Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses

By A. Lans Bovenberg and Lawrence H. Goulder*

Most economies feature levels of public spending that require more tax revenues than would be generated solely from the pollution taxes set according to the Pigovian principle, that is, set equal to marginal environmental damages. As a consequence, tax systems generally rely on both environmental (corrective) and other taxes. However, economists typically have analyzed environmental taxes without taking into account the presence of other, distortionary taxes. The omission is significant because the consequences of environmental taxes depend fundamentally on the levels of other taxes, including income and commodity taxes.

This paper examines how optimal environmental tax rates deviate from rates implied by the Pigovian principle in a second-best setting where other, distortionary taxes are present. Previous investigations of this issue include the partial equilibrium analyses of Dwight R. Lee and Walter S. Misiolek (1986) and Wallace E. Oates (1991), who derive formulas linking the optimal rate for a newly imposed environmental tax to the marginal excess burden from existing taxes. In a general-equilibrium setting, Agnar Sandmo (1975) and Bovenberg and Frederick van der Ploeg (1994) have demonstrated how the well-known "Ramsey" formula for optimal commodity taxes is altered when one of the consumption commodities generates an externality.1

The present paper contributes to the analytical and empirical literature in three ways. First, it extends earlier analytical work on optimal environmental taxation in a general-equilibrium setting by considering pollution taxes imposed on intermediate inputs. This is a useful extension because many actual environmental regulations and taxes affect the costs of intermediate inputs.2 Second, the paper investigates second-best optimal environmental taxes numerically. Here we expand on the analytical work by employing a numerical general-equilibrium model of the United States. The use of a numerical model enables us to employ a realistic specification of taxes and adopt a fairly detailed representation of production and demand. Our paper thus combines the strengths of analytical and numerical approaches: a stylized analytical model uncovers the major mechanisms at play, while a numerical model explores the empirical significance of these mechanisms in a more realistic setting. Despite considerable differences in the complexity of the analytical and numerical models, we find a strong coherence between the two models' results.

The third contribution of the paper is its numerical investigation of optimal environmental tax policies in the presence of realistic policy constraints. The constraints involve either the inability to alter all tax rates (so that much of the initial, suboptimal tax system re-
mains) or the inability to use revenues from environmental taxes in optimal ways. We find that these constraints substantially affect the optimal environmental tax rates.

The paper is organized as follows. Section I develops a stylized general-equilibrium model to uncover the main determinants of optimal environmental taxes in the presence of distortionary taxes. Section II describes the numerical model, and Section III applies this model to evaluate the departures from Pigovian tax rules implied by second-best considerations. The final section offers conclusions.

I. Theoretical Issues and Analytical Results

This section explores analytically how the presence of distortionary taxes affects the optimal setting of environmental taxes on both intermediate inputs and consumption goods. To this end, we extend the model in Bovenberg and Ruud de Mooij (1994) by incorporating intermediate inputs. Output derives from a constant-returns-to-scale production function $F(L, x_c, x_D)$ with inputs not only of labor ($L$) but also of “clean” and “dirty” intermediate goods ($x_c$ and $x_D$, respectively). Output can be devoted to public consumption ($G$), to clean or dirty intermediate inputs, or to household consumption of a “clean” or “dirty” consumption good (denoted by $C_c$ and $C_D$, respectively). Hence, the commodity market equilibrium is given by $F(L, x_c, x_D) = G + x_c + x_D + C_c + C_D$. We normalize units so that the constant rates of transformation between the five produced commodities are unity.

The representative household maximizes utility $U(C_c, C_D, l, G, q) = u(N(H(C_c, C_D), l, G, q))$. Private utility $N(\cdot)$ is homothetic, while commodity consumption $H(\cdot)$ is separable from leisure, $l$. In addition, private utility is weakly separable from the two public goods, environmental quality ($Q$) and (nonenvironmental) public consumption ($G$). These assumptions on utility match the specifications of household behavior in the numerical model (see Section II).

The household faces the budget constraint $C_c + (1 + \tau_D)x_D + (1 - \tau_L)L = (1 - \tau_L)L$, where $\tau_D$ and $\tau_L$ denote, respectively, the tax rates on dirty consumption and labor. Without loss of generality, the tax on clean consumption is assumed to be zero. The labor tax rate and the producer (before-tax) wage $w$ yield the consumption (after-tax) wage, $w_N = (1 - \tau_L)w$.

The government budget constraint is $G = \tau_c x_c + \tau_D x_D + \tau_L C_D + \tau_L w L$, where $\tau_c$ and $\tau_D$ stand for the taxes on clean and dirty intermediate inputs, respectively. Environmental quality, $Q$, deteriorates with pollution, which is directly related to the quantity used of dirty intermediate and dirty consumption goods; thus, $Q = q(x_D, C_D)$, with $\partial q/\partial x_D < 0$. Private decision makers ignore environmental externalities.

To derive the optimal tax rates, we solve the government’s problem of maximizing household utility subject to the government budget constraint and the decentralized optimizing behavior of firms and households. Accordingly, the government chooses values of its four tax instruments $\tau_L, \tau_D, \tau_c$, and $\tau_D$ to maximize:

$$u\left[V(w_N, \tau_D), G, q(x_D, C_D)\right] + \mu(\tau_D x_c + \tau_D x_D + \tau_L C_D + \tau_L w L - G)$$

where $V$ represents indirect private utility and $\mu$ denotes the marginal utility associated with the public goods consumption made possible by one additional unit of public revenue.

Appendix A derives the optimal tax rates. The analysis reveals that the clean intermediate input should not be taxed (that is, $\tau_c = 0$). This is an application of the well-known result of Peter A. Diamond and James A. Mirrlees (1971a, b) demonstrating that, if production exhibits constant returns to scale, an optimal tax system should not distort production.
The optimal tax on the dirty intermediate input is (see Appendix A):

$$\tau_d = \left[ \frac{\partial U}{\partial Q} \left( -\frac{\partial q}{\partial x_d} \right) \right] \frac{1}{\eta}. \tag{2}$$

The term between the large square brackets on the right-hand side of (2) is the marginal environmental damage (MED) from this input. \(\eta \equiv \mu(\partial U/\partial C_c)\) is defined as the ratio of the marginal (utility) value of public revenue to the marginal utility of private income; it is often referred to as the marginal cost of public funds (MCPF). Analogously, the optimal tax on the dirty consumption good is the marginal environmental damage from the use of this good divided by the MCPF (see Appendix A):

$$\tau_d = \left[ \frac{\partial U}{\partial C_c} \right] \frac{1}{\eta}. \tag{3}$$

Equations (2) and (3) indicate how the presence of distortionary taxation affects the optimal environmental tax rate. In general, an optimal pollution tax induces the level of emissions at which the marginal welfare benefit from emissions reductions (MWBE) equals the marginal welfare cost of achieving such reductions (MWCE). In the special case of a first-best world without distortionary taxes, a one-unit reduction in emissions involves a welfare cost corresponding to the loss of tax revenue due to the erosion of the base of the pollution tax—thus the pollution tax rate represents the marginal welfare cost of emissions reductions (MWCE). Hence, in a first-best setting, optimality requires that the pollution tax be set equal to the marginal (environmental) benefit from pollution reduction (or marginal damage from pollution), which is given by the term in the large square brackets in equation (2) or (3). This is the Pigovian tax rate.

The MCPF term in equations (2) and (3) reveals how the presence of distortionary taxes requires a modification of the Pigovian principle. In particular, it shows that the Pigovian rate is optimal if and only if the MCPF is unity. A unitary MCPF means that public funds are no more costly than private funds. The higher the MCPF, the greater the cost of public consumption goods, including the public good of environmental quality. When these goods are more costly, the government finds it optimal to cut down on public consumption of the environment by reducing the pollution tax.

In a second-best world with distortionary taxes, the MCPF is given by (see Appendix A):

$$\eta = \left[ \frac{l - \tau L}{l - \tau L - \theta_L} \right]^{-1}. \tag{4}$$

The MCPF exceeds unity if 1) the uncompensated wage elasticity of labor supply, \(\theta_L\), is positive, and 2) the distortionary tax on labor, \(\tau_L\), is positive (which is required if Pigovian taxes are not sufficient to finance public consumption). Combining equation (4) with equation (2) (or (3)), we find that the presence of distortionary labor taxation reduces the optimal pollution tax below its Pigovian level if and only if \(\theta_L\) is positive. In a second-best world without distortionary taxes, a one-unit reduction in emissions involves a welfare cost corresponding to the loss of tax revenue due to the erosion of the base of the pollution tax—thus the pollution tax rate represents the marginal welfare cost of emissions reductions (MWCE). Hence, in a first-best setting, optimality requires that the pollution tax be set equal to the marginal (environmental) benefit from pollution reduction (or marginal damage from pollution), which is given by the term in the large square brackets in equation (2) or (3). This is the Pigovian tax rate.

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setting, environmental taxes are more costly because they exacerbate the distortions imposed by the labor tax. In particular, by reducing the real after-tax wage, they decrease labor supply if the uncompensated wage elasticity of labor supply is positive. In the presence of a distortionary labor tax, the decline in labor supply produces a first-order loss in welfare by eroding the base of the labor tax. This additional welfare loss raises the overall welfare cost associated with a marginal reduction in emissions. As a result, and in contrast with the first-best case, here the marginal welfare cost of a unit of emissions reduction (MWCE) exceeds the pollution tax rate. Thus, to equate marginal welfare costs and marginal social (environmental) benefits from emissions reduction, the optimal environmental tax must be set below the marginal social benefit, that is, below the Pigovian rate.

II. Basic Features of the Numerical Model

We employ a numerical model of the U.S. economy to examine further the issues of second-best optimal environmental taxation. This model enables us to relax restrictions of the analytical model and thereby assess these issues in a more realistic setting. The additional realism includes greater industry disaggregation, a more detailed treatment of the tax system, and attention to capital (in addition to labor) as a primary factor, which permits attention to dynamic effects. The numerical simulations also allow us to evaluate constrained-optimal environmental tax policies, where the constraints involve either the inability to optimize over all tax rates (so that some prior "imperfections" in the tax system remain) or the inability to optimally recycle revenues from environmental taxes.

A. Components and Behavioral Specifications

The model distinguishes the 13 industries (of which 6 are energy-producing industries) and the 17 consumer products identified in Table 1. In each industry, a nested-CES production structure accounts for substitution between different forms of energy as well as between energy and other inputs. Managers of

Some details on the model's structure and parameters are offered in Appendix B. A more complete description is in Goulder (1992). Miguel Cruz and Goulder (1992) provide data documentation.

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Public consumption that is separable from consumer's choice on leisure and consumption, this literature finds that distortionary labor taxes raise the marginal benefits of public consumption above its direct resource cost if the uncompensated wage elasticity of labor supply is positive.

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Table 1—Industry and Consumer-Good Categories

<table>
<thead>
<tr>
<th>Industries</th>
<th>Consumer goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Coal mining</td>
<td>1. Food</td>
</tr>
<tr>
<td>2. Crude petroleum and natural gas extraction</td>
<td>2. Alcohol</td>
</tr>
<tr>
<td>3. Synthetic fuels</td>
<td>3. Tobacco</td>
</tr>
<tr>
<td>4. Petroleum refining</td>
<td>4. Utilities</td>
</tr>
<tr>
<td>5. Electric utilities</td>
<td>5. Housing services</td>
</tr>
<tr>
<td>7. Agriculture and noncoal mining</td>
<td>7. Appliances</td>
</tr>
<tr>
<td>8. Construction</td>
<td>8. Clothing and jewelry</td>
</tr>
<tr>
<td>11. Miscellaneous manufacturing</td>
<td>11. Services (except financial)</td>
</tr>
<tr>
<td>12. Services (except housing)</td>
<td>12. Financial services</td>
</tr>
<tr>
<td>13. Housing services</td>
<td>13. Recreation, reading, and miscellaneous</td>
</tr>
<tr>
<td>15. Gasoline and other fuels</td>
<td>15. Gasoline and other fuels</td>
</tr>
<tr>
<td>16. Education</td>
<td>16. Education</td>
</tr>
<tr>
<td>17. Health</td>
<td>17. Health</td>
</tr>
</tbody>
</table>
firms choose input quantities and investment levels to maximize the value of the firm. Investment decisions take account of adjustment costs that are a convex function of the rate of gross investment.10

Consumption, labor supply, and saving result from the decisions of a representative household maximizing its intertemporal utility, defined on leisure and overall consumption in each period.11 As in the analytical model of the previous section, the utility function is homothetic and leisure and consumption are weakly separable. Overall consumption in each period is an aggregate of the 17 types of consumer goods, where each consumer good is in turn a composite of a domestic and foreign consumer good of a given type.

Except for oil and gas imports, imported intermediate and consumer goods are imperfect substitutes for their domestic counterparts. Import prices are exogenous in foreign currency, but the domestic-currency price changes with variations in the exchange rate. Export demands are modeled as functions of the foreign price of U.S. exports and the level of foreign income. The exchange rate adjusts to balance trade in each period.

The government's tax instruments include energy taxes, output taxes, the corporate income tax, property taxes, sales taxes, and taxes on individual labor and capital income. In policy experiments, we require that real government spending and real government debt follow the same paths as in the reference case.12

B. Equilibrium and Growth

The model generates paths of equilibrium prices, outputs, and incomes for the United States and the "rest of the world" under specified policy scenarios. All domestic prices in the model are endogenous, except for the domestic price of oil and gas, which is determined by the exogenously specified world oil price.13 The general-equilibrium solution requires that demand equal supply in all markets at all points in time.14 Equilibria are calculated at yearly intervals beginning in the 1990 benchmark year and usually extending to the year 2070.

Economic growth reflects the growth of capital stocks and of potential labor resources. The growth of capital stocks stems from endogenous saving and investment behavior. Potential labor resources are specified as increasing at an exogenous rate.

III. Optimal Environmental Taxes in a Second-Best Setting: Numerical Results

We focus on the policy of a carbon tax. This is a tax on fossil fuels—coal, crude oil, and natural gas—in proportion to their carbon content. Since carbon dioxide (CO2) emissions generally are proportional to the carbon content of these fuels, a tax based on carbon content is effectively a tax on CO2 emissions.

A. Marginal Costs of Emissions Reductions

Our use of the general equilibrium model is summarized in Figure 1. We explain this figure in several steps, starting with the vertical axis of Figure 1B, which shows the carbon tax rate in dollars per ton. As a first step, this tax rate is set exogenously and the model is used to calculate the general equilibrium associated with each value of the tax rate. Each exogenous setting of the carbon tax rate results in a particular

10 The oil and gas industry differs from the 12 other industries in incorporating a nonproduced input, oil and gas reserves. Unit production costs rise as these reserves are depleted. The "synfuels" industry produces a backstop substitute for oil and gas, which permits the model to achieve a steady state despite the waning production of oil and gas. Details on these specifications are in Goulder (1992).

11 The central case value for \( \theta_L \), the uncompensated wage elasticity of labor supply, is 0.15, which is an average of estimates for primary and secondary earners. The sensitivity analysis (Section III.C) considers alternative values.

12 In the reference case (or status quo) simulation, the debt-GNP ratio is constant over time and the government deficit is 2 percent of GNP.


14 Households and producers are assumed to have perfect foresight. Hence the equilibrium in each period depends not only on current prices and taxes but on future levels as well.
percentage reduction in emissions; this relationship is plotted in Figure 1B.\textsuperscript{15} In general, this relationship depends on the use of the carbon tax revenue. However, we find that this curve is virtually identical whether the revenue is returned as a lump sum to households or employed to reduce personal tax rates.\textsuperscript{16} Although the use of

\textsuperscript{15} Obviously, a given tax generates different percentage reductions at different times; we “average” these reductions by calculating the present value of the reductions (over an infinite time horizon). The percentage changes in emissions reductions shown in Figure 1 are the percentage changes in these present values. Present values are calculated using the household’s real after-tax rate of return as the discount rate.

\textsuperscript{16} The difference between returning revenues in lump-sum fashion or through reductions in personal tax rates is virtually undetectable on a graph. For this reason we present only one curve in Figure 1B. The relationships differ
the revenue does not much affect the percentage reduction in emissions, it does affect welfare. As a second step, therefore, we show marginal welfare effects for each use of revenue with separate curves in Figure 1A. The top line in Figure 1A shows the welfare costs when revenues are returned to households in lump-sum fashion, while the middle line in this figure indicates welfare costs when revenues are used to finance cuts in personal income tax rates. Together, Figures 1A and 1B indicate the welfare costs of a given carbon tax under different uses of the revenues. A carbon tax of $11 per ton, for example, implies a reduction in emissions of about 8 percent (Figure 1B). The marginal welfare cost of this emissions reduction is $75 per ton when revenues are returned lump sum (top line of Figure 1A), and just over $25 dollars per ton when revenues are returned through cuts in personal tax rates (middle line of Figure 1A). Figure 1 shows that, for any given carbon tax rate, the marginal welfare costs are lower when revenues are devoted to reductions in marginal tax rates than when revenues are returned in lump-sum fashion. Using revenues to cut personal income tax rates decreases the distortionary costs of the income tax, thereby lowering the cost of this revenue-neutral environmental policy relative to the alternative policy with lump-sum revenue replacement.

B. Optimal Taxes: Departures from Pigovian Rates

1. Lump-Sum Revenue Replacement.— Once the relationships in Figure 1 are calculated and plotted, we can make use of them in the reverse order to calculate the optimal tax rates associated with given assumptions about the marginal environmental benefits from carbon emissions reductions. These optimal tax rates can be compared with the rates endorsed by the Pigovian principle. Consider first the optimal tax rates when revenues are returned lump sum (top line in Figure 1A). Suppose, for example, that the marginal benefits from reductions (or marginal damages from increases) in CO₂ were equal to $75. As discussed in connection with equations (2) and (3) above, the Pigovian principle would call for a carbon tax of the same value. However, the light horizontal and vertical lines in Figure 1A show that marginal benefits are equal to marginal costs when emissions are reduced by about 8 percent; according to Figure 1B, this requires a carbon tax of only $11 per ton. Thus, the optimal carbon tax rate in this case is only a fraction of the marginal environmental benefits.

The information in Figure 1 can be used similarly to derive the optimal rates associated with other values for marginal environmental benefits. Results for a range of marginal benefits are listed in column (3) of Table 2. When revenues are returned lump sum, the optimal carbon tax is always substantially lower than the marginal environmental benefit. Indeed, if marginal environmental benefits are $50 per ton or lower, the optimal carbon tax is negative.¹⁸

2. Revenue Replacement Through Cuts in Marginal Income Tax Rates.— What accounts for these substantial departures from the Pigovian rule? One possible explanation invokes the way revenues are used. Lump-sum replacement of revenues constitutes a suboptimal use of revenues, since a given carbon tax would impose lower welfare costs if revenues were devoted instead to cuts in marginal income tax rates. Would optimal rates closely approximate the marginal environmental benefits if revenues were recycled through cuts in marginal income tax rates? The light horizontal

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¹⁸ The optimal tax is negative because the carbon subsidy is an implicit subsidy to labor and capital which helps offset the distortions to labor and capital markets generated by explicit factor taxes. In this case the subsidy is financed by a nondistorting, lump-sum tax. For analytical treatments of how environmental taxes act as implicit factor taxes, see Bovenberg and de Mooij (1994) and Parry (1995).
TABLE 2—DIFFERENCES BETWEEN PIGOVIAN AND SECOND-BEST TAXES

<table>
<thead>
<tr>
<th>Assumed marginal environmental damages (1)</th>
<th>Realistic Tax System</th>
<th>Optimized Tax System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“Optimal” Pigovian tax (2)</td>
<td>Optimal tax, lump-sum replacement (3)</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>-19</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>-10</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>11</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>28</td>
</tr>
</tbody>
</table>

Notes: All tax rates in 1990 dollars per ton. MCPF\textsubscript{p} denotes the marginal cost of public funds obtained through the personal income tax.

and vertical lines in Figure 1A show that if marginal environmental benefits are $75 per ton, under this form of revenue-replacement marginal costs are equated to marginal benefits when emissions are reduced by about 24 percent; this reduction requires a carbon tax of $52 per ton (Figure 1B). Optimal rates corresponding to different marginal environmental benefits (or damages) are displayed in column (4) of Table 2. When revenues are devoted to reductions in marginal income tax rates, the optimal tax lies midway between the optimal tax under lump-sum replacement and the “optimal” tax prescribed by the Pigovian rule. Thus, while suboptimal (lump-sum) use of revenues explains some of the deviation from Pigovian rates, departures from Pigovian rates remain even when revenues are returned through cuts in marginal income tax rates.

It is useful to compare these results with the ratio, MED/MCPF, which is the optimal rate implied by the analytical model (equation (2) of Section 1). The MCPF depends, in general, on the configuration of all taxes, including whatever carbon taxes are present. We therefore evaluate the MCPF at the new equilibrium after the imposition of the carbon tax.\(^{19}\) A comparison of columns (4) and (6) reveals that the optimal rates from the numerical model are somewhat lower than the rates prescribed by the analytical model.

3. Fully Optimal Tax Policies.—To what might the differences from the “analytical optimum” be attributed? A potential source of this discrepancy is the nature of the numerical model’s benchmark. The analytical model assumes a fully optimized tax system—one in which all tax rates are set optimally. In contrast, the results from column (4) of Table 2 are based on a realistic, suboptimal benchmark reflecting the configuration of taxes in the U.S. economy.\(^{20}\) Thus, the rates in column (4) are constrained optimal tax rates, since the policy involves only incremental changes in other, distortionary taxes to the extent that carbon tax revenues can finance such changes. These simulations do not involve a fully optimized tax system.

To enhance further the comparisons of results across models, we derive the optimal carbon tax in a new, counterfactual scenario in which all taxes are set optimally. Here we develop a configuration of other (noncarbon) taxes that is optimal according to the principles inherent in the analytical model. The optimized configuration of other taxes involves two changes relative to the original benchmark: 1) taxes on intermediate inputs, industry

\(^{19}\) We are grateful to an anonymous referee for having pointed out the importance of measuring the MCPF at the post-tax equilibrium. The MCPF in column (5) of Table 2 (MCPF\textsubscript{p}) applies to funds raised from the personal income tax.

\(^{20}\) An indicator of this suboptimality is the fact that the MCPF differs depending on which tax is employed to raise funds. It is worth noting that we have defined “optimality” strictly in terms of efficiency. Under a broader notion of optimality, differences in MCPF’s need not represent deficiencies in the tax system. For example, to the extent that distributional objectives are realized through uneven factor taxation and associated differences in MCPF’s, these “suboptimal” features may be constructive elements of the tax system.
outputs, and consumer goods are eliminated, and 2) marginal rates of remaining (capital and labor income) taxes are adjusted so that the MCPF is the same for each tax. Since marginal rates of capital and labor taxes depend on the magnitude of the carbon tax (because the carbon tax finances cuts in factor taxes), the MCPF as well is a function of the magnitude of the carbon tax. Thus, the optimal carbon tax and the optimal configuration of other taxes must be determined simultaneously: for each value of marginal environmental benefits from CO₂ reductions, there is an optimal carbon tax and an optimal configuration of labor and capital taxes (that is, a set of factor tax rates that manages to equate the MCPF’s from labor and capital taxes).²¹

Figure 1 and Table 2 include results based on this fully optimized system. Figure 1A shows that in this counterfactual scenario, the marginal welfare costs of given emissions reductions are significantly lower than under the realistic benchmark.²² Correspondingly, in Table 2 the optimal carbon tax associated with given marginal environmental damages is higher than the optimal tax arising in the realistic case. A comparison of columns (7) and (9) of Table 2 shows that the numerical model’s results in this fully optimized case closely approximate the tax rates prescribed by the analytical model.²³,²⁴

C. Sensitivity Analysis

Table 3 indicates the sensitivity of optimal tax rates to key parameters. These simulations involve changes relative to the realistic (as opposed to optimized) tax system. The table reports results based on a posited value of $75 per ton for the marginal environmental benefits from the carbon tax. All results in the table are for simulations in which carbon tax revenues are recycled through cuts in personal income tax rates.

The general result from Table 3 is that, under the range of parameter values considered, the analytical and numerical models call for optimal tax rates below the Pigovian optimum. The analytical optimum is always below the Pigovian optimum because the MCPF consistently exceeds unity. The numerical model’s optimum is always below the prescribed optimum from the analytical model; as discussed above, this reflects the suboptimal nature of the benchmark tax system.

To consider the significance of preexisting taxes, we reduce or increase the marginal rates of all preexisting taxes by 50 percent. The MCPF moves toward unity as the preexisting tax rates are reduced; accordingly, the optimal tax rates from the analytical and simulation models move toward the Pigovian rate of $75/ton.

Higher values for the intertemporal elasticity of substitution in consumption, the uncompensated elasticity of labor supply, or energy

²¹ Thus, in Figure 1A, the lower-most marginal-welfare-cost schedule should be interpreted as the marginal welfare cost of achieving emissions reductions through a fully optimal tax system; the configuration of factor taxes changes with the extent of emissions reductions (although the path of real government spending is the same in all cases). However, we find that the optimal rates for capital and labor taxes change only slightly with changes in the assumed marginal benefits from CO₂ reductions.

²² This reflects two aspects of the realistic benchmark. First, in this benchmark, the MCPF from capital taxes is larger than that from labor taxes. Second, the combination of the carbon tax and a cut in personal income taxes tends to raise the tax burden on capital relative to labor (because the carbon tax component falls primarily on capital). As a consequence, the revenue-neutral policy effectively emphasizes the high MCPF of capital. In the counterfactual, fully optimized tax setting, the MCPF from capital taxes is lower than in the realistic benchmark; thus the welfare costs of carbon taxes are lower as well.

²³ The slight differences between results in the two columns are due to approximation error. While numerical and analytical results virtually match under the optimized benchmark, they differ significantly under the realistic benchmark case (compare results of columns (4) and (6)). The differences under the realistic benchmark stem from the fact that the carbon tax imposes a higher cost in the realistic benchmark than under optimal benchmark conditions (for reasons given in the previous footnote). This implies a lower optimal tax than would be endorsed by the analytical formula, which presumes a fully optimized setting.

²⁴ Column (8) of Table 2 suggests the interconnections between marginal environmental damages and the MCPF in an optimized tax system. With marginal environmental damages of $25 per ton, the MCPF is $1.16. But with higher marginal environmental damages (and a higher optimal value for the carbon tax), the MCPF is somewhat lower, as revenues from the (higher) carbon tax permit lower marginal rates on labor and capital. The MCPF is $1.21 if zero marginal environmental damages are assumed (so that the optimal carbon tax is zero).
TABLE 3—Sensitivity Analysis

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Optimal tax implied by analytical model</th>
<th>Optimal tax from numerical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Central case</td>
<td>1.252</td>
<td>60</td>
</tr>
<tr>
<td>2. Marginal rates for preexisting taxes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowered 50 percent</td>
<td>1.113</td>
<td>67</td>
</tr>
<tr>
<td>Raised 50 percent</td>
<td>1.410</td>
<td>53</td>
</tr>
<tr>
<td>3. Intertemporal elasticity of substitution in consumption:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low (0.33)</td>
<td>1.198</td>
<td>63</td>
</tr>
<tr>
<td>High (0.66)</td>
<td>1.406</td>
<td>53</td>
</tr>
<tr>
<td>4. Uncompensated elasticity of labor supply:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low (0.00)</td>
<td>1.121</td>
<td>67</td>
</tr>
<tr>
<td>High (0.26)</td>
<td>1.398</td>
<td>54</td>
</tr>
<tr>
<td>5. Energy substitution elasticities:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lowered by 50 percent</td>
<td>1.233</td>
<td>61</td>
</tr>
<tr>
<td>Raised by 50 percent</td>
<td>1.366</td>
<td>55</td>
</tr>
</tbody>
</table>

* Marginal environmental benefits are assumed to be $75 per ton. Results for the numerical model are from simulations of a carbon tax with revenue-preserving reductions in marginal rates of the personal income tax.

** These simulations involve changes in \( \nu \), the goods-leisure elasticity of substitution. The central case value of \( \nu \) is 0.77, implying an uncompensated labor supply elasticity of 0.15. \( \nu \) is 0.62 and 0.90 in the low and high elasticity cases. The compensated elasticities in the low, central, and high cases are 0.47, 0.94, and 5.90, respectively.

* In the low (high) elasticity simulation, \( \sigma_{EM} \), the elasticity of substitution between composite energy (E) and composite materials (M), is lowered (raised) in all industries by 50 percent from its central case value of 0.7.

Substitution elasticities raise the potential for distortions in capital, labor, or energy markets from a given configuration of taxes. The numerical model generates a higher MCPF with increases in these elasticities; hence the optimal rates prescribed by the analytical model are lower. Changes in these parameters also induce changes in the optimal rates derived directly from the numerical model. These changes are in the same direction as the changes in optimal rates prescribed by the analytical model.

Under central case values for parameters, the optimal rate from the numerical model is 69 percent of the Pigovian rate. By comparison, the optimal rate is 57 (73) percent of the Pigovian rate under high (low) values for the intertemporal elasticity of substitution in consumption. It is 64 (81) percent of the Pigovian rate under high (low) values for the labor supply elasticity.

IV. Conclusions

This paper has employed analytical and numerical models to examine the general-equilibrium interactions between environmentally motivated taxes and distortionary taxes. The analytical model extends earlier work by examining environmental taxes that, like carbon taxes, apply to intermediate inputs. This model indicates that in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle—even when revenues from environmental taxes are used to cut distortionary taxes.

The numerical simulations support this analytical result. Under central values for parameters, optimal carbon tax rates from the numerical model (when the tax system is fully optimized) are between six and twelve percent below the marginal environmental damages.

Moreover, if the revenue changes from carbon taxes are absorbed through changes in lump-sum transfers (rather than through changes in marginal rates of existing distortionary taxes), apply to intermediate inputs. This model indicates that in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle—even when revenues from environmental taxes are used to cut distortionary taxes.

The numerical simulations support this analytical result. Under central values for parameters, optimal carbon tax rates from the numerical model (when the tax system is fully optimized) are between six and twelve percent below the marginal environmental damages.

In addition, the numerical model shows that in the presence of realistic policy constraints, optimal carbon tax rates are far below the marginal environmental damages—and may even be negative. Simulations based on the U.S. tax system indicate that if policy makers can only incrementally alter existing distortionary taxes (rather than globally optimize the tax system), the optimal carbon tax may be substantially below the marginal environmental damages. Moreover, if the revenue changes from carbon taxes are absorbed through changes in lump-sum transfers (rather than through changes in marginal rates of existing distortionary taxes), apply to intermediate inputs. This model indicates that in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle—even when revenues from environmental taxes are used to cut distortionary taxes.
(A2) and (A3) yield the demands for the two intermediate inputs as functions \((fc, f_d)\) of \(\tau_c\) and \(\tau_d\) and the level of employment:

(A4) \(x_c = f_c(1 + \tau_c; 1 + \tau_d)\)

(A5) \(x_d = f_d(1 + \tau_c; 1 + \tau_d)\).

Substituting (A4) and (A5) into (A1), we can express the producer wage as a function \(\omega\) of \(\tau_c\) and \(\tau_d\):

(A6) \(\omega = \omega(1 + \tau_c; 1 + \tau_d)\)

where

(A7) \(\frac{\partial \omega}{\partial \tau_c} = \frac{-x_c}{L} \quad \text{and} \quad \frac{\partial \omega}{\partial \tau_d} = \frac{-x_d}{L}\).

To find the optimal tax rates, we substitute (A4) and (A5) into equation (1) of Section I to eliminate \(x_c\) and \(x_d\). Maximizing with respect to \(\tau_c\), we obtain the following first-order condition:

(A8) \((\lambda - \mu)L + \mu \left[ \tau_d \frac{\partial C_d}{\partial w_n} + (\tau_d x_c + \tau_c x_d) \frac{1}{L} \frac{\partial L}{\partial w_n} \right] + \frac{\partial U}{\partial Q} \left[ -\frac{\partial Q}{\partial C_d} \frac{\partial L}{\partial w_n} + \frac{\partial Q}{\partial x_d} L \frac{\partial L}{\partial w_n} \right] = 0\)

where we have used \(\frac{\partial U}{\partial w_n} = \lambda L\) (Roy's identity) and \(\lambda = \frac{\partial U}{\partial C_c}\) is the marginal utility of income. Define

(A9) \(\frac{\partial U}{\partial Q} \left( -\frac{\partial Q}{\partial x_d} \right) \mu\)

(A10) \(\frac{\partial U}{\partial Q} \left( -\frac{\partial Q}{\partial C_d} \right) \mu\).

Substitution of (A9) and (A10) into (A8) yields

(A11) \((\lambda - \mu)L + \mu \left[ (\tau_d - \bar{\tau}_d) \frac{\partial C_d}{\partial w_n} + \tau_d \frac{\partial L}{\partial w_n} \right] + \mu [\tau_c x_c + (\tau_d - \bar{\tau}_d) x_d] \times \frac{1}{L} \frac{\partial L}{\partial w_n} = 0.\)

---

25 Nordhaus has pioneered the integration of (environmental) benefits and (nonenvironmental) costs in simulation modeling of carbon taxes.
The first-order condition for maximizing (1) with respect to \( T^c \) is

\[
\frac{\partial U}{\partial T^c} = 0
\]

where we have used (A9) and (A10). Substitution of (A7) and (A11) into (A12) yields:

\[
\frac{\partial f_c}{\partial T^c} + (\tau^c - \tau^c_b) \frac{\partial f_d}{\partial T^c} = 0.
\]

In an analogous way, we derive the first-order condition for \( T^b \) as

\[
\frac{\partial f_c}{\partial T^b} + (\tau^b - \tau^b) \frac{\partial f_d}{\partial T^b} = 0.
\]

(A13) and (A14) together imply \( \tau^c = 0 \) and \( \tau^b = \tau^b_b \).

With (A9), this implies equation (2) of Section I, where \( \eta = \mu(\partial U/\partial C_c) \).

Applying \( \tau^c = 0 \) and \( \tau^b = \tau^b_b \) to (A11) yields

\[
(\lambda - \mu)L = -\mu \left[ (\tau^c_b - \tau^c_b) \frac{\partial C_0}{\partial \omega_w} \right]
\]

By applying \( \tau^c = 0 \) and \( \tau^b = \tau^b_b \) to the first-order condition that results from maximizing (1) with respect to \( \tau^c_b \), we obtain

\[
(\lambda - \mu)C_o = \mu \left[ (\tau^c_b - \tau^c_b) \frac{\partial C_0}{\partial \omega_w} \right] + \gamma \frac{L}{\omega_w}.
\]

Define \( \tau^c_b = \tau^c_b - \tau^b_b \). (A15) and (A16) are modified versions of the familiar Ramsey equations, where the tax on dirty consumption has been replaced by \( \tau^c_b \), the distortionary (or nonenvironmental) component of the tax on dirty consumption. If utility is homothetic and leisure is weakly separable from commodity consumption, (A15) and (A16) can be solved to yield \( \tau^c_b = 0 \), implying \( \tau^b_b = \tau^b_b \).

Substituting \( \tau^c_b \) for \( \tau^c_b \) in (A10), we arrive at equation (3) of Section I. Applying \( \tau^b_b = \tau^b_b = 0 \) to (A15) yields equation (4) of Section I.

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**APPENDIX B:**

**STRUCTURE AND PARAMETER VALUES OF THE NUMERICAL MODEL**

**A. Production Technology**

In each industry \( i \), gross output \( X_i \) is produced using inputs of labor (\( L_i \)), capital (\( K_i \)), and intermediate inputs \( x_{ij} \) (\( j = 1, \ldots, 13 \)). We employ the following nested form

\[
X_i = F_i(L_i, K_i, x_{i1}, \ldots, x_{i13}, I_i)
\]

where \( E_i \) and \( M_i \) represent composites of intermediate inputs of energy and materials, respectively. \( f_i, g_1i, \) and \( g_2i \) are constant-elasticity-of-substitution functions. Hence the function \( f \), for example, can be written as

\[
f_i(\gamma_1, \gamma_2) = \gamma_1 \frac{g_1(\gamma_1)}{g_2(\gamma_2)} = g_1(\gamma_1, \gamma_2) = (\gamma_1 + (1 - \gamma_2) g_2) \gamma_2.
\]

where the industry subscript has been suppressed and where \( \gamma_1, \alpha_1, \) and \( \rho_1 \) are parameters. The parameter \( \rho_1 \) is related to \( \sigma_1 \), the elasticity of substitution between \( g_1 \) and \( g_2 \); \( \rho_1 = (\sigma_1 - 1)/\sigma_1 \). Analogous expressions apply for the functions \( g_1 \) and \( g_2 \).

The second term in equation (B1) represents the loss of output associated with installing new capital (or dismantling existing capital). Per-unit adjustment costs, \( \phi \), are given by

\[
\phi(I/K) = \frac{(\beta/2)(I/K - \delta)^2}{I/K}
\]

where \( I \) represents gross investment (purchases of new capital goods) and \( \beta \) and \( \delta \) are parameters. The parameter \( \delta \) denotes the rate of economic depreciation of the capital stock.

The energy composite \( E \) in equation (B1) is a CES function of the specific energy products of the different energy industries:

\[
E(x_1, x_2 + x_3, x_4, x_5, x_6)
\]

\[
E = E(x_1, x_2 + x_3, x_4, x_5, x_6)
\]

\[
E(x_1, x_2 + x_3, x_4, x_5, x_6)
\]

where

\[
\delta = \begin{bmatrix} x_1, & j = 1 \\
                      x_2 + x_3, & j = 2 \\
                      x_{j+1}, & j = 3, \ldots, 5
\end{bmatrix}
\]

and where \( \sum_{j=1}^5 \alpha_j = 1 \). The subscripts to the \( x_i \)'s in equations (B4a) and (B4b) correspond to energy industries as follows.

---

26 A more comprehensive description of the structure of the model is in Goulder (1992). Detailed documentation of the data and parameters for the model is provided in Cruz and Goulder (1992).
Oil and gas extraction and synthetic fuels combine as one input in the energy composite, reflecting the fact that these fuels are treated as perfect substitutes in production.

Similarly, the materials composite (Mₗ) in equation (B1) is a CES function of the specific materials products of the 7 nonenergy industries:

\[
M = M(x_7, x_8, ..., x_{13})
\]

\[
M = \sum_{i=7}^{13} \alpha_{mi} x_i^{1/\omega}
\]

where \(\sum_{i=7}^{13} \alpha_{mi} = 1\). The subscripts to the \(x_i\)’s in equations (B5a) and (B5b) correspond to materials (nonenergy) industries as follows.

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Energy industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Coal mining</td>
</tr>
<tr>
<td>2</td>
<td>Oil and gas extraction</td>
</tr>
<tr>
<td>3</td>
<td>Synthetic fuels</td>
</tr>
<tr>
<td>4</td>
<td>Petroleum refining</td>
</tr>
<tr>
<td>5</td>
<td>Electric utilities</td>
</tr>
<tr>
<td>6</td>
<td>Processed natural gas</td>
</tr>
</tbody>
</table>

The elements \(x_j\) (\(j = 1, ..., 13\)) in the \(E\) and \(M\) functions are themselves CES composites of domestically produced and foreign made inputs:

\[
x_j = \gamma_j \left[ \alpha_{xj} x_{Dj}^{\sigma_j} + (1 - \alpha_{xj}) x_{Fj}^{\sigma_j} \right]^{1/\sigma_j},
\]

\(j = 1, ..., 13\)

where \(x_{Dj}\) and \(x_{Fj}\) denote domestic and foreign intermediate inputs of type \(j\). The overall nesting of the production system is summarized in Table A1.

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Materials industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Agriculture and mining (except coal mining)</td>
</tr>
<tr>
<td>8</td>
<td>Construction</td>
</tr>
<tr>
<td>9</td>
<td>Metals and machinery</td>
</tr>
<tr>
<td>10</td>
<td>Motor vehicles</td>
</tr>
<tr>
<td>11</td>
<td>Miscellaneous manufacturing</td>
</tr>
<tr>
<td>12</td>
<td>Services (except housing services)</td>
</tr>
<tr>
<td>13</td>
<td>Housing services</td>
</tr>
</tbody>
</table>

In each industry, managers of firms serve stockholders in aiming to maximize the value of the firm. The objective of firm-value maximization determines firms’ choices of input quantities and investment levels in each period of time.

While optimal demands for variable inputs (labor and intermediate inputs) depend only on current prices, optimal investment depends on both present and future prices. In specifying firms’ investment decisions, we adopt the asset price approach of Lawrence H. Summers (1981). The investment decision is fundamentally intertemporal because the firm’s current investment decisions affect future capital stocks and thereby influence future adjustment costs through the function \(\phi(I/K)\) contained in equation (B1). As detailed in Goulder (1992), we assume that managers finance investments through retained earnings, new debt issues, and new share issues, where new share issues represent the marginal source of funds. Optimal investment is a function of tax-adjusted \(q\) (see Goulder, 1992).

C. Household Behavior

Consumption, labor supply, and saving result from the decisions of an infinitely-lived representative household maximizing its intertemporal utility with perfect foresight. The nested structure of the household’s utility function is indicated in Table A2. In year \(t\) the household chooses a path of “full-consumption” \(C\) to maximize

\[
U_t = \sum_{s=1}^{\infty} \left[ (1 + \xi)^{-s} - \frac{\sigma}{\sigma - 1} C_s^{(\sigma - 1)/(\sigma - 1)} \right]
\]

where \(\xi\) is the subjective rate of time preference and \(\sigma\) is the intertemporal elasticity of substitution in full consumption. \(C\) is a CES composite of consumption of goods and services \(\bar{C}\) and leisure \(l\):

\[
C_t = [E_t^{(1-1)/\nu} + \alpha_t^{(1-1)/\nu}]^{1/(\nu-1)}
\]

\(\nu\) is the elasticity of substitution between goods and leisure; \(\alpha_t\) is an intensity parameter for leisure.
### Table A2—Nested Utility Structure

<table>
<thead>
<tr>
<th>Function</th>
<th>Functional form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_t(C, C_t, ..., C_{t-n})$</td>
<td>Constant intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$C_t(l_t)$</td>
<td>CES</td>
</tr>
<tr>
<td>$\bar{C}<em>t(C</em>{t-1}, ..., C_{t-n})$</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td>$\bar{C}<em>{t-i}(CD</em>{t-i}, CF_{t-i})$</td>
<td>CES</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_t$</td>
<td>Intertemporal utility evaluated from period $t$</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Full consumption in period $s$</td>
</tr>
<tr>
<td>$\bar{C}_s$</td>
<td>Overall goods consumption in period $s$</td>
</tr>
<tr>
<td>$l_s$</td>
<td>Leisure in period $s$</td>
</tr>
<tr>
<td>$\bar{C}_{i,s}$</td>
<td>Consumption of composite consumer good $i$ in period $s$</td>
</tr>
<tr>
<td>$CD_{i,s}$</td>
<td>Consumption of domestically produced consumer good $i$ in period $s$</td>
</tr>
<tr>
<td>$CF_{i,s}$</td>
<td>Consumption of foreign produced consumer good $i$ in period $s$</td>
</tr>
</tbody>
</table>

The variable $\bar{C}_s$ in (B10) is a Cobb-Douglas aggregate of 17 composite consumer goods:

$$\bar{C}_s = \prod_{i=1}^{17} \bar{C}_{i,s}^{a_{C,i}}$$

where the $a_{C,i}$ ($i = 1, ..., 17$) are parameters. The 17 types of consumer goods identified in the model are shown in Table 1 of the main text.

Consumer goods are produced domestically and abroad. Each composite consumer good $\bar{C}_i, i = 1, ..., 17$, is a CES aggregate of a domestic and foreign consumer good of a given type:

$$\bar{C}_i = \gamma_\zeta(\alpha_\zeta CD^{\zeta\bar{C}_i} + (1 - \alpha_\zeta) CF^{\zeta\bar{C}_i})^{1/\zeta\bar{C}_i}.$$

In the above equation, $\bar{C}_i$ is the average after-tax return on the household's portfolio of financial capital, $YL$ is after-tax labor income, $GT$ is transfer income, and $\beta$ is the price index representing the cost to the household of a unit of the consumption composite, $C$.

### D. Government Behavior

A single government sector approximates government activities at all levels—federal, state, and local. The main activities of the government sector are purchasing goods and services (both nondurable and durable), transferring incomes, and raising revenue through taxes or bond issue.

1. Components of Government Expenditure. Government expenditure, $G_t$, divides into nominal purchases of nondurable goods and services (GP), nominal government investment (GI), and nominal transfers (GT):

$$G_t = GP_t + GI_t + GT_t.$$

In the reference case, the paths of real government purchases, investment, and transfers all are specified as growing at the steady-state real growth rate, $g$. In simulating policy changes we fix the paths of GP, GI, and GT so that the paths of real government purchases, investment and transfers are the same as in corresponding years of the reference case. Thus, the expenditure side of the government ledger is largely kept unchanged across simulations. This procedure is expressed by

$$GP_t^P / p_{GP,t}^P = GP_t^R / p_{GP,t}^R,$$

$$GI_t^P / p_{GI,t}^P = GI_t^R / p_{GI,t}^R,$$

$$GT_t^P / p_{GT,t}^P = GT_t^R / p_{GT,t}^R.$$

The superscripts $P$ and $R$ denote policy change and reference case magnitudes, while $p_{GP}, p_{GI}$, and $p_{GT}$ are price indices for GP, GI, and GT. The price index for government investment, $p_{GI}$, is the purchase price of the representative capital good. The price index for transfers, $p_{GT}$, is the consumer price index. The index for government purchases, $p_{GP}$, is defined below.

2. Allocation of Government Purchases. GP divides into purchases of particular outputs of the 13 domestic industries according to fixed expenditure shares:

$$\alpha_{G,i} GP_t = GPX_t p_i, \quad i = 1, ..., 13.$$

GPX and $p_i$ are the quantity demanded and price of output from industry $i$, and $\alpha_{G,i}$ is the corresponding expenditure share. The ideal price index for government purchases, $p_{GP}$, is given by

$$p_{GP} = \prod_{i=1}^{13} p_{GP,i}.$$
### Table A3—Parameter Values

**Panel A: Elasticities of substitution in production**

Parameter for substitution margin

<table>
<thead>
<tr>
<th>Producing industry</th>
<th>( \sigma_f )</th>
<th>( \sigma_{s_1} )</th>
<th>( \sigma_{s_2} )</th>
<th>( \sigma_E )</th>
<th>( \sigma_M )</th>
<th>( \sigma_{s_M} )</th>
<th>Domestic-foreign inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Coal mining</td>
<td>0.7</td>
<td>0.80</td>
<td>0.7</td>
<td>1.08</td>
<td>0.6</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>2. Oil and gas extraction</td>
<td>0.7</td>
<td>0.82</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>(infinite)</td>
<td></td>
</tr>
<tr>
<td>3. Synthetic fuels</td>
<td>0.7</td>
<td>0.82</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>(not traded)</td>
<td></td>
</tr>
<tr>
<td>4. Petroleum refining</td>
<td>0.7</td>
<td>0.74</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>2.21</td>
<td></td>
</tr>
<tr>
<td>5. Electric utilities</td>
<td>0.7</td>
<td>0.81</td>
<td>0.7</td>
<td>0.97</td>
<td>0.6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>6. Gas utilities</td>
<td>0.7</td>
<td>0.96</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>7. Agriculture and noncoal mining</td>
<td>0.7</td>
<td>0.68</td>
<td>0.7</td>
<td>1.45</td>
<td>0.6</td>
<td>2.31</td>
<td></td>
</tr>
<tr>
<td>8. Construction</td>
<td>0.7</td>
<td>0.95</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>9. Metals and machinery</td>
<td>0.7</td>
<td>0.91</td>
<td>0.7</td>
<td>1.21</td>
<td>0.6</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>10. Motor vehicles</td>
<td>0.7</td>
<td>0.80</td>
<td>0.7</td>
<td>1.04</td>
<td>0.6</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>11. Miscellaneous manufacturing</td>
<td>0.7</td>
<td>0.94</td>
<td>0.7</td>
<td>1.08</td>
<td>0.6</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>12. Services (except housing)</td>
<td>0.7</td>
<td>0.98</td>
<td>0.7</td>
<td>1.07</td>
<td>0.6</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>13. Housing services</td>
<td>0.7</td>
<td>0.80</td>
<td>0.7</td>
<td>1.81</td>
<td>0.6</td>
<td>(not traded)</td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Parameters of stock effect function in oil and gas industry**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( Z_0 )</th>
<th>( Z )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0</td>
<td>450</td>
<td>1.27</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**Panel C: Utility function parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \xi )</th>
<th>( \sigma )</th>
<th>( \nu )</th>
<th>( \alpha_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.007</td>
<td>0.5</td>
<td>0.77</td>
<td>0.84</td>
</tr>
</tbody>
</table>

\(^a\) This function is parametrized so that \( \gamma_f \) approaches 0 as \( Z \) approaches \( Z \) (see equation (B7)). The value of \( Z \) is 450 billion barrels (about 100 times the 1990 production of oil and gas, where gas is measured in barrel equivalents). \( Z \) is based on estimates from Charles D. Masters et al. (1987). Investment in new oil and gas capital ceases to be profitable before reserves are depleted: the values of \( e_1 \) and \( e_2 \) imply that, in the baseline scenario, oil and gas investment becomes zero in the year 2031.

### REFERENCES


