Reassessing the Diamond/Mirrlees Efficiency Theorem

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Abstract: Diamond and Mirrlees (1971) provide sufficient conditions for a second-best Pareto efficient allocation with linear commodity taxation to require efficient production when a finite set of consumers have continuous single-valued demand functions. This paper considers a continuum economy allowing indivisible goods, other individual non-convexities, and some forms of non-linear pricing for consumers. Provided consumers have appropriately monotone preferences and dispersed characteristics, robust sufficient conditions ensure that a strictly Pareto superior incentive compatible allocation with efficient production results when a suitable expansion of each consumer's budget constraint accompanies any reform which enhances production efficiency. Appropriate cost–benefit tests can identify small efficiency enhancing projects.

1. Introduction

1.1. **Old second-best theory.** During the 1950s, much of the work in welfare economics had become increasingly critical of what had been accomplished during the 1930s and 40s. It was realized that, unless all distortions in the economy could be removed, the usual first-order conditions might all become invalid. This apparently very robust negative conclusion had already been discussed in Samuelson (1947, pp. 252–3), but it culminated in the work of Lipsey and Lancaster (1956).

Consider any distortion in the form of an enforced departure from optimality in at least one first-order condition. Then Lipsey and Lancaster claimed that, in general, every other first-order condition would also have to be violated in order to reach a constrained second-best optimum. Indeed, the irremovable presence of just one fixed distortion, it was suggested, would make it optimal to have other distortions prevail virtually throughout the whole economy.

Lipsey and Lancaster showed that this claim is indeed valid under particular assumptions concerning the nature of the distortion. It implies, of course, that marginal rates of transformation should not be equal for different producers. If so, production should be inefficient.

This negative conclusion has far-reaching implications because, if one can establish the desirability of production efficiency, there are many important corollaries such as gains from freer trade and from adopting a project whose benefits exceed its costs when these are all evaluated at suitable producer prices. All these important results risk losing their validity unless one can rely on the basic proposition that it is desirable to arrange an efficient allocation of inputs and outputs to producers.

1.2. **New second-best theory.** On a personal note, the academic year 1967–8 was when I began my formal study of economics. It was also Jim Mirrlees' last as a lecturer in Cambridge before he moved on to a Professorship at Nuffield College in Oxford. Thus, I was able to benefit from Jim's lectures on optimal growth, optimal taxation, and other topics in public economics. Also, early versions of Diamond and Mirrlees (1971) and of Little and Mirrlees (1968) were made available. These important publications led to a major revolution in public economics and the theory of the second-best — a revolution whose significance I have only recently begun to appreciate much more fully than I did at the time. Nor I do feel alone in my somewhat tardy recognition.

One important and practical implication of the revolution had already been set out quite clearly in Mirrlees (1969), which begins by claiming that shadow prices (or "accounting prices") should be " 'world prices'; or more precisely and more generally, ... prices that might be computed as a guide to particular production decisions, in the public sector and elsewhere". Indeed, the application to project evaluation was extensively explored in Little and Mirrlees (1968, 1974). Their main message was that in a small country with no influence over border prices, projects should have their net outputs of traded goods evaluated at border prices (with other net outputs evaluated at producer shadow prices). This approach to cost–benefit analysis is in marked contrast to Dasgupta, Marglin and Sen (1972), as those authors readily recognize (p. 6). It should also be noted that Little and Mirrlees advocate more complicated procedures for shadow wages and for discount rates, because of market imperfections. These important concerns go beyond the scope of this paper, however.

1.3. **The argument for production efficiency.** A more complete theoretical justification for this Little/Mirrlees approach to cost–benefit analysis comes, however, in a fundamental result due to Diamond and Mirrlees (1971) — see also Mirrlees (1986). In an economy without lump-sum transfers, but with linear taxes or subsidies on each commodity which can be adjusted independently, they were able to show that any second-best optimum of a Paretian social welfare function entails efficient production.

The Diamond/Mirrlees result does rely on a careful treatment of private producer profits which are distributed to consumers as dividends, as Stiglitz and Dasgupta (1971) in particular soon pointed out. Specifically, there is a need to sterilize any adverse effects on consumers' dividend incomes which arise when inefficient producers are shut down — see, for example, Mirrlees (1972), as well as Dasgupta and Stiglitz (1972). Formally, moreover, the Diamond/Mirrlees proof requires that consumers face linear budget constraints, thus excluding the non-linear taxation of income discussed by Vickrey (1945) and Mirrlees (1971). It also assumes a finite set of consumers with continuous single-valued demand functions — as is implied, for instance, if consumers' preferences are all strictly convex. Unlike in standard general equilibrium theory, for the Diamond/Mirrlees argument to hold, it is not enough that each consumer merely have an upper hemi-continuous and convex-valued demand correspondence — though actually their result does hold in this case as well, as can be shown by an argument similar to that used in proving the main theorem of this paper.

Accordingly, the principal aim here is to show how the Diamond/Mirrlees result is much more robust than has generally be recognized. This will be done by confirming its validity when several of the original assumptions have been greatly relaxed. For reasons to be explained shortly, this is much easier to do when, as in the income taxation model of Vickrey and Mirrlees, there is a continuum of consumers.

1.4. **Corporate and public production.** Following the example of Diamond and Mirrlees (1971), the results set out below will be derived in a formal general equilibrium framework, but allowing for an active public sector. In fact, the model used here allows for production which is undertaken by individuals, by corporations, and by the public sector. In the classical first-best setting with unrestricted lump-sum redistribution, aggregate production efficiency over the whole economy is necessary for Pareto efficiency. In a second-best setting, however, aggregate production efficiency over the whole economy may not be desirable because distortionary taxes on transactions between individuals and corporations may well be needed to redistribute real income or to finance public goods in order to reach a second-best Pareto efficient allocation — as they are, for instance, in the Diamond and Mirrlees (1971) framework. Also, shadow prices for the public sector may differ from consumer prices. Nevertheless, the main proposition below concerns the desirability of aggregate production efficiency in the corporate and public sector together, even in the second-best setting considered here.

1.5. **Incentive constraints with a continuum of consumers.** The Lipsey and Lancaster theory of second best was based on *ad hoc* constraints preventing the economy from reaching a first-best allocation. Yet Vickrey (1945) and Samuelson (1947, pp. 247–8) had already emphasized informally how private information makes it practically impossible to arrange a system of optimal lump-sum transfers of the kind that first-best theory requires. Indeed, this impossibility is clearly what motivated Diamond and Mirrlees to examine the implications of replacing lump-sum transfers by linear commodity taxation.

After the important work on incentive compatibility during the 1970s by Hurwicz, Gibbard and others — including Section 3 of Mirrlees (1986), which was originally written in 1977 — it seems quite natural now to impose explicitly the incentive constraints that arise whenever some information is private. Yet the full implications of doing so are hard to analyse when the number of agents is finite — cf. Barberà and Jackson (1995), Córdoba and Hammond (1998). The difficulty is that, when any one consumer manipulates an allocation mechanism by mimicking some different type of consumer, this affects the apparent economic environment, as described by the distribution of consumers' characteristics in the population. Thus, incentive compatible allocations in different economic environments become inextricably linked through incentive constraints. One cannot avoid trading off good allocations in some environments against good allocations in other environments.

With a continuum of agents, on the other hand, the analysis is greatly simplified — as discussed in Hammond (1979, 1987, 1999), Guesnerie (1981, 1995), etc. Indeed, incentive compatibility with a continuum of agents is equivalent to the existence of a common budget set, independent of any private information, which decentralizes the allocation. Moreover, there are fairly obvious conditions under which incentive compatibility actually requires linear commodity taxation without lump-sum transfers — i.e., budget constraints in the form considered by Diamond and Mirrlees. But non-linear pricing is still possible for any good which cannot be freely and anonymously exchanged among consumers. Of course, there may be welfare gains — even potential Pareto gains — from allowing non-linearities, possibly in the form of rationing, into each consumer's budget constraint. Indeed, this is clear from the work of Mirrlees (1971, 1986) himself, as well as Dasgupta and Hammond (1980), Guesnerie and Roberts (1984), Blackorby and Donaldson (1988), and others. On the other hand, there are administrative and other costs which may severely limit the use of fiscal instruments.

With these extensions and limitations in mind, this paper considers those incentive compatible allocations which can be decentralized by budget constraints chosen from within a rather general one parameter family of "piecewise convex" sets. Moreover, as this one parameter increases, each consumer's budget set is assumed to expand — as in one case considered by Diamond and Mirrlees themselves, where the only variable parameter is a non-negative uniform poll subsidy.

Apart from allowing a much simpler treatment of incentive constraints, having a continuum of consumers confers several additional theoretical benefits. There is no reason to restrict attention to convex preferences, to convex consumption or individual feasible sets, to convex budget sets, or even to divisible goods. In principle, the theoretical framework can include individual consumers' decisions to migrate, to make private investments involving set-up costs, to incur fixed transport costs travelling to and from work each day, etc. Allowing such features adds considerably to the practical applicability of the Diamond/Mirrlees efficiency argument. In order to ensure that mean demand is upper hemi-continuous, however, an additional "dispersion" assumption is required, similar to those introduced by Mas-Colell (1977) and Yamazaki (1978, 1981).

1.6. **Outline of paper.** The main theoretical task, accordingly, is to generalize the Diamond/Mirrlees efficiency argument to a model with a continuum of consumers and individual non-convexities. Section 2 lays out the details of a formal model having these features, including indivisible goods. Then Section 3 explores how to distribute to all consumers simultaneously the additional output resulting from a gain in production efficiency in a way that generates an incentive compatible Pareto superior allocation. These possibilities for distributing efficiency gains are assumed to be described by a one-parameter expanding family of "piecewise convex" budget sets. Unfortunately, these specifically exclude income tax schedules whose marginal rate decreases smoothly to zero at the top of the income distribution — i.e., precisely the kind of schedule required for optimality in some economic environments, as discussed by Mirrlees (1971) and also Seade (1977). Nevertheless, though formally excluded, any such schedule can be approximated arbitrarily closely by a piecewise convex budget set, so the restriction seems not too severe.

After setting out the most substantive assumptions, Section 4 derives the continuity and unboundedness properties of the resulting individual demand correspondence. This is followed in Section 5 similar results for the aggregate (or mean) demand correspondence. Once these important preliminary properties have all been established, the main theorem on the desirability of enhanced production efficiency is proved in Section 6. In particular, the existence of an incentive compatible Pareto superior allocation on the frontier of the post-reform production set will be shown. The paper concludes in Section 7, which discusses several important implications of this result when evaluating projects or analysing some other kinds of economic reform.

2. A Formal Model

2.1. **Divisible and indivisible commodities.** Let G denote a non-empty finite set of divisible commodities, and $\mathbb{R}^G$ the associated finite-dimensional Euclidean space. It is important for G to be non-empty so that local satiation can be avoided. Also, let H denote the (possibly empty) finite set of indivisible commodities, whose quantities must belong to $\mathbb{Z}$, the set of integers. Thus, the commodity space is $\mathbb{R}^G \times \mathbb{Z}^H$. Sometimes a vector $x \in \mathbb{R}^G \times \mathbb{Z}^H$ will be written in the partitioned form (x^G, x^H), where $x^G \in \mathbb{R}^G$ and $x^H \in \mathbb{Z}^H$.

A pair $a, b \in \mathbb{R}^G \times \mathbb{Z}^H$ may satisfy one or more of the following four vector inequalities:

$$\begin{aligned}
a \geqq b \quad &\Longleftrightarrow \quad a_j \geq b_j \text{ (all } j \in G \cup H) \\
a > b \quad &\Longleftrightarrow \quad a \geqq b \text{ and } a \neq b \\
a \gg b \quad &\Longleftrightarrow \quad a_j > b_j \text{ (all } j \in G \cup H) \\
a \gg^G b \quad &\Longleftrightarrow \quad a \geqq b \text{ and } a_j > b_j \text{ (all } j \in G)
\end{aligned}$$

Of these, the first three are standard. The last is appropriate when there is a (possibly small) increase in every divisible good, with no decrease in any indivisible good.

2.2. **Consumption and domestic production.** As explained in the introduction, it is assumed that there is a continuum of consumers. Suppose these are labelled by numbers ℓ that are uniformly distributed on the unit interval $[0, 1] \subset \mathbb{R}$ — i.e., the distribution of their labels is described by the Lebesgue measure λ on $[0, 1]$.

Individuals can be distinguished not only by numerical labels, but also by other identifiers such as date of birth. These identifiers can in principle be used to arrange discriminatory prices or taxes and subsidies, such as state pensions paid to those born more than 65 years ago. It is assumed that the relevant identifiers i range over a finite set I. Numerical labels ℓ, however, are assumed to be arbitrary and so not useful as a basis for discrimination of any kind.

Furthermore, individuals have characteristics θ ranging over a specified metric space Θ with a Borel σ-field $\mathcal{F}$ of measurable sets. For obvious reasons, we restrict attention to continuous preferences. Also, so that production efficiency would be desirable in a degenerate economy where all consumers are known to be identical and have the same net demand vector, we assume both free disposal and monotone preferences, though subject to the restriction that quantities of indivisible goods must be integers.

In order to allow for domestic production, it is easier to focus on net trade vectors rather than on consumption and endowment vectors separately. Thus, each consumer is supposed to have a set of feasible net trades. Though these feasible sets may not be convex, especially if there are indivisible goods, at least they should be "piecewise convex" — i.e., the union of a countable collection of convex sets. This is important later on when invoking the dispersion assumption which implies that the demand correspondence is upper hemi-continuous.

2.3. **Consumer characteristics.** Specifically, the first formal assumption is:

Assumption 1: There is a metric space Θ of possible consumer characteristics such that:

1. for each $\theta \in \Theta$, there is a closed set $X_\theta \subset \mathbb{R}^G \times \mathbb{Z}^H$ of feasible net trade vectors, which is equal to the union of a countable collection of closed convex sets $X_\theta(k)$ $(k = 1, 2, \dots)$, each of which has the *restricted free disposal property* that, whenever $x \in X_\theta(k)$ and $x' \geqq x$ with $x' \in \mathbb{R}^G \times \mathbb{Z}^H$, then $x' \in X_\theta(k)$;
2. there exists a uniform lower bound $\underline{x} \in \mathbb{R}^G \times \mathbb{Z}^H$ such that $x \in X_\theta$ for some $\theta \in \Theta$ only if $x \geqq \underline{x}$;
3. if the sequence $\theta_n \in \Theta$ converges to θ, and if $x \in X_\theta$, then there exists a sequence satisfying $x_n \in X_{\theta_n}$ for $n = 1, 2, \dots$ while $x_n \to x$ as $n \to \infty$;
4. for each $\theta \in \Theta$, there is a (complete, reflexive, and transitive) preference ordering $\succsim_\theta$ which is continuous on X_θ and has the *restricted monotonicity*

property that, whenever $x \in X_\theta$ and $x' \geqq x$ with $x' \in \mathbb{R}^G \times \mathbb{Z}^H$, then $x' \succsim_\theta x$, and $x' \succ_\theta x$ if $x' \gg^G x$;

5. if the sequence of triples $(x_n, x'_n, \theta_n) \in (\mathbb{R}^G \times \mathbb{Z}^H)^2 \times \Theta$ converges to (x, x', θ) while satisfying $x_n, x'_n \in X_{\theta_n}$ and $x_n \succsim_{\theta_n} x'_n$ for $n = 1, 2, \ldots$, then $x, x' \in X_\theta$ with $x \succsim_\theta x'$.

Note that, because each component set $X_\theta(k)$ is convex, for each k there must exist a unique vector of integers $z(k) \in \mathbb{Z}^H$ such that $X_\theta(k) \subset \mathbb{R}^G \times \{z(k)\}$. But $X_\theta \cap (\mathbb{R}^G \times \{z\})$ could still fail to be convex for one or more $z \in \mathbb{Z}^H$, in which case it is assumed that $X_\theta \cap (\mathbb{R}^G \times \{z\})$ can be decomposed further into several convex components.

Because the ordering $\succsim_\theta$ is assumed to be reflexive on X_θ, note that

$$X_\theta = \{ x \in \mathbb{R}^G \times \mathbb{Z}^H \mid x \succsim_\theta x \}$$

Hence, each pair $(X_\theta, \succsim_\theta)$ $(\theta \in \Theta)$ consisting of a consumption set $X_\theta \subset \mathbb{R}^G \times \mathbb{Z}^H$ and a monotone continuous preference ordering on X_θ can be identified with the closed graph

$$\Gamma_\theta = \{ (x, x') \in (\mathbb{R}^G \times \mathbb{Z}^H)^2 \mid x \succsim_\theta x' \}$$

of that ordering. Then the space of such closed graphs can be given the metrizable topology of closed convergence. After identifying each $(X_\theta, \succsim_\theta)$ with Γ_θ, the mapping $\theta \mapsto (X_\theta, \succsim_\theta)$ should be continuous. See, for example, Hildenbrand (1974, pp. 18–19 and 96–98) or Mas-Colell (1985, pp. 10–11) for a fuller explanation and further details. In fact, one could take Θ as the whole space of graphs Γ corresponding to pairs $(X, \succsim)$ satisfying parts 1, 2 and 4 of Assumption 1, and then make $\theta \mapsto (X_\theta, \succsim_\theta)$ the identity map. Alternatively, the domain Θ of characteristics that an individual can credibly claim to possess could be a compact subset of this space. In either case, it follows from Hildenbrand that parts 3 and 5 of Assumption 1 are also satisfied. These two parts, however, respectively require only that the correspondence $\theta \twoheadrightarrow X_\theta$ should be lower hemi-continuous, while $\theta \twoheadrightarrow \Gamma_\theta$ should have a closed graph.

2.4. **Potential consumers.** Though the numerical labels $\ell \in [0, 1]$ and identifiers $i \in I$ are assumed to be publicly observable, consumer characteristics $\theta \in \Theta$ will be regarded as private information. That is, the true mapping $\theta(\ell)$ from $[0, 1]$ to Θ

specifying each consumer's private characteristic θ as a function of their label ℓ is completely unknown. Feasible allocations, therefore, can only depend on θ in ways that respect incentive constraints. For this reason, it is natural to consider the entire space $A := [0,1] \times I \times \Theta$ of *potential consumers*, specified by a known numerical label $\ell \in [0,1]$, a known identifier $i \in I$, but an unknown characteristic $\theta \in \Theta$. The space A can be given its obvious product σ-field $\mathcal{A}$, and it will be assumed that the economy can be described by a probability measure α on $(A, \mathcal{A})$. For any $E \in \mathcal{A}$, the measure $\alpha(E)$ should be interpreted as the proportion of individuals whose label, identifier, and characteristic form a triple (ℓ, i, θ) belonging to E. It is assumed that $\alpha(V \times I \times \Theta) = \lambda(V)$ for every Borel set $V \subset [0,1]$. That is, the marginal distribution of numerical labels is the Lebesgue measure, implying that numerical labels are indeed uniformly distributed on $[0,1]$.

The above formulation allows the continuum economy to be interpreted as the limit of an expanding sequence of random finite economies in which there are n consumers whose triples (ℓ, i, θ) of numerical labels, identifiers, and characteristics are independently and identically drawn at random from the probability space $(A, \mathcal{A}, \alpha)$.

3. Distributing Efficiency Gains

3.1. **The status quo.** The main result below shows that, relative to a status quo allocation, any reform which enhances the overall efficiency of production in the combined corporate and public sector can be accompanied by a fiscal reform which will generate a strict Pareto improvement. The status quo allocation is not necessarily an initial allocation; rather, it is what would happen in the absence of any reform. That particular allocation is assumed to be described by some α-integrable mapping $(i, \theta) \mapsto \hat{x}^i_\theta$ from A to $\mathbb{R}^G \times \mathbb{Z}^H$ which depends only on (i, θ).

Define the set

$$\hat{S}^i := \{ \, x \in \mathbb{R}^G \times \mathbb{Z}^H \mid \exists \theta \in \Theta : x = \hat{x}^i_\theta \, \}$$

as the range of the mapping $\theta \mapsto \hat{x}^i_\theta$ as θ varies over Θ, the entire domain of relevant individual characteristics.

The second formal assumption is:

Assumption 2: The range $\hat{S}^i$ is compact.

This will be true if, for instance, the domain Θ of characteristics happens to be compact and the mapping $\theta \mapsto \hat{x}^i_\theta$ is continuous.

3.2. **A Decentralization.** Because each consumer's characteristic $\theta \in \Theta$ is private information, it is natural to assume that the status quo allocation represents the outcome of a *strategyproof mechanism* in the sense that, for all $(i, \theta) \in I \times \Theta$, the *incentive constraint* $\hat{x}^i_\theta \succsim_\theta x$ is satisfied whenever $x \in X_\theta \cap \hat{S}^i$. That is, no potential consumer $(i, \theta) \in I \times \Theta$ is able to manipulate the mechanism determining the relevant net trade vector $\hat{x}^i_\theta$ by finding a better alternative $\hat{x}^i_\eta \in X_\theta \cap \hat{S}^i$ for some $\eta \in \Theta$. Because there is a continuum of agents, so no consumer can influence the apparent distribution α on A, note that the incentive constraints are independent of α.

Say that the set $B^i \subset \mathbb{R}^G \times \mathbb{Z}^H$ satisfies *restricted free disposal* if and only if

$$B^i \subset B^i - (\mathbb{R}^G \times \mathbb{Z}^H) = \{\, x \in \mathbb{R}^G \times \mathbb{Z}^H \mid \exists x' \in B^i : x \leqq x' \,\}$$

Under the assumptions stated so far, the decentralization result presented in Hammond (1979) can be strengthened as follows:

Lemma 1: Any allocation $(i, \theta) \mapsto x^i_\theta$ with compact range

$$S^i := \{\, x \in \mathbb{R}^G \times \mathbb{Z}^H \mid \exists \theta \in \Theta : x = \hat{x}^i_\theta \,\}$$

is strategyproof if and only if it is *decentralizable* in the sense that, for each $i \in I$, there exists a closed *budget set* B^i satisfying restricted free disposal such that $S^i \subset B^i \subset \mathbb{R}^G \times \mathbb{Z}^H$ and also $x^i_\theta \succsim_\theta x$ whenever $x \in X_\theta \cap B^i$.

Proof: Because $S^i \subset B^i$, sufficiency is obvious.

Conversely, suppose that the allocation $(i, \theta) \mapsto x^i_\theta$ is strategyproof. Define $B^i := S^i - (\mathbb{R}^G \times \mathbb{Z}^H)$ as the smallest set that satisfies restricted free disposal while including S^i as a subset. Because S^i is assumed to be compact, the set B^i is easily shown to be closed.

Suppose that $x \in X_\theta \cap B^i$. Then there exists $\eta \in \Theta$ such that $x \leqq x^i_\eta$. Because $x^i_\eta \in \mathbb{R}^G \times \mathbb{Z}^H$, restricted free disposal implies that $x^i_\eta \in X_\theta$ and restricted monotonicity implies that $x^i_\eta \succsim_\theta x$. Because $x^i_\eta \in X_\theta$, the incentive constraints imply that $x^i_\theta \succsim_\theta x^i_\eta$, so $x^i_\theta \succsim_\theta x$ because $\succsim_\theta$ is transitive. $\square$

Of course, this construction typically results in a non-linear budget constraint. Some significant restrictions in such a budget constraint will now be introduced.

3.3. **Expanding budget sets.** Define $\hat{y} := \int_A \hat{x}^i_\theta \, d\alpha$ as the mean net trade vector of all consumers in the status quo allocation. As the mean, note that $\hat{y}$ will generally not have integer components even for indivisible goods. That is, $\hat{y}$ can be a general vector in $\mathbb{R}^G \times \mathbb{R}^H$.

Consider a reform which allows the economy to reach any new mean net trade vector in some (possibly very restricted) aggregate feasible set $Y \subset \mathbb{R}^G \times \mathbb{R}^H$ describing the combined aggregate production possibilities of the corporate and public sector. It will be assumed that Y is closed and allows free disposal — i.e., if $y \in Y$ and $y' \leqq y$, then $y' \in Y$. Also, so that the reform really does enhance production efficiency, it is assumed that Y includes at least one $y \gg \hat{y}$.

In order to convert this reform into an incentive compatible allocation which is strictly Pareto superior for consumers, the budget sets $\hat{B}^i$ that decentralize the status quo allocation $(i, \theta) \mapsto \hat{x}^i_\theta$ must change in order to include at least one strictly preferred net demand vector for each consumer. To make this possible, assume that for each $i \in I$ there is a one-parameter family $B^i(m)$ $(m \geq 0)$ of budget sets, with $B^i(0)$ as the status quo budget set $\hat{B}^i$. Thus, once m has been chosen, each potential consumer with identifier $i \in I$ is constrained to choosing some net trade vector x within the budget set $B^i(m)$. Assume, moreover, that each set $B^i(m)$ is closed, allows free disposal, and that there is an upper bound $\bar{x}(m)$ such that $x \leqq \bar{x}(m)$ whenever $x \in B^i(m)$ with $x \geqq \underline{x}$.

One prominent example of such a one-parameter family of budget sets appears in Diamond and Mirrlees (1971), when the set H of indivisible goods is empty. It is the family $\{ x \in \mathbb{R}^G \mid q\, x \leq m \}$ of linear budget sets, for a fixed consumer price vector $q \gg 0$ and a variable non-negative "poll" subsidy m. An obvious extension of this example is the family $B^i(m) := \{ x \in \mathbb{R}^G \times \mathbb{Z}^H \mid q^i\, x \leq e^i(m) \}$ of budget sets, for fixed consumer price vectors $q^i \gg 0$ and increasing functions $e^i(\cdot)$ of m which can both depend on i. A different example is the family $B^i(m) := \{ x \in \mathbb{R}^G \times \mathbb{Z}^H \mid x - u^i(m) \in B^i \}$ of possibly non-linear budget sets, for a collection of increasing vector functions $u^i : \mathbb{R}_+ \to \mathbb{R}^G_+ \times \mathbb{Z}^H_+$ of m which, for all $i \in I$, satisfy $u^i(0) = 0$ and $u^i(m') \gg^G u^i(m)$ whenever $m' > m$.

Generally, then, it is assumed that:

Assumption 3: For each $i \in I$, there are closed budget sets $B^i(m)$ $(m \geq 0)$ allowing restricted free disposal while satisfying:

1. (expansion) If $m' > m$ then $B^i(m) \subset B^i(m')$, while for every $x \in B^i(m)$ there exists $x' \gg^G x$ such that $x' \in B^i(m')$.
2. (piecewise convexity) Each set $B^i(m)$ is equal to the union $\cup_{j=1}^{\infty} B^i(j,m)$ of a countable family of convex sets $B^i(j,m)$ $(j = 1, 2, \dots)$.
3. (continuity) For each $i \in I$ and each $j = 1, 2, \dots$, there exists a continuous function $\beta^i_j : \mathbb{R}^G \times \mathbb{Z}^H \to \mathbb{R}_+$ such that
$$x \in B^i(j,m) \iff \beta^i_j(x) \leq m$$
and also $\beta^i_j((1-\lambda)x + \lambda x') < \beta^i_j(x')$ whenever $0 < \lambda < 1$ and $x, x' \in \mathbb{R}^G \times \mathbb{Z}^H$ satisfy $\beta^i_j(x) < \beta^i_j(x')$.
4. (bounded budget sets) For each $m \geq 0$ there is an upper bound $\bar{x}(m)$ such that $x \leqq \bar{x}(m)$ whenever $x \in \cup_{i \in I} B^i(m)$ with $x \geqq \underline{x}$.

Here part 2 of Assumption 3 plays a similar role to the comparable part of Assumption 1 requiring each feasible set X_θ to be the union of a countable family of convex sets. Also, the last condition in part 3 is somewhat stronger than merely requiring each β^i_j to be quasi-convex, as already implied by the convexity of each set $B^i(j,m)$. Finally, part 4 plays an obvious role in ensuring that each potential consumer (i, θ) faces a compact constraint set $X_\theta \cap B^i(m)$.

3.4. **Compact constraint sets.** Next, for each $i \in I$, $\theta \in \Theta$ and $m \geq 0$, define the collection of sets
$$H^i_\theta(j,k,m) := X_\theta(k) \cap B^i(j,m)$$
for $j, k = 1, 2, \dots$, as well as the union
$$H^i_\theta(m) := \bigcup_{j=1}^{\infty} \bigcup_{k=1}^{\infty} H^i_\theta(j,k,m) = X_\theta \cap B^i(m)$$

Lemma 2: For each fixed $m \geq 0$, $i \in I$ and $\theta \in \Theta$, the set $H^i_\theta(m)$ is compact and uniformly bounded.

Proof: By Assumption 1, if $x \in X_\theta$, then $x \geqq \underline{x}$. So the boundedness part of Assumption 3 implies that $x \leqq \bar{x}(m)$. Hence, $H^i_\theta(m)$ is uniformly bounded.

Because the sets X_θ and $B^i(m)$ are assumed to be closed, so obviously is $H^i_\theta(m)$, which is therefore compact as a closed and bounded set in $\mathbb{R}^G \times \mathbb{Z}^H$. $\square$

4. Continuous and Unbounded Individual Demand

4.1. **Continuous compensated demand.** For each $i \in I$, define the *demand correspondence* from each pair (θ, m) with $\theta \in \Theta$ and $m \geq 0$ to the set

$$\psi^i_\theta(m) := \{\, x \in H^i_\theta(m) \mid x' \in H^i_\theta(m) \Longrightarrow x \succsim_\theta x' \,\}$$

of optimal net trade vectors within the constraint set $H^i_\theta(m)$. Because $H^i_\theta(m)$ is non-empty and compact, while $\succsim_\theta$ is continuous, the demand set $\psi^i_\theta(m)$ is always non-empty.

In classical demand theory with a Walrasian budget constraint defined by a hyperplane, the compensated demand correspondence is upper hemi-continuous in many situations where the ordinary Walrasian demand correspondence may not be. A similar property applies here, with piecewise convex budget sets.

First, for each $i \in I$ and $x \in \mathbb{R}^G \times \mathbb{Z}^H$, define $\beta^i_*(x) := \inf_{j=1}^{\infty} \beta^i_j(x)$. Then, for each $i \in I$, define the *compensated demand correspondence* from each pair (θ, m) with $\theta \in \Theta$ and $m \geq 0$ to the set

$$\xi^i_\theta(m) := \{\, x \in H^i_\theta(m) \mid x' \succsim_\theta x \Longrightarrow \beta^i_*(x') \geq m \,\}$$

This implies that $\xi^i_\theta(m)$ consists of all the net trade vectors within the constraint set $H^i_\theta(m)$ which are strictly preferred to any other x' in the same set satisfying $\beta^i_*(x') < m$.

The following two lemmas show that the compensated demand correspondence has the same key properties as in the Walrasian case.

Lemma 3: For each $i \in I$, $\theta \in \Theta$, and $m \geq 0$, one has $\psi^i_\theta(m) \subset \xi^i_\theta(m)$.

Proof: Suppose that $x \in \psi^i_\theta(m)$. Evidently $x \in H^i_\theta(m)$.

Consider any $x' \in X_\theta$ with $x' \succ_\theta x$. Then there exists a sequence $x_n \in X_\theta$ with $x_n \gg^G x'$ $(n = 1, 2, \dots)$ such that $x_n \to x'$ as $n \to \infty$. Because Assumption 1 implies that preferences have the restricted monotonicity property and are transitive, for each n it follows that $x_n \in X_\theta$ with $x_n \succ_\theta x$. Then $x \in \psi^i_\theta(m)$ implies that for each n one has $x_n \notin H^i_\theta(m) = X_\theta \cap B^i(m)$. It follows that $x_n \notin B^i(m)$ and so

$\beta^i_j(x_n) > m$ for all j. Because each function β^i_j is continuous, taking the limit as $n \to \infty$ gives $\beta^i_j(x') \geq m$ for all j, so $\beta^i_*(x') \geq m$. This proves that $x \in \xi^i_\theta(m)$. $\square$

Lemma 4: For each $i \in I$, the compensated demand correspondence $(\theta, m) \mapsto\!\!\!\rightarrow \xi^i_\theta(m)$ is upper hemi-continuous throughout the domain of pairs (θ, m) with $\theta \in \Theta$ and $m \geq 0$.

Proof: Consider any sequence of triples (θ_n, m_n, x_n) with $x_n \in \xi^i_{\theta_n}(m_n)$ $(n = 1, 2, \dots)$ where $(\theta_n, m_n) \to (\theta, m)$ as $n \to \infty$. Let m^* denote $\sup_n m_n$. Then $B^i(m_n) \subset B^i(m^*)$ and so $x_n \in X_{\theta_n} \cap B^i(m^*)$ for $n = 1, 2, \dots$. By part 2 of Assumption 1, it follows that each $x_n \geqq \underline{x}$, and so $x_n \leqq \bar{x}(m^*)$ by the boundedness part 4 of Assumption 3. Thus, the sequence x_n is bounded, so has some convergent subsequence with a limit x. From now on, we restrict attention to this convergent subsequence, so $x_n \to x$ as $n \to \infty$. It remains to show that $x \in \xi^i_\theta(m)$.

Consider any $\tilde{x} \in X_\theta$ with $\tilde{x} \succ_\theta x$. By part 3 of Assumption 1, there exists a sequence $\tilde{x}_n \in X_{\theta_n}$ $(n = 1, 2, \dots)$ such that $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$. But then part 5 of Assumption 1 implies that $\tilde{x}_n \succ_{\theta_n} x_n$ for all large n. Otherwise there would exist an infinite subsequence of triples $(\theta_{n_r}, \tilde{x}_{n_r}, x_{n_r})$ such that $x_{n_r} \succsim_{\theta_{n_r}} \tilde{x}_{n_r}$ for $r = 1, 2, \dots$, and taking limits as $r \to \infty$ would give $x \succsim_\theta \tilde{x}$, contradicting $\tilde{x} \succ_\theta x$. Thus, for each large n, because each $x_n \in \xi^i_{\theta_n}(m_n)$, it follows that $\beta^i_*(\tilde{x}_n) \geq m_n$ and so $\beta^i_j(\tilde{x}_n) \geq m_n$ for all $j = 1, 2, \dots$. But β^i_j is continuous, so taking limits as $n \to \infty$ implies that $\beta^i_j(\tilde{x}) \geq m$ for all $j = 1, 2, \dots$.

Finally, consider any $\hat{x} \in X_\theta$ with $\hat{x} \succsim_\theta x$. Then there exists a sequence $\hat{x}_n \in X_\theta$ with $\hat{x}_n \gg^G \hat{x}$ $(n = 1, 2, \dots)$ such that $\hat{x}_n \to \hat{x}$ as $n \to \infty$. Because Assumption 1 implies that preferences have the restricted monotonicity property and are transitive, for each n it follows that $\hat{x}_n \in X_\theta$ with $\hat{x}_n \succ_\theta x$. The previous paragraph shows that for all $j = 1, 2, \dots$ one has $\beta^i_j(\hat{x}_n) \geq m$ for each n. But β^i_j is continuous, so taking limits as $n \to \infty$ implies that $\beta^i_j(\hat{x}) \geq m$ for all $j = 1, 2, \dots$. This proves that $\beta^i_*(\hat{x}) \geq m$, so $x \in \xi^i_\theta(m)$ as required. $\square$

4.2. **Critical parameter values.** First, for each $i \in I$, $\theta \in \Theta$, and $j, k = 1, 2, \dots$, define

$$M^i_\theta(j,k) := \{\, m \geq 0 \mid H^i_\theta(j,k,m) \neq \emptyset \,\}$$

as the set of parameter values allowing the potential consumer (i, θ) to reach the particular convex component $H^i_\theta(j, k, m) = X_\theta(k) \cap B^i(j, m)$ of the constraint set $H^i_\theta(m) = X_\theta \cap B^i(m)$.

Lemma 5: For each $i \in I$, $\theta \in \Theta$, and $j, k = 1, 2, \ldots$, the set $M^i_\theta(j, k)$ is closed.

Proof: Suppose that $m_n \in M^i_\theta(j, k)$ for $n = 1, 2, \ldots$ and $m_n \to m^*$ as $n \to \infty$. For each $r = 1, 2, \ldots$, define $m^*_r := \sup_{n \geq r} m_n$. Evidently $m^*_r \downarrow m^*$ as $r \to \infty$.

By definition of $M^i_\theta(j, k)$, for each $n \geq r$ there exists $x_n \in H^i_\theta(j, k, m_n) \subset H^i_\theta(j, k, m^*_r)$. In particular, $x_n \in X_\theta(k)$. It follows from Assumption 1 that $x_n \geqq \underline{x}$, and then $x_n \leqq \bar{x}(m^*_r)$ for all $n \geq r$ by the boundedness part of Assumption 3. So the sequence x_n is bounded, and must have a convergent subsequence with some limit point x^*. By Assumption 1, the set $X_\theta(k)$ is closed, implying that $x^* \in X_\theta(k)$. Furthermore, $\beta^i_j(x_n) \leq m^*_r$ for all $n \geq r$. Because β^i_j is continuous, taking limits as $n \to \infty$ gives $\beta^i_j(x^*) \leq m^*_r$ for $r = 1, 2, \ldots$, implying that $\beta^i_j(x^*) \leq m^*$ because $m^*_r \downarrow m^*$. It follows that $x^* \in H^i_\theta(j, k, m^*)$ and so $m^* \in M^i_\theta(j, k)$. Hence, the set $M^i_\theta(j, k)$ is closed. $\square$

Next, let $m^i_\theta(j, k) := \min_m M^i_\theta(j, k)$ be the *critical parameter value* which just allows the potential consumer (i, θ) to reach the particular convex component $H^i_\theta(j, k, m)$ of the constraint set $H^i_\theta(m)$. It follows from Lemma 5 that $m^i_\theta(j, k)$ is well defined.

For each $(i, \theta) \in I \times \Theta$, let $C^i_\theta := \cup_{j=1}^\infty \cup_{k=1}^\infty \{ m^i_\theta(j, k) \}$ be the set of all critical parameter values for the potential consumer (i, θ). Clearly, C^i_θ is either finite or countably infinite.

4.3. **Continuous individual demand.** The following lemma shows that the compensated demand $\xi^i_\theta(m)$ of each potential individual (i, θ) coincides with that individual's demand $\psi^i_\theta(m)$ away from critical parameter values $m \in C^i_\theta$.

Lemma 6: For each $i \in I$ and $\theta \in \Theta$ one has $\psi^i_\theta(m) = \xi^i_\theta(m)$ for all $m \notin C^i_\theta$.

Proof: Because of Lemma 3, it is enough to prove that $\xi^i_\theta(m) \subset \psi^i_\theta(m)$ for all $m \notin C^i_\theta$. So suppose that $x \in \xi^i_\theta(m)$, where $m \notin C^i_\theta$. Evidently $x \in H^i_\theta(m)$.

Consider any $x' \in X_\theta$ with $x' \succ_\theta x$. By definition of $\xi^i_\theta(m)$, one must have $\beta^i_j(x') \geq m$ for all $j = 1, 2, \ldots$. Consider any k such that $x' \in X_\theta(k)$ and then any

j such that $m \geq m^i_\theta(j,k)$. The hypothesis $m \notin C^i_\theta$ implies that $m > m^i_\theta(j,k)$. So there exists $\hat{x} \in X_\theta(k) \cap B^i(j,m)$ such that $\beta^i_j(\hat{x}) < m$ and so $\beta^i_j(\hat{x}) < \beta^i_j(x')$.

Let λ_n $(n = 1, 2, \ldots)$ be any sequence of positive scalars satisfying $\lambda_n < 1$ which converges to 0 as $n \to \infty$. Then let $x_n := (1 - \lambda_n)\, x' + \lambda_n\, \hat{x}$ for $n = 1, 2, \ldots$, so $x_n \to x'$. With this construction, each $x_n \in X_\theta(k)$ because $X_\theta(k)$ is a convex component of X_θ. Also, because $\beta^i_j(\hat{x}) < \beta^i_j(x')$, part 3 of Assumption 3 implies that $\beta^i_j(x_n) < \beta^i_j(x')$ for each $n = 1, 2, \ldots$. But for n sufficiently large, continuity of preferences implies that $x_n \succ_\theta x$. By definition of $\xi^i_\theta(m)$, one must have $\beta^i_*(x_n) \geq m$. It follows that $\beta^i_j(x') > \beta^i_j(x_n) \geq \beta^i_*(x_n) \geq m$ and so $\beta^i_j(x') > m$.

The previous two paragraphs show that $\beta^i_j(x') > m$ for all $x' \succ_\theta x$ and all $j = 1, 2, \ldots$. On the other hand, if $x' \in X_\theta \cap B^i(m)$, then there exists j such that $\beta^i_j(x') \leq m$, which is only possible if $x \succsim_\theta x'$. This proves that $x \in \psi^i_\theta(m)$. □

4.4. **Unbounded individual demand.** In the following, let $q \in \mathbb{R}^{G \cup H}_{++}$ denote any fixed strictly positive vector.

Lemma 7: Suppose the sequence m_n $(n = 1, 2, \ldots)$ has the property that, for some potential consumer (i, θ), the sequence $\inf_x \{\, q\,(x - \underline{x}) \mid x \in \psi^i_\theta(m_n)\,\}$ remains bounded as $n \to \infty$. Then the sequence m_n itself must be bounded.

Proof: By hypothesis, there exists a bound K and, for each $n = 1, 2, \ldots$, a net trade vector $x_n \in \psi^i_\theta(m_n)$ such that $q\,(x_n - \underline{x}) \leq K$. Because $q \gg 0$ and because in addition $x_n \in \psi^i_\theta(m_n)$ implies that $x_n \geqq \underline{x}$, the sequence x_n is bounded. So there must exist an upper bound $x^* \in \mathbb{R}^G \times \mathbb{Z}^H$ such that $x_n \ll x^*$ for all $n = 1, 2, \ldots$. Let $\hat{m} := \inf_j \{\, \beta^i_j(x^*) \mid j = 1, 2, \ldots \}$. For any $m > \hat{m}$, there exists j such that $\beta^i_j(x^*) < m$, so $x^* \in B^i(m)$. Because restricted monotonicity implies that $x^* \succ_\theta x_n$, it follows that $x_n \notin \psi^i_\theta(m)$ for all $m > \hat{m}$. Hence, $x_n \in \psi^i_\theta(m_n)$ implies, for all $n = 1, 2, \ldots$, that $m_n \leq \hat{m}$. So the sequence m_n must be bounded. □

5. Continuous unbounded mean demand

5.1. **Continuous mean compensated demand.** Let $\Psi^C(m) := \int_A \xi^i_\theta(m)\, d\alpha$ denote the *mean compensated demand correspondence.*

Lemma 8: On the domain $\mathbb{R}_+$, the mean demand compensated correspondence $m \mapsto\!\!\!\to \Psi^C(m)$ is well-defined, has non-empty convex compact values, and is also upper hemi-continuous.

Proof: By Lemma 4, for each fixed $m \geq 0$, the correspondence $\theta \mapsto\!\!\!\to \xi^i_\theta(m)$ is upper hemi-continuous. It follows that its graph is relatively closed and so measurable. Moreover, the correspondence has uniformly bounded values because $\xi^i_\theta(m) \subset H^i_\theta(m)$, where, by Lemma 2, $H^i_\theta(m)$ is uniformly bounded. Also, the set I is finite. Arguing as in Hildenbrand (1974, pp. 54 and 61), these conditions guarantee that the mean compensated demand correspondence $m \mapsto\!\!\!\to \Psi^C(m)$ is well-defined and has non-empty compact values for all $m \geq 0$. Also, $\Psi^C(m)$ is always convex because the measure α has no atoms.

Suppose that the sequence $m_n \geq 0$ $(n = 1, 2, \ldots)$ converges to m as $n \to \infty$. Suppose also that the sequence $y_n \in \Psi^C(m_n)$ $(n = 1, 2, \ldots)$. By definition, this means that for each $n = 1, 2, \ldots$ there exists a selection $(i, \theta) \mapsto x^i_{n\theta}$ from the correspondence $(i, \theta) \mapsto\!\!\!\to \xi^i_\theta(m_n)$ such that $y_n = \int_A x^i_{n\theta}\, d\alpha$. But $(i, \theta) \mapsto x^i_{n\theta}$ is a selection from the uniformly bounded and compact-valued correspondence $(i, \theta) \mapsto\!\!\!\to H^i_\theta(m_n)$, implying that y_n belongs to $\int_A H^i_\theta(m_n)\, d\alpha$. This in turn is a subset of the compact set $\int_A H^i_\theta(m^*)\, d\alpha$, where $m^* := \sup_n m_n$. It follows that the sequence y_n is bounded, so must have a subsequence converging to a limit $y^* \in \int_A H^i_\theta(m^*)\, d\alpha$.

Now apply Fatou's lemma for finite dimensions (Schmeidler, 1970; Hildenbrand, 1974, Lemma 3, p. 69) to the space $\mathbb{R}^{G \cup H}$. Because the mapping $(i, \theta) \mapsto x^i_{n\theta}$ is uniformly bounded for all n, this lemma implies the existence of an integrable function $(i, \theta) \mapsto x^i_\theta$ such that $y^* = \int_A x^i_\theta\, d\alpha$ and also, for α-almost every (ℓ, i, θ), some subsequence of $x^i_{n\theta}$ converges to x^i_θ. Because $x^i_{n\theta} \in \xi^i_\theta(m_n)$ and, by Lemma 4, each correspondence $m \mapsto\!\!\!\to \xi^i_\theta(m)$ is upper hemi-continuous, it follows that $x^i_\theta \in \xi^i_\theta(m)$ for all (i, θ). But then

$$y^* = \int_A x^i_\theta\, d\alpha \in \int_A \xi^i_\theta(m)\, d\alpha = \Psi^C(m)$$

confirming that $m \mapsto\!\!\!\to \Psi^C(m)$ is upper hemi-continuous. $\square$

5.2. **Dispersion and continuous mean demand.** The demand correspondence $m \mapsto\!\!\!\to \psi^i_\theta(m)$ of each potential consumer (i, θ) may fail to be upper hemi-continuous at any critical value $m \in C^i_\theta$. To avoid these discontinuities having any significance in the aggregate, one can introduce an additional "dispersion" assumption, motivated in part by the *dispersed needs* assumption used in Coles and Hammond (1995). Somewhat similar are Mas-Colell's (1977) assumption that the distribution of individuals' endowment vectors is absolutely continuous w.r.t. Lebesgue measure, and especially Yamazaki's (1978, 1981) *dispersed endowments* assumption.

Indeed, given any $m \geq 0$, define

$$C(m) := \{\, (i,\theta) \in I \times \Theta \mid m \in C^i_\theta \,\} = \cup_{i \in I} \cup_{j=1}^\infty \cup_{k=1}^\infty \{\, m^i_\theta(j,k) \,\}$$

as the set of all potential consumers (i,θ) who have m as a critical parameter value. Then assume:

Assumption 4 (dispersion): For all $m \geq 0$ one has $\alpha([0,1] \times C(m)) = 0$.

Note that, for each $i \in I$ and for $j,k = 1,2,\ldots$, the measure α on $[0,1] \times I \times \Theta$ and the continuous function $\theta \mapsto m^i_\theta(j,k) \in \mathbb{R}_+$ together induce a measure $\sigma^i(j,k)$ defined on the Borel sets $V \subset \mathbb{R}_+$ by

$$\sigma^i(j,k)(V) := \alpha(\{\, (\ell,i,\theta) \in [0,1] \times I \times \Theta \mid m^i_\theta(j,k) \in V \,\})$$

This is the distribution of potential consumers' minimum wealth levels $m^i_\theta(j,k)$ that just make accessible the convex component $H^i_\theta(j,k,m)$ of $H^i_\theta(m)$. Then, as in Mas-Colell (1977), an unnecessarily strong but plausible sufficient condition for Assumption 4 to hold is that each measure $\sigma^i(j,k)$ on $\mathbb{R}_+$ should be absolutely continuous w.r.t. Lebesgue measure — i.e., there should be some integrable density function $\mu^i(j,k)(m)$ on $\mathbb{R}_+$ such that $\sigma^i(j,k)(V) = \int_V \mu^i(j,k)(m)\, dm$ for every Borel set $V \subset \mathbb{R}_+$.

Dispersion has the following important implication:

Lemma 9: On the domain $\mathbb{R}_+$, the mean demand correspondence $m \mapsto\!\!\!\to \Psi(m) := \int_A \psi^i_\theta(m)\, d\alpha$ is well-defined, has non-empty convex compact values, and is also upper hemi-continuous.

Proof: Lemma 6 implies that $\xi^i_\theta(m) = \psi^i_\theta(m)$ for all $(i,\theta) \notin C(m)$. Then the dispersion Assumption 4 implies that for all $m \geq 0$ one has $\xi^i_\theta(m) = \psi^i_\theta(m)$ for α-almost all (ℓ,i,θ). So $\Psi^C(m) = \int_A \xi^i_\theta(m)\, d\alpha = \int_A \psi^i_\theta(m)\, d\alpha = \Psi(m)$ for all $m \geq 0$. The result follows immediately from Lemma 8. $\square$

5.3. **Unbounded mean demand.** The following result confirms that $\Psi(m)$ is unbounded as $m \to \infty$.

Lemma 10: Suppose that the sequence m_n $(n = 1, 2, \dots)$ tends to ∞ as $n \to \infty$. Then, given any fixed $q \in \mathbb{R}^{G \cup H}_{++}$, the sequence $\min_y \{\, q\,(y - \underline{x}) \mid y \in \Psi(m_n)\,\}$ also tends to ∞.

Proof: For $n = 1, 2, \dots$, define $g_n := \inf_y \{\, q\,(y - \underline{x}) \mid y \in \Psi(m_n)\,\}$. But $\Psi(m_n) = \int_A \psi^i_\theta(m_n)\, d\alpha$. From Hildenbrand (1974, Prop. 6, p. 63), it follows that

$$g_n = \int_A \inf_{x^i_\theta} \{\, q\,(x^i_\theta - \underline{x}) \mid x^i_\theta \in \psi^i_\theta(m_n)\,\}\, d\alpha$$

Suppose $m_n \to \infty$. Then Lemma 7 implies that the last integrand tends to ∞ as $n \to \infty$ for all pairs (i, θ), so $g_n \to \infty$. $\square$

The last assumption used in the main theorem is:

Assumption 5: The set $Y(\underline{x}) := \{\, y \in Y \mid y \geqq \underline{x}\,\}$ is bounded.

This expresses the obvious requirement that, because mean net inputs must be bounded in order to satisfy $y \geqq \underline{x}$, mean net outputs must also be bounded.

Lemma 11: Suppose the sequence m_n $(n = 1, 2, \dots)$ has the property that $\Psi(m_n) \cap Y \neq \emptyset$ for all n. Then m_n must be bounded.

Proof: Suppose that $y_n \in \Psi(m_n) \cap Y$ $(n = 1, 2, \dots)$. Because of part 2 of Assumption 1, one has $y_n \geqq \underline{x}$ for all n, so $y_n \in Y(\underline{x})$. By Assumption 5, this implies that y_n is bounded. So therefore is $q\,(y_n - \underline{x})$, for any $q \in \mathbb{R}^{G \cup H}_{++}$. Because $y_n \in \Psi(m_n)$ for each n, the result follows from Lemma 10. $\square$

6. Main Theorem

Finally, the main theorem of the paper:

Theorem: Suppose that Assumptions 1–5 all hold, and in particular, that the status quo allocation $(i, \theta) \mapsto \hat{x}^i_\theta$ can be decentralized by the closed and piecewise convex budget sets $\hat{B}^i = B^i(0)$ $(i \in I)$ which satisfy restricted free disposal within $\mathbb{R}^G \times \mathbb{Z}^H$. Given the status quo mean net trade vector $\hat{y} := \int_A \hat{x}^i_\theta\, d\alpha$, suppose there exists $y \in Y$ such that $y \gg \hat{y}$. Then there exists a strictly Pareto superior allocation which, for some $m^* > 0$ such that $\Psi(m^*)$ includes a boundary point of

Y, is decentralized by the closed and piecewise convex budget sets $B^i(m^*)$ $(i \in I)$ satisfying restricted free disposal.

Proof: Define $m^* := \sup\{\, m \in \mathbb{R}_+ \mid \Psi(m) \cap Y \neq \emptyset \,\}$. By Lemma 11, m^* is finite.

By definition of m^*, there must exist two corresponding sequences m_n and y_n $(n = 1, 2, \dots)$ such that $y_n \in \Psi(m_n) \cap Y$ and $m_n \uparrow m^*$. By Lemma 9, the correspondence $m \twoheadrightarrow \Psi(m)$ is upper hemi-continuous and compact-valued, so the sequence y_n $(n = 1, 2, \dots)$ has a subsequence which converges to some limit $y^* \in \Psi(m^*)$. Because each $y_n \in Y$ and Y is closed, it follows that $y^* \in \Psi(m^*) \cap Y$.

Next, consider any corresponding pair of sequences m_n and y_n $(n = 1, 2, \dots)$ such that $m_n > m^*$ and $y_n \in \Psi(m_n)$ for all n, while $m_n \downarrow m^*$. Then each budget set $B^i(m_n)$ $(i \in I)$ is shrinking, implying that for all $n = 1, 2, \dots$ the mean net trade vector y_n belongs to $\int_A H^i_\theta(m_1)\, d\alpha$. By Lemma 2, this set is bounded. Therefore, the sequence y_n has a subsequence which converges to some limit $\tilde{y}$. Because the correspondence $m \twoheadrightarrow \Psi(m)$ is upper hemi-continuous, it follows that $\tilde{y} \in \Psi(m^*)$. But the definition of m^* implies that $y_n \notin Y$ for all n, so the limit $\tilde{y}$ is not an interior point of Y.

Finally, let $L := [y^*, \tilde{y}]$ denote the closed line segment whose ends are y^* and $\tilde{y}$ respectively. By Lemma 9, $\Psi(m^*)$ must be convex, so $L \subset \Psi(m^*)$. Because Y is closed, the set $Y \cap L$ is compact, and so includes a point y' which is as close as possible to $\tilde{y}$. Of course $y' \in \Psi(m^*) \cap Y$. If $\tilde{y} \in Y$, then $y' = \tilde{y}$. Otherwise $L \setminus Y$ includes a half-open line segment $(y', \tilde{y}]$. In either case, y' must be a boundary point of Y. $\square$

7. Implications for Policy Reform

7.1. **Potential gains from enhanced production efficiency.** Diamond and Mirrlees (1971) chose to emphasize that production efficiency was necessary for a scheme of commodity taxation to maximize any given Paretian Bergson social welfare function. They themselves point out how there should then be no taxation of intermediate goods — including capital held by producers, as discussed by Judd (1999) in particular. The reason, of course, is that otherwise one would introduce unnecessary distortions which reduce efficiency in aggregate production.

Such distortions include those due to import tariffs and other policy measures which protect domestic industry. Production efficiency can only be preserved if import tariffs are restricted to goods that consumers buy directly, and if the tariffs are made equal to the usual consumer taxes on equivalent goods produced domestically. Indeed, such reasoning helps explain the results concerning second-best potential Pareto gains from free trade and from customs unions — as summarized in Hammond and Sempere (1995), for instance.

The proof used by Diamond and Mirrlees, however, actually demonstrates rather more — namely, that any tax scheme leaving the economy at an interior point of the aggregate production set can be Pareto dominated by an alternative tax scheme involving some small change in the relevant tax parameters, and so in the budget constraint facing each consumer. In fact, this process of finding Pareto improvements can continue until the resulting mean demand vector is on the boundary of the aggregate production set. This is what the main theorem above demonstrates. So production efficiency is necessary, not just for optimal taxation in the sense of Diamond and Mirrlees, but for the efficient use of any single tax instrument which generates the kind of one parameter family of budget sets satisfying Assumption 3 in Section 3.

As the argument leading to the main theorem demonstrates, this has the following general implication. Consider any reform which enhances production efficiency, so that the status quo mean net output vector $\hat{y}$ is an interior point of the reformed production set Y. Then the main theorem establishes that the production reform can be accompanied by a tax reform facing consumers with new budget sets $B^i(m^*)$ $(i \in I)$ for some $m^* > 0$ such that the mean net demand set $\Psi(m^*)$ intersects the boundary of Y. Moreover, the result is a strict Pareto improvement.

These results are obviously in the spirit of the Kaldor–Hicks criterion for a potential Pareto improvement. Yet there is a fundamental difference. The original Kaldor–Hicks compensation tests rely on lump-sum transfers from those who gain from the policy reform to those who would otherwise lose. These transfers are generally incentive incompatible, making it impossible to convert the theoretical potential Pareto improvement into actuality. This need to respect incentive constraints, indeed, was the basis of the vigorous disagreement between Kemp and Wan (1986) on one side, and Dixit and Norman (1986) on the other. Only in the case when the status quo is clearly determined by a centrally planned economic system does it seem safe to neglect these incentive constraints — a case suggested by the work of Lau, Qian and Roland (1997) analysing reform in China.

Here, by contrast, incentive constraints have been explicitly recognized, so this kind of obstacle to finding an actual Pareto improvement cannot arise. In Hammond and Sempere (1995) we were able to adapt previous results due to Dixit and Norman (1980, 1986) and Grinols (1981) in particular to demonstrate second-best gains from freer trade. The implication is that in a small country, there should be free trade for producers, and import tariffs for consumers set equal to domestic consumption taxes. We were also able to show that similar gains can be had from customs unions, from enhanced competition between firms, and from adopting on a suitable scale any project whose benefits exceed its costs when these are all evaluated at suitable producer prices.

7.2. **Project Evaluation: A Personal Statement.** At first it might seem that the efficiency result is of no relevance to project evaluation. After all, the purpose of project evaluation is presumably to identify desirable changes in production. This suggests that cost–benefit analysis should be explicitly designed for use when the economy may well remain very far from a second-best optimum, or from any kind of efficient allocation, even after any favourable project has been adopted. Then, with no presumption of production efficiency, it would seem more useful to think of demand or consumer prices rather than supply or producer prices as an aid to identifying favourable projects. That is the basic argument of Hammond (1980).

One major difficulty with this "demand-side" approach, as Sen (1972) pointed out, is that knowing just a project's net output vector is typically insufficient. The project's effect on the economy as a whole, and so ultimately on consumer welfare, depends on other policy measures like tax reform which may be needed to re-equilibrate the economy after that project. After all, a public sector project which earns a large surplus for the government is probably favourable, but its effect on consumers depends crucially on how this surplus is used — what taxes are reduced, beneficial subsidies created, or additional public goods provided. Diewert (1983) shows how a favourable project in an open economy might benefit only foreign consumers and earn a net surplus for the domestic government, without any effect at all through price changes on domestic consumers. That prompted an investigation of how the shadow prices that should be used to evaluate projects depend on precisely what potential tax reform or other balancing policy is seen as most likely to accompany any favourable project — see Hammond (1986), for example.

Later, however, came the realization that some fairly robust results on the gains from trade do not require any kind of second-best optimality or even efficiency after trade liberalization has occurred. Such results do require, of course, that trade liberalization brings about a potential Pareto improvement which can be converted into an actual improvement through some compensation mechanism that transfers real income to those who would otherwise lose. Or better, as in the case of the Dixit and Norman (1980, 1986) scheme for freezing consumer prices, commodity taxes are required to adjust in a way that removes any possibility at all of there being any losers. Now, it turns out that the benefits from trade liberalization in the Dixit/Norman setting are entirely due to improved efficiency in the international organization of production, as countries are led to concentrate on activities where they enjoy a comparative advantage. What is more, as discussed in Hammond and Sempere (1995), the Dixit and Norman scheme allows Pareto gains to result from general reforms which enhance production efficiency, even with limits to redistribution. But really all such results are merely elaborations of the original Diamond/Mirrlees argument. Or, in somewhat more general settings, of the main theorem set out in Section 6 above.

This brings us back to project evaluation, but viewed from a different perspective. Let us readily concede that no single project which is likely ever to be submitted for evaluation will get us anywhere near what could be recognized as a second-best optimum, or even near the economy's production frontier. At best, we seem to be contemplating relatively minor alterations to the economy's projection possibility set — or rather, to a severely constrained set which allows input and outputs to be reallocated among existing firms and production activities, but which does not pretend to describe the full production possibilities taking account of all possible favourable projects.

So, suppose Y is interpreted as this severely constrained set, which must include the status quo net output vector $\hat{y}$. Consider a project in the form of an incremental net output vector $z \in \mathbb{R}^{G \cup H}$, with quantities of each good measured per head of population. Of course, if the status quo net output vector $\hat{y}$ is an interior point of Y, production efficiency can be enhanced merely by moving out to the frontier of Y, without the need for any project. So from now on assume that $\hat{y}$ is an efficient point of Y. This restriction should be regarded as indicating how constrained Y really must be.

If the project z is adopted, then the economy's production set will change from Y to $Y + \{z\}$, reflecting both the project z and the opportunities to rearrange production within the constrained set Y. Or more exactly, since one can choose whether or not to adopt the project, the new production set is $Y \cup (Y + \{z\}) = Y + \{\, 0, z \,\}$. For the project z to enhance production efficiency, therefore, $\hat{y}$ should be an interior point of $Y + \{\, 0, z \,\}$. That is, given that $\hat{y}$ is an efficient point of Y, there must exist some $y \in Y$ for which $y + z \gg \hat{y}$. Equivalently, one must have $z \in Z := \{\hat{y}\} + \mathbb{R}^{G \cup H}_{++} - Y$. Then the Diamond/Mirrlees efficiency argument establishes that the project z allows a potential Pareto improvement.

Viewed in this way, a cost–benefit test should be explicitly designed in order to identify efficiency enhancing projects. Then it can serve as the basis for an iterative planning procedure of accepting successive projects which pass the test. The resulting procedure will then meet some important criteria set out by Malinvaud (1967). That is, a project z should pass a cost–benefit test if and only if it is feasible and enhances production efficiency. Then, provided appropriate adjustments are made to each consumer's budget constraint, each step of the iterative procedure will produce a strict Pareto improvement. Moreover, the iterative procedure stops only when the resulting allocation is at least second-best Pareto efficient, if not a second-best welfare optimum, relative to the production set defined by combining Y with the menu of all available projects.

7.3. **Evaluating small projects.** Unfortunately, however, cost–benefit tests are not especially useful for identifying production efficiency gains in general. The reason is that such tests do not work well unless the project is small and the production set Y is convex or has a smooth boundary. Note that, even if Y does meet these conditions, the new production set $Y + \{\, 0, z \,\} = Y \cup (Y + \{z\})$ will usually be non-convex and have a kinked boundary.

Accordingly, we shall consider instead when $\lambda z \in Z$ for all small $\lambda > 0$, so that adopting the project z on a small enough scale will produce an efficiency gain. In two obvious cases, there will be shadow price vectors $p \in \mathbb{R}^{G \cup H}$ such that cost–benefit tests of the form $p z > 0$ can be used to identify such favourable small projects.

The first case is when Y has a tangent hyperplane at $\hat{y}$.

Proposition 1: Suppose there exists a shadow price vector $p \in \mathbb{R}^{G \cup H}$ with $p > 0$ such that $p y = p \hat{y}$ is the equation of the tangent hyperplane to Y at $\hat{y}$. Then $\lambda z \in Z$ for all small $\lambda > 0$ if $p z > 0$, and only if $p z \geq 0$.

Proof: Suppose that $p z > 0$. Because $p y = p \hat{y}$ is a tangent hyperplane and Y admits free disposal, it follows that for all small $\lambda > 0$ the net output vector $\hat{y} - \lambda z$ is an interior point of Y. So for all small $\lambda > 0$, there exists $y \in Y$ with $y \gg \hat{y} - \lambda z$. It follows that $\lambda z = \hat{y} + u - y$ for some $u \gg 0$ and $y \in Y$, as required for λz to belong to Z.

Conversely, suppose that $\lambda z \in Z$ for all small $\lambda > 0$. Then there exist sequences of scalars $\lambda_k \downarrow 0$ and of vectors $y_k \in Y$ such that $y_k + \lambda_k z \gg \hat{y}$ $(k = 1, 2, \ldots)$. For each k one has $y_k \gg \hat{y} - \lambda_k z$, implying that $\hat{y} - \lambda_k z \in Y$ by free disposal. Because $\lambda_k \downarrow 0$ and $p y = p \hat{y}$ is a tangent hyperplane, it follows that $p z \geq 0$. □

The second case is when Y is convex.

Proposition 2: Suppose that Y is a convex set. Let

$$P(\hat{y}) := \{\, p \in \mathbb{R}^{G \cup H} \setminus \{0\} \mid y \in Y \Longrightarrow p y \leq p \hat{y} \,\}$$

denote the (non-empty) set of price vectors which determine hyperplanes $p y = p \hat{y}$ that support Y at the boundary point $\hat{y}$. Then $\lambda z \in Z$ for all small $\lambda > 0$ if and only if $p z > 0$ for all $p \in P(\hat{y})$.

Proof: Suppose there exists $\lambda > 0$ such that $\lambda z \in Z$. Then there exist $u \gg 0$ and $y \in Y$ such that $\lambda z = \hat{y} + u - y$. Consider any supporting price vector $p \in P(\hat{y})$. Clearly $p > 0$ because Y admits free disposal. So $p u > 0$. It follows that $p (\lambda z) = p (\hat{y} + u - y) > p (\hat{y} - y) \geq 0$, where the last inequality holds because $y \in Y$ and $p \in P(\hat{y})$. This implies that $p z > 0$. Moreover, the same is true for every $p \in P(\hat{y})$, as required.

Conversely, consider the two sets $K := \{\lambda\, z \mid \lambda > 0\,\}$ and Z. Both are non-empty and convex, with 0 as a common boundary point. If they intersect, it must be at some point $\bar{\lambda}\, z$ with $\bar{\lambda} > 0$. Then $\lambda\, z \in K \cap Z$ whenever $0 < \lambda \leq \bar{\lambda}$, so $\lambda\, z \in Z$ for all small $\lambda > 0$. On the other hand, if K and Z are disjoint, then they can be separated by a hyperplane $p\, z' = 0$ through the origin, with $p\, z' \geq 0$ for all $z' \in Z$, but $p\, z' \leq 0$ for all $z' \in K$, which implies that $p\, z \leq 0$. In particular, for all $u \gg 0$ and all $y \in Y$ one has $p\,(\hat{y} + u - y) \geq 0$, so $p\, y \leq p\,(\hat{y} + u)$. This implies that $p\, y \leq p\, \hat{y}$ for all $y \in Y$, so $p \in P(\hat{y})$. In other words, unless $\lambda\, z \in Z$ for all small $\lambda > 0$, there must exist $p \in P(\hat{y})$ such that $p\, z \leq 0$. This is the contrapositive of the desired conclusion. □

Note that the first part of the above proof actually demonstrates:

Corollary: If Y is convex and $z \in Z$, then $p\, z > 0$ for all $p \in P(\hat{y})$.

In other words, when Y is convex, then even if z is a large project, the test $p\, z > 0$ for all $p \in P(\hat{y})$ is necessary for z to increase production efficiency, but may not be sufficient.

A special case is when a small country can trade some commodities at fixed border prices, in which case the relevant marginal rates of substitution are equal to the ratios of these border prices. Hence, for such traded commodities, shadow prices should be set equal to border prices. That is essentially the rationale for the Little/Mirrlees approach to shadow pricing for traded goods.

7.4. **Practical Limitations.** As was noted early on by Stiglitz and Dasgupta (1971), Dasgupta and Stiglitz (1972), as well as by Mirrlees (1972) himself, the Diamond/Mirrlees production efficiency argument faces some practical difficulties when changes in producer prices affect producer profits and so consumer incomes in ways that may be deleterious and hard to correct. Ideally, as assumed in Hammond and Sempere (1995), one would like changes in profit incomes to be sterilized in ways that freeze every consumer's after-tax income, before a uniform poll subsidy is paid to all consumers in order to distribute the benefits of a favourable reform. Presumably a government which can foresee what consumer prices would have been in the absence of any reform and can then identify and tax or subsidize any market transaction between a firm and a consumer in order to freeze consumer prices can also foresee what after-tax profits would have been in the status quo and freeze those also.

An important exception may come in small firms whose affairs are less subject to public scrutiny. Then profit potential in the status quo will often depend on private information, which therefore imposes incentive constraints on a truly feasible allocation. In this paper, however, the case when a small firm is wholly owned by just one consumer poses no problem in the main theorem because we have focused on each consumer's net trade vector. Thus, the inputs and outputs of such a small firm can be traded as if they were the personal demands and supplies of its owner. It is true that overall production efficiency may not be desirable once the production possibilities of small firms are included in the aggregate production set. But it does remain desirable for firms whose transactions with consumers and payments of profits to them can both be effectively monitored and subjected to appropriate taxes and subsidies.

A much more serious limitation is the restriction to a one parameter family of budget sets that are all independent of producer prices. Recall that in the case of linear commodity taxes, this formulation is based on the assumption that all commodity taxes can be varied in ways that sterilize all movements in producer prices, leaving consumer prices at the values they would have reached in the status quo. Such sterilization would seem to face at least two insuperable difficulties. Of these the first is the sheer administrative complexity, even in an age of extraordinarily powerful computes, of having what may be millions of different tax rates on different commodities. Any simplification that reduces the number of tax rates introduces the possibility that changes in some producer prices will not be fully sterilized before being passed on to consumers, some of whom may then be adversely effected. One possible remedy, suggested by the Diamond and Mirrlees (1971) discussion of Pareto improving tax changes, might be to try to "over-sterilize" price increases for goods which consumers buy in order to make sure that no consumer price rises for any such good, with the reverse holding for goods like labour which consumers sell. However, such generous tax reforms may violate the government's budget constraint even after a big increase in production efficiency. And in any case, they certainly introduce new complications which the main theorem of the paper makes no attempt at treating.

The second difficulty appears even more insuperable. This is the problem in an intertemporal economy of predicting what consumer prices would have been in the status quo, without any efficiency enhancing reform. Without such predictions, freezing consumer prices at what they would have been is evidently quite impossible. Even with such predictions, disputes are likely regarding their accuracy, with some consumers claiming that their reasonable expectations have not been met.

7.5. **Concluding Assessment.** Insuperable though these practical difficulties may be, they should be regarded as illustrating how hard it is to please everybody, and how much harder it is to make everybody admit they have been pleased. In other words, insisting on true Pareto improvements is surely excessively restrictive.

Even so, that is no excuse for disregarding the adverse effects that enhanced production efficiency can have on some consumers, such as those whose careers have become closely linked to industries, firms, and techniques of production whose continued survival is incompatible with efficiency. The Diamond/Mirrlees argument for production efficiency relied on being able to make sure that even these consumers would not be adversely effected because, for example, they could continue to supply their labour services for the same after-tax wage as in the status quo. If such compensation is not fully possible in the end, however, that may not by itself justify abandoning the efficiency objective. Instead, it is surely enough to have as an essential part of any efficiency enhancing reform some reasonably generous assistance program designed to re-train workers and to help them deal with the need to adjust their career plans. In particular, one cannot help feeling that sensitive efficiency enhancing reform policies of this kind are likely to do much better than policies which maintain existing inefficiencies in order to placate politically powerful vested interests.

The main lesson to be drawn from the Diamond/Mirrlees efficiency theorem, therefore, may not lie in the formal details. Rather, it is a reminder that the case for enhanced production efficiency may be much more robust than had generally been recognized — certainly more robust than the prior work of Samuelson (1947) or of Lipsey and Lancaster (1956) had suggested, and possible a little more robust than even careful readers of Diamond and Mirrlees (1971) might have supposed.

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