conflicting goals)
  - How will the savings be shared among the participants?

- Dynamic system: Customer demand, supplier capabilities, supply chain roles and relationships and relative market power of the channel members
Quantity Discount Contracts

- Assume the supplier charges $w(q)$ where $w$ is a decreasing function of $q$.
- Let $R(q)$ be the buyer’s (expected) revenue when she stocks $q$ units of the final product.
  - $R(q)$ is a finite, concave function of $q$ with $R(0) = 0$.
  - Assume $R'(0) > c$ (otherwise, production is never profitable).
- Buyer’s (expected) profit: $\Pi_B = R(q) - w(q)q$.
- DSC has the same optimal quantity as CSC if $\Pi_B$ is an affine transformation of $\Pi$.
- Hence, we need

$$
\Pi_B = R(q^*) - w(q^*)q^* = \alpha(R(q^*) - cq^*) \Rightarrow w(q^*) = (1 - \alpha) \left( \frac{R(q^*)}{q^*} \right) + \alpha c.
$$

- Note that $\Pi_S = (1 - \alpha)R(q^*) - (1 - \alpha)cq^* = (1 - \alpha)\Pi$. 

Supply Chain Coordination Revisited

- Quantity discount contracts eliminate double marginalization and coordinate the supply chain.
- There are other contract structures that achieve coordination in such simple settings:
  - Buyback contracts (Pasternack 1985): the buyer can return leftover units to the supplier at the end of the selling season for $b$ per unit where $b < w$.
  - Revenue-sharing contracts (Cachon and Lariviere 2005): the buyer shares a fraction $\alpha < 1$ of her revenues with the supplier.
- Further reading materials:
Introduction

Optimal Contracts Under Complete Information

A Little Bit of Background

Optimal Contracts Under Asymmetric Cost Information

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Dynamic Procurement

Concluding Remarks
The revelation principle is an important concept for designing games when the players have private information.

Consider a seller who wishes to design an auction to maximize his expected revenue.

Specifying the many different auctions the seller should consider could be an enormous task:

- The first-price sealed-bid auction.
- The bidders may have to pay an entry fee.
- Some of the losing bidders might have to pay money, perhaps in amounts that depend on their own and others’ bids.
- The seller might set a reservation price, etc.

The revelation principle shows how to create an incentive compatible game from any game with a Bayesian Nash equilibrium.
Definitions

- Static games in which each player’s only action is to submit a claim about his or her type are called **direct mechanism**.

- A direct mechanism in which it is a Bayesian Nash equilibrium for each bidder to tell the truth is called **incentive-compatible**.
The Revelation Principle: Theorem

**Theorem (Myerson 1979)**

*Any Bayesian Nash equilibrium of any Bayesian game can be represented by an incentive-compatible direct mechanism.*

- The auctioneer does not need to consider every mechanism to find the optimal one.
- He can simply focus on the mechanisms that have truth-telling equilibrium.
- If he finds the optimal mechanism among them, then it is the optimal one among the entire set of feasible mechanisms.
By “represented” we mean that for each possible combinations of players’ types \((t_1, \cdots, t_n)\), the players’ actions and payoffs in the new equilibrium are identical to those in the old equilibrium.

No matter what the original game, the new Bayesian game is always a direct mechanism.

No matter what the original equilibrium, the new equilibrium in the new game is always truth-telling.

The new game is called direct revelation mechanism. Note that, it may have other, undesirable equilibria!!
A local monopolist wine seller can produce wine of any quality \( q \in (0, \infty) \) with a cost of \( C(q) \).

- \( C \) is twice differentiable and strictly convex, that \( C'(0) = 0 \) and \( C'(\infty) = \infty \).

- He charges \( t \) dollars per bottle, i.e., his utility is \( t - C(q) \).

- The consumer is a moderate drinker who plans to buy a bottle of wine.
  - Her utility is \( U = \theta q - t \) where \( \theta \) is a positive parameter that indexes her taste for quality.
  - If she decides not to buy any wine, her utility is 0.

- There are two types of consumers: type 1 or sophisticated and type 2 or coarse. The values for \( \theta \) are such that \( \theta_1 < \theta_2 \); the prior probability that the consumer is of type 1 is \( p \).

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\(^2\)This example is based on Salanié (1997).
If the seller knows the type $\theta_i$ of the consumer, he will solve the following optimization:

$$\max_{q_i, t_i} \quad (t_i - C(q_i))$$

$$\text{s.t.} \quad \theta_i q_i - t_i \geq 0$$

He will offer $q_i = q_i^*$ such that $C'(q_i^*) = \theta_i$ and $\theta_i q_i^* = t_i^*$.

Therefore, he will extract all the surplus, leaving the consumer with zero utility.

This is called first-degree price discrimination ... 

... (and it is generally forbidden by law).
Now, let us consider the original problem: the seller only knows the proportion of coarse consumers $p$.

If he proposes the first-best contracts $(q_1^*, t_1^*)$ and $(q_2^*, t_2^*)$, the sophisticated consumers will not choose $(q_2^*, t_2^*)$ since:

$$\theta_2 q_1^* - t_1^* = (\theta_2 - \theta_1)q_1^* > 0 = \theta_2 q_2^* - t_2^*$$

The two types are not separated any more: Both will choose the low-quality deal.
Now let’s use the Revelation Principle ...

The best pair of contracts can be formulated as follows:

$$\max_{q_1, t_1, q_2, t_2} \{ p(t_1 - C(q_1)) + (1 - p)(t_2 - C(q_2)) \}$$

s.t.

$$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2 \quad (IC_1)$$
$$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 \quad (IC_2)$$
$$\theta_1 q_1 - t_1 \geq 0 \quad (IR_1)$$
$$\theta_2 q_2 - t_2 \geq 0 \quad (IR_2)$$

- The two (IC) constraints are *incentive compatibility* constraints; they state that each consumer prefers the contract that was designed for her.
- The two (IR) constraints are *individual rationality* constraints; they guarantee that each type of consumer accepts her designed contract.
Incomplete Information

- The optimal contract has the following properties:
  1. $(IR_1)$ is active, so $t_1 = \theta_1 q_1$.
  2. $(IC_2)$ is active; i.e., $t_2 - t_1 = \theta_2 (q_2 - q_1)$.
  3. $q_2 \geq q_1$.
  4. We can neglect $(IC_1)$ and $(IR_2)$.
  5. Sophisticated consumers buy the efficient quality: $q_2 = q_2^*$.

- Therefore, $t_1 = \theta_1 q_1$ and $t_2 = \theta_1 q_1 + \theta_2 (q_2^* - q_1)$.

- The quality sold to the coarse consumer is subefficient.

\[
C'(q_1) = \theta_1 - \frac{1 - p}{p}(\theta_2 - \theta_1) < \theta_1.
\]

- That is, the monopolist is paying an **information rent** to the consumer.
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6. Dynamic Procurement
7. Concluding Remarks
Model Definition\(^3\)

- Consider a simple stylized two-stage supply chain with one supplier and one buyer where the buyer (she) buys goods from the supplier (he) and sells them in an end consumer market.
- The supplier's unit cost of production is \( c \).
- Buyer chooses order quantity \( q \) and has (expected) revenue \( R(Q) \) when she stocks \( Q \) units of the final product.
  - \( R(Q) \) is a finite, increasing, and strictly concave function of \( Q \) with \( R(0) = 0 \).
- The buyer and the supplier are risk neutral: each seeks to maximize his/her own expected profit.

The supplier's cost (type) is his private information at the time of contracting.

The buyer does not know \( c \) with certainty, but does knows only that \( c \) has pdf \( f(c) \) with associated cdf \( F(c) \) and support \( \Omega := [c, \bar{c}] \) where \( 0 \leq c < \bar{c} < \infty \).

- \( R'(0) > c \) (otherwise, production is never profitable).

This prior information is common knowledge.

Let \( h(c) := F(c)/f(c) \) and define total cost as

\[
k(c) := c + h(c).
\]
Now let’s use the Revelation Principle ...

- Buyer can implement arbitrarily complex contracts, contingent on the cost reported by the supplier.
- Buyer designs a menu of contracts \( \{ Q(c), t(c) \}_{c \in \Omega} \) that maximizes her (expected) ex-ante payoff from the contract:
  - \( Q(c) \) is the quantity to be procured from the supplier contingent on the cost \( c \) reported by the supplier.
  - \( t(c) \) is the payment from the buyer to the supplier contingent on the cost \( c \) reported by the supplier.
- Supplier selects a contract from this menu by announcing his cost \( c \).
The program that the buyer has to solve to find the optimal menu:

\[
\Pi_B := \max_{Q: \Omega \rightarrow \mathbb{R}^+} \max_{t: \Omega \rightarrow \mathbb{R}} E_c [R(Q(c)) - t(c)] \\
\text{s.t.} \quad t(c) - cQ(c) \geq 0 \quad \forall c \in \Omega \quad (IR_c) \nonumber \\
t(c) - cQ(c) \geq t(\hat{c}) - cQ(\hat{c}) \quad \forall c, \hat{c} \in \Omega \quad (IC_{c\hat{c}}) \nonumber
\]

**Proposition**

*Assume that \( k(c) \) is increasing in \( c \). The buyer’s optimal menu of contracts \( \{Q^c(c), t^c(c)\} \) satisfies:

\[
\{Q^c(c), t^c(c)\} = \left\{ ((R')^{-1}(k(c)))^+, cQ^c(c) + \int_c^c Q^c(\tau)d\tau \right\}.
\]
The Optimal Contract

- Since the buyer does not know the supplier’s cost, the optimal contract does not coordinate the supply chain.
- The buyer has to pay \( k(c) \) per unit for each quantity procured instead of the actual cost \( c \).
- The coordinating quantity is produced only when \( c = \underline{c} \), since \( h(c) = 0 \). Otherwise, the buyer procures less than the coordinating quantity.
- The inefficiency decreases as \( c \) decreases, since \( Q(c) \) is nonincreasing in \( c \).
- The buyer achieves a higher level of coordination when the supplier’s cost is lower by leaving the supplier a higher profit which is simply equal to \( t(c) - cQ(c) = \int_{\underline{c}}^{c} Q(\tau) d\tau \).
- This under-production is an inefficiency arising from asymmetric cost information.
Taxation Principle at Work

The optimal cost-contingent menu of contracts \( \{Q^c(c), t(c)\} \) can easily be transformed to an equivalent quantity-contingent payment \( \{T^c(Q)\} \) as follows:

\[
T^c(Q) = \begin{cases} 
  t^c(c) & \text{if } Q = Q^c(c) \text{ for some } c \in \Omega \\
  0 & \text{otherwise}.
\end{cases}
\]

This equivalence of quantity-contingent and cost-contingent payments is known as the Taxation Principle (Martimort and Stole 2002).

Proposition

The buyer’s optimal quantity-contingent payment, \( T^c(Q) \), is a concave function of \( Q \) for \( Q \in (Q^c[\bar{c}], Q^c[c]) \).
## Wholesale Price Contract vs. Optimal Contract

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Min</th>
<th>Median</th>
<th>90\textsuperscript{th} Percentile</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.997</td>
<td>0.002</td>
<td>0.511</td>
<td>7.382</td>
<td>11.037</td>
</tr>
<tr>
<td>Triangular</td>
<td>1.855</td>
<td>0.001</td>
<td>0.3860</td>
<td>5.9812</td>
<td>15.1980</td>
</tr>
</tbody>
</table>

The parameter values for this numerical study are:

- Demand $D$ is the maximum of zero and a normally distributed random variable with mean $\mu = 100$ and standard deviation $\sigma = \{10, 20, 50, 100\}$.
- We consider both uniform and triangular distribution for the supplier's cost $c$ and specify the minimum cost $c = \mu_2(1 - \triangle)$, maximum cost $\bar{c} = \mu_2(1 + \triangle)$, and, for the triangular distribution, mode $m = \{c, \mu_2, \bar{c}\}$, with mean $\mu_2 = \{0.2, 0.4, 0.8, 1\}$ and cost dispersion $\triangle = \{0.05, 0.1, 0.3, 0.5, 0.7\}$. Note that cost dispersion $\triangle = (\bar{c} - c)/(2\mu_2)$. 
Model Definition

• Consider a simple supply chain with one supplier and one buyer.
• Buyer is a newsvendor:
  • She faces a random demand $D \sim U[\mu - \nu, \mu + \nu]$.
  • Per unit sales price $= \$p$
• Asymmetric mean demand information:
  • $\mu$ of buyer is private information
  • 3 types: High (H), Medium (M) or Low (L)
• Supplier:
  • Unit production cost $= \$k$
  • Different pricing schemes: Wholesale price contract, all-unit discount with 2 prices, all-unit discount with 3 prices
• The buyer and the supplier are risk neutral: each seeks to maximize his/her own expected profit.

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Buyer’s and Supplier’s Decisions

- **Buyer’s order quantity**
  
  (Jucker and Rosenblatt 1985) Under all-unit quantity discounts, the buyer’s optimal order quantity is either at one of the order-up-to levels or at one of the price breaks.

- **Supplier’s pricing decisions**
  
  - Wholesale price contract
    - Supplier finds the optimal wholesale price for the average mean demand type.
  
  - Quantity discount contracts
    - Incentive compatibility of different demand types
    - Price breaks can be used to make a demand type indifferent between buying at a high or low price and to extract rents.
Theoretical Predictions

### Supplier’s optimal decisions

<table>
<thead>
<tr>
<th>Prices</th>
<th>Price breaks</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>One-price</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>Two-price</td>
<td>184</td>
<td>152</td>
</tr>
<tr>
<td>Three-price</td>
<td>200</td>
<td>177</td>
</tr>
</tbody>
</table>

### Buyer’s optimal decisions

<table>
<thead>
<tr>
<th>Procurement quantities</th>
<th>Profit</th>
<th>Total Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_L$</td>
<td>$q_M$</td>
<td>$q_H$</td>
</tr>
<tr>
<td>One-price</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Two-price</td>
<td>8</td>
<td>48</td>
</tr>
<tr>
<td>Three-price</td>
<td>0</td>
<td>84</td>
</tr>
</tbody>
</table>
Human subjects recruited from Stanford student body: 19 subjects at each treatment, 40 periods

Monetary compensation according to game performance

Web-based instructions and quiz before experiment

Implemented in HP Experimental Economics software platform

Experiments conducted at Stanford
Suppliers do not necessarily benefit from quantity discount contracts

- $2\text{-price} > 1\text{-price}$, $3\text{-price} \approx 2\text{-price}$
- All contracts lead to lower supplier profits than theory
Total profits are comparable between WPC and QDC

- 1-price ≈ 2-price ≈ 3-price
- 1-price and 2-price contracts lead to higher total profits than theory
We observe a more equitable distribution of profits.

- 1-price $\approx$ 2-price $\approx$ 3-price
- All contracts lead to higher buyer profits than theory.
Behavioral Observations

- Suppliers set their prices too low
  - All prices are significantly lower than theory
  - Lower prices explain the more equitable distribution of the profits between the supplier and the buyer
- Price breaks do not separate different types of the buyer
  - In theory, the supplier must target medium and high types with the price breaks; occurs only for 33% of instances for high type under the 2-price treatment
- Suppliers understand the dynamics of pricing better than dynamics of separation
  - Sensitivity analysis by optimizing the value of a single contract parameter while keeping others constant
  - Improvements in price breaks lead to the biggest enhancement in suppliers profits
Concluding Remarks

- A nontrivial trade-off between complexity and inefficiency
  - Some complexity is good (2-price $> 1$-price for the supplier)
  - Very complex contracts are not better
- Supplier decisions are significantly different from rational theory predictions
- As the contract complexity increases, human subjects increasingly rely on simple heuristics
- Complexity is a factor in contract design
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Dynamic procurement, i.e., simple wholesale price contracts repeated over time (possibly with different prices), is a commonly observed practice in a vertical channel.

A buyer may prefer to procure goods over time to:
- Manage demand risk
- Spread payments over a period of time
- Minimize potential capacity risks (supplier’s or buyer’s)
- Take advantage of supplier’s decreasing cost over time which may translate to lower prices (e.g., as in the electronics industry).

Dynamic procurement can also serve as a tool to influence future prices.
Model Definition\(^5\)

- There are \(N\) possible periods for procurement before the buyer’s production/selling season begins.
- The supplier’s decisions are the wholesale prices for each period, \(w_n (n = 1, \cdots, N)\).
- The buyer’s decisions are the procurement quantities for each period, \(q_n (n = 1, \cdots, N)\), and the production quantity, \(Q_N\).
- The market is characterized by a linear inverse demand function \(P(Q_N) = a - bQ_N\), where \(a\) is the market potential, \(b\) is the price sensitivity, and \(P(Q_N)\) is the per-unit market price of the product for \(Q_N\).
- The supplier and the buyer maximize their profits.

Sequence of Events

In each period $n$ of the $N$-period game

- Given previous procurement quantities $(q_j, j = 1, \cdots, n - 1)$, the supplier determines the wholesale price $w_n$.
- Given previous procurement quantities $(q_j, j = 1, \cdots, n - 1)$ and the current wholesale price $(w_n)$, the buyer determines her procurement quantity $q_n$.
- In the last period $N$, the buyer chooses her production quantity $Q_N$ and procures extra quantity, if necessary.
- The market clears only once at the end of the $N$-th period; i.e., there is only a single selling opportunity to end consumers.
Double marginalization effect decreases and the efficiency increases and approaches that of the centralized solution.

Even for small values of $N$, dynamic procurement decreases the inefficiency considerably. For example, for $N = 3$, the inefficiency is already less than 10% (compared to 25% for $N = 1$).
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Future Directions

- Opportunities to study various aspects of contracts behaviorally
- Opportunities to study contracts in different industries
- Opportunities to study contracts empirically
Further Reading
