# Contexts for Quantification

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# Natural logic: what we want

Many thanks to Larry, Ulrik for slides!

## Program

Show that significant parts of natural language inference can be carried out in easy, mathematically characterized logical systems.

Whenever possible, to obtain complete axiomatizations, because the resulting logical systems are likely to be interesting.

To work using all tools of fields like proof theory, (finite) model theory, algebraic logic, and complexity theory...

# Introduction

Logic is a cake that can be cut many ways.


This work presents the logical system TIL (for textual inference logic), a formalization of the AI system Bridge developed in PARC.

# Introduction

Previous presentations of TIL described it as a stand alone system, synthetized from what is produced by the software.
This is applied logic.

Much work has gone and has still has to go into the *process* of creating these representations from sentences, but we will not worry about this process (or how to improve it) here.
Want to see TIL now in the context of the work of van Benthem, Pratt-Hartmann and especially Moss, as an extension of traditional syllogistic logic.

# Which logic?

Bridge reads in a sentence $s$ in English and produces a logical knowledge representation $r$ for it.

The collection of all representations forms the system TIL.

TIL was meant to be kept as close as it was sensible to FOL (first-order logic) but from the start we knew that to model natural language sentences we wanted intensional 'concepts' and 'contexts'.. (cf. 'Preventing Existence', FOIS'01)

# An Example: *"Three boys ate pizzas"*

Conceptual Structure:

subconcept(boy-1,[List1])
subconcept(eat-3,[List2])
subconcept(pizza-5,[List3])
role(ob,eat-3,pizza-5)
role(sb,eat-3,boy-1)
role(cardinality-restriction,boy-1,3)
role(cardinality-restriction,pizza-5,pl)

Contextual Structure:

context(t)
instantiable(boy-1,t)
instantiable(eat-3,t)
instantiable(pizza-5,t)
top-context(t)

# TIL has concepts, contexts and roles...

Instead of constants and variables like FOL,
TIL has concepts and subconcepts.
The concept boy-1 is a subconcept of one of the concepts in the list of concepts represented by the synsets in WordNet for boy.

WordNet Search - 3.0 - WordNet home page - Glossary - Help

Word to search for: boy [Search WordNet]

Display Options: (Select option to change) [Change]

Key: "S:" = Show Synset (semantic) relations, "W:" = Show Word (lexical) relations

**Noun**

- S: (n) male child, **boy** (a youthful male person) *"the baby was a boy"; "she made the boy brush his teeth every night"; "most soldiers are only boys in uniform"*
- S: (n) **boy** (a friendly informal reference to a grown man) *"he likes to play golf with the boys"*
- S: (n) son, **boy** (a male human offspring) *"their son became a famous judge"; "his boy is taller than he is"*
- S: (n) **boy** ((ethnic slur) offensive and disparaging term for Black man) *"get out of my way, boy"*

WordNet home page

Similarly for the concepts pizza-5 and eat-3.

# TIL has concepts, contexts and roles...

Concepts in TIL are similar to Description Logic concepts.

They are similar to predicates in FOL,
but are not always unary predicates.

Think of 'eat-3' above as a collection of 'eating' events,
in which other concepts in our domain participate.

Have two kinds of concepts, primitive concepts extracted from an idealized version of *WordNet* and constructed concepts, which are always sub-concepts of some primitive concept.

we assume that our concepts are as fine or as coarse as the sentences that we deal with require.

# TIL has concepts, contexts and roles...

Concepts are related to others via roles. Like

role(ob,eat-3,pizza-5)
role(sb,eat-3,boy-1)

The name of the role in question (ob-ject, sb-subject, agent, patient, theme, etc...) will not matter for us here.

Linguists are used to this 'event semantics'.

Yes, this is similar to description logics, but not exactly one.

Deciding which roles will be used with which concepts is a major problem in computational linguistics.

We bypass this problem by assuming that roles are assigned in a consistent, coherent and maximally informative way.

# TIL has concepts, contexts and roles...

One main difference is contexts and how we use them for e.g. negation.
For example, for the sentence *No boys hopped*.
Conceptual Structure:

role(cardinality-restriction,boy-5,no)
role(sb,hop-6,boy-5)
subconcept(boy-5,[List1])
subconcept(hop-6,[List2])

Contextual Structure:

context(ctx(hop-6)), context(t)
context-lifting-relation(antiveridical,t,ctx(hop-6))
context-relation(t,ctx(hop-6),not-29)
instantiable(boy-5,ctx(hop-6))
instantiable(hop-6,ctx(hop-6))
top-context(t)
uninstantiable(hop-6,t)

# TIL has concepts, contexts and roles...

Contexts here are similar to the formal objects with the same name, discussed by J. McCarthy.

Contexts are used to 'fence-off' concepts and corresponding predicates.

The same mechanism works for negation, disjunction and many intensional concepts, like 'know', 'believe', 'prevent', etc..

Instantiability and uninstantiability are used to deal with quantification and existential import.

# Inference in TIL?

Inference in TIL is very rudimentary.

We can 'drop clauses' like in most event semantics.

From the sentence *Ed walked and Mary talked*
we are able to infer both *Ed walked* and *Mary talked*
by simply forgetting the respective clauses in the original representation.

We can do trivial inferences like identity and we can compose derivations:

$$\frac{}{s \longrightarrow s} \qquad \frac{s \longrightarrow r \quad r \longrightarrow t}{s \longrightarrow t}$$

# How does TIL deal with syllogisms?...

As far as semantics is concerned, very much like Moss' and MacCartney's systems

Example: *All boys are mammals*

Conceptual Structure:

role(cardinality-restriction,boy-2,all(pl))
role(cardinality-restriction,mammal-4,pl)
role(copula-pred,be-3,mammal-4)
role(copula-subj,be-3,boy-2)
subconcept(boy-2,[List1])
subconcept(mammal-4,[List2])

Contextual Structure:

context(t)
instantiable(be-3,t)
instantiable(boy-2,t)
instantiable(mammal-4,t)
top-context(t)

# Semantics

Concepts are interpreted as subsets, so

subconcept(boy-2,[List1])

means that the boys we're talking about
are a subset of all the boys in the universe
in the senses of the word 'boy' in Wordnet.
Similarly the eating concept in our sentence is a subset of the eating events in the universe.

subconcept(eat-3,[List])

This is a bit non-intuitive for proper names, but we still use subsets and not individuals.

This semantics agrees with Moss' description.

# Syllogistic Logic of *All*

Thanks Larry!

Syntax: Start with a collection of unary atoms (for nouns).
The sentences are the expressions *All p are q*
Semantics: A model $\mathcal{M}$ is a collection of sets $M$,
and for each noun p we have an interpretation $[\![p]\!] \subseteq M$.

$$\mathcal{M} \models \textit{All } p \textit{ are } q \qquad \text{iff} \qquad [\![p]\!] \subseteq [\![q]\!]$$

Proof system:

$$\frac{}{\textit{All p are p}} \qquad\qquad \frac{\textit{All p are n} \quad \textit{All n are q}}{\textit{All p are q}}$$

# TIL satisfies the rules for *All*

$$\frac{}{\textit{All } p \textit{ are } p} \qquad \frac{\textit{All } p \textit{ are } n \quad \textit{All } n \textit{ are } q}{\textit{All } p \textit{ are } q}$$

Semantically just transitivity of subset containment.
*All boys are boys* is odd, but not problematic...
For TIL the transitive inference is simply climbing up the concept hierarchy.

$$\frac{\textit{All boys are mammals} \quad \textit{All mammals are animals}}{\textit{All boys are animals}}$$

# Syllogistic Logic of *Some*

The sentences are the expressions *Some p are q*
Semantics: A model $\mathcal{M}$ is a collection of sets $M$,
and for each noun p,q we have an interpretation $[\![p,q]\!] \subseteq M$.

$$\mathcal{M} \models \textit{Some } p \textit{ are } q \qquad \text{iff} \qquad [\![p]\!] \cap [\![q]\!] \neq \emptyset$$

Proof system:

$$\frac{\textit{Some p are q}}{\textit{Some q are p}} \qquad \frac{\textit{Some p are q}}{\textit{Some p are p}} \qquad \frac{\textit{All q are n} \quad \textit{Some p are q}}{\textit{Some p are n}}$$

# TIL satisfies the rules for *Some*

$$\frac{\textit{Some } p \textit{ are } q}{\textit{Some } q \textit{ are } p} \qquad \frac{\textit{Some } p \textit{ are } q}{\textit{Some } p \textit{ are } p} \qquad \frac{\textit{All } q \textit{ are } n \quad \textit{Some } p \textit{ are } q}{\textit{Some } p \textit{ are } n}$$

The semantics is the same as Moss', intersection of subsets.
The inference relation between 'all' and 'some' is outsourced.
Our 'poor man's inference system' called Entailment and Contradiction Detection (ECD), has a table of relationships between 'cardinality-restrictions' postulated.

This is useful if you want to keep your logic options open, no need to decide if empty domains...

# The languages $\mathcal{S}$ and $\mathcal{S}^\dagger$ with noun-level negation

If one adds complemented atoms on top of
the language of All and Some,
with interpretation via set complement: $[\![\overline{p}]\!] = M \setminus [\![p]\!]$,

So if one has

$$\mathcal{S} \left\{ \begin{array}{l} \textit{All } p \textit{ are } q \\ \textit{Some } p \textit{ are } q \\ \textit{All } p \textit{ are } \overline{q} \equiv \textit{No } p \textit{ are } q \\ \textit{Some } p \textit{ are } \overline{q} \equiv \textit{Some } p \textit{ aren't } q \\ \\ \textit{Some non-}p \textit{ are non-}q \end{array} \right\} \mathcal{S}^\dagger$$

Things can get strange, as explained by Moss-Hartmann....

# The logical system for $\mathcal{S}^p$ + names

The language $\mathcal{S}^p$ is the language of All and Some, with no negation, $p$ for positive syllogisms.

$$\frac{}{\textit{All } p \textit{ are } p} \qquad \frac{\textit{Some } p \textit{ are } q}{\textit{Some } p \textit{ are } p} \qquad \frac{\textit{Some } p \textit{ are } q}{\textit{Some } q \textit{ are } p}$$

$$\frac{\textit{All p are n} \quad \textit{All n are q}}{\textit{All p are q}} \qquad \frac{\textit{All n are p} \quad \textit{Some n are q}}{\textit{Some p are q}}$$

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$$\frac{}{J \textit{ is } J} \qquad \frac{\textit{J is M} \quad \textit{M is F}}{\textit{F is J}} \qquad \frac{\textit{J is a p} \quad \textit{J is a q}}{\textit{Some } p \textit{ are } q}$$

$$\frac{\textit{All p are q} \quad \textit{J is a p}}{\textit{J is a q}} \qquad \frac{\textit{M is a p} \quad \textit{J is M}}{\textit{J is a p}}$$

# TIL satisfies $\mathcal{S}^p$ + names

This corresponds to sentences like

*Jon is Jon*,

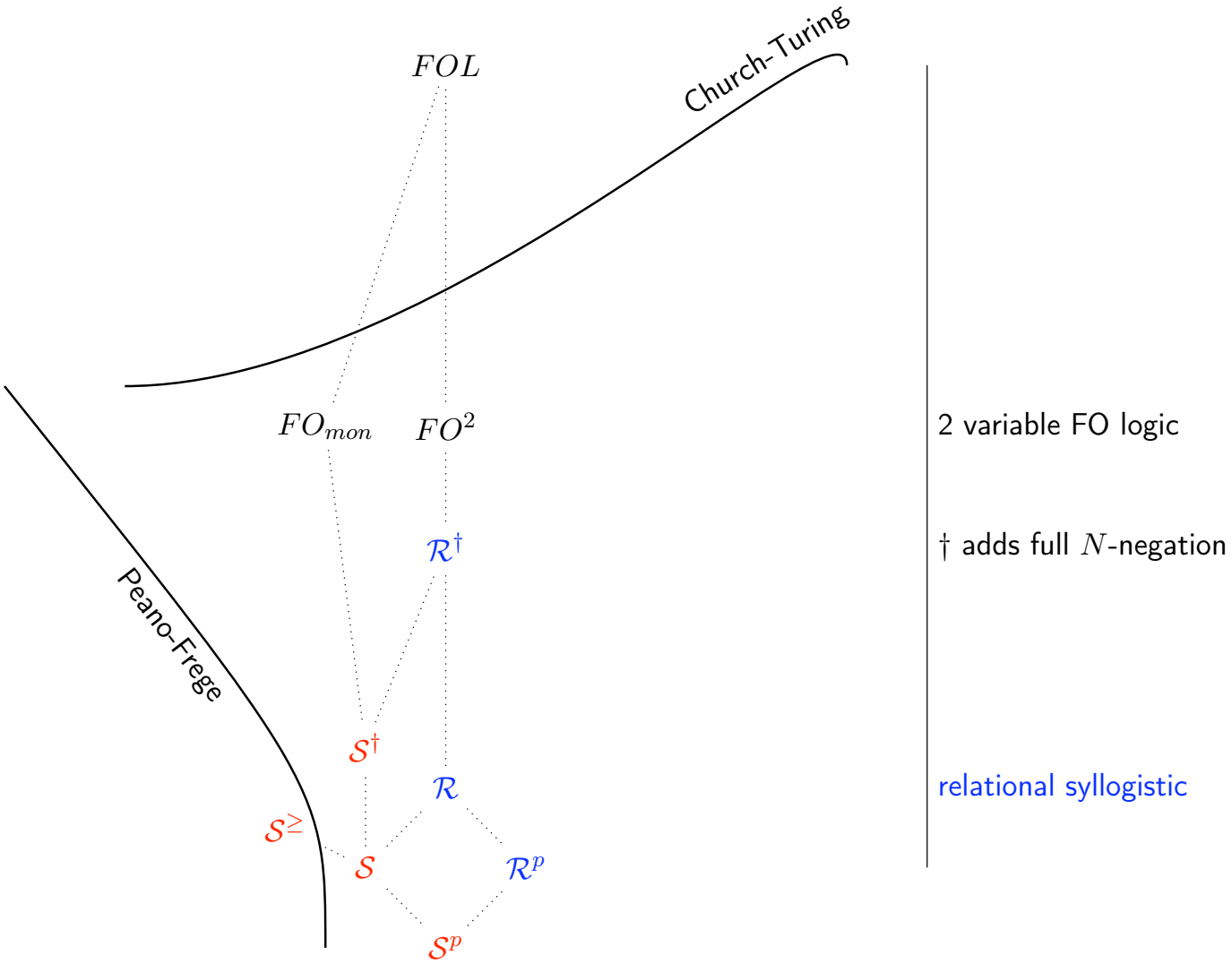
*Jon is Mary* and *Mary is Fred* entails *Fred is Jon*,

*Jon is a man* and *Jon is a doctor* entails *Some men are doctors*,

*All cats are mammals* and *Jon is a cat* entails *Jon is a mammal*

*Mary is a cat* and *Jon is Mary* entails *Jon is a cat.*

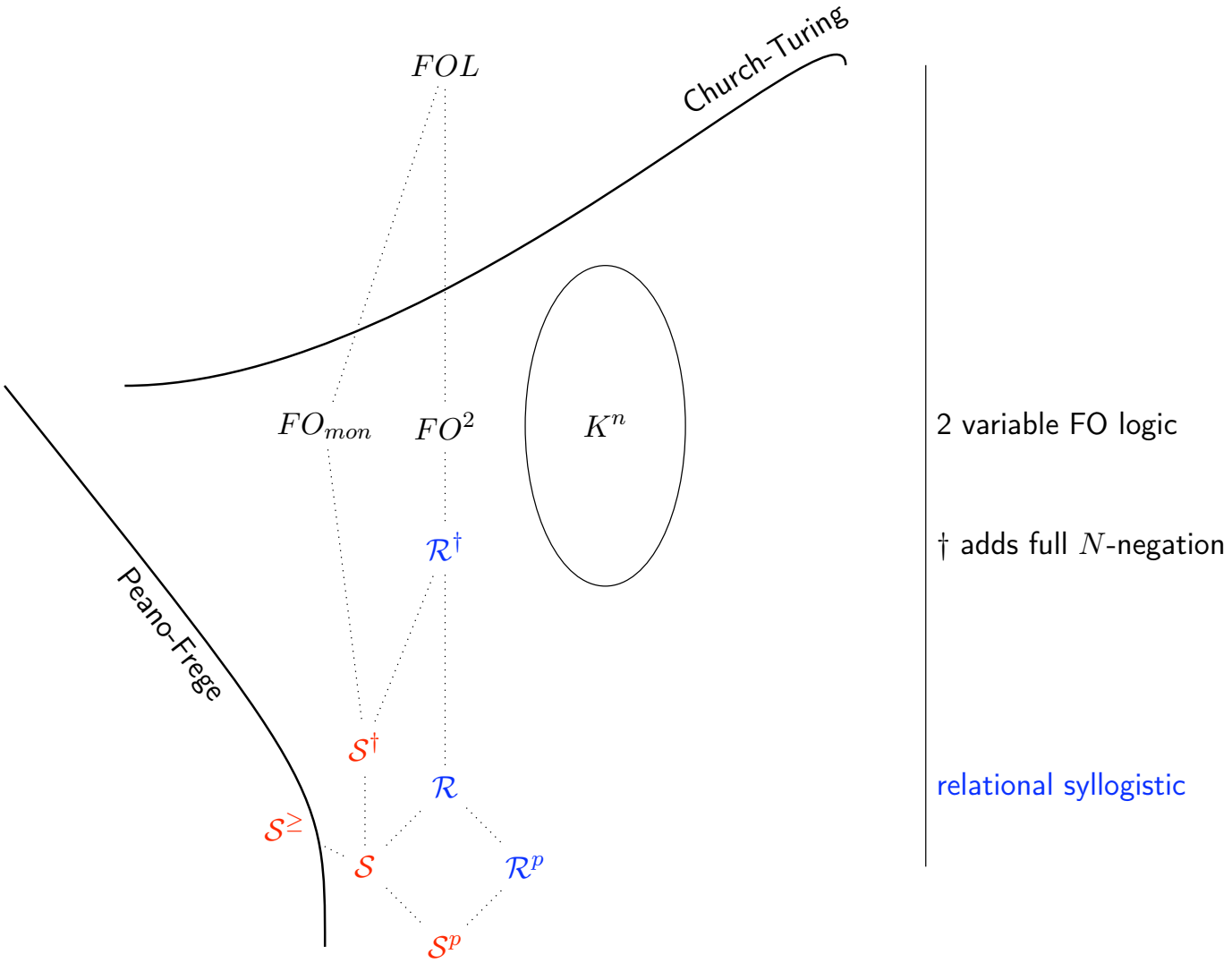
# A picture stolen from Larry...



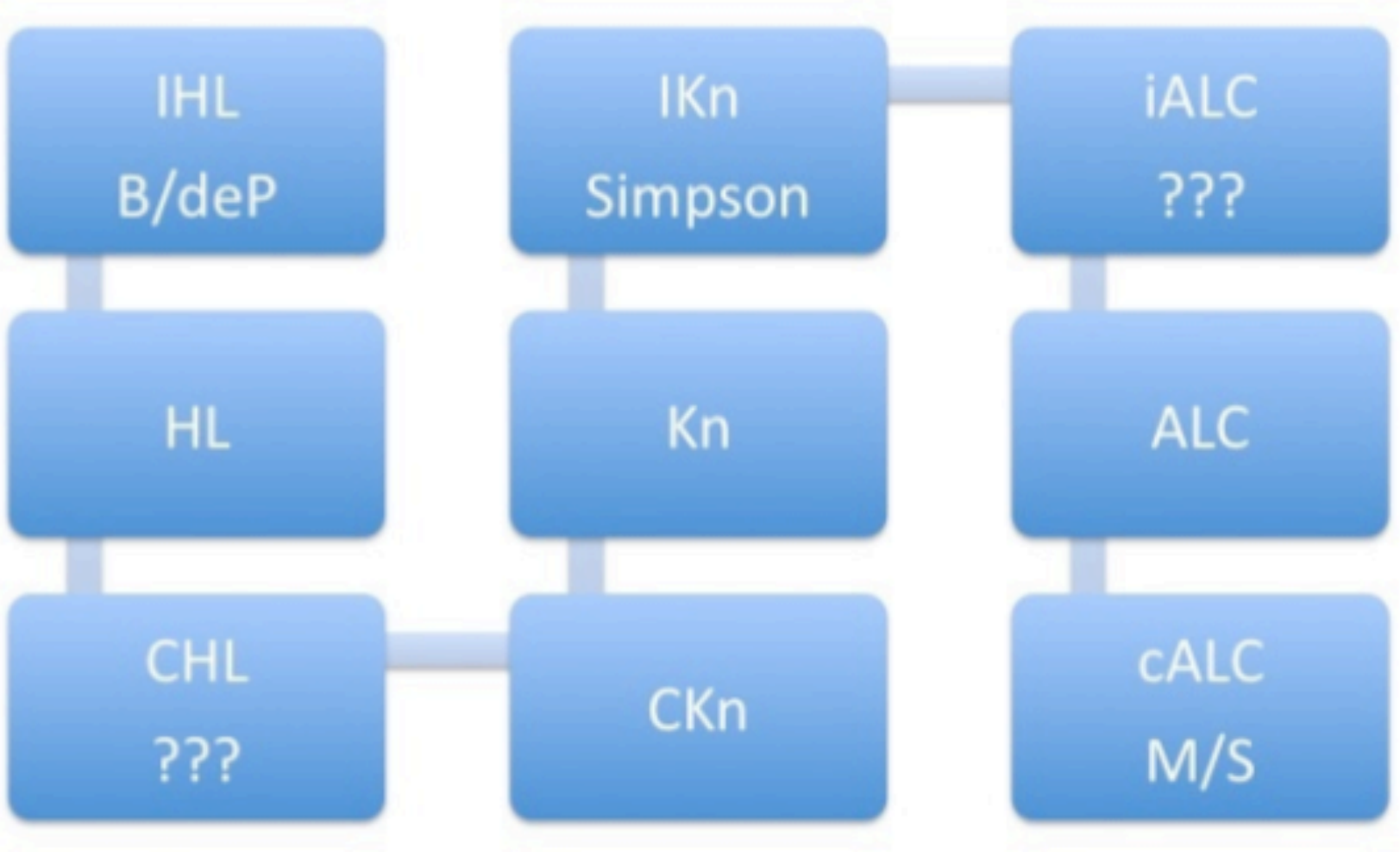
# TIL is not about syllogisms or copula...

Event semantics in general is about transitive,
intransitive, ditransitive, etc. verbs.
TIL should be able to deal with $\mathcal{S}^p$ and $\mathcal{R}^p$ and more...
Noun negation should not present problems,
(outsourced again)
but sentential negation is dealt via contexts.
Contexts look like the higher parts of the picture...
Modal and hybrid logic!

# Another picture stolen from Larry...



# Thinking Differently....



# Avi-Francez'05 sequent system

$$\frac{}{\Gamma, x\colon \mathsf{S} \vdash x\colon \mathsf{S}}$$

$$\frac{\Gamma, x\colon \textit{J is a p} \vdash y\colon \textit{J is a q}}{\Gamma \vdash \lambda x.y\colon \textit{All p are q}}$$

$$\frac{\Gamma \vdash z\colon \textit{All } p \textit{ are } q \quad \Delta \vdash y\colon \mathsf{J\ is\ a\ p}}{\Gamma, \Delta \vdash zy\colon \textit{J is a } q}$$

$$\frac{\Gamma \vdash x\colon \textit{J is p} \quad \Delta \vdash y\colon \textit{J is q}}{\Gamma, \Delta \vdash \langle x, y\rangle\colon \textit{Some p are q}}$$

$$\frac{\Gamma \vdash m\colon \textit{Some p are q} \quad \Delta, x\colon \textit{J is a p}, y\colon \textit{J is a q} \vdash t\colon \mathsf{S}}{\Gamma, \Delta \vdash \mathsf{let}\ \langle x, y\rangle = m\ \mathsf{in}\ t\colon \mathsf{S}}$$

# Conjectures

TIL =? $\mathsf{K}^n(\mathsf{R}^p)$
Must show S maps into TIL properly
Same for R
What about S and R dagger?
TIL maps into $\mathsf{FOL}^-$ (Crouch), but need to do better...
Decidability?
Monotonicity principles?

# Conclusions

TIL defined from representations, linguistic intuitions taken seriously.
Attempts to fit it into the 'Cinderella' shoe of traditional logic.
Gains on the combination logic front, already.
Work is only starting...