

# Ideological Bias and Trust in Information Sources

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## Abstract

We study the role of endogenous trust in amplifying ideological bias. Agents in our model learn a sequence of states from sources whose accuracy is *ex ante* uncertain. Agents learn these accuracies by comparing their own reasoning about the states based on introspection or direct experience to the sources' reports. Small biases in this reasoning can cause large ideological differences in the agents' trust in information sources and their beliefs about the states, and may lead agents to become overconfident in their own reasoning. Disagreements can be similar in magnitude whether agents see only ideologically aligned sources or diverse sources.

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# 1 Introduction

Ideological divisions in society often seem intractable, with those on either side persistently disagreeing about objective facts. In recent years, for example, fervent debates over the validity of global warming, evolution, and vaccination have persisted long after the establishment of a scientific consensus (McIntyre 2018; Jerit and Zhao 2020). Partisans also disagree about which sources can be trusted to provide reliable information about such facts. In the United States, for instance, 75 percent of conservative Republicans say they trust news and information from Fox News, while 77 percent of liberal Democrats say they distrust it (Pew Research Center 2020). Such divisions have deepened even as new media technologies have made information more widely and cheaply available than ever before. The information age has, paradoxically, produced what has been dubbed a “post-truth” era (Keyes 2004).

Such patterns seem at odds with the prediction of many Bayesian (e.g., Blackwell and Dubins 1962) and non-Bayesian (e.g., DeGroot 1974) learning models in which widespread availability and distribution of information leads all agents’ beliefs to converge to the truth. Many possible alternatives have been proposed. However, such accounts generally require that individuals have substantial psychological biases towards cognitive consistency or confirmation (e.g., Lord, Ross, and Lepper 1979; Cotton 1985; Rabin and Schrag 1999; Baliga, Hanany, and Klibanoff 2013) or have limited memory or attention (e.g., Fryer, Harms, and Jackson 2019; Che and Mierendorff 2019).

In this paper, we explore a different possibility, which is that rational Bayesian inference may magnify the influence of even small cognitive biases when agents are uncertain which sources they can trust. Building on insights by Acemoglu, Chernozhukov, and Yildiz (2016) and Sethi and Yildiz (2016), among others, we show that small biases may lead to substantial and persistent divergence in both trust in information sources and beliefs about facts, with partisans on each side trusting unreliable ideologically aligned sources more than accurate neutral sources, and also becoming overconfident in their own judgment. Consistent with evidence suggesting that the magnitude of selective exposure has generally been limited (Gentzkow and Shapiro 2011; Flaxman, Goel, and Rao 2016; Barberá 2020), these patterns arise whether agents selectively view only ideologically aligned sources or are exposed to a diverse range of sources. Increasing the num-

ber of available information sources in such a setting may deepen rather than mitigate ideological differences.

Agents in our model wish to learn about a sequence of unobserved states  $\omega_t \sim N(0, 1)$ , which are drawn independently in each period  $t$ . We think of each period's  $\omega_t$  as capturing a distinct item discussed in the news. In one period this might be the effectiveness of masks at stopping disease transmission, in the next period the extent of fraud in a recent election, in a third period the magnitude of global warming due to human activity, and so on. In each period, each agent  $i$  observes a normally distributed signal  $s_{jt}$  correlated with  $\omega_t$  from one or more information sources  $j$ . We refer to the correlation between  $s_{jt}$  and  $\omega_t$  as the *accuracy* of source  $j$ . We analyze two scenarios, one in which the agent observes exactly one source  $j$  in each period (she “single-homes” in the language of Rochet and Tirole 2003), and another in which she observes all sources  $j$  in each period (she “multi-homes”).

To introduce a political dimension to the model, we assume that the issue in each period  $t$  is associated with an *ideological valence*  $r_t$ . This is a separate state variable which captures the way realizations of that issue map to the political arena. In particular,  $r_t$  is the value of  $\omega_t$  that would be most consistent with conservative ideology and  $-r_t$  is the value of  $\omega_t$  that would be most consistent with liberal ideology. If  $\omega_t$  is the effectiveness of masks,  $r_t$  would be negative (since the conservative position tends to be that masks are ineffective). If  $\omega_t$  is the extent of fraud in the 2020 election,  $r_t$  would be positive (since the conservative position is that fraud was extensive). We allow the information sources  $j$  to have ideological *biases* in the sense that they may be distorted in the direction of  $r_t$ —i.e., the errors in the sources' reports may be correlated with  $r_t$ . The accuracies and biases of the sources are the main persistent state variables that the agent seeks to learn over time.

The final ingredient of our model is each agent's own *reasoning* about the state based on introspection or direct experience. We model this as an observable random variable  $x_{it}$ . In the case of global warming,  $x_{it}$  might be a realization combining information about local temperatures or weather events the agent has experienced directly and/or her understanding of the mechanisms of climate science. In the case of an election,  $x_{it}$  might be a realization combining information about the procedures in place at polling places in her neighborhood and her evaluation of the difficulty of committing fraud. In the case of masks,  $x_{it}$  might be a realization based on the agent's

understanding of the way disease spreads and the mechanisms by which masks could stop it.

Our key assumption is that agents believe their own reasoning to be unbiased, even though this may not be true. In other words,  $x_{it}$  may be systematically correlated with  $r_t$  conditional on  $\omega_t$ , but each agent's prior assigns probability one to the case where this correlation is zero.<sup>1</sup> This prior belief is what distinguishes  $x_{it}$  from the other information sources  $s_{jt}$  in the model. It means that even if an agent knows her own reasoning is noisy, and so has limited information about  $\omega_t$  on its own, she is able to use it as a yardstick to determine the accuracy and biases of the  $s_{jt}$  and so ultimately to learn which sources she can trust. If in fact her reasoning *is* biased, the result will be distorted learning about the accuracy of information sources and, consequently, the states  $\omega_t$ . Our main results characterize the form such distortions take and show that they can in some cases be large even when the magnitude of the agent's bias (i.e., the correlation between  $x_{it}$  and  $r_t$  conditional on  $\omega_t$ ) is small.

Both parts of our key assumption are strongly supported by evidence. Large literatures in psychology and behavioral economics establish mechanisms by which individuals' own reasoning about politically sensitive topics may be subject to ideological bias, including motivated reasoning, selective memory, and availability (Festinger 1957; Tversky and Kahneman 1973; Lord, Ross, and Lepper 1979; Kunda 1990; Eagly et al. 1999). The implication of such biases is that two agents who for exogenous reasons begin with different ideologies and who have access to the same direct experiences—observe the same observations of local weather events, say—might reach conclusions about  $\omega_t$  that are systematically biased toward or away from  $r_t$ . Moreover, substantial evidence also suggests that individuals themselves are not aware of these biases or at a minimum significantly underestimate them (e.g., Pronin et al. 2002; Pronin 2007; Thaler 2022).

Our formal results characterize the limiting distribution of each agent's beliefs about accuracies and states as  $t \rightarrow \infty$ . We show that their beliefs about the accuracies of information sources eventually converge to a limiting distribution, the mean of which we define to be the agent's asymptotic *trust* in the respective sources.

In a benchmark case in which an agent has no ideological bias and there is no uncertainty about the accuracy of her reasoning  $x_{it}$ , her trust is a distribution degenerate at the true accuracies of the

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<sup>1</sup>In an extension, we show that our main results extend to the case where agents entertain the possibility that their reasoning is biased, provided the magnitude of bias in the support of their priors is sufficiently small.

information sources, and her asymptotic beliefs about  $\omega_t$  are the same as if she knew the true data generating process. If the accuracy of the information sources is sufficiently high, her beliefs about  $\omega_t$  are close to correct in each period.

Introducing small biases in an agent's reasoning changes the results of the benchmark case dramatically. An agent with a small conservative bias may come to trust right-leaning sources more than is warranted by their true accuracy and trust unbiased sources less than is warranted. She may become overconfident in the accuracy of her own reasoning (i.e., come to believe that  $x_t$  is more strongly correlated with  $\omega_t$  than it really is). She will generally come to believe that the state  $\omega_t$  is positively correlated with the ideological valence  $r_t$ , and thus begin any period in which she knows  $r_t$  with a conservatively biased prior. All of these effects may be large even if bias is small, provided that the true accuracy of an agent's reasoning is sufficiently low.

To see the intuition for the way small biases are amplified in our model, note that an agent will come to see source  $j$  as more accurate the greater the observed correlation between its report  $s_{jt}$  and her reasoning  $x_{it}$ . This correlation will be zero when  $s_{jt}$  is perfectly uncorrelated with the state and positive when the source is perfectly accurate. But when the reasoning  $x_{it}$  is noisy, the magnitude of the correlation will be small even in the perfectly accurate case. Small differences in observed correlation—such as those that might be induced by a small ideological bias—thus imply large differences in perceived accuracy. An agent who knows her own ability to discern the truth of global warming is limited will view even a weak correspondence between her own conclusions and the reports of an information source as highly diagnostic.

Distortions in the way agents learn about the informational environment can translate into substantial disagreements about the states  $\omega_t$ . We first show how biases affect the accuracy of agents' posterior beliefs about  $\omega_t$ . We then show how the magnitude of disagreement between different agents depends on the accuracy and bias of their reasoning, as well as on those of the observed sources. When agents all observe a common unbiased source, disagreement is generally small even when the agents themselves are biased. When biased sources are introduced to the market, even small biases on the part of agents can lead to large disagreement.

The final section of our main results considers how these findings differ under single and multi-homing. A common intuition is that divergent trust and polarization might be mitigated if agents were exposed to an ideologically diverse set of information sources. We show that it is possible for

multi-homing to have beneficial effects consistent with this intuition, but also that this need not be the case. Multi-homing may leave trust and polarization unchanged, or even exacerbate them.

Two extensions explore the implications of ideological bias for media competition and political behavior. First, we endogenize the choice of bias by media outlets in a sequential positioning game. We find that media competition can lead to greater media bias as well as intensified disagreements among viewers. Second, we show mistrust of motives across ideological divides can arise when agents underestimate both their own and others' biases. Ideological bias in this case can intensify political conflict, leading to costlier battles for power.

Our work contributes to a small but growing set of models in which agents must simultaneously learn about states of the world and the accuracy of sources that provide information about those states. Acemoglu, Chernozhukov, and Yildiz (2016) show that arbitrarily small differences in priors about the signal distribution as well as the state can generate large and persistent disagreements among fully Bayesian agents. Sethi and Yildiz (2016) study the trade-off between learning from well-informed sources, whose signals are precise, and well-understood sources, whose perspectives are well known, and show that long-run behavior is history independent. Cheng and Hsiaw (2022a) study a model in which agents do not integrate information about states and credibility correctly and show that this can lead to persistent polarization. Cheng and Hsiaw (2022b) study a cheap talk game where receivers are uncertain of a state and the sender's type and show that persistent disagreement arises. Liang and Mu (2020) study social learning about information sources over time and focus on failures of learning rather than persistent polarization or disagreement.

Relative to existing models, we highlight and characterize a novel mechanism by which endogenous trust can amplify small behavioral biases. This is related to but distinct from the mechanism in Acemoglu, Chernozhukov, and Yildiz (2016), which focuses on differences in prior beliefs among fully Bayesian agents. It is also distinct from the mechanism in Cheng and Hsiaw (2022a), which is based on a different behavioral bias and does not produce amplification. Our findings that (i) agents end up trusting unreliable but ideologically aligned sources more than accurate neutral sources, (ii) agents become overconfident in their own reasoning, (iii) agents believe that either the conservative or the liberal point of view is closer to the truth on average, (iv) small biases in reasoning are amplified into persistent disagreements about facts, (v) divergent trust and beliefs can arise to a similar extent whether agents selectively view only ideologically aligned sources or

are exposed to a diverse range of sources, and (vi) competition among information sources can deepen rather than mitigate ideological disagreement are all novel relative to the existing literature on simultaneous learning about states and signals.

Other related models attribute belief polarization to behavioral biases but do not consider how endogenous learning about the accuracy of information sources may amplify and alter these biases.<sup>2</sup> Examples include models of confirmation bias (Rabin and Schrag 1999), ambiguity aversion (Baliga, Hanany, and Klibanoff 2013), limited memory (Fryer, Harms, and Jackson 2019), and limited attention (Che and Mierendorff 2019). Bowen, Dmitriev, and Galperti (2021) show that belief polarization can arise in a social network where agents have small misperceptions about the sharing behavior of their neighbors.<sup>3</sup> In contrast to these theories, we build a model in which endogenous trust is the central mechanism, and we assume that agents only have small biases in their reasoning but otherwise have Bayesian learning rules and can process information from an arbitrarily large set of high-quality sources.

Our model contributes to the literature on media bias and competition (Mullainathan and Shleifer 2005; Gentzkow, Shapiro, and Stone 2016). The mechanism by which agents in our model come to trust like-minded sources is closely related to the one explored by Gentzkow and Shapiro (2006). That model is essentially static, however, and does not provide a mechanism by which diverging beliefs or trust can persist over time.<sup>4</sup> Our model also relates to a broader literature on misspecified learning that has recently blossomed, but these models typically do not feature agents who learn underlying states and signal distributions at the same time.<sup>5</sup>

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<sup>2</sup>There are many models wherein initial differences in the interpretation of signals can generate belief polarization (e.g., Dixit and Weibull 2007; Andreoni and Mylovanov 2012; Kondor 2012; Glaeser and Sunstein 2014; Benoit and Dubra 2019). These papers are motivated by a large number of experimental studies (e.g., Lord, Ross, and Lepper 1979) showing that the beliefs of subjects polarized after the presentation of new evidence.

<sup>3</sup>Relatedly, studies have shown that disagreements can persist in models of opinion dynamics in social networks with non-Bayesian learning rules (e.g., Degroot 1974; DeMarzo, Vayanos, and Zwiebel 2003; Golub and Jackson 2010, 2012). Bayesian individuals that only observe posterior beliefs or actions of other individuals may also fail to learn underlying states because they are able only to recall or communicate coarse information (e.g., Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992).

<sup>4</sup>A large empirical literature studies the relationship between media markets and political polarization (Glaeser and Ward 2006; McCarty, Poole, and Rosenthal 2006; Campante and Hojman 2013; Prior 2013). A growing experimental literature studies the link between trust in information sources and political beliefs (Levendusky 2013; Nisbet, Cooper, and Garrett 2015; Benedictis-Kessner et al. 2019; Thaler 2020; Jo 2020).

<sup>5</sup>Berk (1966) provides a general statement that beliefs need not converge in the long run under misspecified learning. Heidhues, Kőszegi, and Strack (2018) study an overconfident agent who becomes misdirected away from the optimal action as she learns about a fundamental. Heidhues, Kőszegi, and Strack (2019) study a model in which agents are overconfident in their own abilities and form disfavored beliefs about the abilities of the members of

The paper proceeds as follows. Section 2 describes the model. Section 3 presents results on trust and polarization. Section 4 analyzes ideology and perceived bias. Section 5 allows for overconfidence. Section 6 considers the multi-homing case. Section 7 presents extensions. Section 8 concludes.

## 2 Model

### 2.1 Setup

Each agent  $i$  learns about a sequence of unobservable states  $\omega_t \sim N(0, 1)$  over time periods  $t = 1, 2, \dots, T$ . Agents may observe signals (i.e., random variables)  $s_{jt}$  from information sources  $j = 1, \dots, J$ , such as media outlets or talkative neighbors. There is also a random variable  $x_{it}$ , which represents information arising from agent  $i$ 's independent *reasoning* about  $\omega_t$  based on logic, introspection, direct experience, or other factors unrelated to  $s_{jt}$ .

In each period, each agent observes her own reasoning  $x_{it}$  followed by at least one signal  $s_{jt}$  and possibly also the ideological valence  $r_t$ . Our baseline analysis focuses on the *single-homing* case, where agents choose one source to observe in each period to minimize their cumulative expected loss over all periods. We later study the *multi-homing* case, where all agents observe all available sources in every period. After observing the signal(s) and her own reasoning, an agent chooses an action  $d_{it} \in \mathbb{R}$ . At the end of the game, the agent receives loss equal to  $\sum_{t=1}^T (d_{it} - \omega_t)^2$ . The true value of  $\omega_t$  is never observed.

The ideological valence  $r_t$  is the conservative position on  $\omega_t$ . We interpret this as the value of  $\omega_t$  most consistent with conservative ideology—e.g., the level of mask effectiveness, election fraud, or global warming severity that a representative conservative ideologue would argue in favor of. In some cases, one could also think of it as the action  $d_{it}$  that a conservative ideologue would want to persuade an agent to take. The liberal position on  $\omega_t$  is  $-r_t$ . In many cases, we expect agents to be aware of how issues like masks and election fraud map to ideology, so we assume that  $r_t$  is observed (formally, at the same time as signals  $s_{jt}$ ). We will also consider the case where  $r_t$

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out-groups from observing signals that are influenced by both individual ability and group-level discrimination. Frick, Iijima, and Ishii (2020) study social learning with misspecification and show that small misspecification can lead to extreme failures of learning. Bohren and Hauser (2021) provide a general framework for the analysis of learning in misspecified models.

is unobserved. This simplifies the analysis in some ways, and may be appropriate for issues like interest rate policy, GATT negotiations, or foreign policy where less politically engaged agents may not know political parties' current stances.

Together,  $\omega_t$ ,  $r_t$ ,  $x_{it}$ , and the  $J$ -vector  $s_t$  of  $s_{jt}$  are jointly normal and are drawn independently over time:

$$\begin{bmatrix} \omega_t \\ r_t \\ x_{it} \\ s_t \end{bmatrix} \sim N(0, \Omega_i)$$

where  $\Omega_{i11} = 1$  and the other elements of  $\Omega_i$  are free parameters.<sup>6</sup>

We define the *accuracy*  $\alpha_j$  of signal  $j$  to be the correlation of  $s_{jt}$  with  $\omega_t$ . Note that this correlation is a sufficient statistic for the value of observing  $s_{jt}$  to an agent who knows  $\Omega_i$  and seeks to learn about  $\omega_t$  because we can rescale all of the variables to have variance one without changing the agent's posterior beliefs. Accuracy will be high when the covariance of  $s_{jt}$  and  $\omega_t$  is large and/or when the variance of the noise in  $s_{jt}$  orthogonal to  $\omega_t$  is low. A signal with higher accuracy  $\alpha_j \geq 0$  has higher precision conditional on  $\omega_t$  in the usual statistical sense, and is more informative according to the Blackwell (1953) ordering.<sup>7</sup>

We define the accuracy  $a_i$  of each agent's reasoning analogously to be the correlation of  $x_{it}$  with  $\omega_t$ . We are primarily interested in the case where the value of  $a_i$  is small, so that the agents' ability to learn  $\omega_t$  through independent reasoning alone is limited, and substantially less than what she could potentially learn through media or other information sources  $s_{jt}$ .

To define bias, let  $\tilde{r}_t$  denote the standardized residual from a regression of  $r_t$  on  $\omega_t$ . We will focus throughout on the case where in fact  $r_t$  is independent of  $\omega_t$ , so  $\tilde{r}_t = r_t / \sqrt{\text{Var}(r_t)}$ . However, for reasons that will become clear below, we will want to allow agents to entertain the possibility

<sup>6</sup>The assumption that  $r_t$ ,  $x_t$ , and  $s_t$  have zero mean is purely to simplify exposition. If the means of these variables were free parameters, and agents' priors on these parameters had full-support, agents would learn the true value of these parameters in the limit as  $t \rightarrow \infty$ . Agents' beliefs about other parameters, and thus the inferences that are relevant for our results, would be unchanged. See Appendix B for details.

<sup>7</sup>To see the first point, note that, conditional on  $\omega_t$ , the random variable  $s_{jt} / [\alpha_j \sqrt{\text{Var}(s_{jt})}]$  is distributed normally with mean  $\omega_t$  and variance  $\frac{1}{\alpha_j^2} - 1$ . Thus, for a decision maker who knows the values of  $\alpha_j$  and  $\text{Var}(s_{jt})$ , observing  $s_{jt}$  is equivalent to observing a normal variable with mean  $\omega_t$  and precision (defined in the usual statistical sense of the inverse variance) increasing in  $\alpha_j$ . To see the second point, note that  $\alpha_j > \alpha_k \geq 0$  implies  $s_{kt}$  is a garbling of  $s_{jt}$ , since we can produce a variable  $s_{kt}^*$  with the same conditional distribution as  $s_{kt}$  by rescaling  $s_{jt}$  and adding random noise orthogonal to  $\omega_t$ .

that  $\omega_t$  and  $r_t$  are correlated. We let  $\gamma$  denote this correlation. In the case where they are correlated, we would not want to say that a signal is biased if it is correlated with  $r_t$  solely through its correlation with  $\omega_t$ . Thus, we define the *bias*  $\beta_j$  of signal  $j$  to be the correlation of  $s_{jt}$  with  $\tilde{r}_t$ . We define the bias  $b_i$  of  $x_{it}$  to be its correlation with  $\tilde{r}_t$ . For ease of exposition, we say that a source  $j$  is *perfectly right-biased* if  $\beta_j = 1$ , *perfectly left-biased* if  $\beta_j = -1$ , and *perfectly accurate* if  $\alpha_j = 1$ . When  $b_i$  and  $\beta_j$  have the same sign, we say that agent  $i$  and source  $j$  are *like-minded*. When they have the opposite sign, we say agent  $i$  and source  $j$  are *opposite-minded*.

We further assume that  $a_i > 0$ , so that  $x_{it}$  is always positively correlated with  $\omega_t$ , that  $x_{it}$  is independent of  $s_t$  conditional on  $\omega_t$  and  $r_t$ , and that both of these restrictions are known by the agents. As shown in Remark 1 below, the conditional independence of  $x_t$  and  $s_t$  implies that the correlations between the observable variables, namely  $r_t$ ,  $x_{it}$ , and  $s_t$ , is fully determined by  $(a_i, b_i, \alpha, \beta, \gamma, \Sigma)$ , where  $\gamma$  is the correlation of  $r_t$  with  $\omega_t$ ,  $\Sigma = \text{corr}(s_t)$ , and  $\alpha$  and  $\beta$  denote the  $J$ -vectors of  $\alpha_j$  and  $\beta_j$  respectively. The independence of  $\omega_t$  and  $\tilde{r}_t$  implies that  $a_i^2 + b_i^2 \leq 1$  and  $\alpha_j^2 + \beta_j^2 \leq 1$ .

Note that in the simple case where the variances of  $x_{it}$  and the elements of  $s_t$  are all one, our setup implies that we can write

$$s_{jt} = \alpha_j \omega_t + \beta_j \tilde{r}_t + \varepsilon_{jt},$$

$$x_{it} = a_i \omega_t + b_i \tilde{r}_t + \eta_{it},$$

for  $\varepsilon_{jt} \sim N(0, 1 - \alpha_j^2 - \beta_j^2)$  and  $\eta_{it} \sim N(0, 1 - a_i^2 - b_i^2)$  residuals orthogonal to both  $\omega_t$  and  $\tilde{r}_t$ .<sup>8</sup>

In the single homing case, each agent chooses a source  $j$  to observe in each period to minimize expected loss. This choice is akin to a multi-arm bandit problem. However, unlike the standard bandit problem, the payoffs from each period are not observed immediately, and the agent observes auxiliary information in the form of  $x_{it}$ . This renders standard solutions to the bandit problem inapplicable. To tractably capture the tradeoff between exploration and exploitation, we assume that all agents follow an  $\varepsilon$ -first (sometimes called an “explore-first”) strategy (Slivkins 2019).<sup>9</sup>

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<sup>8</sup>When the variances are not one, these same expressions hold with  $s_{jt}$  replaced by  $s_{jt}/\sqrt{\text{Var}(s_{jt})}$  and  $x_{it}$  replaced by  $x_{it}/\sqrt{\text{Var}(x_{it})}$ .

<sup>9</sup>This strategy is not optimal but it dramatically increases the tractability of our model by separating the characterization of long-run beliefs from the endogenous choice of which source to observe. A more optimal strategy would incorporate information from sequentially observed signals into the dynamic choices of what sources to observe by

Table 1: Variables and Parameters

$\omega_t$	True state
$r_t$	Ideological valence
$x_{it}$	Agent $i$ 's reasoning
$s_t$	Vector of signals $s_{jt}$ from information sources
$a_i$	Correlation of $x_{it}$ with $\omega_t$
$b_i$	Correlation of $x_{it}$ with the residual of a regression of $r_t$ on $\omega_t$
$\alpha$	Correlation of $s_t$ with $\omega_t$
$\beta$	Correlation of $s_t$ with the residual of a regression of $r_t$ on $\omega_t$
$\gamma$	Correlation of $r_t$ with $\omega_t$
$\Sigma$	Correlation matrix of $s_t$

That is, the agents each observe a random source with uniform probability during the first  $\varepsilon T$  “exploration” periods, where  $\varepsilon \in (0, 1)$ . In the remaining periods, the agent chooses the source that minimizes expected loss, subject to the restriction that she chooses a source for which the posterior mean on  $\alpha_j$  is weakly positive provided such a source is available.<sup>10</sup>

We are interested in agent  $i$ 's beliefs about  $\theta_i = (a_i, b_i, \alpha, \beta, \gamma, \Sigma)$  and  $\omega_t$  during the exploitation phase in the limit where the number of periods is large, so  $T \rightarrow \infty$ . The true value of  $\theta_i$  is denoted  $\theta_{0i}$ , and where it adds clarity we use  $a_{0i}$ ,  $b_{0i}$ ,  $\alpha_{0j}$ ,  $\beta_{0j}$ ,  $\gamma_0$ , and  $\Sigma_0$  to refer to the true values of the individual components. The set of all possible  $\theta_i$  is denoted as  $\Theta_i = \Theta$ . In our main case of interest, the value of  $a_{0i}$  is small and  $\gamma_0 = 0$ .

The correlation matrix for  $(\omega_t, r_t, x_{it}, s_t)$ , which we denote as  $\tilde{\Omega}_i$ , is fully parametrized by  $\theta_i$ .<sup>11</sup> The covariance matrix is thus  $\Omega_i = V_i^{\frac{1}{2}} \tilde{\Omega}_i V_i^{\frac{1}{2}}$ , where  $V_i$  is a diagonal matrix containing the variances of  $(\omega_t, r_t, x_{it}, s_t)$ . We assume that the set of all possible variances  $V_i$  is a compact set  $\mathcal{V}$ . The

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trading off the gains from exploration and exploitation. However, in the limit as  $T \rightarrow \infty$ , it will be in the agent's interest to learn the accuracies as precisely as possible, so we expect long-run beliefs under the epsilon-first strategy to be the same as under a more optimal strategy.

<sup>10</sup>The restriction rules out an agent choosing to observe sources she believes are negatively correlated with the truth over one she believes is positively correlated with the truth; such a strategy could improve the accuracy of beliefs in principle, but we see it as unrealistic from a behavioral point of view. We relax this assumption in Appendix E and show that most of our results are robust to dropping this restriction.

<sup>11</sup>Specifically, given any  $\theta_i$ , the correlation matrix for  $(\omega_t, r_t, x_{it}, s_t)$  is given by

$$\tilde{\Omega}_i(\theta_i) = \begin{bmatrix} 1 & \gamma & a_i & \alpha' \\ \gamma & 1 & a_i\gamma + b_i\sqrt{1-\gamma^2} & \alpha'\gamma + \beta'\sqrt{1-\gamma^2} \\ a_i & a_i\gamma + b_i\sqrt{1-\gamma^2} & 1 & a_i\alpha' + b_i\beta' \\ \alpha & \alpha\gamma + \beta\sqrt{1-\gamma^2} & a_i\alpha + b_i\beta & \Sigma \end{bmatrix}.$$

Note then that  $\Theta$  is the set of all  $\theta \in (0, 1] \times [-1, 1]^{2J+2} \times [-1, 1]^J$  such that  $\tilde{\Omega}(\theta)$  is positive semi-definite and the diagonal entries of  $\Sigma$  are equal to one.

Lebesgue space on  $(\Theta, \mathcal{V})$  is denoted  $((\Theta, \mathcal{V}), \mathcal{L}, v)$ .

**Example. (Masks)** In period  $t$ , the state  $\omega_t$  indexes the effectiveness of masks at preventing the transmission of disease during a pandemic.<sup>12</sup> The ideological valence  $r_t$  is negative: conservatives believe masks are relatively ineffective, while liberals believe they are relatively effective. Note that it is natural to think of the magnitude  $|r_t|$  as a finite number—neither group typically claims that masks are *completely* effective or ineffective. The signals  $s_{jt}$  could include reports on news or social media about scientific studies of mask effectiveness, anecdotes where people caught the disease despite wearing a mask, and so on. The true accuracy of these sources is captured by  $\alpha_j$ . Right-biased sources (with  $\beta_j > 0$ ) might distort information in a negative direction by down-playing evidence that masks are effective (causing the value of  $s_{jt}$  to be pulled closer to  $r_t < 0$ ); left-biased news reports might do the opposite (causing the value of  $s_{jt}$  to be pulled closer to  $-r_t > 0$ ). Reasoning  $x_{it}$  could be information from logical inference, e.g., about the likelihood that masks fully block disease-carrying droplets, and direct observation, e.g., of people in the agents' own social network who either did or did not wear masks and may or may not have caught the disease. The parameter  $a_i$  governs the accuracy of this reasoning, and it might depend on factors like the agent's knowledge of science. The parameter  $b_i$  governs the extent to which the agent's reasoning is systematically distorted toward either the liberal or the conservative position on mask effectiveness.

**Example. (Election)** In period  $t$ , the state  $\omega_t$  indexes the extent of fraud in a recent presidential election. The ideological valence  $r_t$  is positive: conservatives believe fraud was high while liberals believe fraud was low. It is again natural to think of the magnitude  $|r_t|$  as a finite number—neither group claims that *all* votes in the election were fraudulent nor that *no* votes in the election were fraudulent. The signals  $s_{jt}$  could include reports on news or social media about scientific studies of voter fraud, court cases challenging election outcomes, claims made by politicians, and so on. The true accuracy of these sources is captured by  $\alpha_j$ . Right-biased sources (with  $\beta_j > 0$ ) might distort information in a positive direction by emphasizing anecdotes suggesting fraud (causing the value of  $s_{jt}$  to be pulled closer to  $r_t > 0$ ); left-biased news reports might do the opposite by emphasizing

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<sup>12</sup>It might be more intuitive to think of the effectiveness of masks as a bounded random variable. Note that we can accommodate this by interpreting effectiveness as  $\exp[c_0 + c_1 \omega_t] / (1 + \exp[c_0 + c_1 \omega_t])$  so it has support on  $[0, 1]$  and the arbitrary constants  $c_0$  and  $c_1$  determine the location and scale of the distribution of effectiveness within the  $[0, 1]$  interval.

evidence that fraud is rare (causing the value of  $s_{jt}$  to be pulled closer to  $-r_t < 0$ ). Reasoning  $x_{it}$  could be information from logical inference, e.g., about the likelihood that widespread election fraud would be detected, and direct observation, e.g., of the way polling places, mail-in ballots, etc. were administered in the agents' own neighborhood. The parameter  $a_i$  governs the accuracy of this reasoning, and it might depend on factors like the extent to which the agent knows details of the vote-counting process. The parameter  $b_i$  governs the extent to which the agent's reasoning is systematically distorted toward either the liberal or the conservative position on election fraud.

## 2.2 Learning

At the beginning of the first period, agents have an absolutely continuous prior belief  $\mu_0^i$  over  $(\Theta, \mathcal{V})$  with a continuous density with respect to  $v$ . For tractability, we assume that  $\theta_i$  is independent of  $V_i$  under each agent's prior and we let  $\mu_{0,\theta}^i$  and  $\mu_{0,V}^i$  respectively denote agent  $i$ 's prior distribution on  $\theta_i$  and  $V_i$ . Our central assumption is that all  $\theta_i$  in the support of  $\mu_{0,\theta}^i$  have  $b_i = 0$ . In other words, all agents *a priori* believe that their own reasoning  $x_{it}$  is unbiased, even though their true bias may be nonzero. In Appendix C, we show that our main results extend to the case where agents believe their own bias to be  $b_i = b^*$  for some small  $b^* \neq 0$ .

All  $\theta_i$  in the support of  $\mu_{0,\theta}^i$  also have  $a_i \in \mathcal{A}_i$  for some set  $\mathcal{A}_i$ . In our baseline analysis, we assume that agents know the accuracy of their own reasoning, so  $\mathcal{A}_i = \{a_0\}$ . We then extend the model to allow for the possibility that agents could become overconfident. In this case,  $\mathcal{A}_i = (0, a_i^{max}]$  for some  $a_i^{max} \geq a_0$ . We will assume that  $\mu_{0,\theta}^i$  has full support on the subset  $\Theta_i^{prior}$  of  $\Theta$  consistent with the restrictions that  $b_i = 0$  and  $a_i \in \mathcal{A}_i$ .

In our baseline case, agent  $i$  observes both  $x_{it}$  and  $s_{jt}$  for each  $j$  in many periods, and so, in the limit as  $t \rightarrow \infty$ , she learns the  $J$ -vector  $\rho_{is}$  of correlations between  $x_{it}$  and the elements of  $s_t$ . In the case where  $r_t$  is observed, agents also learn the  $J$ -vector  $\rho_{rs}$  of correlations between  $r_t$  and the elements of  $s_t$  and the correlation  $\rho_{ir}$  of  $x_{it}$  and  $r_t$ . Finally, in the case of multi-homing, agents observe all sources in each period, so they additionally learn the correlation matrix  $\Sigma_0$  of the elements of  $s_t$ . We abuse notation and let  $R_{0i}$  denote the true value of the vector of correlations that can be learned by agent  $i$ — $(\rho_{is})$ ,  $(\rho_{is}, \rho_{rs}, \rho_{ir})$ , or  $(\rho_{is}, \rho_{rs}, \rho_{ir}, \Sigma_0)$  depending on the case considered—and we let  $R_i(\theta_i)$  denote the observable subset of the correlations  $\tilde{\Omega}_i$  that would occur under parameters  $\theta_i$ .

Our first result defines agent  $i$ 's limiting beliefs about  $\theta_i$ . We define the *identified set* of parameters in the support of agent  $i$ 's prior that is consistent with correlations  $R_{0i}$  as  $I_i(R_{0i}) = \left\{ \theta_i \in \Theta_i^{prior} : R_i(\theta_i) = R_{0i} \right\}$ . The following proposition shows that agent  $i$ 's beliefs converge asymptotically to this identified set. Because all  $\theta_i$  in the identified set imply the same distribution of observed data, beliefs within the set remain proportional to the prior.

**Proposition 1.** *Suppose the true correlations of the observed data are  $R_{0i}$ . As  $T \rightarrow \infty$ , agent  $i$ 's posterior distribution in any period  $t > \varepsilon T$  converges to a limit  $\mu_{\infty, \theta}^i$  such that for all measurable  $\vartheta \subseteq \mathcal{L}_\Theta$ ,*

$$\mu_{\infty, \theta}^i(\vartheta) = \frac{\mu_{0, \theta}^i(\vartheta \cap I_i(R_{0i}))}{\mu_{0, \theta}^i(I_i(R_{0i}))}.$$

*Proof.* All proofs are in Appendix A.  $\square$

The second result derives the agents' posterior expectations of  $\omega_t$  given their limiting beliefs about  $\theta_i$ . For tractability, we analyze the belief an agent would hold after observing information source(s) but not her reasoning  $x_{it}$  or the ideological valence  $r_t$ . This is similar to analyzing the case where the accuracy  $a_i$  of the agent's reasoning is close to zero and the agents are aware of this. We denote the mean of the agent's posterior at this *ex interim* stage by  $\bar{\omega}_t^i$ . We define agent  $i$ 's expectation of  $\alpha_j$  under  $\mu_{\infty, \theta}^i$ , which we denoted as  $\bar{\alpha}_j^i$ , as agent  $i$ 's *trust* in source  $j$ . The following characterization follows from standard conjugate prior results for the normal distribution.

**Proposition 2.** *Suppose agent  $i$  single homes and her posterior belief on  $\theta_i$  is  $\mu_{\infty, \theta}^i$ . In any period  $t$  where she observes source  $j$ , the mean of her posterior on  $\omega_t$  given  $s_{jt}$  (but not  $x_{it}$  or  $r_t$ ) is*

$$\bar{\omega}_t^i(s_{jt}) = \bar{\alpha}_j^i \tilde{s}_{jt}, \quad (1)$$

where  $\tilde{s}_{jt} = s_{jt} / \sqrt{\text{Var}(s_{jt})}$  is the standardized version of  $s_{jt}$ .

This proposition shows that the mean of agent  $i$ 's posterior on  $\omega_t$  will be proportional to the normalized value of the signal  $s_{jt}$  that she observes, with the constant of proportionality equal to her trust  $\bar{\alpha}_j^i$  in source  $j$ . When the agent believes source  $j$  is unreliable ( $\bar{\alpha}_j^i \approx 0$ ), her beliefs about the true state do not vary much with the content of source  $j$ , and they stay concentrated close to the prior value of zero. When the agent's trust in source  $j$  is high ( $\bar{\alpha}_j^i \approx 1$ ), her beliefs track the content of the source closely, and their variance approaches 1.

Our third result shows that given her limiting belief about  $\theta_i$ , a single-homing agent observes a source that she believes to be most accurate. To be precise, let  $\mathcal{J}_+^i$  denote the set of sources  $j$  such that  $\bar{\alpha}_j^i \geq 0$ , which we assume here to be non-empty. The agent chooses to observe the source from this set that minimizes her expected mean squared error after observing both the source and her own reasoning:<sup>13</sup>

$$\min_{j \in \mathcal{J}_+^i} \mathbb{E} \left[ (d_{ijt}^*(s_{jt}, x_{it}) - \omega_t)^2 \right],$$

where the expectation is taken under the marginal distribution of  $(s_{jt}, x_{it}, \omega_t)$  under  $\mu_{\infty}^i$  and  $d_{ijt}^*(s_{jt}, x_{it})$  is the optimal decision after seeing  $(s_{jt}, x_{it})$ .<sup>14</sup>

**Proposition 3.** *Suppose agent  $i$  single-homes, her posterior belief on  $\theta_i$  is  $\mu_{\infty, \theta}^i$ , and the expected accuracy  $a_i$  of her own reasoning under  $\mu_{\infty, \theta}^i$  is less than one. Then she chooses to observe in each period a source  $j$  for whom her trust  $\bar{\alpha}_j^i$  is maximal (provided some source has  $\bar{\alpha}_j^i \geq 0$ ).*

Proposition 3 shows that, setting aside the degenerate case where an agent believes her own reasoning to be perfectly accurate, she converges to always getting information from the source she trusts most. This result is not entirely trivial. The agent's choice of  $j$  is equivalent to minimizing posterior variance integrating over both the uncertainty in  $(s_{jt}, x_{it}, \omega_t)$  given  $\Omega_i$  and the uncertainty in  $\Omega_i$ . If we were only integrating over the former, this would be a standard case of choosing among signals with known distributions and it would be optimal to choose the one with the highest  $\alpha_j$ . Once there is uncertainty over  $\Omega_i$ , it may not be obvious that the agent would choose the one with the highest expected  $\alpha_j$ , since the whole distribution of the agent's belief about  $\alpha_j$  (and other parameters) could in principle matter for the expected loss. Our proof exploits the fact that the optimal decision rule depends linearly on  $\alpha_j$ .

## 2.3 Benchmark Cases

As shown above, information about source  $j$ 's accuracy  $\alpha_j$  allows the agents to learn about the underlying state  $\omega_t$  from observing  $s_{jt}$ . As shown below in Section 3, agents rely on the observed correlation of reasoning  $x_{it}$  and signal  $s_{jt}$  to learn  $\alpha_j$ . In this section, we discuss benchmark cases

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<sup>13</sup>Here it is unimportant whether the agent observes  $x_{it}$ . The same source  $j$  is chosen whether the agent minimizes her mean squared error after observing both  $x_{it}$  and  $s_{jt}$  or after observing  $s_{jt}$  but not  $x_{it}$ .

<sup>14</sup>If the restriction that  $j \in \mathcal{J}_+^i$  were dropped, the agent instead chooses to observe a source  $j$  for whom  $|\bar{\alpha}_j^i|$  is maximal. In Appendix E, we show that all of our results in subsequent sections are robust to dropping this restriction.

where either no signal or no reasoning is available. In these cases, agents do not endogenously learn  $\alpha_j$ , and small ideological biases in the agents' reasoning are not amplified into large disagreements. For simplicity, we assume here that there is a single source ( $J = 1$ ), agents know the accuracy of their own reasoning ( $\mathcal{A}_i = \{a_0\}$ ), and  $r_t$  is never observed.

## No Signals

First, suppose that agent  $i$  observes only her reasoning  $x_{it}$  in every period, but never observes any source  $j$  or the ideological valence  $r_t$ . In the main case of interest where the accuracy of her reasoning  $a_{0i}$  is small, her posterior beliefs will always be noisy. To see this, assume for the moment that  $x_{it}$  is known to have variance equal to one, and that agent  $i$  knows the accuracy of their own reasoning, so  $\mathcal{A}_i = \{a_{0i}\}$ . Then her posterior mean on  $\omega_t$  after observing  $x_{it}$  is  $\bar{\omega}_t^i = a_{0i}x_{it} = a_{0i}(a_{0i}\omega_t + \eta_{it}) = a_{0i}^2\omega_t + a_{0i}\eta_{it}$ , where  $\eta_t$  is an error term that is normally distributed with mean zero and variance  $1 - a_{0i}^2$ . When  $a_{0i}$  is small,  $\bar{\omega}_t^i$  is a very noisy estimate of  $\omega_t$ .

## No Reasoning; Exogenous Trust

Next, suppose agent  $i$  observes a single source  $j$  every period but never observes her reasoning  $x_{it}$  or the ideological valence  $r_t$ . In this case, her beliefs about the source's accuracy  $\alpha_j$  never change. The mean of her posterior belief about  $\omega_t$  after observing  $s_{jt}$  in any period  $t$  is  $\bar{\omega}_t^i = \bar{\alpha}_j^0 s_{jt}$ , where  $\bar{\alpha}_j^0$  is the expectation of the source's accuracy under the agent's prior  $\mu_0$ .

If  $\bar{\alpha}_j^0$  is zero—roughly speaking, agents think the source is as likely to be a perverse “false news” source as an accurate “true news” source—then agent  $i$ 's posterior belief about  $\omega_t$  would be zero in all periods, no matter what value of  $s_{jt}$  is realized. She would therefore learn nothing from observing  $s_{jt}$ , even if  $s_{jt}$  is in fact highly accurate.

If instead  $\bar{\alpha}_j^0 = \alpha_{0j}$ , so the agents exogenously know the true accuracy of signals, then their posterior belief about  $\omega_t$  will be the same as that of a Bayesian. Furthermore, if source  $j$  is highly accurate,  $\bar{\omega}_t^i$  will be close to the true value  $\omega_t$ . This will be true regardless of the agent's bias  $b_{0i}$  or the source's bias  $\beta_{0j}$ .

A closely related benchmark is one where agent  $i$  does observe  $x_{it}$  but trust is exogenous in the sense that the agent's prior on  $\alpha_j$  is degenerate at  $\bar{\alpha}_j^0$ . Updating from  $s_{jt}$  in this case will be identical to that when  $x_{it}$  is not observed. The only difference is that agent  $i$ 's beliefs about  $\omega_t$

would also be affected by the realization of  $x_{it}$ . In the main case of interest where the accuracy  $a_{0i}$  of  $x_{it}$  is low and the agents know that it is low, the effect of  $x_{it}$  on beliefs will be small, and it will remain true that beliefs do not depend on bias. In this sense, endogenous trust is necessary in our model for bias to produce polarization.

## Towards the Full Model

Now consider an agent  $i$  who observes both  $x_{it}$  and  $s_{jt}$  in every period and whose prior mean is  $\bar{\alpha}_j^0 = 0$ . In this case, her reasoning  $x_{it}$  functions as a reference point that allows her to learn the accuracy of source  $j$ . To see this, note that the agent believes that (1)  $x_{it}$  is positively correlated with  $\omega_t$  (since  $a_i > 0$  with probability one under  $\mu_0^i$ ) and (2)  $x_{it}$  is uncorrelated with  $\tilde{r}_t$  (since  $b_i = 0$  with probability one under  $\mu_0^i$ ). This means that she expects the correlation between  $x_{it}$  and  $s_{jt}$  to be increasing in the source's accuracy  $\alpha_j$ . If she observed a more positive correlation between  $x_{it}$  and  $s_{jt}$  than she would expect under her prior, then she would endogenously revise her estimate of the source's accuracy upward. Having learned about source  $j$ 's accuracy, the agent can then learn about  $\omega_t$  from observing  $s_{jt}$ .

If agent  $i$ 's reasoning is unbiased, her posterior belief on  $\alpha_j$  converges to the true value  $\alpha_{0j}$  as  $t \rightarrow \infty$ . If the source was highly accurate (i.e.,  $\alpha_{0j}$  was equal to one), then in the limit as  $t \rightarrow \infty$ , her posterior mean  $\bar{\omega}_t^i$  approaches the true value  $\omega_t$ . In other words, having access to noisy but unbiased reasoning allows an agent to accurately learn which sources to trust by aggregating over many periods. Unbiased agents can thereby learn the underlying states of the world as if they exogenously knew the accuracy of the information sources and therefore much more precisely than if they had access to only her reasoning. However, as we show in the following sections, endogenous trust can lead small biases in reasoning to become amplified into large disagreements.

## 2.4 Discussion: Reasoning and Bias

What does  $x_{it}$  represent in the real world? As a baseline case, we can think of  $x_{it}$  as information that results from the agent's reasoning and introspection about the likely value of  $\omega_t$ .<sup>15</sup> In the examples above, this could involve thinking about the likelihood that masks fully block disease-

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<sup>15</sup>Note that in this interpretation we do not think of  $x_{it}$  as the agent's posterior belief about the state, but rather as the information arising from reasoning and introspection that produces that posterior belief.

carrying droplets or that widespread election fraud is detected. If  $\omega_t$  relates to economic stimulus policy, agents might reason from first principles about how large the plausible costs and benefits could be. (They might even write down and solve a model!) In writing the result of this reasoning and introspection as a random variable, we think of engaging in reasoning and introspection as analogous to observing the results of a noisy experiment, just as a student trying to solve a math problem in their head will produce a result that is imperfectly correlated with the true value.

There are a large number of well-studied psychological phenomena that could provide a micro-foundation for bias in agents' reasoning and introspection (i.e.,  $b_{0i} \neq 0$ ). Consider an agent who has grown up in a liberal family, benefitted from liberal policies, and taken actions (like voting) consistent with liberal ideology. Such an agent may engage in motivated reasoning (Kunda 1990), distorting her inferences to reach conclusions that support a liberal point of view. She may under-weight arguments or evidence pointing in the conservative direction in order to reduce cognitive dissonance (Festinger 1957). She may be more likely to remember evidence consistent with a liberal view (Eagly et al. 1999). She may tilt her assessment of the credibility of evidence due to confirmation bias (Lord, Ross, and Lepper 1979). She may also live in an environment in which information that supports her position is more "available" in the sense of Tversky and Kahneman (1973) – for example, if she had gone to high quality public schools, she may find it easier to think of the benefits of teachers' unions than their costs. Finally, it may be that evidence that supports the liberal position is more salient in the sense of Bordalo, Gennaioli, and Shleifer (2012). Note that these latter two mechanisms could produce bias even in the case where the agent does not know the value of  $r_t$ —e.g., she might be more likely to reach conclusions favoring teachers' unions even if she is unaware that this is a position typically supported by the Democratic party.

A large body of evidence suggests that individuals themselves are not aware of these kinds of biases or at a minimum significantly underestimate them (e.g., Pronin et al. 2002; Pronin 2007; Thaler 2022).

Another possibility is that  $x_{it}$  captures information about the true value of the state  $\omega_t$  that the agent observes directly. This could be mask behavior or election procedures as in the examples above. It could capture weather events in the agents' locality (when  $\omega_t$  relates to global warming), the agents' own experiences with public schools (when  $\omega_t$  relates to education policy), or the agent's personal economic situation (when  $\omega_t$  relates to economic policy). Such information could

be observed either before or after seeing  $s_{jt}$ . Bias in these observations could arise through many of the same mechanisms as bias in reasoning. For example, an agent from a liberal background might be more likely to remember unusually hot days or unusually severe storms that suggest global warming is severe (Eagly et al. 1999). It could also be that certain kinds of direct observation tend to favor one side or the other inherently and agents fail to adjust for this selection. For example, observing the effort, wages, and impacts of public school teachers may naturally provide more evidence favorable to teachers' unions, and observing polling places where voting fraud is unlikely to be visible might naturally provide more evidence favorable to the view that elections are conducted fairly.

A final possibility is that  $x_{it}$  is the signal of a particular information source that agents believe *a priori* to be unbiased. This might be what their mothers say, or what the Bible says, or what scientists say, or even the report of a particular news source that they begin with extraordinary faith in.

What is crucial for our model is that  $x_{it}$  satisfies two conditions. First, agents believe that it is free from ideological bias ( $b = 0$ ) with probability one. Second, contrary to the agents' beliefs,  $x_{it}$  may in fact be subject to ideological bias ( $b_{0i} \neq 0$ ).

The ultimate source of the agents' biases is left unmodeled in our framework— $b_{0i}$  is an exogenous parameter that we think of as determined by experiences and motivations prior to  $t = 0$ . We show below that non-zero bias leads to distortion in agents' trust in sources and their beliefs about states of the world, and also shapes what we label “ideology”—agent  $i$ 's belief  $\bar{\gamma}^i$  about the correlation between the truth  $\omega_t$  and the ideological valence  $r_t$ . This point might lead to some confusion, as we show how bias can shape ideology, but psychological phenomena like motivated reasoning also provide reasons why ideology would shape bias. A richer model that included this two-way feedback between ideology and bias might well lead to even more dramatic divergence of trust and beliefs. We isolate one direction of causality for analytic tractability, taking bias in reasoning to be a fixed characteristic that may depend on baseline ideology and other traits but does not change as ideology evolves.

It may seem strong to assume that the agent puts such dogmatic *ex ante* faith in her own reasoning or observation. However, if there is *no* source of truth in which she would place significant faith in this sense, the agent would never learn what sources she can trust. More precisely, she

would never be able to reject that any particular source is either perfectly positively correlated, uncorrelated, or perfectly negatively correlated with the state. We demonstrate this result in Appendix F. We also show in Appendix C that our results are robust to allowing agents to entertain the possibility of *small* biases in their own reasoning.

### 3 Trust and Polarization

In this section, we assume there are multiple available sources, so  $J \geq 2$ , but each agent single-homes and so observes only one source in each period. For simplicity, we focus on the case where the ideological valence  $r_t$  is never observed. This means that ideological valence is a latent source of bias in signals and reasoning but is not something agents make inferences from directly. While in reality  $r_t$  is often observed, it is plausible that a politically disengaged person may not know the conservative or liberal positions on a wide range of issues. We also focus on the case where agents know the accuracy  $a_{0i}$  of their own reasoning, so that  $\mathcal{A}_i = \{a_0\}$ . We consider the case where  $r_t$  is observed in Section 4, the case where agents are uncertain about the accuracy of their own reasoning in Section 5, and the case of multi-homing in Section 6.

For ease of exposition, we assume that there are three agents  $i \in \{U, R, L\}$ . All agents have accuracy  $a_{0i} = a_0$ , where  $a_0 > 0$ . Agent  $U$ 's reasoning has no bias, so  $b_{0U} = 0$ . Agent  $R$ 's reasoning has positive bias, while agent  $L$ 's reasoning has negative bias. We set  $b_{0R} = b_0$  and  $b_{0L} = -b_0$ , where  $b_0 > 0$ .

**Assumption 1.** *The support of  $\mu_{0,\theta}^i$  is the set  $\Theta_i^{prior} \subset \Theta$  for which  $b_i = 0$  and  $a_i = a_{0i}$ .*

For agent  $U$ , this is a simple case of Bayesian learning with a correctly specified model. For agents  $R$  and  $L$ , the true parameters of the model lie outside the support of their priors (since  $b_{0i} \neq 0$ ). Our model is thus an example of Bayesian learning under misspecification (Lian 2009).

Because  $r_t$  is not observed, agent  $i$  learns only the correlation  $\rho_{is}$  between  $s_t$  and  $x_{it}$ . This restricts agent  $i$ 's limiting beliefs, since the model requires that  $\rho_{is} = a_i\alpha + b_i\beta$ . The identified set is then  $I_i(R_{0i}) = \left\{ \theta_i \in \Theta_i^{prior} : a_i\alpha + b_i\beta = \rho_{is} \right\}$ .

Misspecified learning can lead to instability or lack of convergence (Berk 1966), but we focus our baseline analysis on parameter values such that the data agents observe do not violate their

model of the world—i.e., the identified set is non-empty. A sufficient condition in the single-homing case is that  $b_0 \leq a_0$  and  $|\alpha_j| + |\beta_j| \leq 1$  for all  $j$ . We relax this assumption when we extend the model to allow overconfidence in Section 5 below.

**Assumption 2.**  $b_0 \leq a_0$  and  $|\alpha_j| + |\beta_j| \leq 1$  for all  $j$ .

We can then show that the identified set  $I_i(R_{0i})$  is non-empty and contains a single value of  $\alpha$ . It does not restrict the values of  $\gamma$  or  $\beta$  because when  $r_t$  is not observed, the observed correlations do not contain information about these parameters.

**Proposition 4.** *Suppose  $r_t$  is never observed and agents single home. Under Assumptions 1 and 2, agent  $i$ 's identified set,  $I_i(R_{0i})$ , is non-empty and consists of all  $\theta_i \in \Theta$  such that  $a_i = a_{0i}$ ,  $b_i = 0$ , and  $\alpha = \frac{\rho_{is}}{a_{0i}}$ .*

Proposition 4 shows that when  $r_t$  is never observed, each agent's beliefs about each source's accuracy is concentrated on  $\alpha_j = \frac{\rho_{ij}}{a_{0j}}$ . The agent believes source  $j$  is more accurate the more correlated source  $j$ 's reports are with  $i$ 's reasoning. For a given level of correlation, the update is larger the less accurate  $i$  believes her own reasoning to be, since when the correlation of  $x_{it}$  with the true state is small,  $x_{it}$  and  $s_{jt}$  can only be substantially correlated if the correlation of  $s_{jt}$  with the true state is large. This result follows from the agent's assumption that her reasoning to be unbiased. Its implications are discussed further below.

## Trust

A large body of evidence shows divergence between the sources trusted by conservatives and the sources trusted by liberals (e.g., Pew Research Center 2014a, 2020), and many have pointed to this as a key factor undermining the media's role in democracy (Gallup and Knight Foundation 2018, 2020). Consistent with this, agents in our model may come to trust biased sources more than unbiased sources. Divergence in trust may be large even when biases are small, provided that the accuracy of the agents' own reasoning is low.

To see this, notice that under Assumptions 1 and 2, Propositions 1 and 4 imply that agent  $i$ 's trust in information source  $j$  is

$$\bar{\alpha}_j^i = \frac{\rho_{ij}}{a_{0i}} = \alpha_{0j} + \frac{b_{0i}\beta_{0j}}{a_{0i}}. \quad (2)$$

Therefore, when the accuracy  $a_{0i}$  of the agents' reasoning is low, small differences in biases  $b_{0i}$  and  $\beta_{0j}$  translate into large differences in trust. Agents come to trust source  $j$  more when their biases are more aligned (i.e.,  $b_{0i}\beta_{0j}$  is positive), and that divergent trust can be extreme even when  $b_{0i}$  is close to zero.

**Corollary 1.** *Suppose  $r_t$  is never observed, agents single home, and Assumptions 1 and 2 hold. Then agent R's (L's) trust in source  $j$  is increasing (decreasing) in the source's bias  $\beta_{0j}$  holding constant the source's accuracy  $\alpha_{0j}$ . In the limit as  $a_0 \downarrow b_0$ , she will come to believe that a perfectly right-biased (left-biased) source is perfectly accurate, and trust it more than any unbiased source with  $\alpha_{0j} < 1$ .*

## Polarization

Substantial literatures document large and growing disagreement between Democrats and Republicans on both policy issues (Pew Research Center 2014b; Boxell, Gentzkow, and Shapiro 2017) and questions of fact (Marietta and Barker 2019). Consistent with these findings, we show that disagreement in our model can be large even when the underlying bias is small and accurate information is widely available.

We focus on the exploitation periods  $t > \varepsilon T$  in the limit as  $T \rightarrow \infty$ , where each agent observes the source  $j$  for which trust is maximal by Proposition 3. We define agents' *expected disagreement* when they observe these sources in such period to be  $\pi = E \left[ \frac{1}{4} (\bar{\omega}_t^R - \bar{\omega}_t^L)^2 \right]$ . Scaling by one-fourth here ensures that  $\pi \in [0, 1]$ .

The expected disagreement of single-homing agents is sensitive to the set of available sources. If all sources are unbiased, expected disagreement will be zero because all agents will come to trust the most accurate source and the trust of  $R$  and  $L$  agents for this source will be identical. However, if highly biased sources are available, disagreement can grow large.

To see this, suppose that all sources have accuracy  $\alpha_{0j} < 1$  and there is one perfectly right-biased source and one perfectly left-biased source. By Corollary 1, agent  $R$ 's trust for the perfectly right-biased source and agent  $L$ 's trust for the perfectly left-biased source are both maximal in the limit as  $a_0 \rightarrow b_0$ . Proposition 3 then implies that each agent observes their like-minded perfectly biased source during the exploitation periods. Expected disagreement then reaches the maximum possible value  $\pi = 1$ .

**Corollary 2.** Suppose  $r_t$  is never observed, agents single home, and Assumptions 1 and 2 hold. Further suppose that all sources have accuracy  $\alpha_{0j} < 1$  and there is at least one perfectly right-biased source and at least one perfectly left-biased source. In the limit as  $a_0 \downarrow b_0$ , expected disagreement is one.

## 4 Ideology and Perceived Bias

We now consider the case where the ideological valence  $r_t$  is observed in every period. Agents are able to learn the vector of correlations  $(\rho_{is}, \rho_{ir}, \rho_{rs})$ . In this case, the results from the previous section are broadly unchanged, but we can additionally derive each agent's beliefs about the correlation of the true states with the ideological valence and the bias of sources.

*Remark 1.* Given parameters  $\theta_i = (a_i, b_i, \alpha, \beta, \gamma, \Sigma)$ , the elements of  $R(\theta_i)$  are given by

$$\begin{aligned}\rho_{is} &= a_i\alpha + b_i\beta \\ \rho_{ir} &= a_i\gamma + b_i\sqrt{1 - \gamma^2} \\ \rho_{rs} &= \alpha\gamma + \beta\sqrt{1 - \gamma^2}.\end{aligned}$$

For simplicity, we continue to focus on parameter values such that the data an agent observes do not violate her model of the world—i.e., the identified set is non-empty. In the current case, this will require  $b_{i0}^2/a_{i0}^2 \leq 1 - \alpha_{0j}^2/(1 - \beta_{0j}^2)$  for all  $i$  and  $j$ . This assumption is more stringent than Assumption 2, because learning the values of  $\rho_{ir}$  and  $\rho_{rs}$  allows agents to reject their models of the world in more cases. We relax this assumption when we extend the model to allow overconfidence in Section 5 below.

**Assumption 2'.**  $b_{i0}^2/a_{i0}^2 \leq 1 - \alpha_{0j}^2/(1 - \beta_{0j}^2)$  for all  $j$ .

Under this assumption, the identified set  $I_i(R_{0i})$  is non-empty.

**Proposition 5.** Suppose  $r_t$  is observed and agents single home. Under Assumptions 1 and 2', agent  $i$ 's identified set,  $I_i(R_{0i})$ , is non-empty and consists of all  $\theta_i \in \Theta$  such that  $a_i = a_{0i}$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ , and  $\beta = \frac{1}{\sqrt{1 - \gamma^2}} (\rho_{rs} - \gamma\alpha)$ .

Comparing Proposition 5 to Proposition 4 reveals three implications of allowing  $r_t$  to be observed: First, the agent's identified set is concentrated on the value  $\gamma = \rho_{ir}/a_i$ . Second, the agent's beliefs about a source's bias  $\beta_j$  is a known function of the observed correlations  $\rho_{ij}$ ,  $\rho_{ir}$ , and  $\rho_{rj}$ . Third, the agent's identified set is still concentrated on the value  $\alpha = \frac{\rho_{is}}{a_i}$  even when  $r_t$  is observed. The implications of these facts are explored below.

## Ideology

A striking feature of political polarization in the US is the significant correlation between citizens' liberal-conservative ideologies and their views across a range of diverse issues (Gentzkow 2016). Consistent with this fact, agents in our model when  $r_t$  is observed may form an *ex ante* conviction that either the conservative or the liberal point of view on *any* issue is likely to be closer to the truth on average. This conviction is captured in agents' limiting beliefs about the correlation  $\gamma$  between  $\omega_t$  and  $r_t$ .

We define an agent's *ideology*  $\bar{\gamma}^i$  to be her limiting posterior mean on  $\gamma$ . We say that agent  $i$ 's ideology is *right-leaning* if  $\bar{\gamma}^i > 0$ . Under Assumptions 1 and 2' and focusing on the case of interest where  $\gamma_0 = 0$ , Proposition 5 implies that agent  $i$ 's ideology is

$$\bar{\gamma}^i = \frac{b_{0i}}{a_0}. \quad (3)$$

Equation (3) implies that if agent  $R$  simply observes the ideological valence  $r_t$ , then her beliefs about  $\omega_t$  become biased toward  $r_t$ , while agent  $L$  comes to believe the opposite. This is because agent  $R$  sees positive correlation between  $r_t$  and  $x_{it}$ , so she comes to believe that conservative views are on average closer to the truth than liberal views.

**Corollary 3.** *Suppose  $r_t$  is observed, agents single home, and Assumptions 1 and 2' hold. Then agent R's (L's) ideology is increasing (decreasing) in her bias  $b_0$ .*

## Perceived Bias

Empirical evidence suggests that most Americans perceive media sources they distrust to be systematically biased (Gallup and Knight Foundation 2018, 2020). Consistent with this, agents in

our model may perceive unbiased sources as biased, and also perceive like-minded sources as less biased than they actually are.

We define an agent's *perceived bias*  $\bar{\beta}_j^i$  of information source  $j$  as the agent  $i$ 's limiting posterior mean on  $\beta_j$ .<sup>16</sup> We say that agent  $i$  perceives a source  $j$  to be *oppositely biased* if  $\text{sign}(\bar{\beta}_j^i) \neq \text{sign}(\bar{\gamma}^i)$ . We say that agent  $i$  perceives a source  $j$  as *less right-biased than it actually is* if  $\bar{\beta}_j^i < \beta_{0j}$ . Proposition 5 implies that:

**Corollary 4.** *Suppose  $r_t$  is observed, agents single home, and Assumptions 1 and 2' hold. Then agents R and L both perceive an unbiased source with  $\alpha_{0j} > 0$  as oppositely biased. They also perceive a like-minded biased source with  $\alpha_{0j} > 0$  as less biased than it actually is.*

### Trust and Polarization when $r_t$ is Observed

Our baseline results on trust (Corollary 1) and polarization (Corollary 2) continue to hold. even when  $r_t$  is observed, by Proposition 5 and since observing  $r_t$  does not change either  $\rho_{is}$  or  $a_i$ . The one proviso is that the limit  $a_0 \downarrow b_0$  can only be consistent with Assumption 2' if  $\alpha_{0j} = 0$  for all  $j$ . When  $a_0$  is bounded away from  $b_0$ , however, it will continue to be the case that trust in perfectly biased sources diverges as  $a_0$  approaches  $b_0$ , and polarization consequently grows large.

## 5 Overconfidence

We now allow for the possibility that agents are uncertain about the accuracy  $a_i$  of their own reasoning. Because this will enable a biased agent to rationalize any signal she might observe via a higher value of  $a_i$ , it allows us to dispense with restrictions on the parameter space (Assumptions 2 and 2'). We continue to assume the ideological valence  $r_t$  is observed in every period.

We now revise Assumption 1 to allow the agent to entertain any  $a_i \in (0, a_i^{max}]$ .

**Assumption 1'.** The support of  $\mu_{0,\theta}^i$  is the set  $\Theta_i^{prior} \subset \Theta$  for which  $b_i = 0$  and  $a_i \in (0, a_i^{max}]$ , where  $a_i^{max} \geq \sqrt{a_0^2 + b_0^2}$ .

Agents' identified sets now may include multiple values of  $a_i$ , and this means they may include multiple values of  $\alpha$ ,  $\beta$ , and  $\gamma$  as well.

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<sup>16</sup>Under Assumptions 1 and 2', the agent's perceived bias of source  $j$  is  $\bar{\beta}_j^i = \beta_{0j} \sqrt{1 - (b_{0i}/a_0)^2} - \frac{a_0 b_{0i} \alpha_{0j}}{a_0^2 \sqrt{1 - (b_{0i}/a_0)^2}}$ .

**Proposition 6.** Suppose  $r_t$  is observed and agents single home. Under Assumption 1', agent  $i$ 's identified set,  $I_i(R_{0i})$ , is non-empty and consists of all  $\theta_i \in \Theta$  such that  $a_i \in [\underline{a}_i, \bar{a}_i^{max}]$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ , and  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , where  $\underline{a}_i = \max_j \sqrt{\zeta_{ij}}$  and  $\zeta_{ij}$  is the population  $R^2$  from a regression of  $x_{it}$  on  $r_t$  and  $s_{jt}$ .

Proposition 6 is similar to Proposition 5, but now the identified set contains a range of values for  $a_i$ , the accuracy of the agent's reasoning. The lower bound  $\underline{a}_i$  depends on the extent to which  $x_{it}$  is correlated with the other data the agent observes. Under the agent's maintained assumption that  $b_i = 0$ , the only way that the variation in  $x_{it}$  can be jointly explained by  $r_t$  and a particular  $s_{jt}$  is if the correlation of  $x_{it}$  with the true state is relatively large. Thus, increasing the maximum  $R^2$  that can be achieved by regressing  $x_{it}$  on  $(r_t, s_{jt})$  increases the lower bound on the possible values of  $a_i$ .

Proposition 6 also shows that an agent's own accuracy  $a_i$  and the accuracy  $\alpha$  of the information sources are not separately identified. Since agents believe that  $b_i = 0$ , we have that  $\rho_{is} = a_i\alpha$  for any  $\theta_i \in I_i(R_{0i})$ . A given value of  $\rho_{is}$  could result from a high value of  $a_i$  and low values of  $\alpha_j$ , or a low value of  $a_i$  and high values of  $\alpha_j$ ; these cannot be distinguished by the observed data. Beliefs within the identified set will therefore remain proportional to the prior.

## Overconfidence

A large literature in psychology, economics, and finance has documented overconfidence in many contexts, with early evidence including the pioneering study of Alpert and Raiffa (1982). Ortoleva and Snowberg (2015) explore in detail the implications of overconfidence for political behavior. While overconfidence is a primitive in their model, the results of this section show that it may arise endogenously as a consequence of other biases in reasoning.<sup>17</sup>

We refer to agent  $i$ 's belief about the accuracy  $a_i$  of her own reasoning as her *confidence*. We say that she is *overconfident* if  $a_i > a_0$  for all  $\theta_i \in I_i(R_{0i})$  and *underconfident* if  $a_i < a_0$  for all  $\theta_i \in I(R_{0i})$ .

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<sup>17</sup>In Ortoleva and Snowberg (2015), agents overestimate the precision of their information because they ignore correlation in the underlying signals they see. This leads overconfident citizens to have excess variance in their posterior beliefs. Overconfidence in our model has the same excess variance implication, but also has a further effect on polarization via endogenous trust.

Proposition 6 shows that agents are never underconfident and will be overconfident if and only if  $\underline{a}_i > a_0$ . The bound  $\underline{a}_i$  is determined by the  $R^2$  values  $\zeta_{ij}$  from regressions of  $x_{it}$  on  $r_t$  and  $s_{jt}$ . Intuitively, agents are overconfident when they observe correlations between  $x_{it}$ ,  $r_t$  and at least one  $s_{jt}$  that are infeasible at the true value  $a_0$  (under the agent's maintained assumption that  $b_i = 0$ ). The proof of Proposition 5 shows that this  $R^2$  is given by

$$\zeta_{ij} = b_{0i}^2 + a_0^2 \left( \frac{\alpha_{0j}^2}{1 - \beta_{0j}^2} \right). \quad (4)$$

Therefore, agents  $R$  and  $L$  are overconfident if and only if

$$\frac{b_{0i}^2}{a_0^2} > 1 - \max_j \left\{ \frac{\alpha_{0j}^2}{1 - \beta_{0j}^2} \right\}. \quad (5)$$

Noting that the fraction in curly braces approaches one as  $\alpha_{0j}^2 + \beta_{0j}^2$  approaches one yields the following result.

**Corollary 5.** *Suppose  $r_t$  is observed, agents single home, and Assumption 1' holds. Then agents are never underconfident. Agent  $U$  is never overconfident. Agents  $R$  and  $L$  are overconfident if (i)  $a_0$  is sufficiently small or (ii) there is some source  $j$  with  $\alpha_{0j}^2 + \beta_{0j}^2$  sufficiently close to one.*

Overconfidence emerges in our model in order to reconcile the agents' dogmatic belief that their reasoning  $x_{it}$  is unbiased with the observed correlations of the sources. No agent is ever underconfident, since it is always possible to rationalize the observed correlations with a sufficiently high value of  $a_i$ . Overconfidence can arise if an agent's bias is large. However, even when an agent's bias is small, she is overconfident if the accuracy of her reasoning  $a_0$  is sufficiently low. She is also overconfident if at least one source in the market is sufficiently accurate. When there is at least one source with  $\alpha_{0j} \neq 0$  and  $\alpha_{0j}^2 + \beta_{0j}^2 = 1$ , agents with any bias are overconfident regardless of the value of  $a_0$ , since  $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$ .

### Trust, Polarization, Ideology, and Perceived Bias in the Presence of Overconfidence

Can divergence in trust, disagreement about  $\omega_t$ , ideology, and perceived bias be large once we allow for overconfidence? The answer is yes if  $a_0$  is small relative to  $b_0$ . To illustrate, we focus on

the case where the upper bound of the agent's prior on  $a_i$  is  $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$ . This is the smallest value for which an agent's identified set is always non-empty, and it approximates a situation where each agent's prior on her own accuracy  $a$  is concentrated on values that are close to the true value  $a_0$ .<sup>18</sup> We further focus on the case where the set of sources is sufficiently rich that it includes each agent's *trust-maximizing* source. These results are not knife-edge; they hold approximately if  $a_i^{max}$  is close to the assumed value and there is an available source that generates sufficiently high trust. More general results for trust and beliefs in the presence of overconfidence are derived in Appendix D.

Because we have relaxed Assumptions 1 and 2, agent  $i$ 's trust-maximizing source is defined by the accuracy and bias  $(\alpha_i, \beta_i)$  that maximizes the agent's trust over all pairs satisfying the feasibility condition  $\alpha_i^2 + \beta_i^2 \leq 1$ . We show in Appendix D that this is

$$(\alpha_i^{max}, \beta_i^{max}) \equiv \left( \frac{a_0}{\sqrt{a_0^2 + b_{0i}^2}}, \frac{b_{0i}}{\sqrt{a_0^2 + b_{0i}^2}} \right).$$

Note that the correlation between  $x_{it}$  and a trust-maximizing source is  $\rho_{isj} = a_0 \alpha_i^{max} + b_{0i} \beta_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$ .

Agent  $U$ 's trust will be maximized by an unbiased source with accuracy  $\alpha_j = 1$  and bias  $\beta_j = 0$ . For agents  $R$  and  $L$ , the trust-maximizing source will be biased. If  $b_0$  is close to  $a_0$ , a biased agent will prefer a source with bias and accuracy close to  $1/\sqrt{2}$ . If the set of available sources includes agent  $i$ 's trust-maximizing source, then in the limit as  $T \rightarrow \infty$ , the single-homing agent observes her trust-maximizing source in all periods  $t > \varepsilon T$  by Proposition 3.

If the trust maximizing source is available, the agent will be maximally overconfident, with confidence degenerate at the maximal value  $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$ . This is because substituting  $(\alpha_i^{max}, \beta_i^{max})$  for  $\alpha_j$  and  $\beta_j$  in equation 4 yields  $\zeta_{ij} = a_0^2 + b_{0i}^2$  which in turn implies  $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$  by Proposition 6. It follows that the agent's trust in the trust-maximizing source will be one, and her posterior will be degenerate at

$$\bar{\omega}_t^i = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t. \quad (6)$$

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<sup>18</sup>In the alternative case where each agent's prior on  $a$  is concentrated on values much larger than  $a_0$ , their initial beliefs about  $a$  are severely mistaken. Furthermore, if  $a_0$  and  $b_0$  are small, then the agent learns that all available sources are very noisy (i.e.,  $\bar{\alpha}_j$  is small for all  $j$ ), and there is limited amplification of bias into divergent trust and disagreement.

We can now apply Proposition 6 to see that trust becomes highly divergent in the limit as  $a_0 \rightarrow 0$ . The fact that overconfidence is maximal means an agent  $R$ 's trust  $\bar{\alpha}_j^R$  for an arbitrary source  $j$  (not necessarily her trust maximizing source) approaches the source's bias  $\beta_j$  in that limit. Her trust for a perfectly right-biased source approaches one. Her trust for a perfectly left-biased source approaches zero. The opposite is true of agent  $L$ . Agents may therefore underestimate the accuracy of unbiased sources and come to trust biased sources strictly more than even a perfectly accurate source.

The agents' ideologies and perceived biases also become highly polarized in the same limit. Agent  $R$  believes that the true state is almost perfectly correlated with  $r_t$ , i.e.  $\bar{\gamma}^R \rightarrow 1$ , and perceives a perfectly accurate source  $j$  to be almost perfectly left-biased. Agent  $L$  correspondingly believes that the true state is almost perfectly correlated with  $-r_t$ , i.e.  $\bar{\gamma}^L \rightarrow -1$ , and perceives a perfectly accurate source to be almost perfectly right-biased.

Expected disagreement increases with the agents' bias, since  $\pi = b_0^2 / (a_0^2 + b_0^2)$ . This will be at least  $1/2$  if  $a_0 \leq b_0$ , and it will approach one in the limit as  $a_0$  approaches zero. Thus, polarization is significant if  $a_0$  is small relative to  $b_0$ .<sup>19</sup>

## 6 Multi-Homing

A common intuition is that divergent trust and polarization could be reduced or eliminated if agents were exposed to an ideologically diverse set of information sources. In this section, we show that it is possible for multi-homing to have beneficial effects consistent with this intuition, but also that this need not be the case. Multi-homing may leave trust and polarization unchanged, or even exacerbate them.

### 6.1 Trust under Multi-Homing

We first consider the agent's limiting beliefs about  $\theta_i$  under multi-homing. In the *multi-homing* case,  $R_{0i} = (\rho_{is}, \rho_{ir}, \rho_{rs}, \Sigma_0)$ , where  $\Sigma$  is the matrix of correlations among elements of  $s_t$ .<sup>20</sup>

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<sup>19</sup>Note that the requirement that the source be trust-maximizing is not knife-edge:  $\pi$  is continuous in  $\alpha$  and  $\beta$ , so the result holds approximately when these are close to the trust-maximizing values.

<sup>20</sup>To avoid singular covariance matrices, we assume that none of  $x_t$ ,  $s_t$ , and  $r_t$  are perfectly correlated with each other, so the vector of true correlations  $R \in \text{int}(\mathcal{R})$ .

**Proposition 7.** Under Assumption 1', agent  $i$ 's identified set under multi-homing,  $I_i(R_{0i})$ , when  $r_t$  is observed, is non-empty and consists of all  $\theta_i \in \Theta$  with  $a_i \in [\underline{a}_i, a_i^{max}]$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ ,  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , and  $\Sigma = \Sigma_0$ , where  $\underline{a}_i = \sqrt{\zeta_i}$  and  $\zeta_i$  is the population  $R^2$  from a regression of  $x_{it}$  on  $r_t$  and the elements of  $s_t$ .

Proposition 7 is similar to Proposition 6 except that in the multi-homing case, the lower bound on  $a_i$  is  $\underline{a}_i$  rather than  $\underline{a}_i$ . Rather than depending on the maximum  $R^2$  value across regressions of  $x_{it}$  on  $r_t$  and various  $s_{jt}$ , the bound now depends on the  $R^2$  from a regression of  $x_{it}$  on  $r_t$  and all the  $s_{jt}$ .<sup>21</sup> Since  $\zeta_i \geq \zeta_{ij}$  for all  $j$ , the lower bound on  $a_i$  is tighter under multi-homing than under single-homing. This reflects the fact that observing the correlation among the different signals in  $s_t$  provides further constraints on the set of parameters that could be consistent with the data.

Since the multi-homing bound on confidence  $\underline{a}_i$  is weakly greater than the single-homing bound  $\underline{a}_i$ , a multi-homing agent's confidence will be weakly greater (in an FOSD sense) than a single-homing agent's confidence. This means that the difference in trust  $|\bar{\alpha}_j^i - \bar{\alpha}_k^i|$  between any two sources can be weakly smaller under multi-homing, and that ideology  $\bar{\gamma}^i$  will tend to be less extreme.

Suppose, for tractability, that the agent has an *a priori* belief that the accuracies or biases of observable external signals (i.e.,  $s_t$  and  $r_t$ ) are independent of her own accuracy.

**Assumption 3.**  $a_i$  and  $(\alpha, \beta, \gamma)$  are independently distributed under the agent's prior  $\mu_0^i$ .

When Assumption 3 holds, each agent's limiting (marginal) posterior distribution on  $a_i$ , which we denote by  $\mu_{\infty,a}^i$ , is the agent's prior marginal distribution on  $a$  except truncated at the lower bound of  $\underline{a}_i$ . It follows that multi-homing may dampen divergent trust and ideology (while also increasing confidence).

**Corollary 6.** Suppose Assumptions 1' and 3 hold. Then for any agent  $i$ , the difference in trust  $|\bar{\alpha}_j^i - \bar{\alpha}_k^i|$  between any two sources and the ideology  $|\bar{\gamma}^i|$  are both weakly smaller under multi-homing.

While such an effect of multi-homing is possible, it need not be large, and it is possible for the limiting posterior  $\mu_{\infty}^i$  of a multi-homing agent to be exactly the same as a single-homing agent's.

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<sup>21</sup>That is,  $\zeta_i = \tilde{\rho}_i' \tilde{\Sigma}^{-1} \tilde{\rho}_i$  where  $\tilde{\rho}_i = \begin{pmatrix} \rho_{ir} \\ \rho_{is} \end{pmatrix}$  and  $\tilde{\Sigma} = \begin{pmatrix} 1 & \rho_{rs}' \\ \rho_{rs} & \Sigma \end{pmatrix}$ .

For example, when at least one source with  $\alpha_{0j} \neq 0$  is located on the frontier,  $\underline{a}_i$  already achieves its maximal value under single-homing, so the limiting posterior under multi-homing is unchanged.

## 6.2 Polarization under Multi-Homing

How does expected disagreement about  $\omega_t$  compare in the single- and multi-homing cases? As shown in this subsection, it is possible for multi-homing to reduce disagreement, but multi-homing may also increase disagreement in some situations.

To demonstrate this, we first characterize the agent's posterior expectation of  $\omega_t$  when her beliefs about  $\theta_i$  are given by the limiting posterior  $\mu_{\infty, \theta}^i$ . Under multi-homing, the average belief  $\bar{\omega}_t^i$  is now a linear function of the observed signals, as shown below.

**Lemma 1.** *Suppose agent  $i$  multi homes and Assumption 1' holds. As  $T \rightarrow \infty$ , in any period  $t > \varepsilon T$ , the mean of her posterior on  $\omega_t$  given  $s_t$  (but not  $x_{it}$  or  $r_t$ ) is*

$$\bar{\omega}_t^i = A_i \rho'_{is} \Sigma^{-1} \tilde{s}_t,$$

where  $\tilde{s}_t$  is the  $J$ -vector of standardized signals  $s_t$  and the amplification factor  $A_i$  is given by

$$A_i = \int_{\underline{a}_i}^{a_i^{\max}} \frac{1}{a} d\mu_{\infty, a}^i(a).$$

We consider a special case wherein each agent may observe three sources with respective biases of  $\beta$ ,  $0$ , and  $-\beta$ , where  $\beta > 0$ . All of these sources are on the *frontier*, so  $\alpha_j^2 + \beta_j^2 = 1$  for all  $j$ . As shown in Lemma 2 below, a multi-homing agent's posterior mean  $\bar{\omega}_t^i$  is the same as that of a single-homing agent who observes her trust-maximizing source. The reason is that a multi-homing agent observing two distinct frontier sources can construct a linear combination of the sources' signals whose value will be equal to the signal of the agent's trust-maximizing source. Appendix G show that we obtain the same result in the limit of a sequence of random markets with non-frontier sources.

**Lemma 2.** *Suppose Assumption 1' holds and there are at least two frontier sources with distinct*

biases. Then the mean of the multi-homing agent's posterior on  $\omega_t$  given  $s_t$  is

$$\bar{\omega}_t^i = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t.$$

It follows, as formalized in Proposition 8 below, that multi-homing does not in general reduce expected disagreement. In fact, multi-homing may make it worse. To see this, let  $\phi_b = \tan^{-1} \left( \frac{b_0}{a_0} \right)$  denote the angle between vectors  $(1, 0)$  and  $(a_0, b_0)$  and let  $\phi_\beta = \sin^{-1}(\beta)$  denote the angle between  $(1, 0)$  and  $(\sqrt{1 - \beta^2}, \beta)$ . If  $\phi_b \in (0, \frac{1}{2}\phi_\beta)$ , all single-homers observe the unbiased source, while the multi-homers' beliefs are the same as would occur if they observed their trust-maximizing source, so multi-homing results in greater disagreement. If instead  $\phi_b \in (\frac{1}{2}\phi_\beta, \phi_\beta)$ , biased single-homers observe a source with more bias than the trust-maximizing source, so multi-homing results in less disagreement.

**Proposition 8.** *Suppose Assumption 1' holds and there are three frontier sources with respective biases  $\beta$ , 0, and  $-\beta$ . Then expected disagreement  $\pi$  is greater under multi-homing than under single-homing if  $\phi_b \in (0, \frac{1}{2}\phi_\beta)$ , but smaller if  $\phi_b \in (\frac{1}{2}\phi_\beta, \phi_\beta)$ .*

## 7 Endogenous Media Bias

What does our model imply about media competition and political behavior? In this extension, we show that media competition can intensify disagreements in a population with ideological biases. In Appendix H, we show that ideological bias results in interpersonal mistrust and creates welfare losses in strategic games of collective decision-making.

To explore how media competition affects ideological disagreement, we endogenize the accuracies and biases of the information sources in a sequential positioning game. We consider a unit mass of agents. The agents are divided into three types  $i \in \{U, R, L\}$ , with accuracies  $a_i$  and biases  $b_i$  defined in Section 2.2. We assume that agents have the same priors as in our full model in Section 5, so Assumption 1' holds. Each type  $i$  has mass  $m_i > 0$  such that  $m_U + m_R + m_L = 1$ . For simplicity, we assume that  $m_L = m_R$ .

A set of  $E$  identical potential entrants sequentially choose whether or not to enter before the first period. If they enter, they may choose any accuracy  $\alpha_{0j}$  and bias  $\beta_{0j}$  on the frontier (i.e.,

$\alpha_{0j}^2 + \beta_{0j}^2 = 1$ ). Prior to entry, each entering outlet observes all preceding entrants' choices of  $(\alpha_{0j}, \beta_{0j})$ . We use subgame perfect equilibrium as our solution concept.

All agents are single-homers who choose a single outlet  $j$  to observe in a given period  $t$ . We focus on media viewership choices during their exploitation period. We assume that agents have beliefs about the accuracies of the outlets during these periods corresponding to the limiting posterior  $\mu_{\infty, \theta}^i$ .<sup>22</sup> If Proposition 3 identifies multiple potential sources to observe, agents randomize between them with equal probability.

We assume that the revenue of a media outlet is increasing in both the mass of viewers that choose it and the trust of its viewers. This is consistent with advertising-supported media where conditional on viewing an outlet a customer spends more time viewing when trust is high. It could also be consistent with paid media where the revenue an outlet can earn from a customer who chooses to view is greater when trust is high.

Let  $\mathcal{J}_i$  be the set of outlets for which an  $i$ -type agent's trust  $\bar{\alpha}_j^i$  is highest. Let  $\xi(\bar{\alpha}_j^i)$  denote revenue per viewer of type  $i$ . We assume that  $\xi(\cdot)$  is positive, strictly increasing, continuously differentiable, and concave, to capture the idea that firms make additional revenue from higher trust, but with declining marginal revenue. Firms also pay an entry cost  $\lambda > 0$ . Each firm  $j$  thus has expected profit:

$$\Pi_j = \sum_{i \in \{U, R, L\}} \mathbf{1}\{j \in \mathcal{J}_i\} \frac{m_i}{|\mathcal{J}_i|} \xi(\bar{\alpha}_j^i) - \lambda,$$

where  $\mathbf{1}\{j \in \mathcal{J}_i\}$  is an indicator for whether outlet  $j$  is in the set of outlets that type- $i$  agents observe, and  $m_i/|\mathcal{J}_i|$  measures the probability of observing  $j$  within that set. Note that both  $\mathcal{J}_i$  and  $\bar{\alpha}_j^i$  are equilibrium outcomes that depend on the accuracy and bias choices of all media outlet entrants.

We can now solve for the media outlets' equilibrium choice of accuracies and bias via backward induction. We first consider outcomes in a monopoly market.

**Proposition 9.** *Suppose there is only one potential entrant ( $E = 1$ ). Then for  $\lambda$  sufficiently low, this firm enters and becomes a monopolist with  $\alpha_j = 1$  and  $\beta_j = 0$ . Biased agents are overconfident,*

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<sup>22</sup>Strictly speaking, this is a behavioral assumption that the agents' inferences about media outlet accuracy do not condition on the equilibrium strategies chosen by the outlets, but only on the signals the outlets produce.

but have expected disagreement  $\pi = 0$ .

Proposition 9 shows that the monopolist becomes a completely accurate and unbiased source of information. Even though the monopolist has a captive audience, it still seeks to capture rising profits from trust. Since it faces a linear trade-off in trust between the  $L$  and  $R$  agents when it adds bias, the optimal choice under equal proportions of  $L$  and  $R$  agents and a concave revenue function  $\xi$  is to simply focus on accuracy instead and choose  $\alpha_{0j} = 1$  and  $\beta_{0j} = 0$ . This results in no expected disagreement in the population, as agents observe a common outlet and have a common level of trust. Note that this trust is still suboptimal, however, and so beliefs are less than perfectly accurate. Note also that this result is not knife edge: If the proportions of  $L$  and  $R$  agents are slightly unequal, the resulting optimal position remains close to unbiased, and confidence, trust, and beliefs remain close to the characterization above.

Turning to the competitive case, we focus on the case where the set of potential entrants is sufficiently large that there are potential entrants who do not find it profitable to enter in equilibrium. We can see from Appendix D (Lemma 7) that sources gain maximum trust from biased agents by choosing those agents' trust-maximizing level of bias. It is then unsurprising that in the case of competition, some sources choose to be biased and successfully retain a large audience.

**Proposition 10.** *For  $\lambda$  sufficiently low and a set of potential entrants  $E(\lambda)$  sufficiently large, all entrant outlets locate at positions on the frontier with  $\beta_{0j} \in \{\beta^L, 0, \beta^R\}$ , where  $\beta^L$  and  $\beta^R$  are the trust-maximizing biases for type  $L$  and type  $R$  agents respectively. At least one outlet chooses each of these positions. Biased agents are overconfident. Furthermore, expected disagreement will be at least  $\pi = \frac{1}{2}$  if  $a_0 \leq b_0$ , and will approach  $\pi = 1$  in the limit as  $a_0 \rightarrow 0$ . Thus, the entry of partisan media leads to greater divergence in beliefs.*

In contrast to the monopoly case, there is now significant disagreement in the population. This stems from their complete faith in the accuracy of like-minded outlets and their undivided attention to such outlets. Their beliefs about  $\omega_t$  are simply degenerate at the signal  $s_{jt}$  of their trust-maximizing source. Since such outlets adopt the trust-maximizing bias and this bias approaches  $\pm 1$  as the ratio of  $b_0$  to  $a_0$  increases, competition can potentially give rise to maximal disagreement and perfectly negatively correlated beliefs. Note these results are not at all dependent on our assumption that  $m_R = m_L$ .

## 8 Conclusion

We present a model to explain why individuals persistently disagree about both objective facts and the trustworthiness of information sources. In contrast to recent theories, we assume that agents have Bayesian learning rules and can process information from an arbitrarily large set of high-quality sources. Agents in our model learn about policy-relevant states by observing signals from information sources whose accuracy is *ex ante* uncertain. Agents learn these accuracies by comparing their own reasoning about the states to the sources' reports.

Our contribution is to characterize how endogenous inference about the accuracy of sources can cause small biases in reasoning to be amplified into significantly divergent beliefs about facts, even when accurate information is commonly observed. Our model generates a large set of novel and sharp predictions about the resulting beliefs: Partisans end up trusting unreliable but ideologically aligned sources more than accurate neutral sources, and become overconfident in their own reasoning. They form a conviction that either the conservative or the liberal point of view is closer to the truth on average, and perceive unbiased sources to be oppositely biased. Divergent trust and beliefs can arise to a similar extent whether agents selectively view only ideologically aligned sources or are exposed to a diverse range of sources. Moving from a monopoly to a competitive market can deepen rather than mitigate ideological disagreement. Mistrust of motives results and leads to inefficient political outcomes.

Taken together, these results highlight the outsized importance of trust in driving ideological differences in society. If individuals' reasoning has even a small amount of bias, then they may learn to trust biased sources, and hence form biased beliefs about facts. For this reason, ideological disagreement can persist even among otherwise Bayesian agents who can process information about an arbitrarily large set of high-quality sources. Reducing selective exposure may therefore fail to redress political polarization. Targeting the underlying drivers of divergent trust – for example, by reducing biases in the population's reasoning through scientific literacy, or increasing the prominence of commonly trusted sources – may yield larger gains.

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# Appendices

## A Proofs

### A.1 Proof of Proposition 1

Let  $D_{it}$  denote the data observed by agent  $i$  in period  $t$  in three cases.

- Case 1. Agent  $i$  single-homes and does not observe  $r_t$ . We let  $D_{it} = (s_{jt}, x_{it})$ .
- Case 2. Agent  $i$  single-homes and observes  $r_t$ . We let  $D_{it} = (s_{jt}, x_{it}, r_t)$ .
- Case 3. Agent  $i$  multi-homes and observes  $r_t$ . We let  $D_{it} = (s_t, x_{it}, r_t)$ .

**Lemma 3.**  $D_{i1}, \dots, D_{it}$  is independent of  $\theta_{0i}$  conditional on  $R_{0i}$  and  $V_{0i}$ .

*Proof.* First consider Case 1. With slight abuse of notation, define  $D_{i\tau}^j = (s_{j\tau}, x_{i\tau})$  to be the  $\tau$ th time  $j$ 's signal and reasoning are observed. The agent knows  $D_{i\tau}^j \sim N(0, \Omega_{0ij})$  for some positive definite  $\Omega_{0ij}$ , where  $\Omega_{0ij} = V_{0ij}^{\frac{1}{2}} R_{0ij} V_{0ij}^{\frac{1}{2}}$ , where  $V_{0ij} = \text{diag}(\Omega_{0ij})$  and  $R_{0ij}$  is the correlation matrix for  $D_{i\tau}^j$ , which in Case 1 is the correlation  $\rho_{ij}$ . Independence across periods then generalizes the result to the entire sequence of data observations. Cases 2 and 3 can be proved in the same way.  $\square$

Let  $P_{Y|X}^i$  denote the posterior distributions of  $Y$  given  $X$  and the prior  $\mu_0^i$ . Then, by Lemma 3, we can see that for all  $\vartheta \in \mathcal{L}_\Theta$ ,  $P_{\theta_i|R_i, V_i, D_1, \dots, D_t}^i(\vartheta) = P_{\theta_i|R_i, V_i}^i(\vartheta)$  and hence that

$$\begin{aligned} P_{\theta_i|D_{i1}, \dots, D_{it}}^i(\vartheta) &= \int_{\mathcal{R}, \mathcal{V}} P_{\theta_i|R_i, V_i, D_{i1}, \dots, D_{it}}^i(\vartheta) dP_{R_i, V_i|D_{i1}, \dots, D_{it}}^i \\ &= \int_{\mathcal{R}, \mathcal{V}} P_{\theta_i|R_i, V_i}^i(\vartheta) dP_{R_i, V_i|D_{i1}, \dots, D_{it}}^i. \end{aligned}$$

We now characterize the limit of  $P_{R_i, V_i|D_{i1}, \dots, D_{it}}^i$ . First consider Case 3, where agent  $i$  multi-homes and observes  $r_t$ . Let  $P_{D_i|R_i, V_i}$  denote the (true) distribution of  $D_{it}$  conditional on  $R_i$  and  $V_i$ . The experiment  $(P_{D_i|R_i, V_i} : R_i \in \mathcal{R}; V_i \in \mathcal{V})$  is Gaussian with known mean zero and known variance, and its parameter space  $(\mathcal{R}, \mathcal{V})$  is compact. It is straightforward to verify the following regularity conditions: (i)  $P_{D_i|R_i, V_i} \neq P_{D_i|(R_i, V_i)'}^i$  for any  $(R_i, V_i) \neq (R_i, V_i)'$ ; (ii) the mapping

$(R_i, V_i) \mapsto P_{D_i|R_i, V_i}$  is continuous in total variation norm; (iii)  $P_{D_i|R_i, V_i}$  has a nonsingular information matrix  $I_{R_{0i}, V_{0i}}$  at  $(R_{0i}, V_{0i})$  (recalling from Remark 1 we focus on  $\theta_0$  such that  $R_{0i} \in \text{int}(\mathcal{R})$ ); (iv)  $(P_{D_i|R_i, V_i} : R_i \in \mathcal{R}; V_i \in \mathcal{V})$  is differentiable in quadratic mean at  $(R_{0i}, V_{0i})$ . Then by van der Vaart (1998) Lemma 10.6 and Theorem 10.1 (the Bernstein-von Mises Theorem), the limit of  $P_{R_i, V_i | D_{i1}, \dots, D_{it}}^i$  as  $t \rightarrow \infty$  is a distribution degenerate at the true correlations  $(R_{0i}, V_{0i})$ .

Next consider Case 1, where agent  $i$  single-homes and does not observe  $r_t$ . Reorder the experimentation periods so those where the agent observes  $(s_{1t}, x_{it})$  occur first, those where the agent observes  $(s_{2t}, x_{it})$  occur second, through those where the agent observes  $(s_{Jt}, x_{it})$ . Denote these respective subsequences of data by  $D_i^1, \dots, D_i^J$ . Since posterior beliefs are invariant to the order of data, this reordering does not affect the limit of  $P_{R_i, V_i | D_{i1}, \dots, D_{it}}^i$ . The logic above implies that as  $T \rightarrow \infty$ , the agent's posterior belief  $P_{R_i, V_i | D_{i1}, \dots, D_{it}}^i$  at the end of the first set of periods converges to a limit whose marginal distribution on  $\rho_{i1}$  is degenerate at the true values of these correlations. Note that for every finite  $t$ ,  $P_{R_i, V_i | D_i^1}^i$  has continuous density on  $(\mathcal{R}, \mathcal{V})$  and so is a valid prior under our model. Applying the same logic again then implies that the agent's posterior belief  $P_{R_i, V_i | D_i^1, D_i^2}^i$  at the end of the second set of periods converges to a limit whose marginal distribution on  $(\rho_{i1}, \rho_{i2})$  is degenerate at the true value of these correlations. Iterating this logic repeatedly shows that  $P_{R_i, V_i | D_i^1, \dots, D_i^J}^i = P_{R_i, V_i | D_{i1}, \dots, D_{it}}^i$  converges to a limit whose marginal distribution  $(\rho_{i1}, \dots, \rho_{iJ})$  is degenerate at the full vector of true correlations  $R_{0i}$ . Case 2 can be proved in the same way.

Finally, note that for all  $\vartheta \in \mathcal{L}_\theta$ ,

$$\begin{aligned}\mu_{\infty, \theta}^i(\vartheta) &= \lim_{T \rightarrow \infty} P_{\theta_i | D_{i1}, \dots, D_{it}}(\vartheta) \\ &= \lim_{T \rightarrow \infty} \int_{\mathcal{R}, \mathcal{V}} P_{\theta_i | R_i, V_i}^i(\vartheta) dP_{R_i, V_i | D_{i1}, \dots, D_{it}}^i \\ &= P_{\theta_i | R_{0i}, V_{0i}}^i(\vartheta) \\ &= \mu_{0, \theta}^i(\vartheta | R_i = R_{0i}) \\ &= \frac{\mu_{0, \theta}^i(\vartheta \cap I_i(R_{0i}))}{\mu_{0, \theta}^i(I_i(R_{0i}))}.\end{aligned}$$

where the third equality uses the convergence of  $P_{R_i, V_i | D_{i1}, \dots, D_{it}}^i$ .

## A.2 Proof of Proposition 2

Given any value of  $\theta_i \in I_i(R_{0i})$ , the state  $\omega_t$  and the (normalized) signals  $\tilde{s}_{jt}$  are jointly distributed

$$\begin{pmatrix} \omega_t \\ \tilde{s}_{jt} \end{pmatrix} \sim N \left( 0, \begin{bmatrix} 1 & \alpha_j \\ \alpha_j & 1 \end{bmatrix} \right).$$

The conditional expectation of  $\omega_t$  given  $\tilde{s}_{jt}$  (under single-homing) is then  $\alpha_j \tilde{s}_{jt}$ , by the properties of the multivariate normal distribution. The desired result follows from taking the expectation over the limiting posterior  $\mu_{\infty, \theta}^i$ .

## A.3 Proof of Proposition 3

The expected loss for a given  $j$  is the same for all  $t$  in the exploitation period, so we can focus on minimizing the single-period loss for a single agent. Thus we can drop the  $i$  and  $t$  subscripts. For notational simplicity, we also focus on the simple case where all variances are known to be one.

The agent solves

$$\min_{j \in \mathcal{J}_+} E_{s_j, x, \omega} \left[ (d_j^*(s_j, x) - \omega)^2 \right],$$

where the expectation is taken under the distribution of  $(s, x, \omega)$  and  $d_j^*(s_j, x)$  is the optimal decision after seeing  $(s_j, x)$ . Note that  $d_j^*(s_j, x) = E_{\omega|s_j, x}[\omega]$ . The law of iterated expectations implies that

$$E_{s_j, x, \omega} \left[ (d_j^*(s_j, x) - \omega)^2 \right] = 1 - E_{s_j, x} \left[ d_j^*(s_j, x)^2 \right].$$

Note that in all cases here expectations are taken under the joint distribution of  $(\Omega, s, x, \omega)$  given  $\Omega \sim \mu_\infty$ . Thus, an expression like  $E_\omega[\cdot]$  refers to the expectation under the marginal distribution of  $\omega$  in that distribution.

Define  $d_j^*(s_j, x, \Omega)$  to be the optimal decision *conditional* on a particular  $\Omega$ . Note that  $d_j^*(s_j, x, \Omega) = E_{\omega|s_j, x, \Omega}[\omega]$ , so we have

$$d_j^*(s_j, x, \Omega) = \begin{pmatrix} a \\ \alpha_j \end{pmatrix}' \begin{bmatrix} 1 & \alpha_j a \\ \alpha_j a & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix} = \begin{pmatrix} a \\ \alpha_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix},$$

where the last line follows from observing that for all  $(a, \alpha_j)$  in the support of  $\mu_\infty$ , we must have  $\alpha_j a = \rho_j$ , where  $\rho_j$  is a constant equal to the empirical correlation observed in the data. Since  $E_{\omega|s_j,x}[\omega] = E_\Omega E_{\omega|s_j,x,\Omega}[\omega] = E_\Omega d^*(s_j, x, \Omega)$ , we have

$$d_j^*(s_j, x) = \begin{pmatrix} \bar{a} \\ \bar{\alpha}_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} x \\ s_j \end{pmatrix}.$$

It follows from some algebraic manipulation that

$$E_{s_j,x} \left[ d_j^*(s_j, x)^2 \right] = \begin{pmatrix} \bar{a} \\ \bar{\alpha}_j \end{pmatrix}' \begin{bmatrix} 1 & \rho_j \\ \rho_j & 1 \end{bmatrix}^{-1} \begin{pmatrix} \bar{a} \\ \bar{\alpha}_j \end{pmatrix} = \frac{\bar{a}^2 - 2\rho_j \bar{a} \bar{\alpha}_j + \bar{\alpha}_j^2}{1 - \rho_j^2}.$$

By Jensen's inequality

$$\bar{\alpha}_j = E_\Omega \left[ \frac{\rho_j}{a} \right] \geq \frac{\rho_j}{\bar{a}},$$

so we have  $\rho_j = c\bar{a}\bar{\alpha}_j$  for some  $c \leq 1$ , holding  $\mu_\infty$  fixed. Therefore, if we compare the expected variance in decisions when the agent observes sources with different trust  $\bar{\alpha}_j$ , holding  $\mu_\infty$  fixed, we have that

$$\frac{\partial E_{s_j,x} \left[ d_j^*(s_j, x)^2 \right]}{\partial \bar{\alpha}_j} = \frac{\partial}{\partial \bar{\alpha}_j} \left[ \frac{\bar{a}^2 - 2c\bar{a}^2\bar{\alpha}_j^2 + \bar{\alpha}_j^2}{1 - c^2\bar{a}^2\bar{\alpha}_j^2} \right] \geq \frac{\partial}{\partial \bar{\alpha}_j} \left[ \frac{\bar{a}^2 - 2\bar{a}^2\bar{\alpha}_j^2 + \bar{\alpha}_j^2}{1 - \bar{a}^2\bar{\alpha}_j^2} \right] > 0,$$

where the first inequality follows from  $c \leq 1$  and the second inequality is strict under the maintained assumption that  $\bar{a} < 1$ . This then implies that for any  $j$  and  $k$  such that  $\bar{\alpha}_j^2 > \bar{\alpha}_k^2$ , we have

$$E_{s_j,x,\omega} \left[ (d_j^*(s_j, x) - \omega)^2 \right] < E_{s_k,x,\omega} \left[ (d_k^*(s_k, x) - \omega)^2 \right]$$

and so the problem is solved by choosing a  $j$  with the highest value of  $\bar{\alpha}_j^2$ .

## A.4 Proof of Proposition 4

Recall that  $I_i(R_{0i}) = \left\{ \theta_i \in \Theta_i^{prior} : a_i \alpha + b_i \beta = \rho_{is} \right\}$ , where any  $\theta_i \in \Theta_i^{prior}$  must give rise to a positive semi-definite correlation matrix  $\tilde{\Omega}(\theta_i)$  for  $(\omega_t, r_t, x_{it}, s_t)$ . Assumption 1 implies that  $\theta_i \in$

$I_i(R_{0i})$  if and only if  $a_i = a_0$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite. Assumption 2 implies that  $|\alpha_{0j}| = |\rho_{ij}/a_{0i}| = |\alpha_{0j} + (b_{0i}/a_{0i})\beta_{0j}| \leq ||\alpha_{0j}| + |\beta_{0j}|| \leq 1$ . We can then pick  $\beta = 0$ ,  $\gamma = 0$ , and assume that  $s_{jt}$  are mutually independent and independent of  $r_t$  and  $x_{it}$  conditional on  $\omega_t$ , so  $\Sigma = \text{Corr}(s_t) = \alpha\alpha' + K$ , where  $K$  is a diagonal matrix with entries equal to  $1 - \alpha_j^2$ . It follows that  $\tilde{\Omega}(\theta_i)$  is positive semi-definite, so  $I_i(R_{0i})$  is non-empty.

## A.5 Proof of Proposition 5

The proof is similar to the proof of Proposition 4, except  $r_t$  is now observed. By Assumption 1, Remark 1, and the definition of  $I_i(R_{0i})$ ,  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i = a_0$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ ,  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite. We first show that the correlation matrix for  $(\omega_t, r_t, s_{jt})$ , i.e.  $\begin{bmatrix} 1 & \tilde{\alpha}_{ij} \\ \tilde{\alpha}_{ij} & \tilde{\Sigma}_j \end{bmatrix}$ , where  $\tilde{\alpha}_{ij} = \begin{pmatrix} \frac{\rho_{ir}}{a_i} \\ \frac{\rho_{ij}}{a_i} \end{pmatrix}$  and  $\tilde{\Sigma}_j = \begin{bmatrix} 1 & \rho_{rj} \\ \rho_{rj} & 1 \end{bmatrix}$ , is positive semi-definite for all  $j$ . This is equivalent to  $1 - \tilde{\alpha}'_{ij}\tilde{\Sigma}_j^{-1}\tilde{\alpha}_{ij} \geq 0$  by standard matrix results (see Boyd and Vandenberghe 2004 Appendix A.5.5). This in turn requires that for all  $j$ ,

$$a_i^2 \geq \zeta_{ij} = \frac{\rho_{ir}^2 + \rho_{xj}^2 - 2\rho_{rj}\rho_{ij}\rho_{ir}}{1 - \rho_{rj}^2}. \quad (7)$$

Algebraic substitution shows that  $\zeta_{ij} = b_0^2 + a_0^2 \left( \frac{\alpha_{0j}^2}{1 - \beta_{0j}^2} \right)$ . Since  $a_i = a_0$ , this condition is guaranteed by Assumption 2'. We can then assume that  $s_{jt}$  are mutually independent and independent of  $x_{it}$  conditional on  $\omega_t$  and  $r_t$ , so  $\Sigma = \alpha\alpha' + \beta\beta' + K$ , where  $K$  is a diagonal matrix with entries equal to  $1 - \alpha_j^2 - \beta_j^2$ . It follows that  $\tilde{\Omega}(\theta_i)$  is positive semi-definite, so  $I_i(R_{0i})$  is non-empty.

## A.6 Proof of Proposition 6

The proof is similar to the proof of Proposition 5, except the agent may become overconfident. By Assumption 1', Remark 1, and the definition of  $I_i(R_{0i})$ ,  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i \leq a_i^{max}$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ ,  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite. By the same logic as the proof of Proposition 5, the last condition is true if and only if  $a_i \geq \underline{a}_i = \max_j \sqrt{\zeta_{ij}}$ . Therefore,  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i \in [\underline{a}_i, a_i^{max}]$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ , and  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ .

## A.7 Proof of Proposition 7

This proof is the same as the proof of Proposition 6, except that agent  $i$  multi-homes. By Assumption 1', Remark 1, and the definition of  $I_i(R_{0i})$ ,  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i \leq a_i^{max}$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ ,  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite. Since the agent believes that  $x_{it} = a_i\omega_t + \eta_{it}$  for some  $a_i \in (0, a_i^{max}]$ ,  $\tilde{\Omega}(\theta_i)$  is positive semi-definite if and only if the covariance matrix for  $(\omega_t, r_t, s_t)$ , given by  $\begin{bmatrix} 1 & \tilde{\alpha}' \\ \tilde{\alpha} & \tilde{\Sigma} \end{bmatrix}$ , where  $\tilde{\alpha} = \begin{pmatrix} \frac{\rho_{ir}}{a_0} \\ \frac{\rho_{is}}{a_0} \end{pmatrix}$  and  $\tilde{\Sigma} = \begin{bmatrix} 1 & \rho'_{rs} \\ \rho_{rs} & \Sigma \end{bmatrix}$ , is positive semi-definite. This holds if and only if  $a_i^2 \geq \zeta_i = \tilde{\rho}_i'\tilde{\Sigma}^{-1}\tilde{\rho}_i$ , where  $\tilde{\rho}_i = \begin{pmatrix} \rho_{ir} & \rho'_{is} \end{pmatrix}'$ , by Boyd and Vandenberghe 2004 Appendix A.5.5. Therefore,  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i \in [\underline{a}_i, a_i^{max}]$ ,  $b_i = 0$ ,  $\alpha = \frac{\rho_{is}}{a_i}$ ,  $\gamma = \frac{\rho_{ir}}{a_i}$ , and  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ , where  $\underline{a}_i = \sqrt{\zeta_i}$ .

## A.8 Proof of Lemma 1

Given any value of  $\theta_i \in I_i(R_{0i})$ , the state  $\omega_t$  and the (normalized) signals  $\tilde{s}_{jt}$  are jointly distributed

$$\begin{pmatrix} \omega_t \\ \tilde{s}_t \end{pmatrix} \sim N \left( 0, \begin{bmatrix} 1 & \alpha' \\ \alpha & \Sigma \end{bmatrix} \right).$$

Under multi-homing, the conditional expectation of  $\omega_t$  given  $s_t$  is then  $\alpha'\Sigma^{-1}\tilde{s}_t$ . By Proposition 7,  $\alpha = \frac{\rho_{is}}{a_i}$ . Furthermore,  $I_i(R_{0i})$  includes all  $a_i \in [\underline{a}_i, a_i^{max}]$ . The desired result then follows from taking the expectation over the limiting posterior  $\mu_{\infty,a}^i$ .

## A.9 Proof of Lemma 2

We first prove the following Lemma.

**Lemma 4.** *Suppose that the signals  $s_{jt}$  are mutually independent conditional on  $\omega_t$  and  $r_t$ . Then in the multi-homing case, we have*

$$\rho'_{is}\Sigma^{-1}\tilde{s}_t = y'Z(Z'Z + K)^{-1}(Z'\varphi_t + \varepsilon_t)$$

and

$$\rho'_{is}\Sigma^{-1}\rho_{is} = y'Z(Z'Z + K)^{-1}Z'y,$$

where  $y = \begin{bmatrix} a_0 & b_{0i} \end{bmatrix}'$ ,  $Z$  is the  $2 \times J$  matrix where the  $j$ th column is  $\begin{bmatrix} \alpha_{0j} & \beta_{0j} \end{bmatrix}'$ ,  $K$  is a diagonal matrix such that the  $j$ th diagonal is  $\kappa_{0j}^2 = 1 - \alpha_{0j}^2 - \beta_{0j}^2$ ,  $\varphi_t = \begin{bmatrix} \omega_t & \tilde{r}_t \end{bmatrix}'$ , and  $\varepsilon_t$  is the  $J$ -vector of  $\varepsilon_{jt} = s_{jt} - \alpha_{0j}\omega_t - \beta_{0j}\tilde{r}_t$ .

*Proof.* The lemma follows from noting that  $\rho_{is} = Z'y$ ,  $\Sigma = Z'Z + K$ , and  $s_t = Z'\varphi_t + \varepsilon_t$ .  $\square$

When all sources are on the frontier and have distinct biases,  $K = 0$  and  $Z$  spans  $\mathbb{R}^2$ , so  $Z(Z'Z + K)^{-1}Z' = I$ . Lemma 4 then implies that  $\rho_{is}'\Sigma^{-1}s_t = a_0\omega_t + b_{0i}\tilde{r}_t$ . Note that the  $R^2$  of the population regression of  $x_{it}$  on  $s_t$  is  $\rho_{is}'\Sigma^{-1}\rho_{is}$ , which is equal to  $a_0^2 + b_{0i}^2$  by Lemma 4. Therefore, the  $R^2$  of the population regression of  $x_t$  on  $s_t$  and  $r_t$  must be weakly greater than  $a_0^2 + b_{0i}^2$ . However, the  $R^2$  from a regression of  $x_{it}$  on  $s_t$  and  $r_t$  cannot exceed the  $R^2$  from a regression of  $x_{it}$  on  $r_t$  and  $\omega_t$ , which is  $a_0^2 + b_{0i}^2$ . Therefore,  $\underline{a}_i = \sqrt{\zeta_i} = \sqrt{a_0^2 + b_{0i}^2}$ . Since  $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$  by assumption, we have that  $A_i = 1/\sqrt{a_0^2 + b_{0i}^2}$ .

Next consider the case with additional non-frontier sources. Let the vector of signals of the frontier sources be  $s_t^F$ . Note that the elements of  $\rho_{is}'\Sigma^{-1}$  are the coefficients from a population regression of  $x_t$  on the elements of  $s_t$ . Further note that  $x_{it} = a_0\omega_t + b_{0i}\tilde{r}_t + \eta_t$  where  $\eta_t$  is orthogonal to  $\varepsilon_{jt}$  for all  $j$ , while each element of  $s_t^F$  is a linearly independent linear combination of  $\omega_t$  and  $r_t$ . Thus  $x_t$  is orthogonal to  $s_{jt}$  conditional on  $s_t^F$  for all non-frontier sources. By the Frisch–Waugh–Lovell theorem, the elements of  $\rho_{is}'\Sigma^{-1}$  corresponding to non-frontier sources must be equal to zero, and the elements of  $\rho_{is}'\Sigma^{-1}$  corresponding to frontier sources are the same as in the case with all frontier sources. We conclude that  $\rho_{is}'\Sigma^{-1}\tilde{s}_t = a_0\omega_t + b_{0i}\tilde{r}_t$ . Furthermore, we can conclude that  $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$  using the same argument as in the previous paragraph.

## A.10 Proof of Proposition 8

In the multi-homing case, there are two or more frontier sources, so  $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$ ,  $\bar{\alpha}_j^i = 1$ , and  $\bar{\omega}_t^i = \alpha_i^{max}\omega_t + \beta_i^{max}\tilde{r}_t$ . Expected disagreement is  $b_0^2/(a_0^2 + b_0^2) = \sin^2(\phi_b)$ . In the single-homing case,  $r_t$  is observed along with a frontier source, so  $\underline{a}_i = \sqrt{a_0^2 + b_{0i}^2}$  and  $\bar{\alpha}_j^i = \frac{a_0\alpha_{0j} + b_{0i}\beta_{0j}}{\sqrt{a_0^2 + b_{0i}^2}}$ . Note that a biased agent observing a bias source results in trust equal to  $\cos(\phi_b - \phi_\beta)$ , while a biased agent observing an unbiased source results in trust equal to  $\cos(\phi_b)$ . If  $\phi_b < \frac{1}{2}\phi_\beta$ , then the agent's trust for the unbiased source is higher, so all single-homers observe the unbiased source. This implies that  $\pi = 0$  under single-homing, while disagreement is positive under multi-homing. If  $\phi_b > \frac{1}{2}\phi_\beta$ ,

then biased single-homers observe the similarly biased source. In this case, disagreement under multi-homing is weakly greater than disagreement under single-homing if and only if  $\sin^2(\phi_b) \geq \cos^2(\phi_b - \phi_\beta) \sin^2(\phi_\beta)$ , which can be shown to be true if and only if  $\phi_b \geq \phi_\beta$  using trigonometric identities and by noting that  $0 < \phi_b, \phi_\beta \leq \pi$ .

## A.11 Proof of Proposition 9

Because all agents will observe the monopolist's signal in every period, the monopolist's profit maximization problem simplifies to choosing accuracy  $\alpha_{0j}$  and bias  $\beta_{0j}$  to maximize

$$\Pi_j = \sum_{i \in \{L, U, R\}} m_i \xi(\bar{\alpha}_j^i) - \lambda,$$

where  $\bar{\alpha}_j^i$  is type- $i$  consumers' trust in the monopolist.

The derivative of trust  $\bar{\alpha}_j^i$  with respect to  $\beta_{0j}$  along the frontier is:

$$\delta^i(\beta_{0j}) = \frac{\partial \bar{\alpha}_j^i}{\partial \beta_{0j}} \Big|_{\alpha_{0j}^2 + \beta_{0j}^2 = 1} = \frac{1}{\sqrt{a_{0j}^2 + b_{0j}^2}} \left( b_{0j} - a_{0j} \frac{\beta_{0j}}{\sqrt{1 - \beta_{0j}^2}} \right).$$

Letting  $m = m_R = m_L$ , the optimal frontier location must satisfy the first order condition that

$$\frac{\partial \Pi}{\partial \beta_{0j}} \Big|_{\alpha_{0j}^2 + \beta_{0j}^2 = 1} = (1 - 2\mu) \xi'(\bar{\alpha}_j^U) \delta^U(\beta_{0j}) + m [\xi'(\bar{\alpha}_j^R) \delta^R(\beta_{0j}) + \xi'(\bar{\alpha}_j^L) \delta^L(\beta_{0j})] = 0. \quad (8)$$

This condition is satisfied at  $\beta_{0j} = 0$  because  $\delta^U(0) = 0$ ,  $\bar{\alpha}_j^R = \bar{\alpha}_j^L$ , and  $\delta^R(\beta_{0j}) = -\delta^L(\beta_{0j})$ . When  $\beta_{0j} > 0$ ,  $\xi'(\bar{\alpha}_j^R) \leq \xi'(\bar{\alpha}_j^L)$  because  $\xi(\cdot)$  is assumed to be concave and  $\bar{\alpha}_j^R \geq \bar{\alpha}_j^L$ . Moreover it is straightforward to show that  $\delta^R(\beta_{0j}) + \delta^L(\beta_{0j}) < 0$  and  $\delta^L(\beta_{0j}), \delta^U(\beta_{0j}) < 0$ . Thus the derivative in equation (8) is strictly negative. Symmetric reasoning shows that this derivative is strictly positive when  $\beta_{0j} < 0$ . Thus, the unique solution is for the monopolist to choose  $\beta_{0j} = 0$  and  $\alpha_{0j} = 1$ .

Since revenues at this position are strictly positive, the monopolist will enter when  $\lambda$  is sufficiently low. Overconfidence for biased agents follows from noting that  $a_i = \sqrt{a_0^2 + b_{0i}^2}$ , and the expected disagreement result follows from noting that  $\bar{\alpha}_j^R = \bar{\alpha}_j^L$ .

## A.12 Proof of Proposition 10

Suppose one of the positions  $\{\beta^L, 0, \beta^R\}$  is not occupied by any outlet. Then a potential entrant  $j$  can enter into this position and become the unique outlet with maximum trust from the associated type of agent. This outlet will have a trust of one and hence positive revenue  $\xi(1)$  from the associated agent type, so entry will be profitable for sufficiently low  $\lambda$ . Let  $\lambda$  be any  $\lambda$  small enough to support positive profit for at least two trust-maximizing outlets in every position earning revenue exclusively from their corresponding agent type, i.e.,<sup>23</sup>

$$\left\{ \lambda \left| \min_{i \in \{U, R, L\}} \frac{1}{2} m_i \xi(1) - \lambda > 0 \right. \right\}.$$

Furthermore, when these positions are occupied, an outlet at any other position earns zero revenue and strictly negative profit since  $\lambda > 0$ . Thus, in any equilibrium all entrants must locate at one of these positions.

It remains to show that an equilibrium exists with at least one outlet in each position. The above result reduces the problem to a standard sequential entry game with three possible locations. Let  $\Pi_L(J_L, J_U, J_R)$  denote the profit earned by an outlet in position  $\beta^L$  in a market with  $J_L, J_U, J_R$  firms in the three positions. We show two properties about  $\Pi_L$  that will be useful later. First, once the other positions (0 and  $\beta^R$ ) have each been occupied by at least one outlet, the specific number of such outlets no longer affect  $\Pi_L$ : for any  $J_L, J_U, J'_U, J_R, J'_R$  such that  $\min(J_U, J'_U, J_R, J'_R) \geq 1$ ,

$$\Pi_L(J_L, J_U, J_R) = \Pi_L(J_L, J'_U, J_R).$$

Moreover, by definition  $\Pi_L$  is strictly decreasing in  $J_L$  and  $\Pi_L(2, J_U, J_R) > 0$  for any  $J_U, J_R$  due to our earlier choice of  $\lambda$ , so there exists a number of potential entrants  $E_L(\lambda) < \infty$  such that  $\Pi_L(E_L, J_U, J_R) < 0$  for any  $J_U, J_R \geq 1$ . Combining this with our preceding result, we obtain the

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<sup>23</sup>Requiring enough profit for two trust-maximizing outlets in every position eliminates edge cases where the market only supports a limited number of outlets. Such outlets could instead choose to pool in one or two of the locations rather than spread across all three.

second property: there exists a unique  $J_L^* \in [2, E_L)$  such that for any  $J_U, J_R \geq 1$ ,

$$\begin{aligned}\Pi_L(J_L^*, J_U, J_R) &\geq 0 \\ \Pi_L(J_L^* + 1, J_U, J_R) &< 0.\end{aligned}$$

This unique  $J_L^*$  is the threshold beyond which entry into position  $\beta^L$  is no longer profitable. Let  $\Pi_U$  and  $\Pi_R$  denote similar objects for positions 0 and  $\beta^R$ , where similar arguments show that these two properties hold as well. Let  $J_U^*$  and  $J_R^*$  denote their counterparts to  $J_L^*$ .

The tuple  $(J_L^*, J_U^*, J_R^*)$  of outlets is an equilibrium if the following conditions hold for  $\Pi_L$ :

$$\begin{aligned}\Pi_L(J_L^*, J_U^*, J_R^*) &\geq 0 \\ \Pi_L(J_L^* + 1, J_U^*, J_R^*) &< 0 \\ \Pi_L(J_L^*, J_U^*, J_R^*) &\geq \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*) \\ \Pi_L(J_L^*, J_U^*, J_R^*) &\geq \Pi_R(J_L^* - 1, J_U^*, J_R^* + 1),\end{aligned}$$

and similar conditions hold for  $\Pi_U$  and  $\Pi_R$ . The first two conditions follow from the definition of  $J_L^*$ . Next, at  $(J_L^*, J_U^*, J_R^*)$  we have  $\Pi_L(J_L^*, J_U^*, J_R^*) \geq 0$  by definition of  $J_L^*$  and  $0 > \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*)$  by definition of  $J_U^*$  (noting that  $J_L^* \geq 2$  and so  $J_L^* - 1 \geq 1$ ). Hence  $\Pi_L(J_L^*, J_U^*, J_R^*) > \Pi_U(J_L^* - 1, J_U^* + 1, J_R^*)$ , giving us the third condition. The fourth condition follows similarly. The same arguments prove that the corresponding conditions hold for  $\Pi_U$  and  $\Pi_R$ . The tuple  $(J_L^*, J_U^*, J_R^*)$  is also the unique equilibrium: by uniqueness of  $(J_L^*, J_U^*, J_R^*)$ , no other candidate tuple can satisfy the first two conditions simultaneously. Thus, at least one outlet chooses each of the positions  $\{\beta^L, 0, \beta^R\}$  in equilibrium.

Applying Corollary 5 shows that agents are overconfident. Expected disagreement follows from equation (11), noting that all outlets have a trust of one and the positions  $\{\beta^L, \beta^R\}$  approach  $\pm 1$  as  $a_0 \rightarrow 0$ .

## B Asymptotic Beliefs when $x_{it}$ , $s_{jt}$ and $r_t$ have Nonzero Means

We show in this appendix that allowing the means of  $x_{it}$ ,  $s_{jt}$ , and  $r_t$  to be non-zero does not affect our main results.

Suppose that

$$\begin{bmatrix} \omega_t \\ r_t \\ x_{it} \\ s_t \end{bmatrix} \sim N(M_i, \Omega_i), \text{ where } M_i = \begin{bmatrix} 0 \\ \bar{r} \\ \bar{x}_i \\ \bar{s} \end{bmatrix}.$$

We assume that the set of all possible means  $M_i$  is a compact set  $M$ . As before, the set of all possible variances  $V_i$  is a compact set  $\mathcal{V}$ . The correlation matrix for  $(\omega_t, r_t, x_{it}, s_t)$ , which we denote as  $\tilde{\Omega}_i$ , is fully parametrized by  $\theta_i$ . With slight abuse of notation, the Lebesgue space on  $(\Theta, \mathcal{M}, \mathcal{V})$  is now denoted  $((\Theta, \mathcal{V}, \mathcal{M}), \mathcal{L}, v)$ .

**Assumption 4.** *At the beginning of the first period, agents have an absolutely continuous prior belief  $\mu_0^i$  over  $(\Theta, \mathcal{V}, \mathcal{M})$  with a continuous density with respect to  $v$ .*

It is straightforward to show that with a slight modification in definitions, our core results continue to hold.

**Proposition 11.** *Under Assumption 4 and the modification that  $\tilde{s}_{jt} = (s_{jt} - \bar{s}_j) / \sqrt{\text{Var}(s_{jt})}$ , Propositions 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 hold.*

*Proof.* We first show the following:

**Lemma 5.** *The sequence of data realizations  $D_{i1}, \dots, D_{it}$  is independent of  $\theta_{0i}$  conditional on  $M_{0i}$ ,  $R_{0i}$  and  $V_{0i}$ .*

*Proof.* First consider Case 1 (as defined in the proof of Proposition 1). With slight abuse of notation, define  $D_{i\tau}^j = (s_{j\tau}, x_{i\tau})$  to be the  $\tau$ th time  $j$ 's signal and reasoning are observed. The agent knows  $D_{i\tau}^j \sim N(M_{0i}, \Omega_{0ij})$  for some positive definite  $\Omega_{0ij}$ , where  $\Omega_{0ij} = V_{0ij}^{\frac{1}{2}} R_{0ij} V_{0ij}^{\frac{1}{2}}$ , where  $V_{0ij} = \text{diag}(\Omega_{0ij})$  and  $R_{0ij}$  is the correlation matrix for  $D_{i\tau}^j$ , which in Case 1 is the correlation  $\rho_{ij}$ . Independence across periods then generalizes the result to the entire sequence of data observations. Cases 2 and 3 can be proved in the same way.  $\square$

By applying Lemma 5 to the proof of Proposition 1, we can prove that, as  $T \rightarrow \infty$ , agent  $i$ 's posterior distribution on  $\theta$  in any period  $t > \varepsilon T$  converges to a limit  $\mu_{\infty, \theta}^i$  that depends only on her prior  $\mu_{0, \theta}^i$  and the correlations  $R_{0i}$ . In other words, Proposition 1 continues to hold under Assumption 4. Then by the properties of the multivariate normal distribution, in any period  $t$  where an agent observes source  $j$ , the mean of her posterior on  $\omega_t$  given  $s_{jt}$  (but not  $x_{it}$  or  $r_t$ ) under  $\mu_{\infty, \theta}^i$  is

$$\bar{\omega}_t^i(s_{jt}) = \bar{\alpha}_j^i \tilde{s}_{jt}, \quad (9)$$

where  $\tilde{s}_{jt} = (s_{jt} - \bar{s}_j) / \sqrt{Var(s_{jt})}$  is the new standardized version of  $s_{jt}$ . This proves that Proposition 2 holds under Assumption 4 and the modified  $\tilde{s}_{jt}$ . The remaining propositions can be shown to hold under Assumption 4 and the modified  $\tilde{s}_{jt}$  by applying the modified Propositions 1 and 2 to the respective proofs of Propositions 3, 4, 5, 6, 7, 8, 9 and 10.  $\square$

## C Trust and Polarization when Agents Believe their own Bias is Nonzero

We consider a case where agents place a dogmatic prior on  $b_i = b^*$  for some  $b^* \neq 0$ . We follow the same setup as those in Section 3, namely that there exist multiple available sources, but agents are single-homing and ideological valence  $r_t$  is never observed. As in Section 3, we make the following assumptions, with the first having been modified to incorporate the dogmatic prior on  $b_i$ :

**Assumption 5.** *The support of  $\mu_{0, \theta}^i$  is the set  $\Theta_i^{prior} \subset \Theta$  for which  $b_i = b^*$  and  $a_i = a_{0i}$ .*

**Assumption 6.**  *$b_0 \leq a_0$  and  $|\alpha_j| + |\beta_j| \leq 1$  for all  $j$ .*

The following proposition (originally Proposition 4) only requires minor modification.<sup>24</sup><sup>25</sup>

**Proposition 12.** *Suppose  $r_t$  is never observed and agents single home. Under Assumptions 5 and 6, agent  $i$ 's identified set,  $I_i(R_{0i})$ , is non-empty and consists of all  $\theta_i \in \Theta$  such that  $a_i = a_{0i}$ ,*

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<sup>24</sup>The results from Section 2 do not rely on the agent's prior on  $b_i$ .

<sup>25</sup>Unlike in the original proposition, the presence of a nonzero  $b^*$  here means that the identified set may sometimes exclude extreme values of  $\beta$ . The specific bounds are not necessary for the results that follow, and we do not derive explicit expressions for them.

$b_i = b^*$ ,  $\alpha = \frac{\rho_{is} - \beta b^*}{a_i}$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite, where  $\tilde{\Omega}(\theta_i)$  is the correlation matrix for  $(\omega_t, r_t, x_{it}, s_t)$ .

*Proof.* The proof for Proposition 4 (reproduced with only minor modification below) applies the expressions from Assumption 5 and then shows that the identified set is non-empty by picking (among other parameters)  $\beta = 0$ . The same argument can be used to prove the new setting, as the degenerate prior on  $b_i = b^*$  has no effect in the case with  $\beta = 0$ .

Recall that  $I_i(R_{0i}) = \left\{ \theta_i \in \Theta_i^{prior} : a_i\alpha + b_i\beta = \rho_{is} \right\}$ , where any  $\theta_i \in \Theta_i^{prior}$  must give rise to a positive semi-definite correlation matrix  $\tilde{\Omega}(\theta_i)$  for  $(\omega_t, r_t, x_{it}, s_t)$ . Assumption 5 implies that  $\theta_i \in I_i(R_{0i})$  if and only if  $a_i = a_0$ ,  $b_i = b^*$ ,  $\alpha = \frac{\rho_{is} - \beta b^*}{a_i}$ , and  $\tilde{\Omega}(\theta_i)$  is positive semi-definite. Assumption 6 implies that  $|\alpha_{0j}| = |\rho_{ij}/a_{0i}| = |\alpha_{0j} + (b_{0i}/a_{0i})\beta_{0j}| \leq ||\alpha_{0j}| + |\beta_{0j}|| \leq 1$ . We can then pick  $\beta = 0$ ,  $\gamma = 0$ , and assume that  $s_{jt}$  are mutually independent and independent of  $r_t$  and  $x_{it}$  conditional on  $\omega_t$ , so  $\Sigma = \text{Corr}(s_t) = \alpha\alpha' + K$ , where  $K$  is a diagonal matrix with entries equal to  $1 - \alpha_j^2$ . It follows that  $\tilde{\Omega}(\theta_i)$  is positive semi-definite, so  $I_i(R_{0i})$  is non-empty.  $\square$

Under Assumptions 5 and 6, Propositions 1 and 12 imply the agent's trust in information source  $j$  is<sup>26</sup>

$$\bar{\alpha}_j^i = \frac{\rho_{ij} - \bar{\beta}_j^i b^*}{a_{0i}} = \frac{a_{0i}\alpha_{0j} + b_{0i}\beta_{0j} - \bar{\beta}_j^i b^*}{a_{0i}} = \alpha_{0j} + \frac{b_{0i}\beta_{0j} - \bar{\beta}_j^i b^*}{a_{0i}},$$

where  $\bar{\beta}_j^i$  denotes agent  $i$ 's expectation of  $\beta_j$  under  $\mu_{\infty, \theta}^i$ .

**Proposition 13.** Suppose  $r_t$  is never observed, agents single home, and Assumptions 5 and 6 hold. Then given any prior  $\mu_{0, \theta}^i$ , there exists  $b_{max}^* > 0$  such that for all  $b^* \in [-b_{max}^*, b_{max}^*]$ , agent  $R$ 's ( $L$ 's) trust in source  $j$  is increasing (decreasing) in the source's bias  $\beta_{0j}$  holding constant the source's accuracy  $\alpha_{0j}$ . Additionally, in the limit as  $b^* \rightarrow 0$  and then as  $a_0 \downarrow b_0$ , she will come to believe that a perfectly right-biased (left-biased) source is perfectly accurate, and trust it more than any unbiased source with  $\alpha_{0j} < 1$ .

*Proof.* Without loss of generality, let  $i$  be an  $R$ -agent. For the first result, note the derivative of her

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<sup>26</sup>For any  $\theta_i \in I_i(R_{0i})$ , by Proposition 12  $\alpha_j^i = \frac{\rho_{ij} - \beta_j^i b^*}{a_{0i}}$ . The expression for trust  $\bar{\alpha}_j^i$  is then obtained by taking the expectation over the limiting posterior  $\mu_{\infty, \theta}^i$ .

trust  $\bar{\alpha}_j^i$  with respect to in a source  $j$ 's bias  $\beta_{0j}$  holding constant its accuracy  $\alpha_{0j}$  is:

$$\frac{\partial}{\partial \beta_{0j}} \bar{\alpha}_j^i \propto b_{0i} - \left( \frac{\partial}{\partial \beta_{0j}} \bar{\beta}_j^i \right) b^*.$$

For any continuous prior  $\mu_{0,\theta}^i$ ,  $\frac{\partial}{\partial \beta_{0j}} \bar{\beta}_j^i$  is bounded above by some finite  $\bar{M}_j$  and below by some finite  $\underline{M}_j$  for the closed interval  $\beta_{0j} \in [-1, 1]$ . First, consider any  $b^* > 0$ . If  $\bar{M}_j \leq 0$  then the second term above is always strictly negative. If  $\bar{M}_j > 0$ , then for any  $b^* < b_{0i}/\bar{M}_j$  we have  $b_{0i} > \bar{M}_j b^* \geq \left( \frac{\partial}{\partial \beta_{0j}} \bar{\beta}_j^i \right) b^*$ . In both cases, we have the desired result of  $\frac{\partial}{\partial \beta_{0j}} \bar{\alpha}_j^i > 0$  for all  $\beta_{0j}$ . A similar argument with  $\underline{M}_j$  proves the result for  $b^* < 0$ . Hence we can set  $b_{max}^* = \min \{ \min_j |b_{0i}/\bar{M}_j|, \min_j |b_{0i}/\underline{M}_j| \}$  to guarantee the result for all  $b^* \in [-b_{max}^*, b_{max}^*]$  and across all sources  $j$ .

For the second result, the  $R$ -agent's trust with respect to a perfectly right-biased source is  $\bar{\alpha}_j^i = \frac{b_{0i} - \bar{\beta}_j^i b^*}{a_{0i}}$ . Since  $\bar{\beta}_j^i$  is bounded within  $[-1, 1]$  for any prior, taking the limit as  $b^* \rightarrow 0$  and then  $a_0 \downarrow b_0$  gives  $\bar{\alpha}_j^i = 1$ . For any unbiased source with  $\alpha_{0j} < 1$ , the agent's trust is  $\bar{\alpha}_j^i = \alpha_{0j} - \frac{\bar{\beta}_j^i b^*}{a_{0i}}$ , which approaches  $\alpha_{0j} < 1$  in the limit as  $b^* \rightarrow 0$ .  $\square$

**Corollary 7.** Suppose  $r_t$  is never observed, agents single home, and Assumptions 5 and 6 hold. Further suppose that all sources have accuracy  $\alpha_{0j} < 1$  and there is at least one perfectly right-biased source and at least one perfectly left-biased source. In the limit as  $b^* \rightarrow 0$  and then  $a_0 \downarrow b_0$ , expected disagreement is one.

*Proof.* Follows from Proposition 13.  $\square$

## D Trust and Disagreement with Overconfidence

In this section, we derive general results for trust and beliefs in the presence of overconfidence, as discussed in Section 5. Here we drop the simplifying assumption that  $a_i^{max} = \sqrt{a_0^2 + b_{0i}^2}$ , and instead assume that  $a_i^{max} \geq \sqrt{a_0^2 + b_{0i}^2}$ , as in Assumption 1'.

**Lemma 6.** Under Assumption 1', agent  $i$ 's trust in information source  $j$  is

$$\bar{\alpha}_j^i = A_i \rho_{ij} = A_i (a_0 \alpha_{0j} + b_{0i} \beta_{0j}),$$

where the amplification factor  $A_i$  is given by

$$A_i = \int_{\underline{a}_i}^{a_i^{\max}} \frac{1}{a} d\mu_{\infty,a}^i(a). \quad (10)$$

*Proof.* This follows from combining Propositions 1 and 6.  $\square$

Lemma 6 shows that, as before, small differences in biases  $b_0$  and  $\beta_{0j}$  translate into large differences in trust, provided the amplification factor  $A_i$  is large. The amplification factor  $A_i$  is typically large whenever  $a_0$  and  $b_{0i}$  are small. To see this, suppose that the posterior marginal probability density function on  $a_i$  is nonincreasing. It follows then that  $A_i \rightarrow \infty$  as  $\underline{a}_i \rightarrow 0$ . In other words,  $A_i$  is large if  $\underline{a}_i \leq \sqrt{a_0^2 + b_{0i}^2}$  is small and the agent's posterior places sufficient weight on values of  $a_i$  close to  $\underline{a}_i$ .

**Lemma 7.** *The accuracy and bias  $(\alpha_i, \beta_i)$  that maximizes the agent  $i$ 's trust over all pairs satisfying the feasibility condition  $\alpha_i^2 + \beta_i^2 \leq 1$  is given by*

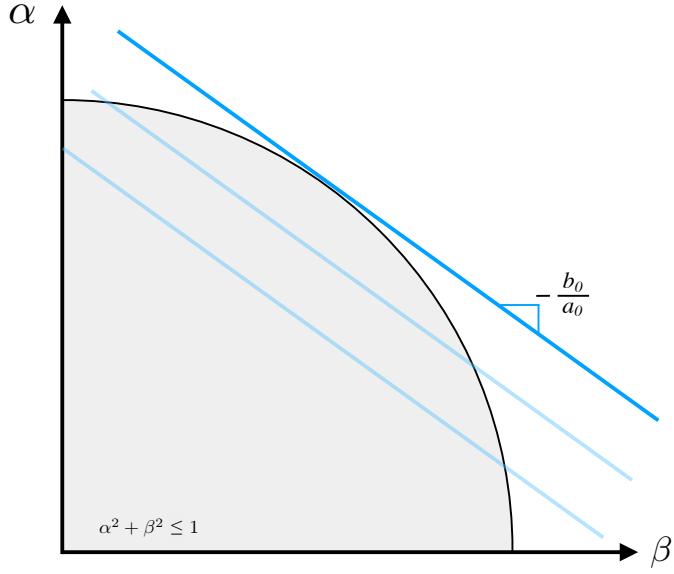
$$(\alpha_i^{\max}, \beta_i^{\max}) \equiv \left( \frac{a_0}{\sqrt{a_0^2 + b_{0i}^2}}, \frac{b_{0i}}{\sqrt{a_0^2 + b_{0i}^2}} \right).$$

*Proof.* By Lemma 6, we choose  $(\alpha_i, \beta_i)$  to maximize  $A_i(a_0\alpha_i + b_{0i}\beta_i)$  subject to  $\alpha_i^2 + \beta_i^2 \leq 1$ . Standard constrained optimization techniques yield the result.  $\square$

For intuition, Figure 1 provides a graphical illustration of the forces that determine trust in our model. The gray shaded area shows the set of all feasible signals—i.e., the  $(\alpha, \beta)$  satisfying the constraint that  $\alpha^2 + \beta^2 \leq 1$ . The curved boundary of this area is defined as the set of *frontier sources* which have maximum possible accuracy given their bias. The blue lines in the figure plot the set of *iso-trust curves*: combinations of  $\alpha$  and  $\beta$  that yield the same trust. The slope of these lines is  $-\frac{b_0}{a_0}$ . Sources that fall on higher iso-trust curves are trusted more. From this graphical analysis, it is immediately apparent that the trust-maximizing source  $(\alpha^{\max}, \beta^{\max})$  will be the point on the frontier tangent to the iso-trust curves.

**Lemma 8.** *In the limit as  $T \rightarrow \infty$ , whenever a single-homing agent  $i$  observes source  $j$  in any*

Figure 1: Iso-trust curves



period  $t > \varepsilon T$ , the mean of the agent's posterior on  $\omega_t$  given  $s_{jt}$  is

$$\bar{\omega}_t^i(j) = \bar{\alpha}_j^i \tilde{s}_{jt} = A_i \rho_{ij} \tilde{s}_{jt}, \quad (11)$$

where  $\tilde{s}_{jt} = s_{jt} / \sqrt{\text{Var}(s_{jt})}$  is the standardized version of  $s_{jt}$ .

*Proof.* This follows from combining Lemma 6 with Proposition 2.  $\square$

## E Trust and Polarization when Agents may Observe Perverse Sources

This appendix drops the restriction that a single-homing agent in any period  $t > \varepsilon T$  chooses among sources that she believes to have *nonnegative* accuracy. We instead assume the following:

**Assumption 7.** *In any period  $t > \varepsilon T$ , a single-homing agent chooses to observe a source among all available sources to minimize her expected mean squared error after observing the source and her own reasoning.*

**Lemma 9.** *Under Assumption 7, if all available sources are frontier sources (such that  $\alpha_{0j}^2 + \beta_{0j}^2 = 1$ ) with positive accuracy (i.e.,  $\alpha_{0j} > 0$ ) and symmetric biases (i.e., for any source with  $\beta_{0j} = \beta$*

where  $\beta \in (-1, 1)$ , there exists a source  $k$  with  $\beta_{0k} = -\beta$ ), then biased agents will observe a like-minded source in all periods  $t > \varepsilon T$ .

*Proof.* This lemma follows from applying our assumption that  $a_{0i} > 0$  to Equation (2) and Lemma 6, and then applying the logic of the proof of Proposition 3.  $\square$

**Lemma 10.** *Under Assumption 7, if a trust-maximizing source exists, the agent observes a source with  $\bar{\alpha}_j^i = 1$  in all periods  $t > \varepsilon T$ .*

*Proof.* This lemma follows immediately from our assumption that  $a_{0i} > 0$ , which implies that if a trust-maximizing source exists, there exists a source such that  $\bar{\alpha}_j^i = 1$  and there can be no source such that  $\bar{\alpha}_j^i = -1$ .  $\square$

The above two lemmas imply that all main propositions except Proposition 3 continue to hold.

**Proposition 14.** *Suppose Assumption 7 holds. Propositions 1, 2, 4, 5, 6, 7, 8, 9 and 10 hold. However, if agent  $i$  single-homes, her posterior belief on  $\theta_i$  is  $\mu_{\infty, \theta}^i$ , and the expected accuracy  $a_i$  of her own reasoning under  $\mu_{\infty, \theta}^i$  is less than one, then she chooses to observe in each period a source  $j$  for whom  $|\bar{\alpha}_j^i|$  is maximal.*

*Proof.* Propositions 1, 2, 4, 5, 6, and 7 do not depend on the choice of sources to observe during the exploitation phase, so their proofs are unaffected by Assumption 7. Proposition 8 can be shown by applying Lemma 9, since sources are assumed to be symmetric. For Proposition 9, there is only one source, so the agent's choice does not matter. Proposition 10 can be shown by applying Lemma 10, since outlets will still choose the trust-maximizing positions. The last sentence follows from the proof of Proposition 3.  $\square$

## F Asymptotic Learning without Reasoning

Can agent  $i$  learn about  $\omega_t$  if the agent does not observe any information source that she *ex ante* believed to be unbiased? As discussed in Section 2.4 and shown in this section, learning about  $\omega_t$  from  $s_t$  is not possible if reasoning  $x_{it}$  were not available. Since  $\rho_{is}$  and  $\rho_{ir}$  are not observed in this case, the distribution of observable data is given by  $R_{0i} = \rho_{rs}$  alone. The identified set  $I_i(R_{0i})$  consistent with observed data  $\rho_{rs}$  contains a wide range of parameter values, including  $\alpha_j = 1$  for

some source  $j$ , or  $\alpha_j = -1$  for the same source  $j$ . The agent thus cannot rule out the extreme possibilities that any of the sources is perfectly positively correlated, uncorrelated, or perfectly negatively correlated with the true state.

Without reasoning  $x_{it}$ , the agent's posterior mean on  $\omega_t$  may always be zero regardless of what signals are available. This occurs whenever the agent's prior are  $(\alpha, \gamma)$ -symmetric, meaning that  $\mu_{0,\theta}^i(\vartheta) = \mu_{0,\theta}^i(\vartheta')$  for all measurable  $\vartheta \subseteq \mathcal{L}_\Theta$ , where  $\vartheta' = \{(a, -\alpha, b, \beta, -\gamma, \Sigma) \mid (a, \alpha, b, \beta, \gamma, \Sigma) \in \vartheta\}$ . Intuitively, the agent's average belief about  $\omega_t$  does not change after observing  $s_t$  if she believes *a priori* that the correlation of  $\omega_t$  with any observable source (i.e. any element of  $s_t$  or  $r_t$ ) is zero in expectation.

**Proposition 15.** *Suppose agent  $i$  does not observe  $x_{it}$  in any period, but still observes  $r_t$ . Under Assumption 1', agent  $i$ 's identified set,  $I_i(R_{0i})$ , includes  $\theta_i \in \Theta$  such that  $\alpha_j = z$ , for any source  $j$  and any  $z \in [-1, 1]$ . Furthermore, the mean of the agent's posterior on  $\omega_t$  given  $s_t$  in any period  $t > \varepsilon T$  is zero in the limit as  $T \rightarrow \infty$  if the agent's prior is  $(\alpha, \gamma)$ -symmetric.*

*Proof.* Consider the multi-homing case first. Take any  $z \in [-1, 1]$  and  $j \in \{1, \dots, J\}$ . Define  $\theta_i$  as follows. Set  $a = a_0$ ,  $b = 0$ ,  $\alpha_j = z$ ,  $\alpha_k = z\Sigma_{jk}$  for all  $k \neq j$ ,  $\gamma = z\rho_{rj}$  and  $\beta = \frac{1}{\sqrt{1-\gamma^2}}(\rho_{rs} - \gamma\alpha)$ . It is immediate that  $\rho_{rs} = \alpha\gamma + \beta\sqrt{1-\gamma^2}$  (as required by Remark 1). Furthermore, note that  $\theta_i$  corresponds to a well-defined correlation matrix for the joint distribution of  $(\omega_t, r_t, s_t)$ .<sup>27</sup> Therefore, we have that  $\theta_i \in I_i(R_{0i})$ . The same  $\theta_i$  works in the single-homing case, which only requires a well-defined correlation matrix for the unit-normal joint distribution of  $(\omega_t, r_t, s_{kt})$  for each  $k$ .

By Propositions 1 and the properties of the multivariate normal distribution, the mean of the multi-homing agent's posterior on  $\omega_t$  given  $s_t$  is

$$\frac{\int_{I_i(R_{0i})} \alpha' \Sigma^{-1} \tilde{s}_t d\mu_{0,\theta}^i(\theta_i)}{\mu_{0,\theta}^i(I_i(R_{0i}))},$$

---

<sup>27</sup>This correlation matrix is given by

$$\bar{\Omega} = \begin{bmatrix} 1 & \gamma & \alpha' \\ \gamma & 1 & \rho'_{rs} \\ \alpha & \rho_{rs} & \Sigma \end{bmatrix}.$$

Setting  $\gamma = z\rho_{rj}$ ,  $\alpha_j = z$  and  $\alpha_k = z\Sigma_{jk}$  for all  $k \neq j$ , where  $z \in [-1, 1]$ , corresponds to supposing that  $\omega_t = z\tilde{s}_{jt} + (1-z)e_t$ , where  $e_t \sim N(0, 1)$  and is independent of  $(r_t, s_t)$ . Since it follows that  $\omega_t \sim N(0, 1)$ ,  $\bar{\Omega}$  is well-defined.

where  $I_i(R_{0i}) = \left\{ \theta_i \in \Theta : \tilde{\alpha}' \tilde{\Sigma}^{-1} \tilde{\alpha} \leq 1; b = 0 \right\}$ . Under single-homing, the analogous mean is

$$\frac{\int_{I_i(R_{0i})} \alpha_j \tilde{s}_{jt} d\mu_{0,\theta}^i(\theta_i)}{\mu_{0,\theta}^i(I_i(R_{0i}))},$$

where  $I_i(R_{0i}) = \left\{ \theta \in \Theta : \tilde{\alpha}_j' \tilde{\Sigma}_j^{-1} \tilde{\alpha}_j \leq 1 \forall j; b = 0 \right\}$ .<sup>28</sup> Since  $\mu_{0,\theta}^i$  is  $(\alpha, \gamma)$ -symmetric, both integrals above are zero.  $\square$

## G Large Market of Non-frontier Sources

This section considers a situation wherein there are many non-frontier sources that together provide a relatively “large” quantity of information about both  $\omega_t$  and  $r_t$ , as discussed in Section 6. In particular, suppose that the number of sources is large, the signals  $s_{jt}$  are mutually independent conditional on  $\omega_t$  and  $r_t$ , and there is at least a minimal amount of diversity in their biases. We formalize this notion of a “large and diverse” set of sources as follows.

**Definition 1.** A sequence of *random markets* is indexed by  $J = 1, 2, \dots, \infty$ . Random market  $J$  has  $J$  sources, indexed by  $j = 1, \dots, J$ , each with accuracy and bias  $(\alpha_{0j}, \beta_{0j})$  drawn i.i.d. from some distribution  $F$ . The sources’ signals  $s_{jt}$  are mutually independent conditional on  $\omega_t$  and  $r_t$ . Furthermore, under  $F$ ,

1. Both  $\alpha_{0j} \neq 0$  and  $\beta_{0j} \neq 0$  have nonzero probability;
2.  $\alpha_{0j}$  and  $\beta_{0j}$  are not perfectly correlated; and
3.  $\alpha_{0j}^2 + \beta_{0j}^2 < 1$  with probability one.

As shown in Proposition 16 below, a multi-homing agent’s posterior mean  $\bar{\omega}_t^i$  in a large random market is the same as a single-homing agent when she observes her trust-maximizing source. The reason is that a multi-homing agent in a random market can construct a linear combination of the sources’ signals whose value will approach the signal of the agent’s trust-maximizing source, as in Lemma 2.

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<sup>28</sup>See the proof of Propositions 6 and 7 for the definitions of  $\tilde{\alpha}_j$ ,  $\tilde{\Sigma}_j$ ,  $\tilde{\alpha}$  and  $\tilde{\Sigma}$ .

**Proposition 16.** Suppose Assumption 1' holds. Then the mean of the multi-homing agent's posterior on  $\omega_t$  given  $s_t$  under  $\mu_{\infty, \theta}^i$  in the probability limit of a sequence of random markets is

$$\bar{\omega}_t^i = \alpha_i^{max} \omega_t + \beta_i^{max} \tilde{r}_t.$$

*Proof.* By Lemma 1, it suffices to show that  $\lim_{J \rightarrow \infty} \rho_{is}' \Sigma^{-1} \tilde{s}_t = a_0 \omega_t + b_{0i} \tilde{r}_t$  and  $\lim_{J \rightarrow \infty} A_i = 1/\sqrt{a_0^2 + b_{0i}^2}$ . We use notation developed in Lemma 4, and let  $Q = ZK^{-1}Z'$ ,  $d_{\alpha\alpha} = \frac{1}{J} \sum_{j=1}^J \alpha_{0j}^2 / \kappa_{0j}^2$ ,  $d_{\beta\beta} = \frac{1}{J} \sum_{j=1}^J \beta_{0j}^2 / \kappa_{0j}^2$ , and  $d_{\alpha\beta} = \frac{1}{J} \sum_{j=1}^J \alpha_{0j} \beta_{0j} / \kappa_{0j}^2$ . It follows that  $Q = J \begin{bmatrix} d_{\alpha\alpha} & d_{\alpha\beta} \\ d_{\alpha\beta} & d_{\beta\beta} \end{bmatrix}$ . Next let  $W = Q(I+Q)^{-1}$ . By Woodbury's matrix identity, we can write  $Z(Z'Z+K)^{-1} = (I-W)ZK^{-1}$ .

It is easy to check that

$$W = \begin{bmatrix} \frac{1}{J} d_{\alpha\alpha} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 & -\frac{1}{J} d_{\alpha\beta} \\ \frac{1}{J^2} + \frac{1}{J} d_{\alpha\alpha} + \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 & \frac{1}{J^2} + \frac{1}{J} d_{\alpha\alpha} + \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 \\ -\frac{1}{J} d_{\alpha\beta} & \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 \\ \frac{1}{J^2} + \frac{1}{J} d_{\alpha\alpha} + \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 & \frac{1}{J^2} + \frac{1}{J} d_{\alpha\alpha} + \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2 \end{bmatrix}.$$

It is also easy to check that  $Z(Z'Z+K)^{-1}Z' = (I-W)Q = W$ . By Definition 1,  $E[\alpha_{0j}^2/\kappa_{0j}^2]$ ,  $E[\beta_{0j}^2/\kappa_{0j}^2]$ , and  $E[\alpha_{0j}\beta_{0j}/\kappa_{0j}^2]$  exist and are finite. Therefore,  $d_{\alpha\alpha}$ ,  $d_{\beta\beta}$ , and  $d_{\alpha\beta}$  converge in probability in the limit as  $J \rightarrow \infty$  by the weak law of large numbers. Furthermore, Definition 1 implies that  $\alpha_{0j}$  and  $\beta_{0j}$  are not linearly dependent, so neither are  $\alpha_{0j}/\sqrt{\kappa_{0j}^2}$  and  $\beta_{0j}/\sqrt{\kappa_{0j}^2}$ . By Cauchy-Schwarz, we have that  $E[\alpha_{0j}\beta_{0j}/\kappa_{0j}^2]^2 < E[\alpha_{0j}^2/\kappa_{0j}^2]E[\beta_{0j}^2/\kappa_{0j}^2]$ . This implies that  $\text{plim}(d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}^2) > 0$ . Therefore,  $Z(Z'Z+K)^{-1}Z' = W \rightarrow_p I$ .

Further algebraic manipulation shows that

$$y'Z(Z'Z+K)^{-1}\varepsilon_t = \frac{(a_0 d_{\beta\beta} + b_0 d_{\alpha\beta}) \left( \frac{1}{J} \sum_{j=1}^J \frac{\alpha_{0j}}{\sqrt{\kappa_j^2}} \tilde{\varepsilon}_{jt} \right) + (a_0 d_{\alpha\beta} + b_0 d_{\alpha\alpha}) \left( \frac{1}{J} \sum_{j=1}^J \frac{\beta_{0j}}{\sqrt{\kappa_j^2}} \tilde{\varepsilon}_{jt} \right)}{\frac{1}{J^2} + \frac{1}{J} d_{\alpha\alpha} + \frac{1}{J} d_{\beta\beta} + d_{\alpha\alpha} d_{\beta\beta} - d_{\alpha\beta}^2} + o_p(1)$$

where  $\tilde{\varepsilon}_{jt} = \varepsilon_{jt}/\sqrt{\kappa_j^2} \sim N(0, 1)$  are mutually independent across  $j$  as well as independent of  $\alpha_{0j}$  and  $\beta_{0j}$ . Therefore, by the weak law of large numbers,  $y'Z(Z'Z+K)^{-1}\varepsilon_t \rightarrow_p 0$ . It immediately follows from Lemma 4 that  $\rho_{is}' \Sigma^{-1} \tilde{s}_t = y'Z(Z'Z+K)^{-1}(Z'\varphi_t + \varepsilon_t) \rightarrow_p y'\varphi_t = a_0 \omega_t + b_{0i} \tilde{r}_t$ .

Next note that the  $R^2$  of the population regression of  $x_{it}$  on  $s_t$  is  $\rho_{is}' \Sigma^{-1} \rho_{is}$ . By Lemma 4 and

the first paragraph in this proof,  $\rho'_{is}\Sigma^{-1}\rho_{is} = y'Z(Z'Z+K)^{-1}Z'y \rightarrow_p y'y = a_0^2 + b_{0i}^2$ . Therefore, the  $R^2$  of the population regression of  $x_{it}$  on  $s_t$  and  $r_t$  also converges in probability to  $a_0^2 + b_{0i}^2$ . This implies that  $\underline{a}_i \rightarrow_p \sqrt{a_0^2 + b_{0i}^2}$ . By Assumption 3,  $\mu_{\infty,a}^i = \mu_{0,a}^i$ , so  $A_i \rightarrow_p 1/\sqrt{a_0^2 + b_{0i}^2}$ .  $\square$

## H Mistrust of Motives and Partisan Conflict

This appendix shows how ideological bias can lead to mistrust of motives across ideological divides and results in intensified conflict in political settings. This exploration is motivated by studies that show rising numbers of Americans hold negative views towards people on the other side of the partisan divide, for example, seeing them as unintelligent and selfish (Iyengar, Sood, and Lelkes 2012; Iyengar et al. 2019), with potentially important consequences such as reducing the efficacy of government (Hetherington and Rudolph 2015).<sup>29</sup>

We augment our model by adding an observable policy decision  $d_t$  to be made by one of two agents,  $R$  and  $L$ , and allow for ulterior motives  $B$  in decision making. We then characterize the agents' beliefs about the others' motive  $B$  when the agents assume both their and others' biases are  $b = 0$  when in fact  $b \neq 0$ . This assumption is deliberately stark to illustrate that if people underestimate the extent to which others' reasoning is biased, they may attribute their behavior to biased motives instead. More precisely, we show that agents mistakenly learn that  $B \neq 0$  even when in fact  $B = 0$ .

The setup is as follows. Suppose Assumption 1' holds and all agents are single-homers observing their trust-maximizing source  $(\alpha_i^{max}, \beta_i^{max})$  in some period  $t > \varepsilon T$  in the limit as  $T \rightarrow \infty$ . After observing the sources' signals in some period  $t$ ,  $R$  makes an observable policy decision  $d_t$  to maximize the social welfare function, given by  $-(\omega_t - d_t)^2$ .

We assume that agents fail to appreciate both their own and others' ideological bias and believe that  $b_i = 0$  for all agents  $i$ . Consequently, they believe that others have the same belief about the state  $\omega_t$  as they do. At the same time, agents entertain the possibility that others may have ulterior motives. Specifically, we assume that  $L$  believes that  $R$  maximizes  $-(\omega_t + B_R r_t - d_t)^2$ , where  $B_R$  parameterizes  $R$ 's ulterior motive and may not be equal to zero. Under these assumptions,

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<sup>29</sup>Relatedly, Ortoleva and Snowberg (2015) and Levy and Razin (2015) explore how correlation neglect and resulting overconfidence impact polarization and political behavior.

it is immediate that people who underestimate the extent to which others' *reasoning* is biased attribute observed behavior to biased *motives* instead.

If  $L$  observes  $R$ 's decision  $d_t$  in any period when  $r_t \neq 0$ , then  $L$  infers that  $B_R = 2b_0/\sqrt{a_0^2 + b_0^2} > 0$ . Similarly, if  $R$  were to observe  $L$ 's decisions,  $R$  would also conclude that  $L$  had an ulterior motive  $B_L = -2b_0/\sqrt{a_0^2 + b_0^2} < 0$ . In other words, mistrust of motives arises when well-meaning agents fail to see how ideological bias colors inference about facts by both themselves and others. The magnitude of mistrust in other's motives is increasing in ideological bias  $b_0$  of the agents.

It follows that the political behavior of well-meaning agents with ideological bias can mimic that of self-interested agents with actual conflicts of interest. For example, suppose that the above two agents engage in a contest for the power to decide  $d_\tau$  for some  $\tau > t$  after learning about the bias in each other's preferences. Tullock (1980) provides an elemental model of such a contest.  $R$  and  $L$  simultaneously invest in "arms" to obtain decision-making power, where the probability that  $R$  has power to decide  $d_\tau$  depends on  $R$ 's stock of arms relative to  $L$ 's. In  $L$ 's eyes, the payoff of obtaining decision-making power is zero when  $B_R = 0$ , since the two agents would choose the same decision. However, the gain from winning the contest becomes positive if  $L$  either perceives  $R$  to have a nonzero ulterior motive  $B_R$  or believes  $R$ 's inference about  $\omega_t$  to be biased. The symmetric Nash equilibrium therefore has positive expenditures on arms, even though in equilibrium the contestants have the same probabilities of winning as if neither had spent anything.

Other types of inefficient strategic behavior also arise from conflicts of interest in elemental game theoretic models of organizational behavior, including costly signaling, signal jamming, obfuscation and uninformative cheap talk (see Gibbons, Matouschek, and Roberts 2013). Ideological differences may therefore lead to welfare losses from uninformative communication across ideological divides, poor decision-making, as well as inefficient expenditures in the battle for power.

## I Finite-time Evolution of Beliefs

This appendix studies the finite-time evolution of beliefs and speed of convergence by presenting numerical simulations. Recall that each period's state variable  $\omega_t$  represents a different topic in the news—the efficacy of surgical masks in preventing the spread of COVID-19 in one period, the efficacy of Vitamin D in reducing the severity of COVID-19 infections in another period, and so on.

Here we characterize how quickly agents form beliefs about whether information sources provide accurate information about the states. Here we characterize how quickly agents form beliefs about whether information sources provide accurate information about the states.

For simplicity, we focus on two agents  $i \in \{R, U\}$ . Both agents have reasoning  $x_{it}$  with the accuracy  $a_{0i} = a$ , where  $a > 0$ , but different biases,  $b_{0R} = b$  and  $b_{0U} = 0$ , where  $b > 0$ . We assume that both agents *a priori* believe that  $b_i = 0$  and  $a_i = a$ . There are two sources  $j \in \{R, U\}$ , where  $(\alpha_R, \beta_R) = (\frac{1}{2}, \frac{1}{2})$  and  $(\alpha_U, \beta_U) = (1, 0)$ . The sources and the agents' reasoning have unit-normal distributions.

Each agent  $i$  observes both  $x_{it}$  and  $s_{jt}$  for the same source  $j$  in each period  $t$ . Let the covariance matrix of the observed signals  $(x_{it}, s_{jt})$  be denoted by  $\Sigma_{ij} = \begin{bmatrix} \sigma_i^2 & \rho_{ij} \\ \rho_{ij} & \sigma_j^2 \end{bmatrix}$ . Each agent has a prior  $f(\Sigma_{ij})$ , where

$$\Sigma_{ij} = \text{diag}(V_{ij}) \times \Omega_{ij} \times \text{diag}(V_{ij}),$$

$\Omega_{ij}$  is a correlation matrix, and  $V_{ij}$  is a vector of coefficient scales. We use an Lewandowski-Kurowicka-Joe (LKJ) prior with parameter  $\eta = 1$  for the correlation matrix. Setting  $\eta = 1$  implies that the density is uniform over all correlation matrices. We use a log normal prior with parameters  $\mu = 0$  and  $\nu = 0.0001$  for the scale. Furthermore, the prior is truncated for all values  $\Sigma_{ij}$  such that

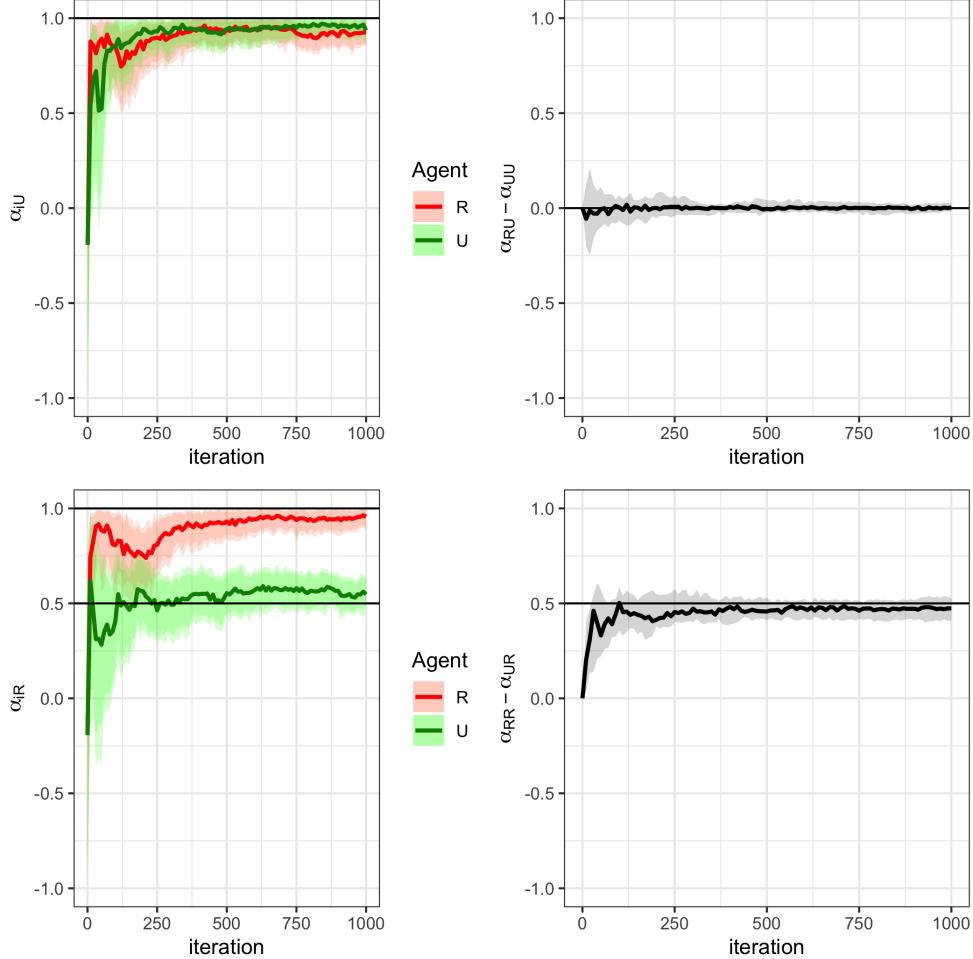
$$\alpha_{ij} = \frac{1}{a} \frac{\rho_{ij}}{\sqrt{\sigma_i^2 \sigma_j^2}} \notin [-1, 1].$$

We use the software platform STAN to simulate 1000 draws from the agent  $i$ 's posterior distribution in each period  $t$ .

Figure 2 shows belief evolution when  $a = b = \frac{1}{2}$ . On the left, we display the posterior distribution for  $\alpha_{ij}$  for a single sequence of signal realizations. On the right, we display the median difference in posterior mean between agents  $R$  and  $U$  across multiple sequences of signal realizations. The top two panels show that the trust of agents  $R$  and  $U$  in an unbiased source quickly converges towards the asymptotic value of one., while the difference in posterior mean converges to zero. The bottom two panels show that the trust of agents  $R$  and  $U$  in a  $R$ -biased source quickly diverges.

Figure 3 shows the case where  $a = b = \frac{1}{5}$ . In this case, the agent's reasoning is noisier,

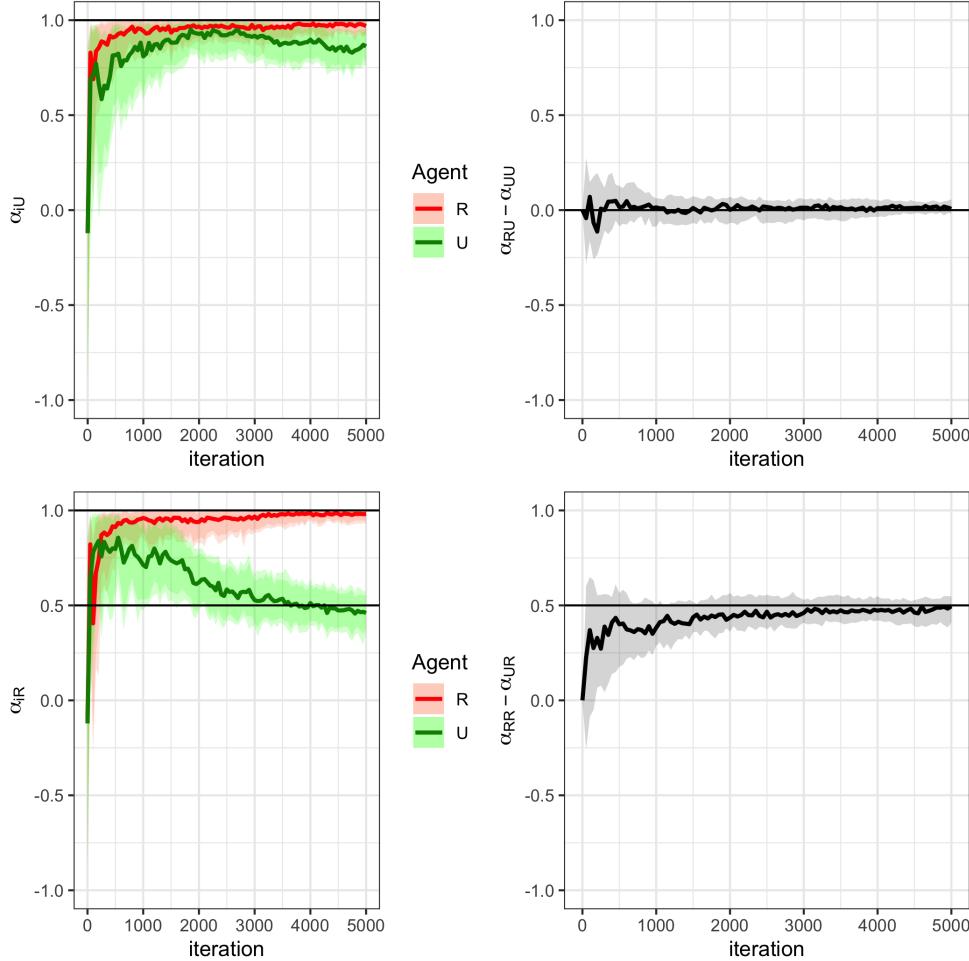
Figure 2: Finite-time belief evolution when  $a = b = \frac{1}{2}$



Notes: The upper left panel show the simulated posterior distribution of  $\alpha_{ij}$  for agents  $i = R, U$  observing an unbiased source  $j = U$  for single sequence of signal realizations. We assume that  $a = b = \frac{1}{2}$ . The thick lines indicate the posterior median. The lighter bands are for the [0.05, 0.95] percentiles and the darker bands are for [0.10, 0.90] percentiles. The horizontal line denotes the asymptotic values, respectively. In the upper right panel, the thick black line shows the median of the difference in posterior means between the two agents for 25 sequences of simulated signal realizations with different randomization seeds. The bands are for [0.20, 0.80] percentiles. The lower panels show the simulated posteriors when agents observe an biased source  $j = U$  instead of an unbiased source.

so convergence is slower. However, in all of the above cases, the beliefs converge towards the asymptotic beliefs that we analytically derive.

Figure 3: Finite-time belief evolution when  $a = b = \frac{1}{5}$



Notes: The upper left panel show the simulated posterior distribution of  $\alpha_{ij}$  for agents  $i = R, U$  observing an unbiased source  $j = U$  for single sequence of signal realizations. We assume that  $a = b = \frac{1}{5}$ . The thick lines indicate the posterior median. The lighter bands are for the [0.05, 0.95] percentiles and the darker bands are for [0.10, 0.90] percentiles. The horizontal line denotes the asymptotic values, respectively. In the upper right panel, the thick black line shows the median of the difference in posterior means between the two agents for 25 sequences of simulated signal realizations with different randomization seeds. The bands are for [0.20, 0.80] percentiles. The lower panels show the simulated posteriors when agents observe an biased source  $j = U$  instead of an unbiased source.