Online Appendix to:

What matters in school choice tie-breakings? How competition guides design

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In this Online Appendix we provide simulation results to test the robustness of our findings.

A Computational experiments

This section presents simulations results that complement our theoretical results. We first consider markets with complete preference lists for students and varying capacities for schools. After that, we consider markets with short preference lists, and finally, tiered markets where a subset of of schools are preferred by all students over the rest of schools.

A.1 Numerical results for our model

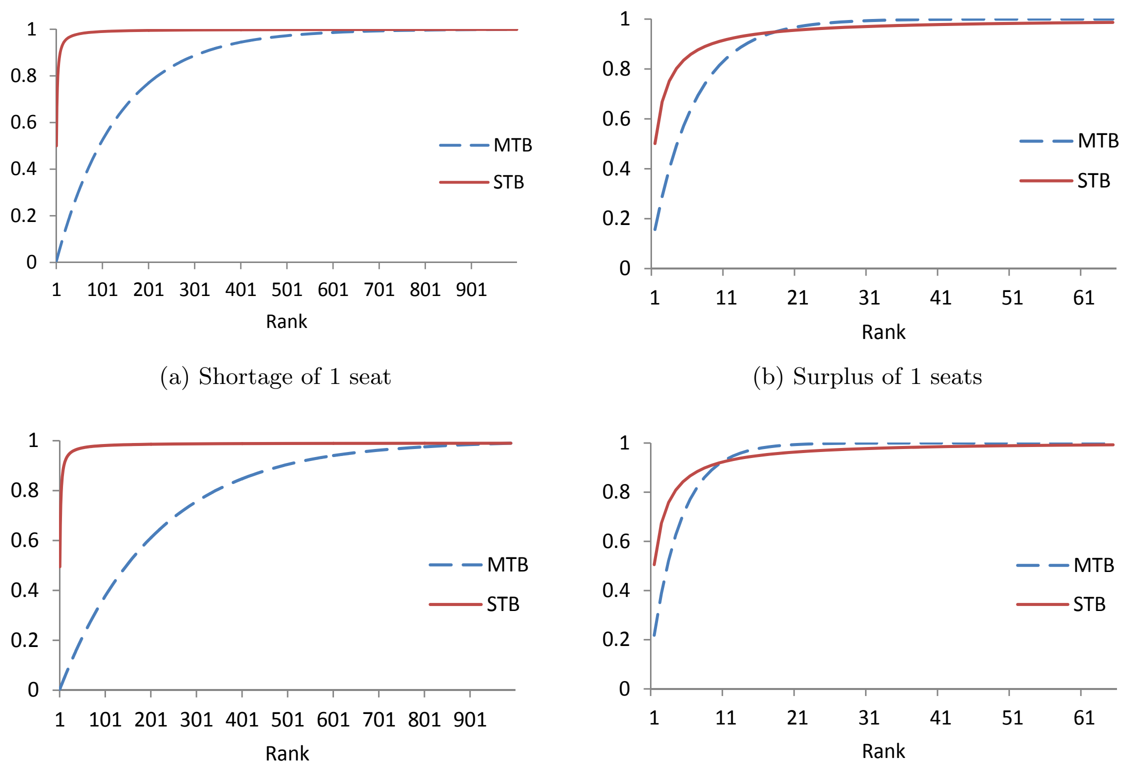
The first computational experiments illustrates the effect of the imbalance in the market on the students' rank distributions under STB and MTB and the relationship between the two. For each instance that we consider,[1] we sample realizations by drawing complete preference lists uniformly at random and independently for each student. In addition, under MTB, for each market realization we draw a complete order over students for each school, independently and uniformly at random. Under STB, for each market realization we draw a single order over students uniformly at random. Then, we compute the student optimal stable matching. The plots and the tables that we present here are generated by taking average over several (between 100 to 1000) samples for each instance.[2]

Figure 1 shows the cumulative rank distribution under each tie-breaking rule in a market with 1000 students. We consider instances with a small imbalance of either 1 or 10 seats, i.e. four

[1] An instance contains the information regarding market characteristics (size, capacities, list length), and the choice of tie-breaking rule.

[2] The number of samples were chosen large enough so that increasing this number changes the numerical results insignificantly.

different instances with 1000±1 and 1000±10 seats. Each school has one seat (capacity 1). Observe that when there is a shortage of seats (left panel), the rank distribution under STB stochastically dominates the rank distribution under MTB. When there is a surplus of seats (right panel), there is no stochastic dominance.



(a) Shortage of 1 seat

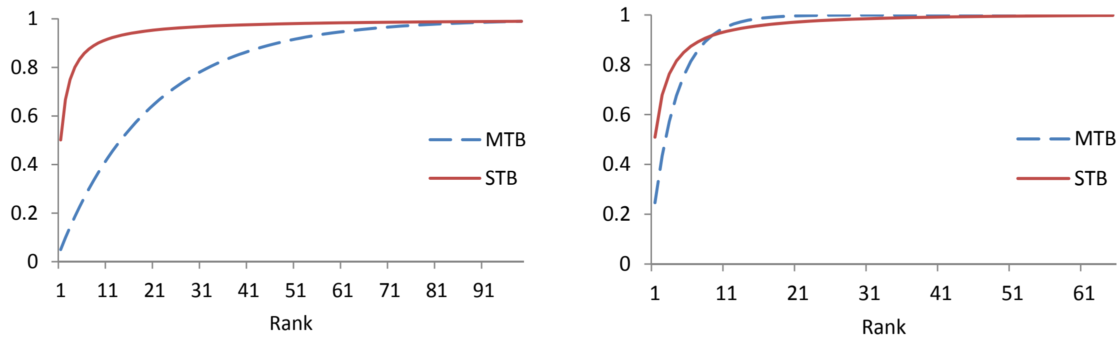
(b) Surplus of 1 seats

(c) Shortage of 10 seats

(d) Surplus of 10 seats

Figure 1: The cumulative rank distributions under MTB and STB in random market with 1000 students. Panels 1a and 1c plot the distributions in markets with a shortage of 1 and 10 seats, respectively. Panels 1b and 1d plot the distributions in markets with a surplus of 1 and 10 seats, respectively. The dashed and solid lines indicates the rank distributions under MTB and STB, respectively.

Figure 2 illustrates similar findings for a market with only 100 students, unit capacities, and a shortage or surplus of a single seat.



(a) Shortage of seats

(b) Surplus of seats

Figure 2: The rank distribution under MTB and STB in random market with 100 students with a shortage (left) and surplus (right) of one seat. The dashed and solid lines indicates the rank distribution under MTB and STB, respectively.

Table 1 reports the expected average rank and expected social inequity (or the variance of a student's rank) for markets with varying imbalances and each school has a single seat. Observe that the variance of the rank is larger under MTB (than under STB) when there is a shortage of seats and that the variance increases significantly in this case as the shortage grows from 1 to 10. Furthermore, notice that the variance of the rank is smaller under MTB when there is a surplus of seats.

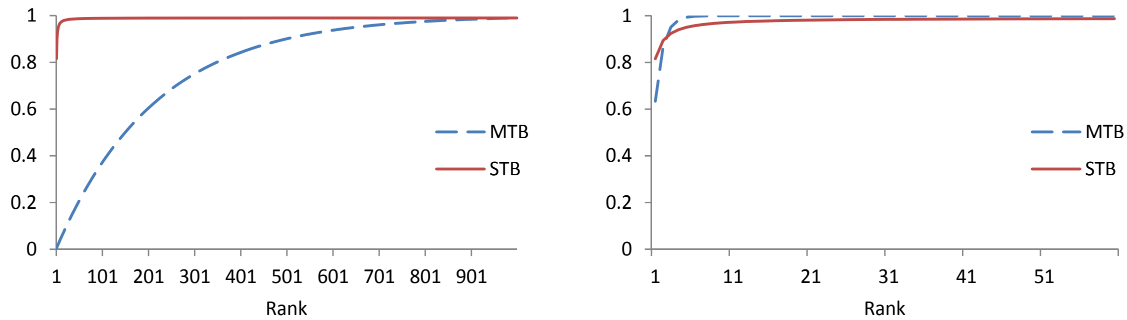
m \ $n-m$		-10	-1	1	10
100	$\mathcal{Ar}(\mu_{\text{STB}})/\mathcal{Ar}(\mu_{\text{MTB}})$	2.52/2.54	3.78/4.1	4.14/29.5	4.23/19.79
	$\mathcal{Si}(\mu_{\text{STB}})/\mathcal{Si}(\mu_{\text{MTB}})$	9.47/3.87	49.8/12.6	69.6/516.9	78.2/322.9
1000	$\mathcal{Ar}(\mu_{\text{STB}})/\mathcal{Ar}(\mu_{\text{MTB}})$	4.53/4.59	6/6.46	4.14/203.5	6.5/136.8
	$\mathcal{Si}(\mu_{\text{STB}})/\mathcal{Si}(\mu_{\text{MTB}})$	144.4/16.51	628.9/35.7	69.6/35780	947/18300

Table 1: Average rank and social inequity under under STB and MTB in the student optimal stable matching for different markets. A student's most preferred rank is 1 and larger rank indicates a less preferred school.

A.2 Robustness to large imbalances and capacities

This section presents simulation results to examine the effect of different imbalances as well as capacities on the random assignments under MTB and STB. We find that for all markets with a shortage of seats, the rank distribution under STB stochastically dominates the one under MTB.

Figure 3 shows the rank distribution under each tie-breaking rule in markets with 10000 students. Each school has 10 seats, and there is a total imbalance of 100 seats.



(a) Shortage of seats (b) Surplus of seats

Figure 3: The rank distribution under MTB and STB in a random market with 1000 schools where each school has 10 seats and in total there is a shortage (left) or surplus (right) of 100 seats. The dashed and solid lines indicates the rank distribution under MTB and STB, respectively.

Table 3 reports the expected average rank and social inequity for eight markets with imbalances 1 or 100 and school capacities are either 5 or 10. All schools have the same capacity in each instance; we denote this capacity by q.

m (q) \ $n - qm$		-100	-1	1	100
1000 (5)	$\mathcal{A}r(\mu_{\mathsf{STB}})/\mathcal{A}r(\mu_{\mathsf{MTB}})$	1.77/1.77	2.74/2.94	2.86/112	2.86/234.9
	$\mathcal{S}i(\mu_{\mathsf{STB}})/\mathcal{S}i(\mu_{\mathsf{MTB}})$	7.36/1.37	213.6/5.8	280/12429	289.2/44348
1000 (10)	$\mathcal{A}r(\mu_{\mathsf{STB}})/\mathcal{A}r(\mu_{\mathsf{MTB}})$	1.57/1.57	2.15/2.25	2.19/104.2	2.19/206.8
	$\mathcal{S}i(\mu_{\mathsf{STB}})/\mathcal{S}i(\mu_{\mathsf{MTB}})$	6.29/0.9	134.7/2.844	166.7/10851	36773/167.5

Table 2: Average rank and social inequity under under STB and MTB in the student optimal stable matching for different markets. A student's most preferred rank is 1 and larger rank indicates a less preferred school.

Figure 4 shows the ratio between $\mathcal{S}i(\mu_{\mathsf{STB}})$ to $\mathcal{S}i(\mu_{\mathsf{MTB}})$ in a market with 10000 students, unit capacities, and the surplus of seats varying from 100 to 1000. Observe that the ratio decreases as the surplus grows because the larger the surplus, the more students will receive their top choices.

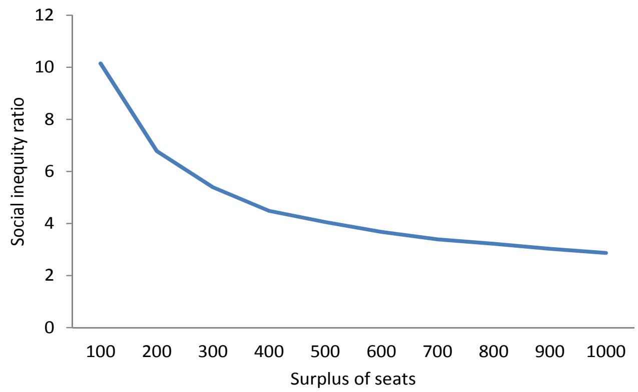


Figure 4: The ratio between $\mathcal{S}i(\mu_{\mathsf{STB}})$ to $\mathcal{S}i(\mu_{\mathsf{MTB}})$ in a random market with 10000 students, unit capacities, and a surplus of seats. (the x-axis denotes the surplus of schools)

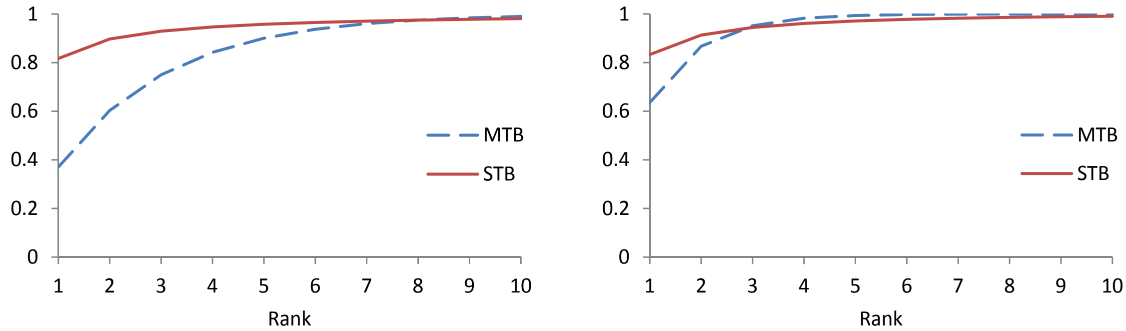
A.3 Short preference lists

In this section, we present simulations to illustrate the effect of shortening the students' preference lists on our results.

Figure 5 presents the rank distribution in random market with 1000 schools, each with capacity of 10. In addition there are either 10100 or 9900 students, each of which ranks independently uniformly at random 10 schools. (Note that we considered the same instance with complete preference lists in Appendix A.2, Table 3). When there is a shortage of seats and the preference lists are complete, our simulations reveal that the rank distribution under STB stochastically dominates the rank distribution under MTB; when the preference lists are short, stochastic dominance "almost" holds.

Shortening the lists reduces competition among students (see Ashlagi et al. (2015)), which impacts the market balance, i.e. whether students are "effectively" on the long side or the short side of the market. Therefore, whether there is a surplus or shortage in the market, as the preference lists become shorter, the crossing point of the rank distributions moves to the left (if the crossing happens at all).[3] In overdemanded markets, shortlists and large capacities act as two forces pushing in opposite directions: the former reduces competition and the latter increases it: When the capacities are large in an overdemanded market, MTB creates a much harsher competition relative to when the capacities are small, i.e. rejection chains become longer. On the other hand, under STB, a rejection reveals much more information about the rejected student's priority number, and thus, that student is less likely to initiate rejection chains. Consequently, as the capacities increase, the crossing point moves to the right (if crossing happens at all).

[3] The extreme case is when the list length is 1, where both distributions become identical.



(a) Shortage of seats (b) Surplus of seats

Figure 5: The rank distribution under MTB and STB in random market with 1000 schools each with 10 seats and in total there is a shortage (left) or surplus (right) of 100 seats. Each student ranks 10 schools. The dashed and solid lines indicates the rank distribution under MTB and STB, respectively.

m (q) \ $n - qm$		-100	100
1000 (10)	$\mathcal{Ar}(\mu_{\text{STB}})/\mathcal{Ar}(\mu_{\text{MTB}})$	1.36/1.57	1.4/2.6
	$\mathcal{Si}(\mu_{\text{STB}})/\mathcal{Si}(\mu_{\text{MTB}})$	1.24/0.89	1.44/3.59

Table 3: Average rank and social inequity under under STB and MTB in the student optimal stable matching for different markets. A student's most preferred rank is 1 and larger rank indicates a less preferred school.

A.4 Comparison to a hybrid tie-breaking rule

This section provides simulation results for two different tiered markets where some schools are considered as *top* schools and others are considered as *bottom* schools. In these markets every student prefers every top school to every bottom school and the preferences within a tier are drawn independently uniformly at random. Motivated by our findings, we compare three tie-breaking rules: (i) STB, (ii) MTB, and (iii) HTB (Hybrid Tie-Breaking rule), which is tie-breaking rule that uses a single preference list for all of the top schools and an independently drawn preference list for each bottom school.

Example: unit capacity Figure 6 shows the rank distribution under the three tie-breaking rules in a market with 1000 students and 1000 schools, each with unit capacity. We consider 100 schools to be the top schools. Notice that up to rank 100, the STB and HTB plots coincide and are above the MTB plot. Conditioning on being above the 100 rank, the MTB and HTB coincide and note that there is no stochastic dominance in this range.

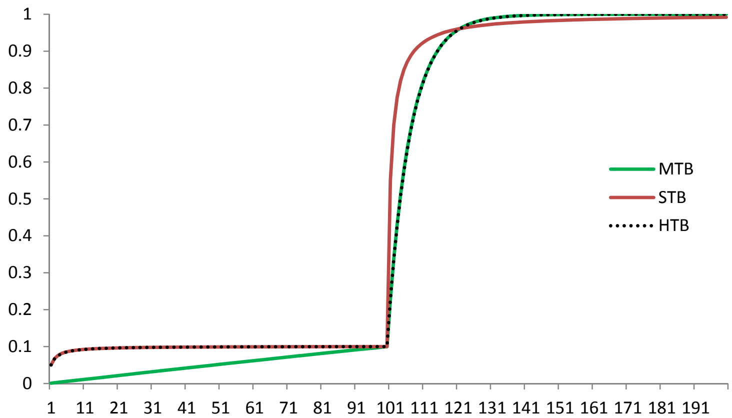


Figure 6: Students' rank distribution under STB, MTB and HTB. The market consists of $n = m = 1000$ and 100 top schools.

We list down the expected average rank and social inequity under the three tie-breaking rules below.

$$\mathbb{E}\left[\mathcal{Ar}(\mu_{\mathsf{STB}})\right] \approx 96.23 \qquad \mathbb{E}\left[\mathcal{Ar}(\mu_{\mathsf{MTB}})\right] \approx 101.48 \qquad \mathbb{E}\left[\mathcal{Ar}(\mu_{\mathsf{HTB}})\right] \approx 96.97$$
$$\mathbb{E}\left[\mathcal{Si}(\mu_{\mathsf{STB}})\right] \approx 1752.81 \qquad \mathbb{E}\left[\mathcal{Si}(\mu_{\mathsf{MTB}})\right] \approx 422.40 \qquad \mathbb{E}\left[\mathcal{Si}(\mu_{\mathsf{HTB}})\right] \approx 1005.34.$$

Example: large capacity Figure 7 shows the rank distribution under the three tie-breaking rules in a market with 1000 students, 26 schools each with capacity 50. We consider 5 schools to be the top schools. Observe the same patterns as in the previous example.

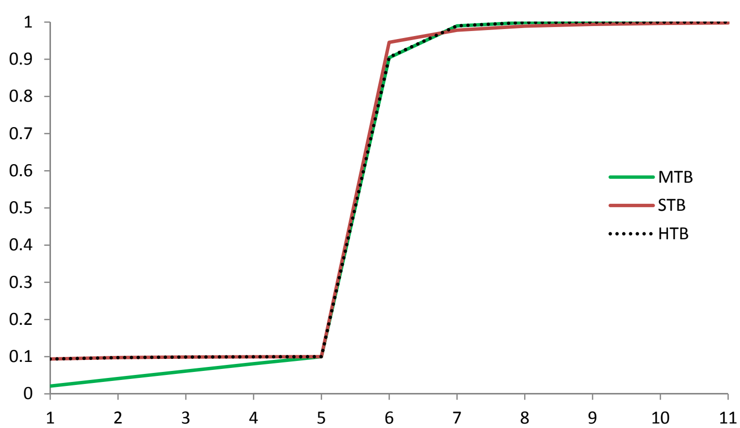


Figure 7: Students' rank distribution under STB, MTB and HTB. The market consists of 1000 students, 26 schools each with 50 seats and 5 top schools.

We list down the expected average rank and social inequity under the three tie-breaking rules below.

$$\mathbb{E}\left[\mathcal{A}r(\mu_{\mathsf{STB}})\right] \approx 5.61 \qquad \mathbb{E}\left[\mathcal{A}r(\mu_{\mathsf{MTB}})\right] \approx 5.80 \qquad \mathbb{E}\left[\mathcal{A}r(\mu_{\mathsf{HTB}})\right] \approx 5.61$$
$$\mathbb{E}\left[\mathcal{S}i(\mu_{\mathsf{STB}})\right] \approx 2.60 \qquad \mathbb{E}\left[\mathcal{S}i(\mu_{\mathsf{MTB}})\right] \approx 1.21 \qquad \mathbb{E}\left[\mathcal{S}i(\mu_{\mathsf{HTB}})\right] \approx 2.39.$$

References

Itai Ashlagi, Afshin Nikzad, and Assaf I Romm. Assigning more students to their top choices: A tiebreaking rule comparison. *Available at SSRN 2585367*, 2015.