Review of Economic Studies (2003) **70**, 439–460 0034-6527/03/00170439$02.00

Collusion, Exclusion, and Inclusion in Random-Order Bargaining

ILYA SEGAL
Stanford University

First version received May 2000; *final version accepted September* 2001 (*Eds.*)

This paper examines the profitability of three types of integration in a cooperative game solved by a random-order value (*e.g.* the Shapley value). Collusion between players i and j is a contract merging their resources in the hands of one of them, say i. This contract can be represented as a combination of exclusion, which lets i exclude j's resource but not use it himself, and inclusion, which lets i use j's resource but not exclude j from it. This representation yields a third-difference condition on the characteristic function that determines the profitability of collusion, generalizing existing results for specific games. Namely, collusion is profitable [unprofitable] when the complementarity of the colluding players is reduced [increased] by other players.

1. INTRODUCTION

The effect of integration by a group of economic agents on their bargaining with other agents has long been a subject of economic inquiry. While old conventional wisdom held that size confers a bargaining advantage (see, *e.g.* Galbraith (1952), Scherer and Ross (1990, Chapter 14)), theoretical analysis, starting with Aumann (1973), demonstrated that this is not always the case. Since then, numerous papers have examined the bargaining effect of integration in specific settings, with applications to horizontal integration (Gardner (1977), Guesnerie (1977), Legros (1987), Chipty and Snyder (1999)), vertical integration (Stole and Zwiebel, 1998), unionization (Horn and Wolinsky (1988), Stole and Zwiebel (1996*a*)), and proxy agreements in voting games (Haller, 1994).[1]

Despite the extensive body of literature, no general conditions have been found for integration to be advantageous or disadvantageous in bargaining. Furthermore, the results obtained in specific settings appear to be in conflict with each other. For example, Horn and Wolinsky (1988) and Stole and Zwiebel (1996*a*) find that workers bargaining with an indispensable firm gain by forming a union when they are substitutable to each other, and lose when they are complementary. The accompanying intuition is that integration by complementary players reduces their ability to "hold up" others, while integration by substitutable players eliminates detrimental "competition" among them. However, this intuition is contradicted by examples where the substitutable owners of identical resources lose by forming a monopoly (see Aumann (1973), Postlewaite and Rosenthal (1974), Gardner (1977), Guesnerie (1977), Legros (1987) and Subsection 7.3 below). Aumann (1973) found such examples "disturbing by their utter lack of pathology".

Another shortcoming of the literature is that it has not given much thought to the definition of integration. For the most part, integration has been modeled as a *collusive* contract that merges

1. Disadvantageous strategic effects of integration have also been found in noncooperative oligopolistic games—see, *e.g.* Salant *et al*. (1988). However, the cooperative bargaining effects examined in the present paper appear to be of a different nature.

the resources of a group of players in the hands of one "proxy" player. However, as observed by Hart and Moore (1990), integration affects only the ownership of physical assets, but not that of inalienable human assets. The owner of a physical asset obtains the right to exclude other agents from it, but not to use their human assets. In particular, the ownership of a physical asset that is indispensable to agents' human assets is equivalent to an *exclusive* contract on the human assets (as in Segal and Whinston, 2000) rather than to a collusive contract. Yet another kind of integration, considered by Haller (1994), gives one player the right to use another player's resource, but not to prevent the latter player from using it himself. Such contracts will here be called *inclusive* (Haller (1994) calls them "associations"). Given this variety of plausible contractual forms, it is important to understand how the effects of different kinds of integration compare to each other.

The present paper answers these questions by deriving simple general conditions for collusive, exclusive, and inclusive contracts to be advantageous or disadvantageous to a coalition of players in a transferable-utility cooperative game. The game is solved by a random-order value (Weber, 1988), which gives each player his expected marginal contribution to the set of preceding players in various orderings of players, according to some probability distribution over orderings. The random-order value in which all orderings are equally likely is the Shapley value, but the general case allows different players to have different bargaining power against various coalitions. The paper derives simple conditions that are necessary and sufficient for contracts to be profitable (or unprofitable) for all possible probability distributions over orderings.

The simple intuition behind these conditions for the case of contracting between two players, i and j, in an n-player game is illustrated in Figure 1. Consider two particular orderings of players: in ordering 1 player i arrives before player j, while in ordering 2 the two players are switched. Suppose the two players write an exclusive contract giving player i the right to exclude player j from dealing with other players. The effect of this contract is shown in Figure 1(a). In ordering 1, player i is already present when player j arrives. In this case, player j contributes his resource immediately, and the contract has no effect on either player's marginal contribution. In ordering 2, on the other hand, the contribution of player j's resource is delayed until the arrival of player i. Thus, the marginal contribution of player j's resource is now evaluated relative to a larger coalition. This increases the joint marginal contribution of the two players if and only if player j is *complementary* to other players *in the absence of player* i.

Now consider an inclusive contract between the players, under which player i can use player j's resource himself, but cannot prevent player j from using it independently. The effect of this contract is depicted in Figure 1(b). The contract does not affect the players' marginal contributions in ordering 2. In ordering 1, on the other hand, the contract allows player i upon arrival to bring forward player j's resource. Thus, the marginal contribution of player j's resource is now evaluated relative to a smaller coalition. This increases the joint marginal contribution of the two players if and only if player j is *substitutable* to other players *in the presence of player* i.

Finally, a collusive contract gives full control of both players' resources to one "proxy" player. For definiteness, suppose it is player i who becomes the proxy player, and player j becomes a dummy player upon ceding his resource (when players i and j are treated symmetrically in the random-order value, their joint value does not depend on which of them becomes the proxy).[2] Under this contract, player i always brings player j's resource with him, regardless of whether he arrives before or after player j. Therefore, as shown in Figure 1(c),

2. This describes the models of integration (syndication, cartelization, unionization) used in most of the literature referenced above. The dummy players who gave up their resources are usually ignored—for example, the Shapley value of a colluding coalition of k players can be calculated using the Shapley formula for $n - k + 1$ players. However, it is convenient for the present argument to keep the dummy player in, so that the number of players in the game is preserved.

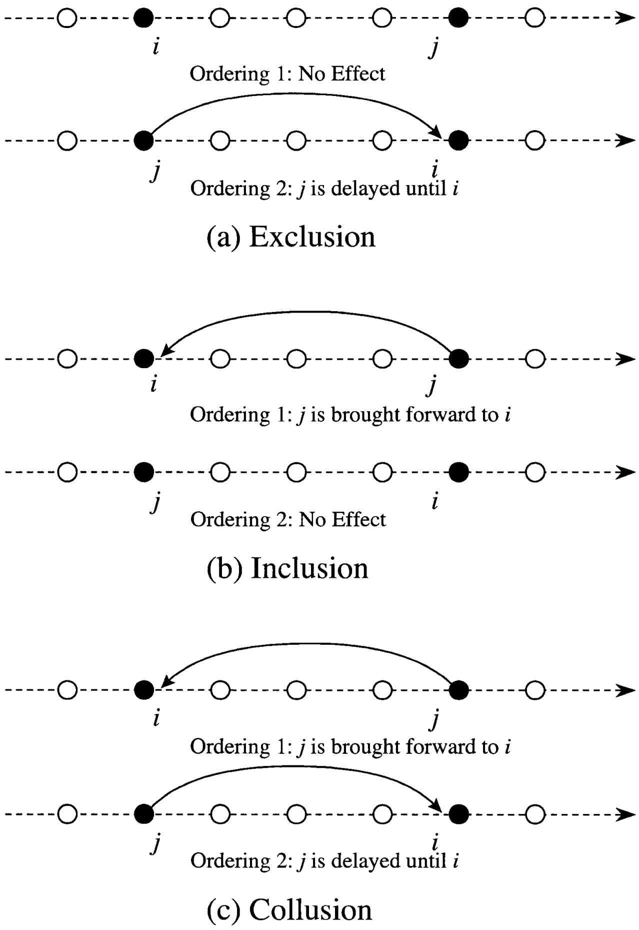

FIGURE 1

The effects of integration contracts

the collusive contract is equivalent to the inclusive contract in ordering 1, and to the exclusive contract in ordering 2. The profit from collusion therefore equals the sum of profits from exclusion and inclusion.

When orderings 1 and 2 have equal probabilities, this observation yields a simple third-difference condition for the profitability of the collusive contract. The condition reflects how the complementarity of player j with other players is affected by player i. For example, when player i is a dummy, the gain from exclusion exactly offsets the loss from inclusion (or vice versa), and collusion does not change the two players' joint payoff. On the other hand, when player i reduces [increases] the complementarity between player j and other players, the gain from exclusion exceeds [falls short of] the loss from inclusion, and collusion is profitable [unprofitable]. Another interpretation of the same third difference is that it reflects how the complementarity between the colluding players i and j is affected by other players. Collusion is profitable [unprofitable] if this complementarity is reduced [increased] by other players.

The third-difference condition explains Aumann's paradox for the Shapley value, by demonstrating that the profitability of collusion depends not on the substitutability of colluding players *per se*, but on whether this substitutability is increased by other players. In Stole and Zwiebel (1996*a*), substitutable workers gain from collusion because their bargaining opponent is an indispensable firm that enhances their substitutability. More generally, however, substitutability between the owners of a resource may be reduced by the increased availability of other resources. It is precisely in such cases that Guesnerie (1977), Gardner (1977), and Legros (1987) find examples of disadvantageous monopolies.

While the obtained sufficient conditions for the profitability of different forms of integration generalize many existing results, one might wonder whether they could be generalized even further. In fact, the conditions are shown to be the most general possible provided that one insists on conclusions that are robust to the probability distribution over players' orderings in the random-order value.[3] In other words, these conditions are the right ones to use when one is agnostic about the players' relative bargaining abilities. Extending this logic to contracts involving more than two players, the paper identifies simple conditions for contracts among members of a coalition $S \subset M$ to benefit or hurt coalition M, which are robust both to the probability distribution over players' orderings and to the set of contracting players S from M.

The present analysis is directly applicable to situations in which one coalition has an institutional advantage in prior contracting. For example, members of this coalition can meet before other players arrive at the scene. A more difficult question concerns equilibrium coalitional structures that will emerge if all coalitions have equal ability to enter *ex ante* contracts. Hart and Kurz (1983) define several concepts of stability of collusion structures for such settings. However, general characterization of stable collusion structures proves to be impossible: instead, Hart and Kurz (1984) only provide such characterization for several special classes of games. While the present paper addresses a less ambitious question of whether a *given* coalition wants to collude, it provides a general answer to this question. Furthermore, this answer proves to be useful for understanding some aspects of coalitional stability. For example, it provides a sufficient condition for a game to be proof to collusion by *any* coalition, which generalizes a condition suggested by Hart and Kurz (1984).[4]

A few aforementioned papers on integration use solution concepts other than random-order values. For example, Aumann (1973) uses the core, and Legros (1987) and Horn and Wolinsky (1988) use the nucleolus.[5] While there is no general characterization of the profitability of integration under these bargaining solutions, several examples are examined here to illustrate the similarities and differences in conclusions. Identification of general integration properties of the core and the nucleolus remains an important open problem.

The remainder of this paper is organized as follows. Section 2 formulates and interprets the concepts of TU cooperative games, random-order values, contracting within the cooperative framework, and the notions of complementarity used in the paper. Sections 3, 4, and 5 examine exclusion, inclusion, and collusion, respectively. Section 6 addresses the question of stability of collusion structures and provides a sufficient condition for collusion-proofness. Section 7 illustrates the application of general results to several specialized settings considered in the existing literature. Section 8 considers integration in the nucleolus and in the core. The

3. In the case of collusion, attention is restricted to distributions that treat the colluding players symmetrically. Otherwise the effect of collusion would depend on which of the colluding players becomes the proxy player.

4. Another strand of the literature on endogenous coalition formation considers the formation of *ex ante* links among players that are prerequisite for the realization of coalitional surplus in *ex post* bargaining (see, *e.g.* Aumann and Myerson, 1988). This differs from the present approach in two important ways. First, contracting in this literature affects the value of the grand coalition. Second, while an incomplete graph of links can give player i the power to exclude player j (when i is j's only link to other players), it can never give player i the power to include player j or to act as player j's proxy.

5. More precisely, Horn and Wolinsky (1988) consider a noncooperative bargaining game with outside options, whose subgame-perfect equilibrium implements the nucleolus allocation.

Conclusion discusses the forms integration can take in reality, points out some limitations of the present approach, and suggests some directions for future research.

2. SETUP

2.1. *The game*

Consider a transferable-utility cooperative game with the set of players $N = \{1, \ldots, n\}$. The game is described by its characteristic function $v : 2^N \to \mathbb{R}$, where $v(S)$ is called the *worth* of coalition $S \subset N$. In accordance with the standard convention, $v(\emptyset) = 0$.

The cooperative game will be interpreted as derived from an underlying economy with quasilinear preferences. Agents in this economy own resources, which can be combined to generate surplus. The worth of a coalition is the maximum surplus achievable by combining its members' resources.[6] In reality, agents' resources can be of two kinds: standard economic goods ("physical resources") and technologies or abilities that are unique to individual agents ("human resources"). The feasibility of the "integration" contracts considered below may depend on whether they involve physical or human resources: while physical resources (such as apples) can be transferred with a long-term contract, human resources (such as an agent's unique ability to produce or enjoy apples) may not be. Indeed, most countries have laws against indentured servitude, which give rise to "inalienability of human capital". This point will be important for the application of formal results, as discussed further in the Conclusion.

2.2. *Complementarity and differences*

As suggested in the Introduction, the analysis will use notions of complementarity between players in game v. These notions can be described using difference operators (previously used by Itsiishi (1993, p. 29)). For each player $i \in N$, define the *difference operator* Δ_i by[7]

$$[\Delta_i v](S) = v(S \cup i) - v(S \backslash i) \text{ for all } S \subset N.$$

(The square brackets will be omitted when this causes no ambiguity.) This expression represents the marginal contribution of player $i \in N$ to coalition $S \subset N$. Note that it is defined so as not to depend on whether coalition S includes player i.

The *second-order difference operator* Δ^2_{ij} for a pair of players $i, j \in N$ is defined as a composition of first-difference operators: $\Delta^2_{ij} v = \Delta_i[\Delta_j v]$. Note that it does not depend on the order of taking differences: for $i \neq j$,

$$\Delta^2_{ij} v(S) = v(S \cup i \cup j) + v(S \backslash i \backslash j) - [v(S \backslash j \cup i) + v(S \backslash i \cup j)] = \Delta^2_{ji} v(S).$$

Note also that $\Delta^2_{ii} v(S) \equiv 0$.

The second difference $\Delta^2_{ij} v(S)$ expresses player i's effect on player j's marginal contribution to coalition S (or vice versa). Thus, it describes the complementarity of the two players in coalition S. This measure of complementarity can be related to Topkis's (1998) concepts of supermodularity and increasing differences in the set inclusion ordering over coalitions. For example, it is well known (see, *e.g.* Itsiishi (1993, Theorem 2.1.3)) that $\Delta^2_{ij} v(S) \geq 0$ for all $i, j \in N$ and all $S \subset N$ if and only if game v is supermodular (or "convex", in the traditional terminology). This describes the complementarity of all players from N to each other.

6. This framework also allows a "public" resource (technology) that is available to all coalitions, as in Hart and Mas-Colell (1996). Such a public resource can generate non-superadditive games, since the union of two coalitions cannot duplicate it.

7. The one-element set $\{i\}$ will be written as i when this creates no ambiguity.

Complementarity between players from two distinct sets A and B can be described in a similar way. Specifically, it can be seen that $\Delta^2_{ij} v(S) \geq 0$ for all $i \in A$, $j \in B$, and $S \subset A \cup B$ if and only if $v(X \cup Y)$ has increasing differences in $(X, Y) \in 2^A \times 2^B$. Using these observations, the complementarity and substitutability conditions identified below could be restated using Topkis's concepts of supermodularity and increasing differences.

As suggested in the Introduction, the analysis of collusion will use the third-order difference operator, defined as $\Delta^3_{ijk} v = \Delta_i[\Delta^2_{jk} v]$ for $i, j, k \in N$. The operator again does not depend on the order of taking differences. $\Delta^3_{ijk} v(S)$ can be interpreted as the effect of player k on the complementarity between players i and j in coalition S.

2.3. *Solution concept*

The cooperative game is solved by a random-order value (Weber, 1988), which can be defined using the following notation. Let $\mathbf{\Pi}$ denote the set of orderings of N. $\mathbf{\Pi}$ can be treated as the group of permutations of $N = \{1, \ldots, n\}$, the product of two permutations being defined as their composition.[8] Let $\pi(i)$ denote the rank of player $i \in N$ in ordering $\pi \in \mathbf{\Pi}$. Also, let $\pi^i = \{j \in N : \pi(j) \leq \pi(i)\}$ denote the set of players that come before player i in ordering π, including i himself.

Let $\mathcal{P}(\mathbf{\Pi}) = \{\alpha \in \mathbb{R}^{\mathbf{\Pi}}_+ : \sum_{\pi \in \mathbf{\Pi}} \alpha_\pi = 1\}$ denote the set of probability distributions over $\mathbf{\Pi}$. Each probability distribution $\alpha \in \mathcal{P}(\mathbf{\Pi})$ gives rise to a random-order value $f^\alpha(v)$ that assigns each player $i \in N$ a payoff of

$$f_i^\alpha(v) = \sum\nolimits_{\pi \in \mathbf{\Pi}} \alpha_\pi \Delta_i v(\pi^i).$$

The joint value of a group $M \subset N$ of players will be denoted by $f_M^\alpha(v) = \sum_{i \in M} f_i^\alpha(v)$.

Random-order values can be characterized axiomatically with the linearity, dummy, monotonicity, and efficiency axioms (Weber, 1988). (Adding the symmetry axiom yields the Shapley value, in which all orderings are equally likely.) Alternatively, random-order values can be given a noncooperative foundation. Gul (1989), Hart and Mas-Colell (1996), and Stole and Zwiebel (1996*b*) suggest different noncooperative bargaining games that give rise to the Shapley value. Asymmetric versions of their games would give rise to asymmetric random-order values.

One of the simplest noncooperative games generating random-order values can be described as follows: after nature chooses an ordering $\pi \in \mathbf{\Pi}$ at random according to the distribution α, n bargaining stages follow, numbered in reverse order. At each stage k, the agent i whose rank is k makes an offer to the set $\pi^i \backslash i$ of preceding agents. The offer specifies a division of surplus $v(\pi^i)$ among the coalition π^i. If the offer is rejected by at least one agent from $\pi^i \backslash i$, the proposer leaves and the game proceeds to stage $k-1$, otherwise the game ends and the proposed division is implemented. In any subgame-perfect equilibrium following a realized ordering π, each player i receives the payoff $\Delta_i v(\pi^i)$. Therefore, the expected payoff of each player i is his random-order value $f_i^\alpha(v)$.

2.4. *Contracts*

Before the cooperative bargaining game begins, a coalition of players can enter into an integration contract. Following the "property rights" approach of Hart and Moore (1990), such contracts are modeled as reallocating players' control rights over resources (assets), thereby changing

8. The group 17 is called a symmetric group of degree n (see, *e.g.* Kargapolov and Merzljakov (1979)).

the characteristic function of the cooperative game. A key assumption of the "property rights" approach is that the bargaining procedure itself, and therefore the solution concept, are *not* affected by integration. The usual justification for this assumption is that the bargaining procedure is given by the players' innate bargaining abilities.[9]

For simplicity assume that each agent owns a single indivisible resource. An integration contract can then be represented by an *access structure* $A : 2^N \to 2^N$, where $A(S) \subset N$ is the set of agents whose resources are accessible to coalition $S \subset N$.[10] This contract gives rise to a new characteristic function game vA, defined by $[vA](S) = v(A(S))$ for all $S \subset N$. (The square brackets will be omitted when this causes no ambiguity.)

This paper will examine the incentive of a given coalition $M \subset N$ to enter a given contract A, using lump-sum side transfers to compensate its members. This incentive obtains by comparing the joint value of members of M before and after the contract, $f_M^\alpha(v)$ and $f_M^\alpha(vA)$. The lump-sum transfers themselves will play no role in the analysis.

Only contracts A that do not affect the resources accessible to the grand coalition (*i.e.* $A(N) = N$) will be considered. For such contracts,

$$f_M^\alpha(vA) + f_{N\backslash M}^\alpha(vA) = v(N) = f_M^\alpha(v) + f_{N\backslash M}^\alpha(v). \tag{1}$$

Therefore, a contract is profitable to coalition M if and only if it imposes a negative externality on the complementary coalition $N\backslash M$. This simple observation will prove useful for analyzing the effects of integration.

3. EXCLUSION

This section considers exclusive contracts that give one player $i \in N$ the right to exclude members of $J \subset N$, but not to use their resources without their permission. The access structure E_i^J resulting from such a contract is given by

$$E_i^J(S) = \begin{cases} S & \text{if } i \in S, \\ S\backslash J & \text{otherwise.} \end{cases}$$

One interpretation of the contract is that it makes the resource of each $j \in J$ "jointly owned" in the sense of Hart and Moore (1990): both players i and j have veto power over it.[11]

It is convenient to start by examining the externality that a bilateral exclusive contract E_i^j has on a player $k \in N\backslash\{i, j\}$. For this purpose, consider the contract's effect on player k's marginal contribution in different orderings π. This contribution is affected only in the orderings in which player k comes after player j but before player i. In those orderings, the arriving player k finds player j excluded by player i who has not arrived yet, hence k's marginal contribution is reduced by $\Delta_k v(\pi^k) - \Delta_k v(\pi^k\backslash j) = \Delta_{jk}^2 v(\pi^k)$. Taking expectation over all orderings yields the effect on player k's payoff:

$$f_k^\alpha(vE_i^j) - f_k^\alpha(v) = -\sum\nolimits_{\pi\in\mathbf{\Pi}:\pi(j)<\pi(k)<\pi(i)} \alpha_\pi \Delta_{jk}^2 v(\pi^k). \tag{2}$$

The term $\Delta_{jk}^2 v(\pi^k)$ reflects the complementarity of players j and k in coalition π^k. Note that this coalition does not include player i for any ordering π appearing in the summation.

9. In particular, the integrating players do not acquire the ability to communicate or negotiate outside of the fixed bargaining game. Hart (1995) justifies this assumption as follows: "...It would be *too* easy to obtain a theory of the costs and benefits of integration if it were supposed that the bargaining process changes under integration".

10. This coincides with Hart and Moore's (1990) concept of *control structure*, the only difference being that a resource can be accessible to both a coalition and its complement (as in an inclusive contract).

11. An equivalent situation obtains when player i acquires a physical asset that is indispensable to the human assets of players from J.

Therefore, when player j is complementary [substitutable] to player k in the absence of player i, the bilateral exclusive contract E_i^j has a negative [positive] externality on player k. If this is true for all players $k \in N\backslash\{i, j\}$, then, by the constant-sum condition (1), the contract must be profitable [unprofitable].[12]

This conclusion can be extended to multilateral exclusives by representing them as compositions of bilateral exclusive contracts: $E_i^J = \prod_{j \in J} E_i^j$. Specifically, a sufficient condition for contract E_i^J to benefit [hurt] a coalition M including i and J is that all "excluded" players from J be complementary [substitutable] to all "outside" players from $N\backslash M$ in the absence of the "excluding" player i. This condition is also necessary for exclusion to benefit [hurt] coalition M in a "robust" way, if two kinds of robustness are required. The first kind is across all random-order values, reflecting a lack of knowledge of the players' relative bargaining powers. The second kind of robustness, required when $|J| > 1$, is that all contracts $E_i^{J'}$ excluding subsets $J' \subset J$ affect the profits of coalition M in the same direction.[13] These results are summarized as follows:

Proposition 1. *Let $M \subset N$, $i \in M$, and $J \subset M$. The following three statements are equivalent:*

(i) $f_M^\alpha(vE_i^j) \geq [\leq]\ f_M^\alpha(v)$ *for all $j \in J$ and all $\alpha \in \mathcal{P}(\mathbf{\Pi})$.*

(ii) $\Delta_{jk}^2 v(S) \geq [\leq]\ 0$ *for all $j \in J$, $k \in N\backslash M$, and $S \subset N$ such that $i \notin S$.*

(iii) $f_M^\alpha(vE_i^{J'})$ *is nondecreasing [nonincreasing] in $J' \subset J$ for all $\alpha \in \mathcal{P}(\mathbf{\Pi})$.*

Proof. Consider the case of profitable contracts (the proof for the other case is similar). Since (iii) $\Rightarrow$ (i) trivially, the proof proceeds by showing that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii).

(i) $\Rightarrow$ (ii): Suppose in negation that (i) holds yet there exist $j \in J$, $k \in N\backslash M$, and $S \subset N\backslash i$ such that $\Delta_{jk}^2 v(S) < 0$. Take an ordering $\hat{\pi} \in \mathbf{\Pi}$ such that

$$\hat{\pi}(S\backslash\{j,k\}) < \hat{\pi}(j) < \hat{\pi}(k) < \hat{\pi}(i) < \hat{\pi}(N\backslash S\backslash\{i,j,k\}),$$

and let

$$\alpha_\pi = \begin{cases} 1 & \text{if } \pi = \hat{\pi}, \\ 0 & \text{otherwise.} \end{cases}$$

Then (2) implies that

$$f_l^\alpha(vE_i^j) - f_l^\alpha(v) = \begin{cases} -\Delta_{jk}^2 v(\hat{\pi}^k) & \text{if } l = k, \\ 0 & \text{if } l \in N\backslash\{i,j,k\}. \end{cases}$$

Using (1), $f_M^\alpha(vE_i^j) - f_M^\alpha(v) = \Delta_{jk}^2 v(\hat{\pi}^k) = \Delta_{jk}^2 v(S) > 0$, which contradicts (i).

(ii) $\Rightarrow$ (iii): Take any $J' \subset J$, $j \in J\backslash J'\backslash i$, and $k \in N\backslash M$. Say that a contract A *does not affect* player k if $A(S \cup k) = A(S) \cup k$ for all $S \subset N$. It is easy to see that in this case, $\Delta_k[vA](S) = \Delta_k v(AS)$. In particular, since the contract $E_i^{J'}$ does not affect players j and k, $\Delta_{jk}^2[vE_i^{J'}](S) = \Delta_{jk}^2 v(E_i^{J'} S) \geq 0$ by (ii) for all $S \subset N\backslash i$. Therefore, application of (1) and (2)

12. It is interesting to note that the effect of contract E_i^j does not depend on player i's role in the game. In particular, it is the same if player i is a dummy player. The feature that contracting with a dummy player can provide strategic commitment in bargaining is not specific to this model. For example, in Aghion and Bolton (1987) and Marx and Shaffer (1999), a buyer contracts with the incumbent seller to extract bargaining surplus from an entrant, even when the incumbent seller has a very high cost and is therefore a dummy.

13. While this kind of robustness is less important in applications, it is imposed to generate a condition that is economically meaningful and easy to verify.

to game $vE_i^{J'}$ yields

$$f_M^{\alpha}(vE_i^{J' \cup j}) - f_M^{\alpha}(vE_i^{J'}) = \sum_{k \in N \setminus M} \sum_{\pi \in \mathbf{\Pi}: \pi(j) < \pi(k) < \pi(i)} \alpha_\pi \Delta_{jk}^2 [vE_i^{J'}](S) \geq 0. \quad \|$$

Condition (i) says that all bilateral contracts in which player i excludes a player $j \in J$ benefit coalition M for all random-order values. In this case, condition (ii) must hold, *i.e.* the excluded players (members of J) must be complementary to the outsiders (members of $N \setminus M$) in the absence of the excluding player i. Furthermore, this complementarity condition implies condition (iii), which says that for all random-order values, the more players from J are excluded by player i, the better off is coalition M. Finally, condition (iii) immediately implies condition (i).

One can also consider contracts in which more than one player can exclude others. For example, for $Q, J \subset N$, consider the contract

$$E_Q^J(S) = \begin{cases} S & \text{if } Q \subset S, \\ S \setminus J & \text{otherwise.} \end{cases}$$

In this contract, any member of Q can exclude each member of J. In particular, when $Q = J$, this describes a mutually exclusive contract, which is equivalent to "joint ownership" of the players' resources in the sense of Hart and Moore (1990). Formula (2) can be used to show that any such contract E_Q^J with $Q, J \subset M$ benefits [hurts] coalition M for all random-order values if and only if $\Delta_{jk}^2 v(S) \geq [\leq]\ 0$ for all $j \in J$, $k \in N \setminus M$, and $S \subset N$ such that $M \not\subseteq S \cup j$.

4. INCLUSION

This section considers inclusive contracts that give one player $i \in N$ the right to use the resources of players from $J \subset N$, but not to prevent them from using their resources on their own. The access structure resulting from such a contract is given by

$$I_i^J(S) = \begin{cases} S & \text{if } i \notin S, \\ S \cup J & \text{otherwise.} \end{cases}$$

Similar contracts were first considered by Haller (1994).[14]

Following the approach of the previous section, it is convenient to start by examining the externality that a bilateral inclusive contract I_i^j has on a player $k \in N \setminus \{i, j\}$. For this purpose, consider the contract's effect on player k's marginal contribution in a given ordering π. The contribution is affected only in the orderings in which player k comes before player j but after player i. In those orderings, the arriving player k finds player j included by player i who has already arrived, hence player k's marginal contribution is increased by $\Delta_k v(\pi^k \cup j) - \Delta_k v(\pi^k) = \Delta_{jk}^2 v(\pi^k)$. Taking expectation over orderings yields the contract's effect on player k's payoff:

$$f_k^{\alpha}(vI_i^j) - f_k^{\alpha}(v) = \sum_{\pi \in \mathbf{\Pi}: \pi(i) < \pi(k) < \pi(j)} \alpha_\pi \Delta_{jk}^2 v(\pi^k). \qquad (3)$$

The term $\Delta_{jk}^2 v(\pi^k)$ reflects the complementarity of players j and k in coalition π^k. Note that this coalition includes player i for all orderings π that appear in the summation. Therefore, when player j is complementary [substitutable] to player k in the presence of player i, the bilateral inclusive I_i^j has a positive [negative] externality on player k. If this is true for all players $k \in N \setminus \{i, j\}$, then, by the constant-sum condition (1), the contract must be unprofitable [profitable].

14. One might dispute the realism of inclusive contracts on the grounds that they give rise to unstable games (*e.g.* the game vI_i^J may not be superadditive and may have an empty core). In any case, however, inclusive contracts provide a useful step for the analysis of collusion.

This conclusion can be extended to multilateral inclusive contracts, by representing them as compositions of bilateral inclusive contracts: $I_i^J = \prod_{j\in J} I_i^j$. Specifically, a sufficient condition for contract I_i^J to benefit [hurt] a coalition M including i and J is that all "included" players from J be substitutable [complementary] to all "outside" players from $N\backslash M$ in the presence of the including player i. This condition is also necessary for inclusion to benefit [hurt] coalition M in a way that is robust to both the random-order value and the choice of included players $J' \subset J$:

Proposition 2. *Let $M \subset N$, $i \in M$, and $J \subset M$. The following three statements are equivalent:*

(i) $f_M^\alpha(vI_i^j) \geq [\leq]\ f_M^\alpha(v)$ *for all $j \in J$ and all $\alpha \in \mathcal{P}(\mathbf{\Pi})$.*

(ii) $\Delta_{jk}^2 v(S) \leq [\geq]\ 0$ *for all $j \in J$, $k \in N\backslash M$, and $S \subset N$ such that $i \in S$.*

(iii) $f_M^\alpha(vI_i^{J'})$ *is nondecreasing* [*nonincreasing*] *in $J' \subset J$ for all $\alpha \in \mathcal{P}(\mathbf{\Pi})$.*

The proof of Proposition 2 parallels that of Proposition 1 and is omitted. Condition (i) says that all bilateral contracts in which player i includes a player $j \in J$ benefit coalition M for all random-order values. In this case, condition (ii) must hold, *i.e.* the included players (members of J) and the outsiders (members of $N\backslash M$) must be substitutable in the presence of the including player i. Furthermore, this substitutability implies condition (iii), which says that for all random-order values, the joint surplus of coalition M is higher the more players from J are included by player i. Finally, condition (iii) immediately implies condition (i).

One can also consider contracts where more than one player can include others. For example, for $Q, J \subset N$, consider the contract

$$I_Q^J(S) = \begin{cases} S & \text{if } Q \subset N\backslash S, \\ S \cup J & \text{otherwise.} \end{cases}$$

In this contract, any member of Q can include each member of J. In particular, when $Q = J$, this describes a mutually inclusive contract, which Haller (1994) calls an "association". Formula (3) can be used to show that any such contract I_Q^J with $Q, J \subset M$ benefits coalition M for all random-order values if and only if $\Delta_{jk}^2 v(S) \leq [\geq]\ 0$ for all $j \in J$, $k \in N\backslash M$, and $S \subset N$ such that $S \not\subseteq N\backslash M \cup j$.

5. COLLUSION

A collusive contract gives one player $i \in N$ full control over the resources of a group of players $J \subset N$. Formally, this contract results in the access structure

$$C_i^J(S) = \begin{cases} S \cup J & \text{if } i \in S, \\ S\backslash J & \text{otherwise.} \end{cases}$$

Note that this contract can be thought of as a composition of exclusion and inclusion. In particular, for coalitions S containing i, $C_i^J(S) = I_i^J(S)$ and $E_i^J(S) = S$. Similarly, for coalitions S that do not contain i, $C_i^J(S) = E_i^J(S)$ and $I_i^J(S) = S$. In both cases,

$$vC_i^J(S) - v(S) = [vE_i^J(S) - v(S)] + [vI_i^J(S) - v(S)],$$

i.e. the effect of collusion on the characteristic function equals the sum of the effects of exclusion and inclusion. Since random-order bargaining solutions are linear in the characteristic function, this implies:

Lemma 1. *For $i, k \in N$, $J \subset N$, and $\alpha \in \mathcal{P}(\mathbf{\Pi})$,*

$$f_k^\alpha(vC_i^J) - f_k^\alpha(v) = [f_k^\alpha(vE_i^J) - f_k^\alpha(v)] + [f_k^\alpha(vI_i^J) - f_k^\alpha(v)].$$

As in the previous sections, the effect of collusion can be derived from the externality that a bilateral collusion contract C_i^j has on players $k \in N \backslash \{i, j\}$. Using Lemma 1, this externality can be obtained by adding the external effects of exclusion E_i^j and inclusion I_i^j, given by (2) and (3) respectively:

$$f_k^\alpha(vC_i^j) - f_k^\alpha(v) = -\sum_{\substack{\pi \in \mathbf{\Pi} \\ \pi(j) < \pi(k) < \pi(i)}} \alpha_\pi \Delta_{jk}^2 v(\pi^k) + \sum_{\substack{\pi \in \mathbf{\Pi} \\ \pi(i) < \pi(k) < \pi(j)}} \alpha_\pi \Delta_{jk}^2 v(\pi^k).$$

The first sum contains orderings in which k comes after j but before i, while the second sum contains orderings in which k comes after i but before j. The two sums can be combined by changing the summation variable π in the second sum. Specifically, letting (i, j) denote the transposition of players i and j, the externality can be rewritten as

$$\begin{aligned} f_k^\alpha(vC_i^j) - f_k^\alpha(v) &= -\sum_{\substack{\pi \in \mathbf{\Pi} \\ \pi(j) < \pi(k) < \pi(i)}} \alpha_\pi \Delta_{jk}^2 v(\pi^k) \\ &\quad + \sum_{\substack{\tilde{\pi} \in \mathbf{\Pi} \\ \tilde{\pi}(j) < \tilde{\pi}(k) < \tilde{\pi}(i)}} \alpha_{\tilde{\pi} \circ (i,j)} \Delta_{jk}^2 v((\tilde{\pi} \circ (i, j))^k) \\ &= \sum_{\pi \in \mathbf{\Pi}: \pi(j) < \pi(k) < \pi(i)} [\alpha_{\pi \circ (i,j)} \Delta_{jk}^2 v(\pi^k \cup i) - \alpha_\pi \Delta_{jk}^2 v(\pi^k)], \end{aligned}$$

using the fact that $(\pi \circ (i, j))^k = \pi^k \backslash j \cup i$ for all orderings in the last sum.

This expression demonstrates that the effect of collusion depends on the relative likelihood of orderings in which player i comes after player j and those in which the two players are switched. Henceforth, attention is restricted to random-order values that are symmetric with respect to the colluding agents. Formally, let $\mathbf{\Pi}_S$ denote the subgroup of $\mathbf{\Pi}$ consisting of permutations that permute only members of $S \subset N$, and hold all the players from $N \backslash S$ fixed. A probability distribution $\alpha \in \mathcal{P}(\mathbf{\Pi})$ is *S-symmetric* if $\alpha_{\pi \circ \tau} = \alpha_\pi$ for all $\tau \in \mathbf{\Pi}_S$. (For example, the Shapley value is the unique N-symmetric random-order value.) An additional reason to focus on random-order values that are symmetric with respect to the colluding players is that this ensures that the gain from collusion does not depend on which player becomes the proxy. (Otherwise, the colluding players would gain more by putting resources in the hands of a player with a greater bargaining power.)

For $\{i, j\}$-symmetric random-order values, the above display can be rewritten as

$$f_k^\alpha(vC_i^j) - f_k^\alpha(v) = \sum_{\pi \in \mathbf{\Pi}: \pi(j) < \pi(k) < \pi(i)} \alpha_\pi \Delta_{ijk}^3 v(\pi^k). \tag{4}$$

To understand the formula intuitively, recall that the exclusive contract E_i^j has a negative externality on player k when the excluded player j is complementary to player k *in the absence of player i*, while the inclusive contract I_i^j is has a negative externality on player k when the included player j is substitutable to player k *in the presence of player i*. The net effect on player k depends on how the complementarity between players j and k is affected by player i, which is captured by the third-difference term $\Delta_{ijk}^3 v(\pi^k)$. Another interpretation of the same third difference is that it reflects how the complementarity between the colluding players i and j is affected by player k. Formula (4) implies that when player k increases [reduces] the complementarity between the colluding players i and j, collusion imposes a positive [negative] externality on him. If this is true for all players $k \in N \backslash \{i, j\}$, then, by the constant-sum condition (1), collusion between i and j is unprofitable [profitable].

This conclusion can be extended to multilateral collusive contracts by representing them as compositions of bilateral collusive contracts: $C_i^J = \prod_{j\in J} C_i^j$. Specifically, a sufficient condition for collusion among members of J to benefit [hurt] a coalition M including J for all J-symmetric random-order values is that the complementarity among members of J is reduced [increased] by "outside" players from $N\backslash M$. This condition is also necessary for the profitability conclusion when certain kinds of robustness are required, both to the choice of random-order values and to the choice of colluding players from J:

Proposition 3. *Let $J \subset M \subset N$. If*

(i) *For all $i, j \in J$, $f_M^\alpha(vC_i^j) \geq [\leq]\ f_M^\alpha(v)$ for all $\{i, j\}$-symmetric $\alpha \in \mathcal{P}(\mathbf{\Pi})$, then*

(ii) $\Delta_{ijk}^3 v(S) \leq [\geq]\ 0$ *for all $i, j \in J, k \in N\backslash M, S \subset N$.*

In turn, (ii) implies

(iii) *$f_M^\alpha(vC_i^{J'})$ is nondecreasing [nonincreasing] in $J' \subset J$ for all $i \in J$ and all J-symmetric $\alpha \in \mathcal{P}(\mathbf{\Pi})$.*

Proof. Consider the case of profitable contracts (the proof for the other case is similar).

(i) ⇒ (ii): Suppose in negation that (i) holds yet $\Delta_{ijk}^3 v(S) > 0$ for some $i, j \in J, k \in N\backslash M$, and $S \subset N$. Take an ordering $\hat{\pi} \in \mathbf{\Pi}$ such that

$$\hat{\pi}(S\backslash\{i, j, k\}) < \hat{\pi}(j) < \hat{\pi}(k) < \hat{\pi}(i) < \hat{\pi}(N\backslash S\backslash\{i, j, k\}).$$

Let

$$\alpha_\pi = \begin{cases} 1/2 & \text{if } \pi = \hat{\pi} \text{ or } \pi = \hat{\pi} \circ (i, j), \\ 0 & \text{otherwise.} \end{cases}$$

Note that α is $\{i, j\}$-symmetric by construction. (4) implies that

$$f_l^\alpha(vC_i^j) - f_l^\alpha(v) = \begin{cases} \frac{1}{2}\Delta_{ijk}^3 v(\hat{\pi}^k) & \text{if } l = k, \\ 0 & \text{if } l \in N\backslash\{i, j, k\}. \end{cases}$$

Then by (1), $f_M^\alpha(vC_i^j) - f_M^\alpha(v) = -\frac{1}{2}\Delta_{ijk}^3 v(\hat{\pi}^k) = -\frac{1}{2}\Delta_{ijk}^3 v(S) < 0$, which contradicts (i).

(ii) ⇒ (iii): Take any $J' \subset J$, $i \in J$, $j \in J\backslash J'\backslash i$, and $k \in N\backslash M$. Since the contract $C_i^{J'}$ does not affect j, k (see the proof of Proposition 1),

$$\begin{aligned} \Delta_{ijk}^3[vC_i^{J'}](S) &= \Delta_{jk}^2[vC_i^{J'}](S \cup i) - \Delta_{jk}^2[vC_i^{J'}](S\backslash i) \\ &= \Delta_{jk}^2 v(C_i^{J'}(S \cup i)) - \Delta_{jk}^2 v(C_i^{J'}(S\backslash i)) \\ &= \Delta_{jk}^2 v(S \cup J' \cup i) - \Delta_{jk}^2 v(S\backslash(J' \cup i)) \leq 0, \end{aligned}$$

where the inequality obtains because by assumption $\Delta_{ljk}^3 v(S) \leq 0$ for all $l \subset J' \cup i$ and all $S \subset N$. Applying (1) and (4) to game $vC_i^{J'}$ yields

$$f_M^\alpha(vC_i^{J'\cup j}) - f_M^\alpha(vC_i^{J'}) = -\sum\nolimits_{k\in N\backslash M} \sum\nolimits_{\pi\in\mathbf{\Pi}:\pi(j)<\pi(k)<\pi(i)} \alpha_\pi \Delta_{ijk}^3[vC_i^{J'}](S) \geq 0. \quad \|$$

Condition (i) says that any bilateral collusion contract between members of $J \subset M$ benefits coalition M for all random-order values that are symmetric with respect to the colluding players. In this case, the third-difference condition (ii) must hold. One interpretation of this condition is that the complementarity of the colluding players (members of J) is reduced by the outside players (members of $N \backslash M$). This property in turn implies condition (iii), which says that the more members of J collude, the better off is M for all J-symmetric random-order values.[15]

6. STABILITY OF COLLUSION STRUCTURES [16]

So far the paper has considered the incentives of a *given* coalition to integrate. An important further question concerns the equilibrium contracts that emerge if *all* coalitions have the ability to integrate. For the case of collusive integration, this question has been posed by Hart and Kurz (1983, 1984, henceforth HK), who proposed several concepts of coalitional stability, and examined their implications for several specific classes of games. This section shows how the preceding analysis can be applied to the study of equilibrium collusion structures. For the sake of comparison to HK's results, attention is restricted to the Shapley value.

HK consider coalitional structures that form a partition $\mathfrak{B} \subset 2^N$ of the set N of players, and use Owen's (1977) value to assign payoffs to all players under any such structure. The joint Owen value of a coalition $M \in \mathfrak{B}$ equals its Shapley value in the game in which each coalition from $\mathfrak{B}$ is represented by one proxy player, which corresponds to collusion in this paper's terminology. (It also postulates how the gains from collusion are divided *within* each $M \in \mathfrak{B}$.) Using the Owen value, HK call a coalition structure $\mathfrak{B}$ stable if no coalition $S \subset N$ wants to deviate by colluding.[17]

The simplest question in this framework is under what conditions the collusion structure consisting of singletons (*i.e.* $\mathfrak{B} = \{\{k\} : k \in N\}$) is stable, *i.e.* no coalition of players finds it profitable to collude. A game v with this property will be called *collusion-proof*. Proposition 3 implies that the following is a sufficient condition for collusion-proofness:

$$\Delta^3_{ijk} v(S) \geq 0 \text{ for all } i, j, k \in N \text{ and all } S \subset N. \tag{5}$$

Note that condition (5) is fairly strong—for example, for superadditive games it implies convexity (supermodularity):

Proposition 4. *For a superadditive game v, (5) implies that $\Delta^2_{ij} v(S) \geq 0$ for all $S \subset N$ and all $i, j \in N$.*

Proof. By induction on S. First, by superadditivity, $\Delta^2_{ij} v(\emptyset) = v(\{i, j\}) - v(\{i\}) - v(\{j\}) \geq 0$. Second, if $\Delta^2_{ij} v(S) \geq 0$ and $k \in N \backslash S$, then, by the definition of Δ^3_{ijk},

$$\Delta^2_{ij} v(S \cup k) = \Delta^2_{ij} v(S) + \Delta^3_{ijk} v(S) \geq 0. \quad \|$$

Recall from Shapley (1971) that in a convex game the set of random-order values coincides with the core. Thus, for superadditive games, condition (5) implies that all random-order values are in the core, but it is substantially stronger (which can already be seen in three-player games).

It is interesting to compare condition (5) to the sufficient condition for collusion-proofness suggested by HK, which is based on unanimity games. A T-unanimity game u_T (for $T \subset N$) is

15. In contrast to Propositions 1 and 2, condition (iii) of Proposition 3 does not imply condition (i) when $|J| > 2$. Indeed, condition (iii) only requires collusion to be profitable for J-symmetric random-order values, and does not ensure that it is profitable for all $\{i, j\}$-symmetric random-order values with $\{i, j\} \subset J$.

16. I am grateful to Michel Le Breton for suggesting the idea of this section.

17. The profitability of such deviation may depend on the behaviour of the remnants of the coalitions from $\mathfrak{B}$ deserted by the deviators. Different assumptions about such behaviour give rise to the four notions of stability examined by HK.

described by the characteristic function

$$u_T(S) = \begin{cases} 1 & \text{if } T \subset S, \\ 0 & \text{otherwise.} \end{cases}$$

It is well known that any game v can be uniquely represented as a linear combination of unanimity games: $v = \sum_{T \subset N} \gamma_T u_T$. HK prove that v is collusion-proof if all coefficients γ_T with $|T| \geq 3$ are non-negative in this representation.

Condition (5) is more general than HK's condition. To see this, observe that for a T-unanimity game,

$$\Delta^3_{ijk} u_T(S) = \begin{cases} 1 & \text{if } \{i, j, k\} \subset T \subset S \cup \{i, j, k\}, \\ 0 & \text{otherwise.} \end{cases}$$

Using this and the linearity of the third-difference operator,

$$\Delta^3_{ijk} v(S) = \sum\nolimits_{T \subset N} \gamma_T \Delta^3_{ijk} u_T(S) = \sum\nolimits_{T \subset N:\{i,j,k\} \subset T \subset S \cup \{i,j,k\}} \gamma_T \geq 0$$

whenever $\gamma_T \geq 0$ for all $|T| \geq 3$. Thus, any game satisfying the HK condition also satisfies condition (5) for collusion-proofness. To see that the converse is not true, consider the four-player game[18] given by $v = -\frac{1}{2} u_N + \sum_{T \subset N:|T|=3} u_T$, which fails the HK condition by construction but satisfies (5).

Proposition 3 also has implications for the stability of other collusion structures. For example, it implies that a sufficient condition for a collusion structure $\mathfrak{B}$ to be immune to the breakup of a coalition $M \in \mathfrak{B}$ is that $\Delta^3_{ijk} v(S) \leq 0$ for all $i, j \in M$, $k \in N \backslash M$, and all $S \subset N$. (This is relevant for HK's concept of "γ-stability", which assumes that desertion by any player from M would dissolve M into singletons.) What the present approach does not allow is analysis of deviations that cut across existing coalitions—for example, a deviation in which some players from M break away and collude with some players from $N \backslash M$. Indeed, the profitability of such a deviation depends on the allocation of gains from collusion within M, which is not modelled here (but is modelled by the Owen value).

7. APPLICATIONS

This section applies the general results to some special games that have been considered in the literature. To rule out trivial cases, assume that all games in this section have at least three players.

7.1. *Games with indispensable players*

Some applied papers examine the bargaining effect of integration in games in which one or several players are indispensable. For example, the labour bargaining models of Stole and Zwiebel (1996*a*, 1998) and Horn and Wolinsky (1988) assume that a firm is indispensable to its workers. The literature on vertical contracting often assumes that a single buyer is indispensable to the sellers (see, *e.g.* Aghion and Bolton (1987), Segal and Whinston (2000)).

Formally, a player $p \in N$ is *indispensable* to a player $d \in N$ if $\Delta_d v(S) = 0$ whenever $p \notin S$. In this case, d will be called *idiosyncratic* to p (similar terminology is used by Hart and Moore (1990)). In this situation, the second difference between p and d reduces to a first difference—player d's marginal contribution, which is non-negative in economic games:

$$\Delta^2_{pd} v(S) = \Delta_d v(S \cup p) \geq 0. \tag{6}$$

18. The two conditions are equivalent in three-player games, in which the above display implies that $\Delta^3_{123} v(S) = \gamma_N$ for all $S \subset N$.

In words, an indispensable player is always complementary to his idiosyncratic player. Together with Proposition 1, this implies that *a contract that excludes players who are indispensable or idiosyncratic to a given player has a negative externality on this player.* This generalizes the finding of Segal and Whinston (2000) (earlier discovered by Aghion and Bolton (1987) in a setting with asymmetric information) that an exclusive contract can be used by an incumbent seller and a buyer to extract surplus from an entrant who is idiosyncratic to the excluded buyer.[19] This also implies that exclusive contracts among workers who are idiosyncratic to a firm hurt the firm and thereby raise the workers' joint surplus. As for inclusion, (6) together with Proposition 2 implies that *a contract that includes players who are indispensable or idiosyncratic to a given player has a positive externality on this player.*

As for collusion, if p is indispensable to d, (6) implies that the third difference among p, d, and a third player f reduces to the second difference:

$$\Delta^3_{pdf} v(S) = \Delta^2_{df} v(S \cup p). \tag{7}$$

Together with Proposition 3, (7) has three implications for the profitability of collusion. First, in application to collusion between players d and f, it implies that *collusion between two complementary [substitutable] players helps [hurts] players who are indispensable to either colluding player*. This is consistent with the finding of Stole and Zwiebel (1996*a*) that substitutable [complementary] workers extract more [less] bargaining surplus from an indispensable firm by forming a collusive union. Similarly, in Chipty and Snyder (1999), a merger by substitutable [complementary] buyers increases [reduces] their bargaining surplus against an indispensable seller.

Next, in application to collusion between players p and f, (7) implies that *a player helps* [*hurts*] *his idiosyncratic players by colluding with someone who is complementary* [*substitutable*] *to them*. For example, in the labour bargaining model of Stole and Zwiebel (1998), firm p's idiosyncratic workers benefit from its merger with another firm, f, which is assumed to be complementary to them. Since the same argument applies in their model to firm f's workers, the merger has positive externalities on all workers, and is therefore unprofitable.[20] This argument clarifies the role of Stole and Zwiebel's (1998) assumptions for their result. For example, the result is reversed if the firms are instead substitutable to each other's workers (*e.g.* if joint production allows a labour-saving technology). The result can also be reversed if workers are not idiosyncratic to firms and can work for either firm (as in the two-sided market model in Subsection 7.3 below, where horizontal collusion may be profitable).

The third implication of (7) concerns the effect of collusion between players p and d: *A player helps [hurts] his complements [substitutes] by colluding with his indispensable player*. For example, if a seller colludes with an indispensable buyer, this extracts surplus from the sellers of substitutable goods, but leaves more surplus to the sellers of complementary goods.

7.2. *Apex games*

Suppose that there exists an "apex" player $p \in N$ such that

$$v(S) = \begin{cases} 1 & \text{if either } p \in S \text{ and } S \backslash p \neq \emptyset, \text{ or } S = N \backslash p, \\ 0 & \text{otherwise.} \end{cases}$$

In words, a coalition has worth 1 if it includes either the apex player and at least one other ("minor") player, or all minor players. All other coalitions have worth zero. Such games were

19. Along with this "bargaining effect", Segal and Whinston (2000) examine investment effects of exclusive contracts, which are not modelled here.

20. Along with this "bargaining effect", Stole and Zwiebel (1998) also examine the employment effect of mergers, which is not modelled here.

first described by von Neumann and Morgenstern (1944) and studied by Hart and Kurz (1984). They are considered here because they provide a rare example where both exclusive and inclusive contracts by a group of players (here minor players $M = N \setminus p$) are profitable.

It can be calculated that for a minor player $j \in M$,

$$\Delta^2_{pj} v(S) = \begin{cases} 1 & \text{if } S \subset \{j, p\}, \\ -1 & \text{if } N \setminus \{j, p\} \subset S, \\ 0 & \text{otherwise.} \end{cases}$$

In words, a minor player is complementary to the apex player when no one else is present, substitutable when everybody else is present, and does not interact with the apex player otherwise. Since the apex player is complementary to minor players in the absence of (at least) one minor player $i \in M$, *exclusive contracts by minor players benefit them*. At the same time, since the apex player is substitutable to minor players in the presence of (at least) one minor player $i \in M$, *inclusive contracts by minor players also benefit them*.

Since the gain from collusion equals the sum of those from exclusion and inclusion, *collusion by minor players is more profitable than either inclusion or exclusion separately*. Indeed, collusion by all minor players makes the apex player a dummy player, and thus leaves him with a zero payoff. In contrast, under either exclusive or inclusive contracts by minor players, the apex player preserves a positive marginal contribution to some coalitions, and thus obtains a positive payoff for generic random-order values.

7.3. *Two-sided markets*

Suppose that the set N of players is partitioned into two subsets, M_1 and M_2, and that each player from M_i is endowed with one unit of factor x_i ($i = 1, 2$). The worth of a coalition is a function of the coalition's total endowment of the two factors. Formally, if $X_1 \subset M_1$ and $X_2 \subset M_2$, then $v(X_1 \cup X_2) = \phi(|X_1|, |X_2|)$. Horizontal mergers (*i.e.* mergers by owners of one factor) in such a model have been considered by Gardner (1977) and Legros (1987).

For simplicity, we restrict attention to production functions ϕ that are smooth and exhibit diminishing marginal productivity and constant returns to scale. Under these assumptions, the two factors must be Edgeworth complements, *i.e.* $\partial^2\phi(x_1, x_2)/\partial x_1 \partial x_2 \geq 0$.[21] According to Propositions 1 and 2, this implies that *horizontal exclusion contracts are profitable, while horizontal inclusion contracts are not*.

As for horizontal collusion among owners of factor x_1, according to Proposition 3, its profitability depends on the third derivative $\partial^3\phi(x_1, x_2)/\partial x_1^2 \partial x_2$. To simplify matters, following Gardner (1977), assume that ϕ has a constant elasticity of substitution:

$$\phi(x_1, x_2) = (x_1^{-\beta} + x_2^{-\beta})^{-1/\beta},$$

where $\beta > -1$ (the elasticity of substitution is $\sigma = 1/(1 + \beta)$). A simple calculation yields

$$\operatorname{sign} \frac{\partial^3\phi(x_1, x_2)}{\partial x_1^2 \partial x_2} = \operatorname{sign}\left[\frac{\beta}{1+\beta} - \left(\frac{x_1}{x_2}\right)^{\beta}\right]. \tag{8}$$

When $\beta \leq 0$ (which corresponds to $\sigma \geq 1$, *i.e.* the factors are more substitutable than under the Cobb–Douglas technology), the sign is always negative, hence, according to Proposition 3, collusion by owners of x_1 is always profitable. On the other hand, when $\beta > 0$ (*i.e.* $\sigma < 1$), the sign depends on the factor ratio x_1/x_2, hence a robust conclusion on the profitability of collusion

21. Indeed, differentiation of the Euler identity ($x_1 \partial\phi/\partial x_1 + x_2 \partial\phi/\partial x_2 \equiv \phi$) with respect to x_2 yields $\partial^2\phi/\partial x_1 \partial x_2 = -(x_2/x_1)\partial^2\phi/\partial x_2^2 \geq 0$.

among members of M_1 cannot be made. This is consistent with the findings of Gardner (1977, Propositions 4 and 5).

A more definitive prediction for the case where $\beta > 0$ can be obtained by restricting attention to the Shapley value, in which all orderings are equally likely. To avoid complicated calculations, consider the case where there are many players on each side of the market. By the Law of Large Numbers, the factor ratio x_1/x_2 in a randomly drawn coalition will then approximate the ratio in the overall population, $|M_1|/|M_2|$.[22] Thus, (8) suggests that when $\beta > 0$, *collusion by owners of a factor is profitable when this factor is sufficiently abundant, and unprofitable when it is sufficiently scarce*.

To relate this conclusion to the third-difference condition of Proposition 3, note that when $\beta > 0$, the marginal product of a factor approaches zero when it is very abundant, and approaches one when it is very scarce. Thus, the factor's owners are not substitutable with each other in both extreme cases, but they are substitutable when the market is more balanced. Therefore, the substitutability of owners of a very abundant factor is reduced by increasing the supply of the other factor, and so collusion by these owners is profitable. On the other hand, the substitutability of owners of a very scarce factor is increased by *reducing* the supply of the other factor, and so collusion by these owners is unprofitable.

For a simple illustration, consider the market with Leontief technology, corresponding to the limiting case of $\beta \to \infty$ (and $\sigma \to 0$). In a large (formally, "atomless") market, the Shapley value gives all surplus to the owners of the scarce factor. To see this, observe that in a random ordering, by the Law of Large Numbers, an owner of the abundant factor with a very large probability encounters coalitions in which his factor is abundant, and thus has a zero marginal contribution. On the other hand, if, say, factor 1 is monopolized, in a random ordering the monopoly will with a large probability encounter a coalition that has some factor 2 but less than $|M_1|$ of it, thus receiving a share of total surplus that is strictly between zero and one (the rest is extracted by factor 2 owners arriving afterwards). Thus, collusion results in a more even allocation of surplus, which is advantageous to the abundant factor but disadvantageous to the scarce factor. (The same conclusion is obtained by Legros (1987), although he focuses on the nucleolus solution concept, discussed in Subsection 8.1 below.)

8. OTHER BARGAINING SOLUTIONS

This section offers a preliminary discussion of the degree to which this paper's results extend to the prenucleolus and core solution concepts. The discussion provides a comparison with the results of Legros (1987) and Horn and Wolinsky (1988) on collusion in the nucleolus, and to the literature on disadvantageous monopolies in the core initiated by Aumann (1973).

8.1. *The* (*Pre*)*nucleolus*

The prenucleolus is a simple variation on Schmeidler's (1969) nucleolus solution concept that does not force the allocation to be individually rational. (The two solution concepts coincide and belong to the core whenever it is nonempty—see, *e.g.* Maschler (1992).) To define the prenucleolus, for each payoff vector $x \in \mathbb{R}^N$ and each coalition $S \subset N$, define the *excess* of coalition S at x by

$$e(S; x) = v(S) - \sum_{i \in S} x_i.$$

22. In the limit with a continuum of "atomless" players, this intuition gives rise to Aumann and Shapley's (1974) "diagonal formula" for the Shapley value.

Furthermore, for each payoff vector $x \in \mathbb{R}^N$, define a vector $\theta(x) \in \mathbb{R}^{2^N}$ to be the permutation of the excess vector $(e(S;x))_{S\subset N}$ in which the excesses of different coalitions are arranged in nonincreasing order. $\theta(x)$ is *lexicographically smaller* that $\theta(y)$ if for some integer q, $\theta_r(x) = \theta_r(y)$ for all $r < q$ and $\theta_q(x) < \theta_q(y)$. The prenucleolus solution $f^{pn}(v) \in \mathbb{R}^N$ of the game v is the payoff vector x that minimizes $\theta(x)$ on the set of efficient payoff vectors, $\{x \in \mathbb{R}^N : \sum_{i\in N} x_i = v(N)\}$, in the lexicographic order (the minimizer can be shown to be unique).

It is easy to see that the results on the profitability of exclusion and inclusion do not extend to the prenucleolus.[23] At the same time, somewhat unexpectedly, the third-difference condition for the profitability of collusion *does* extend to the prenucleolus in three-player games:

Proposition 5. *With* $n = 3$, *if* $\Delta^3_{123}v(N) \le [\ge]\ 0$, *then* $f^{pn}_{\{i,j\}}(vC^j_i) \ge [\le]\ f^{pn}_{\{i,j\}}(v)$ *for all* $i, j \in N$.

Proof. Let $x = f^{pn}(v)$, and for definiteness let $\{i,j\} = \{1,2\}$. The gain from collusion between players 1 and 2 can be expressed through excesses $e(1,2;x)$ and $e(3;x)$. For this purpose, note that

$$f^{pn}_{12}(v) = x_1 + x_2 = v(1,2) - e(1,2;x).$$

On the other hand, after collusion C^2_1 and elimination of the dummy player 2, the prenucleolus in the resulting two-player game coincides with the Nash bargaining solution, hence

$$\begin{aligned} f^{pn}_{12}(vC^2_1) &= f^{pn}_1(vC^2_1) = vC^2_1(1) + \tfrac{1}{2}[vC^2_1(N) - vC^2_1(1) - vC^2_1(3)] \\ &= v(1,2) + \tfrac{1}{2}[v(N) - v(1,2) - v(3)] = v(1,2) - \tfrac{1}{2}[e(1,2;x) + e(3;x)], \end{aligned}$$

where the last equality follows from the efficiency of x. Subtracting the previous equation yields

$$f^{pn}_{12}(vC^2_1) - f^{pn}_{12}(v) = \tfrac{1}{2}[e(1,2;x) - e(3,x)]. \tag{9}$$

Using linearity of the third difference operator, observe that

$$\Delta^3_{123}e(N;x) = \Delta^3_{123}v(N).$$

Since x is an efficient payoff vector, $e(N;x) = e(\emptyset;x) = 0$, and the above display yields

$$e(1;x) + e(2;x) + e(3;x) - [e(1,2;x) + e(2,3;x) + e(1,3;x)] = \Delta^3_{123}v(N). \tag{10}$$

The result can now be proven by contradiction. Suppose in negation that $\Delta^3_{123}v(N) \ge 0$ and $f^{pn}_{12}(vC^2_1) > f^{pn}_{12}(v)$. The latter implies by (9) that $e(1,2;x) > e(3;x)$. Then (10) implies

$$e(1;x) + e(2;x) > e(2,3;x) + e(1,3;x). \tag{11}$$

But then $e(2;x) > e(1,3;x)$, $e(1;x) > e(2,3;x)$, or both. Without loss of generality let $e(2,x) > e(1,3;x)$. Consider an efficient payoff vector $(x'_1, x'_2, x'_3) = (x_1, x_2 + \varepsilon, x_3 - \varepsilon)$. With $\varepsilon > 0$ sufficiently small,

$$\begin{gathered} e(3;x) < e(3;x') < e(1,2;x') < e(1,2;x), \\ e(1,3;x) < e(1,3;x') < e(2;x') < e(2;x), \\ e(1;x') = e(1;x),\ e(2,3;x') = e(2,3;x). \end{gathered}$$

23. For a simple counterexample, consider the three-player game with $v(N) = 10$, $v(2) = v(1,2) = 1$, $v(2,3) = 2$, and $v(1) = v(3) = v(1,3) = 0$. The prenucleolus of this game is $(3,4,3)$. The exclusive contract E^2_1 sets the values of coalitions $\{2\}$ and $\{2,3\}$ to zero, and the prenucleolus of the resulting game is $(10/3, 10/3, 10/3)$. Thus, the exclusive E^2_1 reduces the joint payoff of players 1 and 2, despite the complementarity of players 2 and 3 in all coalitions.

This implies that $\theta(x')$ is lexicographically smaller than $\theta(x)$, which contradicts the assumption that $x = f^{pn}(v)$.

Now suppose in negation that $\Delta^3_{123}v(N) \leq 0$ and $f^{pn}_{12}(vC^j_i) > f^{pn}_{12}(v)$. The latter implies by (9) that $e(1, 2; x) < e(3; x)$. Then (10) implies

$$e(1; x) + e(2; x) < e(2, 3; x) + e(1, 3; x). \tag{12}$$

But then $e(2; x) < e(1, 3; x)$, $e(1; x) < e(2, 3; x)$, or both. Without loss of generality let $e(2, x) < e(13, x)$. Consider an efficient payoff vector $(x'_1, x'_2, x'_3) = (x_1, x_2 - \varepsilon, x_3 + \varepsilon)$. With $\varepsilon > 0$ sufficiently small,

$$e(1, 2; x) < e(1, 2; x') < e(3; x') < e(3; x),$$
$$e(2; x) < e(2; x') < e(1, 3; x') < e(1, 3; x),$$
$$e(1; x') = e(1; x), e(2, 3; x') = e(2, 3; x).$$

This implies that $\theta(x')$ is lexicographically smaller than $\theta(x)$, which contradicts the assumption that $x = f^{pn}(v)$. ‖

Proposition 5 can be related to the findings of Horn and Wolinsky (1988). While they consider a noncooperative bargaining game with outside options, its equilibrium allocation coincides with the prenucleolus. Since their game has three players (two workers and a firm), Proposition 5 establishes that the profitability of collusion hinges on the sign of the third difference. Specifically, since the firm is indispensable, the argument in Subsection 7.1 implies that collusion is advantageous when the workers are substitutable and disadvantageous when they are complementary, which is consistent with Horn and Wolinsky's (1988) results.

Unfortunately, no intuition is currently available for Proposition 5. (Recall that the intuition in the Introduction for the third-difference condition is based on the results for exclusion and inclusion, which do not extend to the prenucleolus.) Another problem is that Proposition 5 does not extend to larger games, in which the third-difference condition of Proposition 3 no longer characterizes the profitability of collusion in the prenucleolus.[24] On the other hand, in some such games, collusion properties of the nucleolus do resemble those of the Shapley value (as, *e.g.* in Legros (1987)). Characterization of collusion properties of the (pre)nucleolus is an important direction for future research.

8.2. *The core*

The problem of disadvantageous integration was first analyzed in the context of the core (Aumann, 1973). The early literature defined "syndication" by a group M of players as an agreement that "no proper subset of them will form a coalition with traders outside the group, so that only the group as a whole will enter into broader coalitions" (Gabzewicz and Dreze, 1971). Formally, syndication modifies the solution concept (*i.e.* which coalitions are allowed to block) rather than the characteristic function. However, in the core of games with transferable utility, syndication is equivalent to collusion, which merges the resources of coalition M in the hands of one player, with possible side payments among the members of M. Since syndication reduces the

24. The following four-player counterexample has been constructed by David Miller:

S	$i \in N$	1, 2	1, 3	1, 4	2, 3	2, 4	3, 4	1, 2, 3	1, 2, 4	1, 3, 4	2, 3, 4	N
$v(S)$	0	0	2	1	2	6	2	5	8	2	8	10

It can be calculated that $f^{pn}(v) = (4/3, 9/2, 4/3, 17/6)$ and $f^{pn}(vC^2_1) = (6, 0, 1, 3)$. Note that players 1 and 2 gain 1/6 by colluding, even though $\Delta^3_{123}v(S) = \Delta^3_{124}v(S) = 1 > 0$ for all $S \subset N$.

set of allowable blocking coalitions, it can only expand the set of core allocations. The question is whether the expansion is in a direction that is advantageous or disadvantageous to coalition M.

First note that in a smooth quasilinear economy (such as the two-sided market of Subsection 7.3) that is large (formally, "atomless"), collusion must affect the core in an advantageous way. Indeed, the unique core allocation in such an economy is the competitive equilibrium, in which each player i receives his marginal contribution to the grand coalition, $\Delta_i v(N)$ (as shown, *e.g.* in Ostroy (1980), Makowski (1980)). But even after collusion by a coalition M, no player $i \in N\backslash M$ can receive more than $\Delta_i v(N)$ in the core, for otherwise coalition $N\backslash i$ would block the allocation. Therefore, collusion cannot have positive externalities, and so it cannot be unprofitable. This suggests that Aumann's (1973) examples of "disadvantageous monopolies" in the core of large economies rely critically on wealth effects (which are absent here). This contrasts with the Shapley value, under which disadvantageous monopolies arise in large quasilinear economies, as shown in Subsection 7.3.[25]

Disadvantageous collusion in the core can arise in *small* quasilinear economies, as has been demonstrated by Postlewaite and Rosenthal (1974) with an example of a two-sided market with Leontief technology. (Legros (1987) finds a similar example using the nucleolus solution concept, which is always contained in the core.) Other than the examples mentioned here, little is known about the profitability of collusion in the core, and further research is called for.

9. CONCLUSION

While previous research has examined the effect of collusion in several specific bargaining settings, it has failed to provide a general intuitive condition for when collusion is profitable and when it is not. This paper provides such a condition for the case of Shapley value bargaining by representing collusion as a composition of exclusion and inclusion. Exclusion is profitable when the excluded players are complementary to outside players in the absence of the excluding player, while inclusion is profitable when the included players are substitutable with outside players in the presence of the including player. The gain from collusion obtains as a sum of those from exclusion and inclusion, and depends on a third difference of the characteristic function: it is positive [negative] when the complementarity among colluding players is reduced [increased] by outside players.

A second contribution of this paper lies in pointing out that different types of integration have different bargaining effects. This calls for a closer look at the forms integration takes in the real life. In particular, the feasibility of various forms of integration depends on whether economic agents own physical or human assets. As one example, consider the bargaining of workers with an indispensable firm. Stole and Zwiebel (1996*a*) and Horn and Wolinsky (1988) model a workers' union as a collusive agreement, which is advantageous when workers are substitutable and disadvantageous when they are complementary. However, in reality workers cannot relinquish control over their inalienable human assets to the union, as a collusive agreement would require. Instead, a workers' union is better described as an exclusive agreement, under which the union can prevent its members from working, but cannot force them to work against their will. As shown in Section 7, such an agreement always benefits workers as a whole, regardless of whether they are complementary or substitutable to each other.

The same point applies to the analysis of mergers. For example, Stole and Zwiebel (1998) find that a "collusive" merger is disadvantageous in their setting, which is consistent with this

25. This comparison is consistent with the finding of Einy *et al.* (1999) that syndicates in large quasilinear economies are treated less favourably under the Shapley value than under the nucleolus, which is a selection from the core.

paper's general results. However, a collusive merger of two firms' assets is not feasible when the assets in question include inalienable human assets. In the extreme case where the acquired firm's physical assets and human assets are indispensable to each other, the merger can be described as an exclusive contract on the human assets. The discussion in Subsection 7.1 implies that such an exclusive contract is profitable in the setting of Stole and Zwiebel (1998), since it extracts surplus from the acquired firm's idiosyncratic workers. The study of mergers by firms whose human and physical assets are only partially complementary would require extending the present framework to contracts that split assets initially owned by one player. While such extension is beyond the scope of this paper, the present analysis suggests that the profitability of such mergers will depend on the relative importance of the human and physical assets of the acquired firm, as well as on the degree of their complementarity.

Of course, the three forms of integration considered here do not exhaust all possible contractual arrangements. In this sense, they are "incomplete contracts", and the players may gain by using more complex contracts. For example, if a contract could be made contingent on the realization of the random ordering of players, optimal integration would take the form of "selective intervention", which reallocates assets only in those orderings in which this raises the contracting players' joint marginal contribution. Even if the ordering is not verifiable directly, "selective intervention" may still be achievable with a message-contingent contract. To take a simple example, if a player has the right to opt out of an exclusive contract by paying stipulated damages, his optimal decision of whether to exercise the option may depend on the realized ordering of players. Of course, if integration with "selective intervention" is feasible, non-integration can never be strictly optimal. A restriction to incomplete contracts is therefore important for the analysis of the costs and benefits of integration, just as it is in other "property rights" models of integration, such as Hart and Moore (1990).

In conclusion, note that this paper's model of bargaining has some important limitations. First, as discussed in Section 8, its results do not fully extend to nonlinear bargaining solutions, such as the nucleolus and the core. This is not surprising in light of the recent finding that the investment effects of ownership identified by Hart and Moore (1990) in the Shapley bargaining model are reversed with nonlinear bargaining solutions (deMeza and Lockwood (1998), Chiu (1998)). It is thus important to examine how the bargaining effects of various forms of integration depend on the bargaining solution, and whether any robust predictions can be made for wide classes of games or solutions. Second, the cooperative model of bargaining does not properly represent settings with externalities. Neither the characteristic function representation of bargaining, nor the more general partition function representation contain enough relevant information in such settings when agents may enter more than one contract at the same time. Not only may contracting outcomes be inefficient in this case, but, as shown in Segal (1999), the nature of the inefficiency may depend crucially on the fine details of the contracting process.

Acknowledgements. I am grateful to Hans Haller, Oliver Hart, Michel Le Breton, Jonathan Levin, Mark Machina, Andrew Postlewaite, Roberto Serrano, Lars Stole, Steven Tadelis, Michael Whinston, Eyal Winter, Jeffrey Zwiebel, and participants of seminars at Berkeley, Penn, Princeton, Stanford, UBC, University of Chicago, University of Toronto, Yale, and the Franqui Conference on the Economics of Contracting for valuable comments. I also thank the editor, Mark Armstrong, and two anonymous referees for useful suggestions. I am indebted to Federico Echenique, David Miller, and Luis Rayo for excellent research assistance. Last but not least, the hospitality of the Hoover Institution for War, Revolution, and Peace and financial support from the National Science Foundation are gratefully acknowledged.

REFERENCES

AGHION, P. and BOLTON, P. (1987), "Contracts as a Barrier to Entry", *American Economic Review*, **77**, 388–401.

AUMANN, R. J. (1973), "Disadvantageous Monopolies", *Journal of Economic Theory*, **6**, 1–11.

AUMANN, R. J. and MYERSON, R. M. (1988), "Endogenous Formation of Links Between Players and of Coalitions: An Application of the Shapley Value", in A. Roth (ed.) *The Shapley Value* (Cambridge: Cambridge University Press).
AUMANN, R. J. and SHAPLEY, L. (1974) *Values of Non-atomic Games* (Princeton: Princeton University Press).
CHIPTY, T. and SNYDER, C. M. (1999), "The Role of Firm Size in Bilateral Bargaining: A Study of the Cable Television Industry", *Review of Economics and Statistics*, **81**, 326–340.
CHIU, S. Y. (1998), "Noncooperative Bargaining, Hostages, and Optimal Asset Ownership", *American Economic Review*, **88**, 882–901.
DEMEZA, D. and LOCKWOOD, B. (1998), "Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm", *Quarterly Journal of Economics*, **113**, 361–386.
EINY, E., MORENO, D. and SHITOVITZ, B. (1999), "The Asymptotic Nucleolus of Large Monopolistic Market Games", *Journal of Economic Theory*, **89**, 186–206.
GABZEWICZ, S. and DREZE, J. (1971), "Syndicates of Traders in an Exchange Economy", in H. W. Kuhn and E. P. Szego (eds.) *Differential Games and Related Topics* (Amsterdam: North-Holland).
GALBRAITH, J. K. (1952) *American Capitalism: The Concept of Countervailing Power* (Boston: Houghton Mifflin).
GARDNER, R. (1977), "Shapley Value and Disadvantageous Monopolies", *Journal of Economic Theory*, **16**, 513–517.
GUESNERIE, R. (1977), "Monopoly, Syndicate, and Shapley Value: About Some Conjectures", *Journal of Economic Theory*, **15**, 235–251.
GUL, F. (1989), "Bargaining Foundations of Shapley Value", *Econometrica*, **57**, 81–95.
HALLER, H. (1994), "Collusion Properties of Values", *International Journal of Game Theory*, **23**, 261–281.
HART, O. D. (1995) *Firms, Contracts, and Financial Structure* (Oxford: Clarendon Press).
HART, O. D. and MOORE, J. (1990), "Property Rights and the Nature of the Firm", *Journal of Political Economy*, **98**, 1119–1158.
HART, S. and KURZ, M. (1983), "Endogenous Formation of Coalitions", *Econometrica*, **51**, 1047–64.
HART, S. and KURZ, M. (1984), "Stable Coalition Structures", in M. J. Holler (ed.) *Coalitions and Collective Action* (Wuerzburg: Physica-Verlag) 235–258.
HART, S. and MAS-COLELL, A. (1996), "Bargaining and Value", *Econometrica*, **64**, 357–380.
HORN, H. and WOLINSKY, A. (1988), "Worker Substitutability and Patterns of Unionisation", *Economic Journal*, **98**, 484–497.
ITSIISHI, T. (1993) *The Cooperative Nature of the Firm* (Cambridge: Cambridge University Press).
KARGAPOLOV, M. I. and MERZLJAKOV, J. I. (1979) *Fundamentals of the Theory of Groups* (New York: Springer-Verlag).
LEGROS, P. (1987), "Disadvantegeous Syndicates and Stable Cartels: The Case of the Nucleolus", *Journal of Economic Theory*, **42**, 30–49.
MAKOWSKI, L. (1980), "A Characterization of Perfectly Competitive Economies with Production", *Journal of Economic Theory*, **22**, 208–221.
MARX, L. M. and SHAFFER, G. (1999), "Predatory Accomodation: Below-Cost Pricing Without Exclusion in Intermediate Goods Markets", *RAND Journal of Economics*, **30**, 22–43.
MASCHLER, M. (1992), "The Bargaining Set, Kernel, and Nucleolus", in R. J. Aumann and S. Hart (eds.) *Handbook of Game Theory*, Vol. I (Amsterdam: Elsevier) 591–665.
OSTROY, J. M. (1980), "The No-Surplus Condition as a Characterization of Perfectly Competitive Equilibrium", *Journal of Economic Theory*, **22**, 183–207.
OWEN, G. (1977), "Values of Games with A Priori Unions", in R. Henn and O. Moeschlin (eds.) *Essays in Mathematical Economics and Game Theory* (New York: Springer-Verlag) 76–88.
POSTLEWAITE, A. and ROSENTHAL, R. W. (1974), "Disadvantageous Syndicates", *Journal of Economic Theory*, **9**, 324–326.
SALANT, S. W., SWITZER, S. and REYNOLDS, R. J. (1988), "Losses from Horizontal Merger: The Effect of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", in A. F. Daughety (ed.) *Cournot Oligopoly: Characterization and Applications* (Cambridge: Cambridge University Press).
SEGAL, I. (1999), "Contracting with Externalities", *Quarterly Journal of Economics*, **114**, 337–88.
SEGAL, I. and WHINSTON, M. (2000), "Exclusive Contracts and Protection of Investments", *RAND Journal of Economics*, **31**, 603–633.
SCHERER, F. M. and ROSS, D. (1990) *Industrial Market Structure and Economic Performance,* 3rd edition (Boston: Houghton-Mifflin).
SCHMEIDLER, D. (1969), "The Nucleolus of a Characteristic Function Game", *SIAM Journal of Applied Mathematics*, **17**, 1163–1170.
SHAPLEY, L. (1971), "Cores of Convex Games", *International Journal of Game Theory*, **1**, 11–26.
STOLE, L. and ZWIEBEL, J. (1996*a*), "Organizational Design and Technology Choice under Intrafirm Bargaining", *American Economic Review*, **86**, 195–222.
STOLE, L. and ZWIEBEL, J. (1996*b*), "Intra-firm Bargaining under Non-binding Contracts", *Review of Economic Studies*, **63**, 375–410.
STOLE, L. and ZWIEBEL, J. (1998), "Mergers, Employee Hold-up, and the Scope of the Firm: An Intrafirm Bargaining Approach to Mergers" (Mimeo, University of Chicago), `http://gsblas.uchicago.edu/research/papers/finbarg.html`.
TOPKIS, D. M. (1998) *Supermodularity and Complementarity* (Princeton: Princeton University Press).
VON NEUMANN, J. and MORGENSTERN, O. (1944) *Theory of Games and Economic Behavior* (Princeton: Princeton University Press).
WEBER, R. J. (1988), "Probabilistic Values for Games", in A. Roth (ed.) *The Shapley Value* (Cambridge: Cambridge University Press).