NATURAL LOGIC, PAST AND FUTURE 04/08/2011, CSLI Stanford

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1 Many different sources, often rediscovered

Philosophy, logic, linguistics, computer science, cognitive neuroscience, ...

2 The shift from classical to modern logic

De Morgan's example: *"All horses are animals. So, all horse tails are animal tails."*
Supposed to show the inadequacy of traditional logic: binary relations are essential.
First-order: $\forall x\,(Hx \to Ax) \Rightarrow \forall x\,((Tx\ \&\ \exists y\,(\mathbf{H}y\ \&\ Rxy)) \to (Tx\ \&\ \exists y\,(\mathbf{A}y\ \&\ Rxy)))$.
Semantic *monotonicity*: if $\boldsymbol{M}, s \models \varphi(P)$, $P^{\boldsymbol{M}} \subseteq Q^{\boldsymbol{M}}$, then $\boldsymbol{M}, s \models \varphi(Q)$.
Syntactic *positive occurrence*: even number of negations, or *atoms* / & / ∨ / ∀ / ∃.
Lyndon's Theorem: A first-order formula $\varphi(P)$ is semantically monotone in P iff
$\varphi(P)$ is equivalent to a formula whose only occurrences of P are positive.
Direction from right to left also holds in higher-order logics – and natural language.

3 Distribution in traditional logic was quite powerful

Van Eijck, Sanchez, Hodges: "Dictum de Omni (↑) et Nullo (↓)". Traditional logic accounts for relational expressions! Difficulty: systematic account in complex expressions/quantifier iteration. Modern Traditionalists: Sommers' syllogistic alternatives to first-order logic.

4 Top-down and bottom-up perspectives

Pegging traditional logic as 'monadic first-order logic' is misleading. It was used for top-down analysis of language, like we do in practice: large chunks remain unanalyzed and can be very complex. Bottom-up translation into full formal syntax is a modern idea. The natural level to study might even be *hybrids* of natural language plus formal notation, cf. math.

5 One key mechanism: monotonicity in natural language

Ladusaw around 1980: negative polarity items triggered by 'downward contexts'.
Generalized Quantifier Theory: $Q\,AB$. Upward, but also downward entailment:

$Q\,AB,\ A' \subseteq A\ /\ Q\,A'B$ (e.g., "All" is ↓*MON*↑)

Some further basic properties of determiner expressions in natural language:

Conservativity $Q\,AB$ iff $Q\,A(B \cap A)$

Variety if $A \neq \emptyset$, then $Q\,AB$ for some B, and $\neg Q\,AC$ for some C.

Theorem The quantifiers "All", "Some", "No", "Not All" in the Square of Opposition are the only ones satisfying Conservativity, Double Monotonicity, and Variety.

Survey of possible patterns: Peters & Westerståhl's recent "Quantifiers" book (2006).
Other properties of specific determiners: e.g., *Symmetry*: $Q\,AB$ iff $Q\,BA$.

6 The Dutch 'natural logic' program in the 1980s

E.g., van Benthem, 1986, *Essays in Logical Semantics*, Reidel, Dordrecht. More details: 1987, 'Meaning: Interpretation and Inference', *Synthese* 73:3, 451-470. Develop system of inference directly on natural language surface forms, and see how that cuts the cake differently from the syntax of first-order logic. Border: where is *FOL* really needed?

7 Syntactic structure and computation: the monotonicity calculus

Iterated quantifiers: $Q_1A\ R\ Q_2B$. Monotonicity (and Conservativity) w.r.t. both Q_1, Q_2. Scholastics: new inference patterns for combinations. Frege: single quantifiers + recursion. 'Innocent' surface natural logic still needs modern logical-linguistic methods. For instance:

Categorial Grammar & *Monotonicity Calculus*. Compute positive/negative occurrences in tandem with syntactic analysis of an expression. The four main ideas summarized:

> Specific information about lexical items: e.g. "Every" has type $(e^-, (e^+, t))$.
>
> General effects of composition: function head: positive, lambda body: positive.
>
> Arguments get inferentially transparent if the functor has a +/–marked argument.
>
> Marking along unbroken strings of marked items: + + = – – = +   + – = – + = –

Illustration: compute all the markings in "No mortal man can slay every dragon."

Clean version in Boolean *lambda calculus*: J. van Benthem, 1991, *Language in Action*, Elsevier, A'dam. Positive occurrence implies upward monotonicity ('soundness'). Lyndon 'completeness': is every semantically monotone occurrence positive up to equivalence? Yes in single-bind fragment for Lambek Calculus – still open for type theory in general. For a related practical format, compare Muskens' lambda calculus tableau system.

Note: Border-line first-order/higher-order is totally irrelevant for this view of inference.

8 Further issues in natural logic today

* What is the *proper abstraction level* for the Monotonicity Calculus? We give inferential information a 'free ride' on the syntactic construction. How far does this grammaticalizability (Stanley) go? (Ed Stabler's answer seemed to be: 'free rides' are features that can be added while staying inside the same level of the grammar hierarchy, say *CF*.)

* *Further features in natural reasoning* that might allow for fast inferential access? E.g., Conservativity as domain or role restriction, $Q_1A\ R\ Q_2B$ iff $Q_1A\ R{\cap}(AxB)\ Q_2B$. Cheap algebraic principles, such as the earlier Symmetry ('conversion'), and so on. The work by MacCartney, Icard and others at the Workshop has further examples.

* Extend to other implications: similar systems for natural *default reasoning*? Monotonicity reasoning works by standard inclusions, say from some fixed hierarchy. What about default implications, and projecting their inferential behavior upwards in sentences?

One simple model: Horn-clause fragments of classical implication + conditional logic.

* Subsystems can be simple, but we may lose it all when we have to *combine* them.
  Putting it together: how do inferential subsystems (anaphora, time) combine/cooperate?
  This is known to be a very delicate problem in the field of 'combining logics'.
  Simple combinations of simple decidable logics can even be undecidable.

* Drop the construction: natural inference from *ambiguous expressions*?

9 Back to the scholastics after all

Very interesting new development. Logics for iterated quantifier languages (first-order, but also higher-order) with limited iteration, sometimes without full recursion. Completeness and complexity: the talks by Larry Moss and Ian Pratt showed where this program stands.

10 Interfaces with computer science

* Serious analysis: many natural logic calculi come with anecdotal complexity claims.
  Does worst-case complexity help? What are the 'naturally occurring' inferences?

* Many low-complexity fragments exists in computational logic (modal logics, description logics, a still-growing list): systematic comparison with natural logics for language?

* What a weak base can carry: weak computational logics often remain decidable when adding generalized quantifiers or fixed-point operators. Also in natural logic?
  Intriguing: monotonicity underpinning for recursive definitions $P\boldsymbol{x} \leftrightarrow \varphi(P^+)(\boldsymbol{x})$.

11 Cognitive science

* Natural logics with extra devices (variables) might still model natural hybrid practices.

* Complexity versus human 'difficulty": some research starting in cognitive psychology.

* Conscious vs. unconscious versions. Neuroscience experiments (Geurts, Nijmegen).

12 History once more (Mohist patterns, China, 5th century B.C., Liu & Zhang 2007)

Monotonicity in Mohist logic: "A white horse is a horse. To ride a white horse is to ride a horse."
"A cart is a wooden object. To ride a cart is not to ride a wooden object."

13 One bridge too far: natural logic and cognitive models?

Usual view: natural logics are sound but incomplete. First question: if we only take some natural subset of the rules, would the system be complete for some 'rougher semantics', closer to mental models that we use? (Similar point by Kamp on generalized quantifiers.)

Also use for *unsound* and incomplete calculi (say, Sommers' elegant tricky rules)? Analogy: belief revision. We form fast but maybe false beliefs, but this is efficient when kept in check by another key cognitive ability: *self-correction*, revision when proven wrong. In that line, we should pair natural logic with efficient mechanisms of inference-driven *natural revision*.