

THE CORE AND EQUILIBRIUM THROUGH THE LOOKING-GLASS*

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“The rule is, jam to-morrow and jam yesterday – but never jam *to-day*.”

“It must come sometimes to ‘jam to-day’,” Alice objected.

“No, it can’t,” said the Queen. “It’s jam every *other* day: to-day isn’t any other day, you know.”

“I don’t understand you,” said Alice. “It’s dreadfully confusing!”

“That’s the effect of living backwards,” the Queen said kindly: “it always makes one a little giddy at first - ”

“Living backwards!” Alice repeated in great astonishment. “I never heard of such a thing!”

“—but there’s one great advantage in it, that one’s memory works both ways.”

(from Alice’s conversation with the White Queen, in Ch. V, “Wool and Water”, of Lewis Carroll, *Through the Looking-Glass*.)

“Indeed in Cambridge it is now *de rigueur* for economists as well as logicians to pretend to derive their inspiration from Lewis Carroll.”

(from R. B. Braithwaite, “Lewis Carroll as Logician”, *Mathematical Gazette*, Vol. XVI, 1932.)

I. HUMPTY DUMPTY SAT ON A WALL

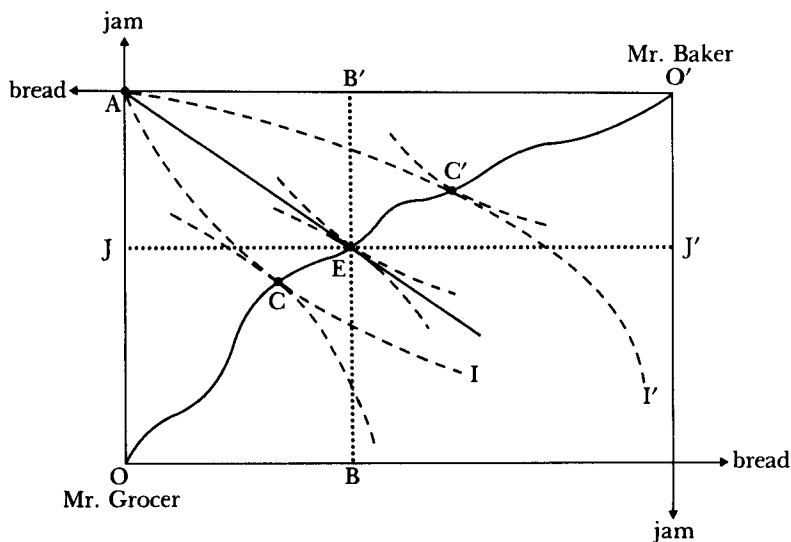
Alice wandered into a great big hall. It had rows and rows of seats, and many of those seats were filled with strange creatures. There in front of them all was Humpty Dumpty, who was sitting on a wall and drawing on an even bigger wall behind him. It seemed that all the creatures were trying to copy Humpty Dumpty’s picture. As he drew, Humpty Dumpty was speaking as well. He seemed to be saying something very important.

Alice tried first of all to make sense of Humpty Dumpty’s picture, but she found it very difficult. It was full of straight lines, curves, letters, and strange squiggles. But then she found that she could read one of the squiggles. It said, “jam”. Encouraged, she looked again rather harder, and discovered that she could read even more, and so, being a patient and diligent girl, she copied down Humpty Dumpty’s picture, and this is what she drew:

* Written while the author was a Research Fellow at the Australian National University. Readers will appreciate that this is situated in a part of the world where, if clocks go round forwards, then the sun goes round backwards. I am grateful to Robert Hawkins, a Cambridge Economics graduate, for remembering where he had found jam tomorrow, and predicting where I should find jam yesterday. My debts to “Lewis Carroll” and Frank Hahn are obvious.

** Who learned much of what economics he knows from Cambridge economists, mainly in Cambridge.

FIGURE 1



What Humpty Dumpty was saying was also rather difficult to understand. But Alice was persistent, and tried to write down as much of what Humpty Dumpty said as she could. She wrote:-

1. This is an Edgeworth box.
2. Mr Grocer starts with OA units of jam.
3. Mr Baker starts with $O'A$ units of bread.
4. A is the initial allocation of bread and jam.
5. Any point in the box represents a feasible allocation of bread and jam.
6. AI is Mr Grocer's indifference curve through A .
7. AI' is Mr Baker's indifference curve through A .
8. OO' is the contract curve, along which Mr Grocer's and Mr Baker's indifference curves have common tangents. It is the set of Pareto efficient allocations.
9. CC' is the core; the set of Pareto efficient allocations which neither trader can block by himself.
10. E is a competitive equilibrium, because the common tangent to Mr Baker's and Mr Grocer's indifference curves at E is the line AE passing through the initial allocation A .
11. AE is the competitive equilibrium price line.
12. In competitive equilibrium, Mr Grocer gets OB units of bread and OJ units of jam; Mr Baker gets $O'B'$ units of bread and $O'J'$ units of jam.

There followed an especially long and difficult argument which poor Alice was quite unable to grasp. But she did write down the conclusion, which was as follows:-

13. As the number of identical bakers and grocers tends to infinity, the core shrinks to E , the competitive equilibrium; all bakers get the same amounts of bread and jam to eat, and so do all the grocers.

Alice was feeling quite lost. But, right at the end, Humpty Dumpty suggested two things which the creatures should try to read. They were the following:-

F. Y. Edgeworth, *Mathematics Psychics*

E. Malinvaud, *Lectures in Microeconomic Theory*, Chapters 6 and 7.

Then Humpty Dumpty said, "Well, next week, I want to discuss what happens when we have jam today and jam tomorrow, and the . . ."

But his last words were drowned as all the creatures stampeded towards the doors.

Alice left too, and went off to find the readings which Humpty Dumpty had suggested, because she was very anxious to understand what he had so clearly thought to be important.

II. HUMPTY DUMPTY HAD A GREAT FALL

"When *I* use a word," Humpty Dumpty said, "it means just what I choose it to mean—neither more nor less."

"The question is," said Alice, "whether you *can* make words mean so many different things."

"The question is," said Humpty Dumpty, "which is to be master—that's all."

(from Alice's conversation with Humpty Dumpty, in Ch. VI, "Humpty Dumpty", of Lewis Carroll, *Through the Looking-Glass*.)

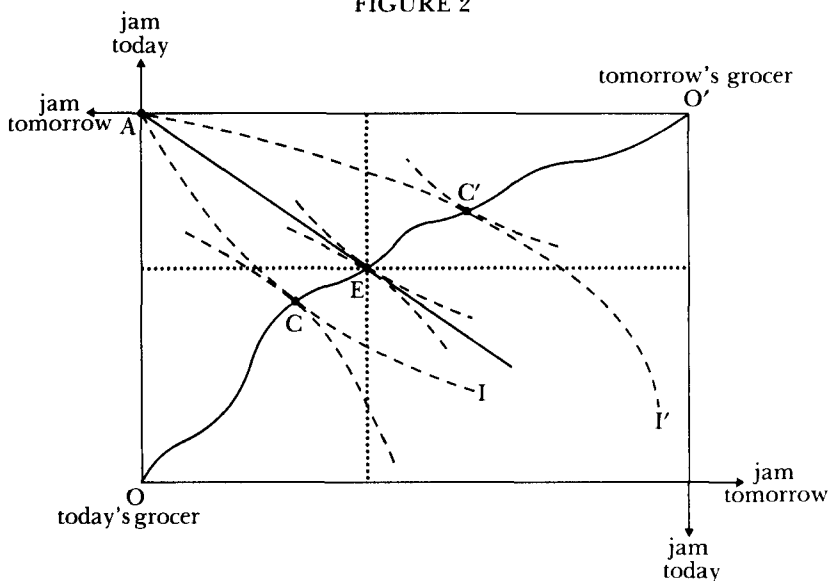
"The definition which I want to adopt is the following: an economy is in equilibrium when it generates messages which do not cause agents to change the theories they hold or the policies which they pursue."

(from p. 25 of F. H. Hahn, *On the Notion of Equilibrium in Economics* (an Inaugural Lecture) (Cambridge University Press, 1973).)

Next week had come, and Alice was most excited. She had read what Humpty Dumpty had recommended most carefully, and thought she understood it. At least, she understood all except the point about the core shrinking to the competitive equilibrium. For that, she thought that she would have to read the paper by Debreu and Scarf in the Marschak *Festschrift* edited by C. B. McGuire and R. Radner, but she had not yet been able to find the time. Now she was looking forward to hearing what Humpty Dumpty would have to say about jam today and jam tomorrow.

After reminding all the creatures that he was using box diagrams, this was the box diagram which Humpty Dumpty drew on the large wall behind him:

FIGURE 2



Really, the box diagram was just the same as before; only some of the labelling was different. And the story was much the same too. But Alice soon stopped listening to what Humpty Dumpty was saying. There was one question which seemed to her to be crucial, and she was quite unable to think of anything except this question.

“What happens tomorrow? What happens tomorrow? What *does* happen tomorrow?”

What is more, and what made Alice even more confused and worried, was that she thought she knew perfectly well what *would* happen tomorrow. Her thoughts were whirling through her head, and she sat where she was as though she were in a trance.

Suddenly, all the creatures got up and some of them jostled and pushed past her on their way to get out. The room had emptied and the hustle was over. Alice could wait no longer.

“What happens tomorrow?”, she burst out. “Tell me what happens tomorrow,” she demanded.

“What happens tomorrow?” repeated Humpty Dumpty, questioning the question.

“Yes, I want to know who gets the jam tomorrow.”

“Oh, I see. But the box diagram up here shows you that,” Humpty Dumpty said, proudly pointing to what was, for him, a most elegantly drawn diagram.

“No, it doesn’t; at least, I don’t think it does. What it does show me is what today’s grocer *thinks* he might get. He *thinks* he is going to get some jam tomorrow in exchange for the jam today he gives up, but can he be *sure*?”

“Of course he can; unless tomorrow’s grocer agrees to give up some jam tomorrow, today’s grocer can block by refusing to give up any of his jam today.”

“But today’s grocer has to give up *his* jam first,” retorted Alice. “The most he can get is a *promise* of jam tomorrow. And what is it that makes tomorrow’s grocer keep his promise?”

Humpty Dumpty looked thoughtful, and there was silence for a moment before Alice went on.

“Anyway, let’s look at the core tomorrow. In tomorrow’s economy, the *only* good is jam tomorrow. And tomorrow’s grocer has all of it. Today’s grocer has nothing to exchange when tomorrow comes. So, in the core of tomorrow’s economy, tomorrow’s grocer keeps all the jam tomorrow.

“What is more, if today’s grocer foresees this, then what he will do is to keep all his jam today. He has nothing to gain by giving any of it up, after all.”

Humpty Dumpty then said, “But that would not be an equilibrium. Both grocers are better off if jam today is exchanged for jam tomorrow. Both grocers can see that, and so they will find some way of ensuring that exchange does occur.”

“But I don’t see how they can possibly ensure that exchange will occur. It seems obvious that tomorrow’s grocer will just take any jam today he happens to be offered, and yet he will provide no jam tomorrow at all. That is, unless somebody else—a government, or something like that—*makes* him give up some of his jam tomorrow. But you have no government in your Edgeworth box; you just have the two grocers. Anyway, you used the word ‘equilibrium’ a moment ago. Now what did you mean by that?”

Because Alice had not yet been told about Frank Hahn’s inaugural lecture, there was now a rather lengthy digression, in which Humpty Dumpty explained what “equilibrium” meant. It seemed that Alice picked up this subtle notion of equilibrium quite quickly, because she was soon looking at what equilibrium meant in the Edgeworth box economy which they had been discussing. She said:

“So, in today’s economy, the core is an equilibrium, and in the core of today’s economy, Pareto efficient exchange occurs. Equilibrium means that there is no tendency for the grocers to move away from a point of the core, nor for them to change their theory of the world they find themselves in. That seems to be the claim you are making. In particular, today’s grocer has a theory that he will receive some jam tomorrow in exchange for the jam today he gives up. No information generated within today’s economy can make him change his theory. And yet, as I see it, his theory happens to be wrong.”

“What do you mean by ‘wrong’?” asked Humpty Dumpty. “After all, the theories which agents hold in equilibrium must always be logically consistent theories, otherwise they would be silly theories.”

“Yes, of course: I can see that. But today’s grocer’s theory is not wrong in a logical sense. Actually, his theory is logically faultless. It could even be right, if tomorrow’s grocer happens to be someone who keeps promises, and if he makes the promise to hand over some jam tomorrow. But it happens that tomorrow’s grocer is *not* going to hand over any jam tomorrow. He is not going to, even though he is someone who never breaks a promise; he simply refuses to make a promise he knows he won’t honour. So today’s grocer has a theory which happens to be wrong, even though it is logically consistent. Doesn’t that make a difference to the equilibrium?”

“But if today’s grocer foresees that he won’t get any jam tomorrow, then he can always refuse to give up any jam today.”

“All right. But now you are allowing today’s grocer to be in equilibrium even when no jam at all is traded, so the final allocation is inefficient. This contradicts what you were saying previously.”

“True,” said Humpty Dumpty, who looked very unhappy, very thoughtful and very puzzled.

Alice suddenly chuckled. A mischievous thought had struck her, which she was able to suppress no longer.

“Are you in equilibrium on your wall?” she asked.

Humpty Dumpty frowned and then laughed uproariously.

“I know that I can stay on this wall as long as I take enough care to keep my balance, and provided some wicked person doesn’t come along to push me off.”

“So that is your theory, is it?” said Alice. “You are in equilibrium because you think you are. But are you really sure you will always be careful enough to keep your balance? Are you sure that your theory which enables you to think you are in equilibrium will always remain valid?”

“My theory is that I will be in equilibrium as long as I am careful enough, and I *will* be careful enough.”

“But are you *really* sure?” asked Alice, almost aggressively.

At this point, the argument started to degenerate, so that Alice was quite unable to remember what had been said when she thought about it afterwards. She could remember Humpty Dumpty becoming more and more excited, and waving his arms about more and more vigorously. And she could remember the words:

“ . . . I *will* be care . . . Aaaargh!”

Humpty Dumpty had finally flung one of his arms too far backwards, in mid-sentence, and then, very very slowly, he started to slide backwards off the wall. Alice rushed forward to try to catch him, but there was nothing she was able to hang on to. So poor Humpty Dumpty rolled out of her grasp, and, as Alice turned away, quite unable to watch any longer, Humpty Dumpty fell to the ground with a great crash.

III. ALL THE KING’S HORSES AND ALL THE KING’S MEN COULDN’T PUT HUMPTY DUMPTY TOGETHER AGAIN. BUT THE WHITE QUEEN DID MANAGE TO SAVE HIS ARGUMENT.

“It’s a poor sort of memory that only works backwards.”

(The White Queen, to Alice, in Ch. V, “Wool and Water,” of Lewis Carroll, *Through the Looking-Glass*.)

Alice had been brought before the White Queen. She was full of remorse over Humpty Dumpty’s great fall, and said very simply and very sincerely, “I’m very sorry. I should never have talked to Humpty Dumpty as I did. If I hadn’t, he’d still be happily sitting on his wall. But now . . . oh, how horrible!”, and Alice hid her face in her hands.

“Hmm,” said the White Queen. “Are you really sorry? Didn’t your memory tell you how Humpty Dumpty would fall off his wall when you argued with him like that?”

“No,” said Alice, “How could I *remember* something that hadn’t even happened yet?”

“I don’t understand you”, said the White Queen. “Speak plainly, will you!”

“I mean that I never knew that Humpty Dumpty was going to fall off his wall until he did. If I *had* known how upset he’d become, then, of course, I’d never have started that terrible argument.”

“You mean that you can’t remember what is yet to happen? In fact, do you really mean that you can only remember what is in the past?”

“Of course”, replied Alice. “What else is memory, anyway?”

“How amazing!” said the White Queen. “I don’t remember hearing anybody talk like you are doing ever before, or, indeed, ever after. Nor, come to think of it, could I remember that you *were* going to talk like that before you did. Otherwise, of course, I wouldn’t have been so amazed. So what are we to make of it all?”

“I don’t know any better than you,” said Alice, “but I wonder whether I really understand you. Do you really remember what is *yet* to happen?”

“Of course. And I can now remember that this conversation will become increasingly repetitive unless we change the subject fast. What was it that sparked off your argument with Humpty Dumpty, anyway?”

Alice was relieved to be able to get back to something she understood, and explained with some alacrity Humpty Dumpty’s box diagram, with its jam today and jam tomorrow, and with today’s grocer and tomorrow’s grocer. She explained what was meant by the core. Then she started to argue why allocations in the core could never come about.

Suddenly the White Queen interrupted. “But surely today’s grocer can remember that tomorrow’s grocer is not going to give him any jam tomorrow, and so he does not give up any of his jam today.”

“You assume that today’s grocer has a memory that runs forward, I take it”, Alice suggested.

“Oh no, I do not *assume* it; I *remember* it”, retorted the Queen, most indignantly.

“Oh, I see.” Alice had been rather taken aback by the vehemence of the Queen. But she was starting to understand how the Queen’s mind was working. “Yes, if today’s grocer remembers *forwards*, and if tomorrow’s grocer remembers *backwards*, I can see now that the core does make perfect sense. But if we have three grocers, yesterday’s, today’s and tomorrow’s, then today’s grocer has to remember both ways, it seems.”

“And whatever is wrong with that?” protested the Queen. “You know, you’re the first person I ever remember speaking to who cannot remember both ways. And I am not sure that I even remember meeting *you*. You are so confusing that I am beginning to think that you must be part of a bad dream. In fact, you must be a dream, otherwise, of course, I would remember everything you were going to say.”

But Alice had given up listening by now, and was wrapped in her own thoughts. “Yes, the core *does* make sense if the grocers have memories which work both ways. And Humpty Dumpty would not have fallen off his wall, I suppose, if he had remembered falling off his wall when he was about to wave his arms about the way he did. Or if I had remembered his falling off the wall as a result of the argument I started. And, now, let me think, what was it Humpty Dumpty said he meant by an “equilibrium”. Something about agents never receiving messages which caused them to change either their theories or their actions. Well, I suppose that an agent whose memory ran both ways would never hold a theory which could become wrong in the course of time. Then any agent who had no reason to change either his theory, or the action based on that theory, would really be in a persistent and everlasting equilibrium. Yes, everything really fits into place now. It really is most interesting; except, of course, that the world I know is not like that. The people I know do not have memories which work both ways. It seems a pity really; the world might be a better place if we could all remember forwards as well as backwards. It could save us from all manner of stupidities. Except that it would be terrible never to be

able to experience a nice surprise, or, indeed, to know that something dreadful was going to happen to us that we could not avoid.”

IV. A POST SCRIPTUM

Alice returned through the looking-glass, much improved by her experience. She was avid to learn whether anybody had previously noticed the sort of problem which had led to poor Humpty Dumpty falling off his wall. After much searching, she did find a few instances of people noticing how the set of possible equilibria tomorrow could affect the set of possible equilibria today. She found an important aspect of the problem recognised in chapter XIV of Thomas Hobbes' *Leviathan*. Some ideas of apparent great relevance were examined in sections 2 and 3 of R. Selten's article, "A Simple Model of Imperfect Competition, where 4 are Few and 6 are Many", published in the *International Journal of Game Theory*, Vol. 2, 1973. She also found a great deal of interest in a series of recent papers by M. Shubik, notably "Money, Trust and Equilibrium Points in Games in Extensive Form", published in the *Zeitschrift für Nationalökonomie*, 1974. I am unable to say what she made of an article entitled "Charity: Altruism or Cooperative Egoism?" published in E. S. Phelps (ed.), *Altruism, Morality, and Economic Theory* (Russell Sage, 1975) or of a paper entitled "On Dynamic Games with an Infinite Sequence of Players" presented at the European Congress of the Econometric Society, Budapest, 1972.