

Property Rights*

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1 Introduction

Every organization, whether a firm, a nonprofit organization, or a society, must confront two basic problems. The first is the creation of incentives for efficient behavior among its members. The second is the efficient allocation among those members of the resources available to and produced by the organization. These problems are closely related, because the rule for allocating resources generally affects individuals' incentives.

Organizations have many ways of trying to achieve these goals. Sometimes they use contracts. These may explicitly specify behavior or attempt to indirectly encourage desirable behavior through incentive pay. Other times they allocate decision rights to parties and leave them considerable discretion. In this chapter, we focus on one particular instrument:

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the allocation of property rights over assets.

The basic concept of a property right is relatively simple: A property right gives the owner of an asset the right to the use and benefits of the asset, and the right to exclude others from them. It also, typically, gives the owner the freedom to transfer these rights to others. Roman law referred to these elements as *usus* (the right to use), *abusus* (the right to encumber or transfer), and *fructus* (the right to the fruits). The American jurist Oliver Wendell Holmes put it this way,

But what are the rights of ownership? They are substantially the same as those incident to possession. Within the limits prescribed by policy, the owner is allowed to exercise his natural powers over the subject matter uninterfered with, and is more or less protected in excluding other people from such interference. The owner is allowed to exclude all, and is accountable to no one but him.¹

Thus, property rights over an asset can be defined as a bundle of decision rights involving the asset (also called *entitlements* in the legal literature), which provide rights to take certain actions (“rights of access”) and to prevent others from taking certain actions (“rights of exclusion”), including the right to take the profit generated by use of the asset and to prevent others from doing so, often called “profit rights” or “cash flow rights” in the literature.²

This chapter surveys some theories of the optimal allocation of property rights. The problem of optimal allocation of property rights differs from the classical economic problem

¹Holmes (1881, p. 246), quoted in Grossman and Hart (1986).

²In practice, however, complications can arise in the notion of a property right. First, these rights of ownership are not always bundled together. For example, some stockholders of a firm may own a share of its profits, but may not have the right to vote on the use of the firm’s assets. Similarly, an individual may possess the right to use an asset, such as a community garden or lake, but may not have the right to exclude others from doing so. Lastly, in some cases an owner may not possess the right to transfer his ownership rights to others, as with the prohibition against slavery. In addition, property rights are in practice often held collectively. For example, no single shareholder in a firm may be able to use the firm’s assets as he sees fit. Yet, a majority of the shareholders, should they reach an agreement, can do so.

of optimal allocation of goods in an important way: As suggested by the above definitions, a central feature of property rights is that while they may influence economic allocations, many of the details of the allocation are left for future specification by the agents, either unilaterally or by negotiation with each other. For example, *usus* leaves it up to the owner to decide how exactly the asset should be used. *Abusus* leaves it to the owner in negotiation with other agents to decide who the asset should be transferred to and on what terms. In this sense, property rights are simple (or “incomplete”) contracts that establish a status quo but leave discretion for economic agents to fill in the gaps. This incompleteness is, of course, inevitable in the real world, because it may be difficult to list all possible deployments of an asset in advance or all the relevant variables on which the deployment should be contingent, and specifying such a contingent rule would be very costly. As a result, as Grossman and Hart (1986) note, property rights confer *residual rights of control* to the owner of an asset: the owner is entitled to the use and fruits of the asset except insofar as he has contractually agreed to limits on those rights (say, by transferring them to others).

To fix ideas and foreshadow the models discussed in this chapter, consider the following framework in which property rights may influence subsequent economic outcomes:

1. An allocation of property rights $\hat{x} \in \hat{X}$ is chosen;
2. Agents individually choose some noncontractible actions $a \in A$;
3. Uncertainty $\theta \in \Theta$ is realized;
4. Agents can renegotiate the initial property rights allocation, specifying a new outcome $x \in X$, where $\hat{X} \subseteq X$ (outcomes in $X \setminus \hat{X}$ cannot be specified contractually ex ante, but are contractible ex post). Should the agents not reach an agreement, then the initial property rights remain unchanged so $x = \hat{x}$. Each agent i 's utility is $v_i(x, a, \theta) + t_i$, where t_i is a monetary transfer.³

³This framework could be enriched in many respects without altering the fundamental conclusions. For

As an example, suppose the only ex ante contractible decision $z \in Z$ is ownership of assets, while ex post in addition the parties can contract on decisions $q \in Q$ (e.g., cooperation in the use of the assets). Then, letting $q^\circ \in Q$ denote the default that arises when no ex post agreement is reached, we have $\hat{X} = Z \times \{q^\circ\}$, while $X = Z \times Q$. Alternatively, in an R&D setting, intellectual property rights may be contractible for existing discoveries but not for future ones until they occur.

The framework encompasses many models of property rights in the literature. These differ in which of the variables are actually present (are there noncontractible actions a ? are there decisions that are noncontractible ex ante but contractible ex post?) and the nature of the ex post negotiation process (is bargaining perfectly efficient? is it imperfect due to asymmetric information about θ or due to agreements involving only subsets of the agents?). They also differ in how property rights matter. In some models, property rights are rights to asset returns (*fructus*) that directly affect payoffs $U_i(\cdot)$, and are possibly valued differently by different agents. In others, property rights are “control rights” that determine who takes noncontractible actions (*usus*), but leave payoffs as a function of the chosen actions unchanged (e.g., all returns may reflect private benefits). To formally capture this latter case in the framework we can allow each agent to take the action in question, but have only the choice of the asset’s owner (as determined by \hat{x}) matter for payoffs (see Remark 1 in Section 2.2).

In the simplest settings, the effect of property rights \hat{x} is simply to allocate assets to agents, who derive a direct private benefit from them, and there are neither noncontractible actions a nor ex post negotiations. In this case, an optimal property rights allocation is very simple: it gives assets to the agents who value them most in expectation. That is, \hat{x} is set

example after Stage 4 the agents may be able to choose noncooperative actions, in which case the functions $v_i(x, a, \theta)$ incorporate Nash equilibrium choices of such actions. These actions may become contractible in Stage 4 (i.e., parties may be able to specify them with an allocation from $X \setminus \hat{X}$) or they may never be contractible.

to maximize $\mathbb{E}[\sum_i v_i(\hat{x}, \theta)]$. Thus, setting optimal property rights in this case is just like choosing an optimal allocation of goods in an exchange economy. (Of course, if externalities are present, so agents care about others' possession of assets in addition to their own, then this must be taken account of in determining the optimal allocation.) We call these cases of *Simple Allocation*.

The case of Simple Allocation, however, fails to capture property rights' role as an incomplete contract, because it does not envision any activities after the initial property rights allocation is specified. Of greater interest for the study of organizations are settings in which property rights affect payoffs not just directly but also through their effects on subsequent noncontractible actions and/or negotiated decisions.

One simple benchmark of this type arises when ex post negotiation is always efficient (i.e., yields a surplus-maximizing outcome) and all outcomes of importance can be contractually specified before they occur (there are no actions a). In this case, as the celebrated *Coase Theorem* [Coase (1960)] points out, the initial specification of property rights \hat{x} is irrelevant for efficiency of the final outcome, which is always first-best, and affects only the distribution of the first-best surplus among the agents.⁴

To move beyond the Coasian irrelevance result in this framework either (1) ex post renegotiation must not be fully efficient or (2) the agents must choose noncontractible actions a . In the case of Simple Allocation, for example, initial property rights matter because ex post bargaining is impossible. In this chapter, we focus on models in which the allocation of property rights matters because of the presence of subsequent noncontractible actions and/or imperfect negotiations, but in which their optimal allocation amounts to more than a case

⁴Note, however, that when utilities are not quasilinear (there are wealth effects) final outcomes will typically depend on the property rights specified in stage 1. Still, the final outcome will be on the ex post Pareto frontier. In this survey, we will ignore wealth effects. For the role of property rights in the presence of wealth effects/credit constraints, see e.g., Legros and Newman (1996). Note also that in this case, initial property rights may have ex ante efficiency consequences due to insurance considerations.

	Noncontractible Actions?	
	NO	YES
Efficient Bargaining	Coasian Benchmark	Section 3 (Hold-up models)
No Bargaining	Simple Allocation Benchmark	Section 2 (Tragedy of the Commons, Adaptation, and Incentives models)
Imperfect Bargaining	Section 4 (Asymmetric Information / Bilateral Contracting)	

Table 1: Alternative Assumptions about Bargaining and Actions

of Simple Allocation. Table 1 organizes these models and their relation to the Coasian and Simple Allocation benchmarks.

We begin in Section 2 by examining settings in which agents choose noncontractible actions a but no bargaining occurs. Thus, the setting is essentially static, and an optimal property rights allocation must take account of the effect of the property rights allocation on agents’ choices of noncontractible actions. Discussions of property rights in such static settings – in the guise of the famous *Tragedy of the Commons* – were the first writings in the economics literature to consider how property rights can improve economic efficiency. Static settings appear in the more recent literature in what Gibbons (2005) has called adaptation models of the firm [e.g., Hart and Holmstrom (2010), Baker et al. (2010)], as well as in incentive models such as Holmstrom and Milgrom (1991,1994).⁵ We examine when (and how) the first-best can be achieved in such settings, as well as the nature of distortions and second-best property rights when the first best cannot be achieved. Throughout, we

⁵Gibbons uses the term “adaptation” in reference to Simon (1951), in which authority is granted ex ante to one individual (the “boss”) to make a decision following the resolution of some uncertainty. This fits into our framework in Section 2 by viewing that individual’s action a as the choice of a decision rule which describes his ex post action as a function of the state θ .

emphasize the role of property rights in ameliorating inefficiencies caused by externalities.

In Section 3, we extend Section 2's model of noncontractible actions by introducing perfect ex post renegotiation (although some results hold with imperfect renegotiation). In this context, the noncontractible actions are typically interpreted as long-term investments that are subject to hold-up in the subsequent bargaining. These *hold-up* models, whose use for examining the optimal allocation of property rights began with the seminal contribution of Grossman and Hart (1986), have been a workhorse of much of organizational economics over the last 20 years.⁶ We discuss the nature of optimal property rights in these settings, including how they depend on the nature of agents' investments, the characteristics of assets, and the form of bargaining. We show how the analysis of hold-up models can make use of the insights from Section 2, by applying them to a new form of externalities – bargaining externalities. We also provide here a discussion of the use of more complicated mechanisms than simple property rights, which relates to discussions of the foundations of incomplete contracts.

In Section 4, we suppose instead that renegotiation does occur but is imperfect. We examine two reasons for imperfect renegotiation and their implications for optimal property rights: First, we suppose that θ is not public information – pieces of it are observed privately by different agents. With such asymmetric information, renegotiation may be hindered by agents' incentives to misrepresent θ , incentives that can depend on the initial property rights. We then suppose that the impediment to bargaining is that the agents cannot all get together and write a grand contract. Rather, contracting is bilateral, which can create externalities on other parties. These models can be viewed as possible formalizations of Williamson's (1971) focus on integration as a means of affecting ex post haggling costs.⁷

⁶They have also been used extensively in other fields, such as industrial organization and corporate finance.

⁷Gibbons (2005) instead formalizes ex post haggling costs as arising from influence costs. Influence cost models are closest to the static models we consider in Section 2, although with the presence of learning and

The analysis of optimal property rights that follows can be viewed either as prescriptive (normative) or descriptive (positive). If perfect (Coasian) bargaining regarding property rights occurs at the ex ante stage 1 (even if bargaining is imperfect at the ex post stage 4), then the parties will arrive at an optimal initial property rights allocation.⁸ On the other hand, the same factors that may cause imperfections in negotiation at the ex post stage (such as private information or the inability to write grand contracts) may also be present at the ex ante stage, in which case the parties may fail to achieve an optimal allocation of property rights on their own.⁹ In such cases, legal rules can improve efficiency by setting the optimal property rights allocation as the default.

While our chapter focuses on property rights, the variable \hat{x} in our framework could in principle represent not only an allocation of property rights but also any other kind of ex ante contractible choices facing the organization. These could include, for example, the allocation of decision rights that are not linked to assets, as in Aghion et al. (2004) or Baker et al. (2008). It could also include compensation schemes, as in Holmstrom and Milgrom (1991) and (1994). That said, the results we discuss here depend on the assumed properties of $v_i(\cdot)$, which depend on the interpretation of \hat{x} . [For example, in the literature on hold-up models, which we discuss in Section 3, seminal works such as Hart and Moore (1990) assume that increases in \hat{x} (greater ownership of assets) increase the marginal returns to a (investments), which is motivated by their interpretation of \hat{x} as asset ownership and a as an investment in asset-specific human capital.]

Finally, the scope of our discussion is limited by space considerations, and we omit a number of topics that could naturally fall within a survey of the effects of property rights on the addition of “signal-jamming” as a noncontractible action.

⁸Although, unlike in the Coase Theorem, the outcome need not be first-best – it may be second-best given subsequent contracting imperfections.

⁹ An additional reason why ex ante bargaining may be imperfect is that agents may not know at the ex ante stage who all the relevant parties are. (For example, a pharmaceutical firm may not know ex ante all of the potential discoverers of new drugs.)

organizations. For example, in Section 2's discussion of static settings, we omit discussion of a set of interesting recent papers in which agents' actions include communication, and this communication affects how the holders of property or decision rights choose to exercise their rights based on their information [e.g., Aghion and Tirole (1997), Alonso et al. (2008), Rantakari (2008)]. We also discuss in only a limited way the interaction between property rights and other incentive instruments [e.g., Holmstrom and Milgrom (1991) and (1994)]. The effects of property rights with relational contracts [as in Baker, Gibbons, and Murphy (2002)] are also not considered. We also do not discuss the extensive, but less formal, literature on transaction cost economics. These topics are covered in this Handbook, however, in the chapters by Bolton and Dewatripont, Gertner and Scharfstein, Gibbons and Henderson, Malcolmson, and Tadelis.

2 Static Models of Property Rights, Externalities, and Efficiency

The importance of property rights for incentives has long been recognized, as illustrated in the following passage from an early 19th century text:

Suppose that the earth yielded spontaneously all that is now produced by cultivation; still without the institution of property it could not be enjoyed; the fruit would be gathered before it was ripe, animals killed before they came to maturity; for who would protect what was not his own...?

Mrs. J.H. Marcet

Conversations in Political Economy, 3rd. Edition (1819, pp. 60-1)¹⁰

Indeed, commentary on the incentive effects of property rights appear in Aristotle's writings:

¹⁰Quoted in Baumol and Oates (1988, 28n).

[T]hat which is common to the greatest number has the least care bestowed upon it. Everyone thinks chiefly of his own, hardly at all of the common interest; and only when he is himself concerned as an individual.

(*Politics*, Book II, Chapter III, 1261b)

When behavior generates externalities, as in the quotes above, efficiency may call for allocating an asset to an individual or individuals who do not derive the greatest benefit from it, to induce desirable behavior. In this section, we explore this trade-off in a simple static setting. We first illustrate these effects in a classic model of property rights, the *Tragedy of the Commons*, and then derive some more general results about the efficiency effects of property rights. These results turn out to apply as well to a variety of dynamic settings, as we will see in Section 3.

2.1 An Example: The Tragedy of the Commons

We begin by considering the classic Tragedy of the Commons, updated with a modern “organizations” spin. Suppose there are two managers, denoted $i = 1, 2$, who each try to meet their clients’ consulting needs. There is a single asset, a data center. Access to the data center enables a manager to better serve his clients. Each manager i can exert some effort to meeting client needs, denoted by $a_i \in [0, \bar{a}_i] \subset \mathbb{R}$. Access to the data center, determined by the specification of property rights, is captured by the variables $(\hat{x}_1, \hat{x}_2) \in \{0, 1\}^2$. Manager i has access to the data center if $\hat{x}_i = 1$, and does not if $\hat{x}_i = 0$. Private ownership gives the owner access to the data center and the ability to exclude all others from access to it, and given the negative externalities we assume below, the owner always exercises this right of exclusion. So private ownership by manager 1 results in $(\hat{x}_1, \hat{x}_2) = (1, 0)$; private ownership by manager 2 leads to $(\hat{x}_1, \hat{x}_2) = (0, 1)$. In contrast, with common ownership, both managers have the right to use the data center and neither has a right to exclude the other, so $(\hat{x}_1, \hat{x}_2) = (1, 1)$.

Given access (\hat{x}_1, \hat{x}_2) and efforts (a_1, a_2) , manager i 's payoff is given by the function

$$\hat{x}_i V_i(a_i, \hat{x}_{-i} a_{-i}) - C_i(a_i),$$

where $V_i(\cdot) \geq 0$ is the gross benefit that manager i receives if he has access to the data center, and $C_i(\cdot)$ is his cost of effort, normalized so that $C_i(0) = 0$. The function $V_i(\cdot)$ is nonincreasing in its second argument, reflecting the fact that when manager $-i$ has access to the data center (so $\hat{x}_{-i} = 1$), his increased use may lower manager i 's payoff. [Note that $V_i(0, a_{-i})$ may be positive, and that the externality arises in this example only from manager $-i$'s effort, not his access per se. We can imagine that certain low urgency uses of the data center such as billing are not generated by managerial effort, create no externalities, and provide a benefit for serving clients regardless of congestion effects; in contrast, urgent uses are generated by a manager's effort, and both create and are subject to congestion externalities.] We assume that these benefits are private so that they cannot be transferred through a contract and that each manager's effort is noncontractible. Thus, ownership is the only thing that is contractible. Finally, due to the urgency of the clients' needs, access to the data center cannot be renegotiated.¹¹

Since a manager generates an externality only if he exerts effort, there is always a (first-best) efficient outcome in which both managers have access to the data center. Indeed, if $V_i(0, a_{-i}) > 0$ for all a_{-i} and $i = 1, 2$, using the data center without effort (and thus no urgent tasks) generates a strictly positive payoff for a manager regardless of the other manager's level of effort, which implies that *all* first-best outcomes involve giving both managers access. When benefits and costs are differentiable, (interior) efficient effort levels (a_1^*, a_2^*) then satisfy $\partial V_i(a_i^*, a_{-i}^*)/\partial a_i + \partial V_{-i}(a_{-i}^*, a_i^*)/\partial a_i = C_i'(a_i^*)$ for $i = 1, 2$. In contrast, (interior) *equilibrium* effort levels with common ownership (a_1°, a_2°) satisfy $\partial V_i(a_i^\circ, a_{-i}^\circ)/\partial a_i = C_i'(a_i^\circ)$ for $i = 1, 2$ because the managers ignore the external effects of their actions. If $V_i(\cdot)$ is independent of

¹¹A manager's value of access could be uncertain ex ante. Since we assume that the urgency of the tasks precludes renegotiation of access ex post, we can think of V_i in the text as manager i 's expected payoff.

a_{-i} for $i = 1, 2$, so that externalities are absent, the equilibrium is efficient. This would be the case if there was no congestion (e.g., a large enough data center). But if $\partial V_{-i}(\cdot)/\partial a_i < 0$, then any interior equilibrium is inefficient.

In some cases, there is also a first-best outcome in which only one manager has access to the data center. This occurs, for example, when $a_2^* = 0$ and $V_2(0, a_1) = 0$ for all a_1 , and can arise when manager 2's cost of effort is very high or his benefit from fulfilling urgent needs is very low, and he has no non-urgent needs for the data center. Since there are no externalities under private ownership, private ownership by manager 1 then achieves the first-best.

Which property rights allocation maximizes aggregate surplus when the first best requires giving access to both managers but externalities are present? In that case, the optimal (second-best) property rights allocation involves a trade-off: common ownership allows both managers to benefit from the data center, but introduces externalities, while private ownership denies the benefit to one manager but avoids externalities. As an illustration, consider a situation with symmetric managers where $V_i(a_i, \hat{x}_{-i}a_{-i}) = v(a_i) - e(\hat{x}_{-i}a_{-i})$ with $e(0) = 0$, and $C_i(a_i) = c(a_i)$ for $i = 1, 2$. In that case, common ownership is optimal if and only if $v(a^\circ) - c(a^\circ) \geq 2e(a^\circ)$, that is, if the direct benefit of granting a second manager access exceeds the externalities that this access creates.

2.2 A General Static Model

We now consider what can be said more generally about the optimal allocation of property rights in a static setting. Suppose there is a set \mathcal{N} of N agents, each of whom chooses an action $a_i \in A_i$. The profile of actions is $a = (a_1, \dots, a_N) \in A = \prod_i A_i$. Before actions are chosen, a property rights allocation $\hat{x} \in \hat{X}$ is specified. (For example, \hat{x} may be a vector of zeros and ones indicating, for each asset and individual, whether the individual has access

to the asset.) Agent i has quasi-linear utility of the form $U_i(\hat{x}, a) + t_i$.¹²

The aggregate surplus given property rights \hat{x} and action profile a is therefore $S(\hat{x}, a) = \sum_i U_i(\hat{x}, a)$, and the first-best outcomes are denoted

$$O^* = \arg \max_{\hat{x} \in \hat{X}, a \in A} S(\hat{x}, a).$$

It is also useful to define the efficient actions given some fixed property rights $\hat{x} \in \hat{X}$ as $A^*(\hat{x}) = \arg \max_{a \in A} S(\hat{x}, a)$ and the efficient allocation of property rights given some fixed actions $a \in A$ as $\hat{X}^*(a) = \arg \max_{\hat{x} \in \hat{X}} S(\hat{x}, a)$. If actions were fixed at a so that incentive effects were not an issue, the optimal property rights would be an element of $\hat{X}^*(a)$, reflecting the principle that the individual or individuals who most value access to an asset should have it. We also let $\hat{X}^* = \left\{ \hat{x} \in \hat{X} : (\hat{x}, a) \in O^* \text{ for some } a \in A \right\}$ be the property rights allocations that can arise in a first-best outcome.

Given an allocation of property rights \hat{x} , the agents choose their actions noncooperatively to maximize their individual payoffs. The resulting Nash equilibrium set is

$$A^\circ(\hat{x}) = \left\{ a \in A : a_i \in \arg \max_{a'_i \in A_i} U_i(\hat{x}, a'_i, a_{-i}) \text{ for all } i \right\}. \quad (1)$$

We will be interested in studying the (second-best) optimal assignment of property rights, which takes into account not only the property rights' direct effect on payoffs, but also their effect on agents' actions.

Remark 1 *In some cases property rights specify control rights, which give agents the rights to take certain actions. As mentioned in the introduction, the model can formally capture these cases in the following way: Suppose there are K actions to be chosen: $\alpha = (\alpha_1, \dots, \alpha_K)$, where $\alpha_k \in \mathcal{A}_k$ for each $k = 1, \dots, K$, which result in payoffs $\mathcal{U}_i(\alpha)$ to each agent i . A control rights allocation specifies $\hat{x}_{ik} \in \{0, 1\}$ for each agent i and each action k , with $\hat{x}_{ik} = 1$ when*

¹²In terms of the framework in the introduction, the important assumption here is that there is no bargaining stage 4. Although, strictly speaking, the model introduced here has no uncertainty θ , we can think of the functions $U_i(\cdot)$ as specifying expected payoffs (averaging over realizations of θ).

agent i has the right to choose action k , and $\sum_i \hat{x}_{ik} = 1$ for each k . To incorporate this setting in our general framework, we endow all agents i with the same action set $A_i = \prod_{k \in K} \mathcal{A}_k$, and let $a_i = (a_{i1}, \dots, a_{ik}) \in A_i$ describe the agent's plan for all the actions, with the actual implemented actions being $\alpha(\hat{x}, a) = (\sum_i \hat{x}_{ik} a_{ik})_{k=1}^K$. The resulting payoffs take the form $U_i(\hat{x}, a) = \mathcal{U}_i(\hat{x}, \alpha(\hat{x}, a))$. When property rights do not affect these payoffs directly but only through implemented actions α [i.e., when we can write $U_i(\hat{x}, a) = \mathcal{U}_i(\alpha(\hat{x}, a))$], we will call them “pure control rights.”

The model includes the Tragedy of the Commons as a special case. The following two examples, drawn from the recent literature, also illustrate the model. In the first, property rights affect only payoffs. In the second, property rights affect not only payoffs, but also who takes certain actions, as in Remark 1.

Example 1 (Agency Model) Consider a principal-agent setting where two individuals pursue a project. Individual P is the “principal” and takes no actions while individual A is the “agent” and has a noncontractible action choice $a \in A$. As in Holmstrom and Milgrom (1991) and Demski and Sappington (1991), there is an indivisible asset that may be owned by either the principal or the agent, and we let $\hat{x} \in \{0, 1\}$ denote the agent's ownership share. We let $V_i(a)$ denote the asset's value to individual i . The project also generates inalienable returns to the principal equal to $Y(a)$, while the agent's private cost of taking action a is $C(a)$. (This private “cost” may include private benefits as well as costs, so may be positive.) In some applications the agent's actions move Y and the V_i 's in the same direction (e.g., Y and the V_i 's are the project's current and future profits, respectively, both of which are enhanced by greater agent effort) while in other cases they move in opposite directions (e.g., increasing current project returns depletes the asset). Because these returns are private, ownership of the asset is the only thing that is contractible and is the only mechanism for

providing incentives.¹³ The principal's payoff is

$$U_P(\hat{x}, a) = Y(a) + (1 - \hat{x})V_P(a)$$

while the agent's is

$$U_A(\hat{x}, a) = \hat{x}V_A(a) - C(a).$$

In one well-studied case, the parties' different values for the asset are attributed entirely to their differing risk aversion and the randomness of the asset's returns. Namely, if individual i has constant absolute risk aversion with coefficient $r_i \geq 0$, and the asset's returns are normally distributed with mean $\mu(a)$ and variance $\sigma^2(a)$ then his certainty equivalent asset value equals $V_i(a) = \mu(a) - \frac{1}{2}r_i\sigma^2(a)$.¹⁴

In this example, efficiency calls for the individual i with the higher value $V_i(a)$ to own the asset. However, note that incentive effects may call for a different asset ownership \hat{x} because ownership may affect the agent's action: $A^\circ(\hat{x}) = \arg \max_{a \in A} [\hat{x}V_A(a) - C(a)]$.

Example 2 (Coordination Model of Firm Scope) This example is based on Hart and Holmstrom (2010). In this setting, property rights allocate control rights as in Remark 1 but also affect payoffs directly. For each of two production units indexed by $k = 1, 2$, an action in $\{0, 1\}$ is to be chosen, with action 1 being the "coordination" action and action 0 being the "noncoordination" action. There are three agents and two possible property rights considered: nonintegration, in which the action for each unit $k = 1, 2$ is chosen by agent $k = 1, 2$ (the unit's manager/owner), and integration, in which both actions are chosen by agent 3 – the owner of the integrated firm. If the coordination action is chosen for both units, the "coordinated outcome" $c = 1$ results, and otherwise the "noncoordinated outcome" $c = 0$ results.

¹³In Holmstrom and Milgrom (1991) returns $Y(a)$ are also verifiable so incentive schemes can be based on them as well.

¹⁴Many papers study the case where the asset is divisible then under risk aversion fractional ownership $\hat{x} \in [0, 1]$, is typically optimal, and interpret \hat{x} as the strength of the agent's incentives. While our analysis extends straightforwardly to this case, for simplicity, we restrict attention to indivisible assets.

To fit this setting into the general framework, as in Remark 1, we let the action set of manager 3 be $A_3 = \{0, 1\}^2$, where $a_{3k} \in \{0, 1\}$ is his planned action for unit $k = 1, 2$, and let the action set for each agent $i = 1, 2$ be $A_i = \{0, 1\}$, with $a_i \in \{0, 1\}$ being his planned action for his unit i . Letting $\hat{x} \in \{0, 1\}$, where $\hat{x} = 1$ stands for integration and $\hat{x} = 0$ for nonintegration, the resulting outcome is $c(\hat{x}, a) = (1 - \hat{x})a_1a_2 + \hat{x}a_{31}a_{32}$.

Hart and Holmstrom (2010) assume that in addition to control rights over unit k , its owner receives its alienable profit flows $v_k(c)$. In addition, each unit manager $i = 1, 2$ receives private benefits $w_i(c)$ from his unit. Coordination is assumed to reduce the unit managers' private benefits, so $w_i(1) \leq w_i(0)$ for $i = 1, 2$. These assumptions yield the following payoffs:

$$\begin{aligned} U_i(\hat{x}, a) &= (1 - \hat{x})v_i(c(\hat{x}, a)) + w_i(c(\hat{x}, a)) \text{ for } i = 1, 2, \\ U_3(\hat{x}, a) &= \hat{x}[v_1(c(\hat{x}, a)) + v_2(c(\hat{x}, a))]. \end{aligned}$$

The total surplus in this setting is

$$S(\hat{x}, a) = v_1(c(\hat{x}, a)) + v_2(c(\hat{x}, a)) + w_1(c(\hat{x}, a)) + w_2(c(\hat{x}, a)).$$

Note that since property rights here allocate control rights and cash rights that are valued equally by all agents, their effect on the total surplus is entirely through the implemented actions.¹⁵

Examples 1 and 2 illustrate what might be called cases of “pure cash rights” and “pure control rights.” Pure cash rights allocate the benefits of an asset which are valued identically by all agents. For instance, they arise in Example 2, and also in Example 1 when the two

¹⁵Unlike in our discussion, Hart and Holmstrom (2010) assume that an agent i may feel “aggrieved” when he gets less than his maximal possible payoff, and that such aggrievement causes agent i to lower agent j 's payoff through “shading” that induces a loss for agent j proportional to the level of aggrievement. Under nonintegration, for example, each manager $i = 1, 2$'s payoff is then $\hat{U}_i(0, a) = U_i(0, a) - \theta[\max_{\hat{a}} U_{-i}(0, \hat{a}) - U_{-i}(0, a)] = U_i(0, a) + \theta U_{-i}(0, a) - \theta \max_{\hat{a}} U_{-i}(0, \hat{a})$ for some $\theta > 0$.

agents have equal values for the asset [$V_A(a) = V_P(a)$ for all a]. Pure control rights, as defined in Remark 1, alter who takes actions without changing agents' payoffs conditional on the implemented actions. These were present in Example 2. Note that in cases of pure cash rights and pure control rights, the first best can be achieved for any property rights allocation provided that the appropriate actions are implemented: formally, we have $\hat{X}^* = \hat{X}$. This feature of these cases will play a role in some of our characterizations of optimal property rights.

2.3 Achieving Efficiency

We begin with a result identifying conditions under which a first-best outcome can be achieved. The basic idea is that efficiency can be sustained in equilibrium if there is a first-best outcome at which harmful externalities are absent.

Definition 1 There are no harmful externalities at $(\hat{x}, a) \in \hat{X} \times A$ if $U_i(\hat{x}, a) \leq U_i(\hat{x}, a_i, a'_{-i})$ for all i and all a'_{-i} .

Proposition 1 *If there are no harmful externalities at some first-best outcome $(\hat{x}^*, a^*) \in O^*$, then $a^* \in A^\circ(\hat{x}^*) \subseteq A^*(\hat{x}^*)$; that is, given property rights \hat{x}^* , action profile a^* is sustained in a Nash equilibrium, and every Nash equilibrium outcome (\hat{x}^*, a°) given property rights \hat{x}^* is efficient [and thus $(\hat{x}^*, a^\circ) \in O^*$]. Moreover, every Nash equilibrium a° given property rights \hat{x}^* results in the same (first-best) payoff for each player.^{16,17}*

¹⁶It is clear from the proof that for the first result [$a^* \in A^\circ(\hat{x})$], the nonharmful externalities assumption can be weakened to requiring that for each j and a_j , $\sum_{i \neq j} U_i(\hat{x}^*, a_j, a_{-j}^*) \geq \sum_{i \neq j} U_i(\hat{x}^*, a^*)$ (i.e., that *unilateral* deviations have no harmful externalities on the *aggregate* payoff of the other agents).

¹⁷The last result can be viewed as generalizing the payoff equivalence of Nash equilibria in zero-sum games. Indeed, in such games, any outcome is efficient, and there are no nonharmful externalities at any Nash equilibrium.

Proof. For all j and $a_j \in A_j$,

$$\begin{aligned}
U_j(\hat{x}^*, a^*) &= S(\hat{x}^*, a^*) - \sum_{i \neq j} U_i(\hat{x}^*, a^*) \\
&\geq S(\hat{x}^*, a_j, a_{-j}^*) - \sum_{i \neq j} U_i(\hat{x}^*, a_j, a_{-j}^*) \\
&= U_j(\hat{x}^*, a_j, a_{-j}^*).
\end{aligned}$$

where the inequality holds because $a^* \in A^*(\hat{x}^*)$ and any externalities are nonharmful. Hence, $a^* \in A^\circ(\hat{x}^*)$.

Next, for any $a^\circ \in A^\circ(\hat{x}^*)$

$$U_i(\hat{x}^*, a^\circ) \geq U_i(\hat{x}^*, a_i^*, a_{-i}^\circ) \geq U_i(\hat{x}^*, a^*) \text{ for each agent } i, \quad (2)$$

where the first inequality follows from Nash equilibrium and the second inequality from non-harmful externalities at (\hat{x}^*, a^*) . Summing over agents yields $\sum_i U_i(\hat{x}^*, a^\circ) \geq S(\hat{x}^*, a^*)$. Since the reverse inequality also holds due to $(\hat{x}^*, a^*) \in O^*$, we have $\sum_i U_i(\hat{x}^*, a^\circ) = S(\hat{x}^*, a^*)$, so $a^\circ \in A^*(\hat{x}^*)$. Equation (2) then implies that $U_i(\hat{x}^*, a^\circ) = U_i(\hat{x}^*, a^*)$ for all i . ■

Consistent with this result, recall that in our Tragedy of the Commons example, common ownership is efficient if, because the data center capacity is large, congestion externalities are absent. Similarly, whenever there is a first-best outcome involving private ownership (so that only one manager has access to the data center), private ownership – which involves no externalities – leads to a first-best outcome.¹⁸ In the Agency Model (Example 1) when $Y(a) \equiv 0$ and $V_A(a) \geq V_P(a)$ for all a , there is a first-best outcome in which externalities are absent in which the asset is owned by the agent. For example, this applies to the classical incentive model when the agent is risk-neutral, since selling the asset to him achieves optimal

¹⁸Note also that, according to Proposition 1, in both of these cases efficiency would be preserved if, in addition, the individuals could take actions (make “investments”) that enhanced their own value from using the data center or reduced their cost of effort, since such actions generate no externalities.

risk-sharing and at the same time lets the agent internalize any externalities from his actions. Proposition 1 tells us that agent ownership sustains the first best in such cases.

Recall that when property rights consist of pure control rights and/or pure cash rights, we have $\hat{X}^* = \hat{X}$, i.e., any property rights allocation can arise in the first best. (This property will also hold in dynamic settings considered in the next section in which any property rights allocation is renegotiated towards an ex post efficient allocation after actions are chosen.) For such cases, Proposition 1 establishes that any property rights allocation $\hat{x} \in \hat{X}$ such that there are no harmful externalities at (\hat{x}, a^*) for some $a^* \in A^*(\hat{x})$ ensures first-best payoffs [since (\hat{x}, a^*) is a first-best outcome]. For example, if property rights allocate pure cash flows and pure control rights, and their transfer to a single owner eliminates all externalities – as in the traditional view of vertical integration – it yields the first best. More generally, it suffices to eliminate only *harmful* externalities. For instance, in the Coordination Model of Firm Scope (Example 2), integration eliminates harmful externalities at coordinated actions, since the owner’s deviation would raise the managers’ private benefits. On the other hand, nonintegration eliminates harmful externalities at noncoordinated actions. So, Proposition 1 tells us that the first best is sustained by integration when coordination is efficient, and by nonintegration when noncoordination is efficient.

2.4 Externalities and Distortions

When harmful externalities are instead present at all first-best outcomes, efficiency typically cannot be achieved. As intuition suggests, equilibrium actions will be distorted in directions that generate harmful externalities. To formalize this point, we assume that each action space A_i is a partially ordered set, and define:

Definition 2 *Actions generate positive (negative) externalities at \hat{x} if, for all i , all $j \neq i$, and all $a_{-i} \in A_{-i}$, $U_j(\hat{x}, a)$ is nondecreasing (nonincreasing) in a_i .*

First we show that if a single agent has a one-dimensional action choice that generates positive (negative) externalities, in equilibrium he will choose an action that is too low (high) relative to the efficient level:

Proposition 2 *Suppose that only agent i takes actions, he has a one-dimensional action choice $a \in \mathbb{R}$, and his action choice generates positive (negative) externalities at \hat{x} . Then $A^\circ(\hat{x}) \leq (\geq) A^*(\hat{x})$ in the strong set order.¹⁹*

Proof. Consider the case of positive externalities. Define the function

$$\Psi_i(\hat{x}, a, \lambda) \equiv U_i(\hat{x}, a) + \lambda_i \sum_{j \neq i} U_j(\hat{x}, a) = (1 - \lambda_i)U_i(\hat{x}, a) + \lambda_i S(\hat{x}, a), \quad (3)$$

where $\lambda_i \in [0, 1]$. Observe that $A^*(\hat{x}) = \arg \max_{a \in A} \Psi_i(\hat{x}, a, 1)$ and $A^\circ(\hat{x}) = \arg \max_{a \in A} \Psi_i(\hat{x}, a, 0)$.

Positive externalities imply that the function $\Psi_i(\cdot)$ has increasing differences in (a, λ_i) .²⁰ The result follows by Topkis's Monotonicity Theorem [see Milgrom and Roberts (1990)]. The result for negative externalities follows similarly. ■

As an illustration, in the Agency Model, if $A \subset \mathbb{R}$ and both $Y(a)$ and $Y(a) + V_P(a)$ are nondecreasing in a (greater effort raises Y by more than any asset value deterioration for the principal), Proposition 2 tells us that the agent's choice of a will be weakly too low (in the strong set order) under any ownership structure.

The result also implies that when many agents choose actions and/or actions are multi-dimensional, the equilibrium level of any single dimension that generates positive (negative)

¹⁹The strong set order comparison $A^\circ(\hat{x}) \leq (\geq) A^*(\hat{x})$ means that for all $a^\circ \in A^\circ(\hat{x}) \setminus A^*(\hat{x})$, $\bar{a} \in A^\circ(\hat{x}) \cap A^*(\hat{x})$, and $a^* \in A^*(\hat{x}) \setminus A^\circ(\hat{x})$, we have $a^\circ \leq \bar{a} \leq a^*$ ($a^\circ \geq \bar{a} \geq a^*$). If we know that the action has *strictly* positive (negative) externalities at \hat{x} , in the sense that $U_j(a, \hat{x})$ is *strictly* increasing decreasing in a_i for all $j \neq i$ and all $a_{-i} \in A_{-i}$, then we can derive the stronger conclusion that $a^\circ \leq (\geq) a^*$ for all $a^\circ \in A^\circ(\hat{x})$ and $a^* \in A^*(\hat{x})$. This follows from the Monotone Selection Theorem [see Milgrom and Shannon (1994)].

²⁰A function $f(x, y, z)$ has increasing differences in (x, y) if

$$f(x', y', z) - f(x, y', z) \geq f(x', y, z) - f(x, y, z)$$

for all $(x', y') \geq (x, y)$. See Milgrom and Roberts (1990).

externalities will be too low (high) holding fixed all of the other dimensions. To get a result comparing the entire equilibrium action profile with the first-best action profile, we need to make assumptions on the interactions among different dimensions, so that we can apply the theory of supermodular games (Milgrom and Roberts, 1990):

Definition 3 Actions are strategic complements at property rights allocation \hat{x} if A_i is a compact lattice for each agent i , and $U_i(\hat{x}, a_i, a_{-i})$ is continuous in a , supermodular in a_i , and has increasing differences in (a_i, a_{-i}) .²¹

Proposition 3 Suppose that at a given property rights allocation $\hat{x} \in \hat{X}$, actions are strategic complements, and the total surplus $S(\hat{x}, a)$ is supermodular in a . Then there exist smallest and largest elements of the sets $A^\circ(\hat{x})$ and $A^*(\hat{x})$, denoted respectively by $(\underline{a}^\circ, \bar{a}^\circ)$ and $(\underline{a}^*, \bar{a}^*)$. If, in addition, actions generate positive (negative) externalities at \hat{x} , then $\underline{a}^\circ \leq \underline{a}^*$ ($\bar{a}^\circ \geq \bar{a}^*$).

Proof. Consider the case of positive externalities. For all i , consider again function (3), but now where $a = (a_i, a_{-i})$. Observe first that because $S(\hat{x}, a)$ is continuous and supermodular in a , there exist smallest and largest elements of the set $A^*(\hat{x})$, denoted $(\underline{a}^*, \bar{a}^*)$ [see Milgrom and Roberts (1990)]. Next, observe that $A^\circ(\hat{x})$ is the set of Nash equilibria of the game in which each agent i 's payoff function is $\Psi_i(\hat{x}, a_i, a_{-i}, 0)$, while $A^*(\hat{x})$ is a subset of the set of Nash equilibria of the game in which each agent i 's payoff function is $\Psi_i(\hat{x}, a_i, a_{-i}, 1)$. The functions $\Psi_i(\cdot)$ are supermodular in a_i and, due to positive externalities, have increasing differences in a_i and λ_i , as well as in a_i and a_{-i} . The existence of smallest and largest elements of the set $A^\circ(\hat{x})$, denoted $(\underline{a}^\circ, \bar{a}^\circ)$, follows from Theorem 5 in Milgrom and Roberts (1990). The Corollary of Theorem 6 in Milgrom and Roberts (1990) implies that the smallest Nash equilibrium action profile when $\lambda_i = 1$ for all i is larger than the smallest Nash equilibrium

²¹In particular, A_i is a compact lattice when $A_i = \prod_{l=1}^{L_i} A_{il}$, where each A_{il} is a compact subset of \mathbb{R} . A function $f(a)$ is supermodular in $a \in \mathbb{R}^m$ if it has increasing differences in every pair (a_k, a_l) of variables.

action profile when $\lambda_i = 0$ for all i (\underline{a}°). Since the set of maxima of $S(a, \hat{x})$ are a subset of the set of Nash equilibria of the game when $\lambda_i = 1$ for all i , it must be that $\underline{a}^\circ \leq \underline{a}^*$. The result for negative externalities follows similarly. ■

In particular, when there is a unique equilibrium action profile given property rights \hat{x} [i.e., $A^\circ(\hat{x}) = \{a^\circ\}$], and actions generate positive (negative) externalities, Proposition 3 implies that equilibrium actions a° are weakly below (above) those in any efficient outcome given property rights \hat{x} .

The result implies, for instance, that efforts are weakly too high under common ownership in the Tragedy of the Commons example when each $V_i(\cdot)$ function has increasing differences in its two arguments. In the Coordination Model of Firm Scope, it implies that coordination is too low under nonintegration when $v_i(1) + w_i(1) \geq v_i(0) + w_i(0)$ for $i = 1, 2$ so that externalities are positive (of course, this result is immediate in any case).

The result also applies to cases with multi-dimensional actions. For example, suppose that in the Agency Model the agent has a two-dimensional effort choice $a = (a_1, a_2) \in A_1 \times A_2 \subset \mathbb{R}^2$, and that the principal's profit flow and the asset values are given by the nondecreasing functions $Y(a_1)$, $V_P(a_2)$, and $V_A(a_2)$. Thus, externalities are positive for any ownership share \hat{x} . If $C(a_1, a_2)$ has increasing differences in (a_1, a_2) , so the two dimensions of agent effort (increasing flow profit Y and asset values V_P, V_A) are strategic complements, then the result implies that for any \hat{x} both dimensions are weakly too low.

Finally, Propositions 2 and 3 do not rule out the possibility that an equilibrium action profile could be efficient. However, if payoff functions are differentiable and the marginal externalities are strict, using the approach of Edlin and Shannon (1998) it can be shown that there is necessarily inefficiency,

2.5 Second-best Property Rights Allocations

When there are unavoidable externalities and therefore inefficiency, what can be said about the welfare effect of the property rights allocation, and about optimal (second-best) property rights allocations? We define second-best best outcomes as optimal outcomes subject to the constraint that agents choose their actions noncooperatively:²²

$$O^{**} = \arg \max_{\hat{x} \in \hat{X}, a \in A^\circ(\hat{x})} S(\hat{x}, a).$$

We also let $\hat{X}^{**} = \left\{ \hat{x} \in \hat{X} : (\hat{x}, a) \in O^{**} \text{ for some } a \in A \right\}$ be the property rights allocations that can arise in a second-best outcome.

We first observe that since actions are too low (high) with positive (negative) externalities, any change in property rights that is both directly beneficial and encourages (discourages) those actions is welfare-enhancing:

Lemma 1 *Consider two property rights allocations \hat{x} and \hat{x}' and a pair of Nash equilibria resulting from these property rights allocations, $a \in A^\circ(\hat{x})$ and $a' \in A^\circ(\hat{x}')$. If actions generate positive (negative) externalities at \hat{x}' , $S(\hat{x}', a) \geq S(\hat{x}, a)$, and $a' \geq (\leq) a$, then $S(\hat{x}', a') \geq S(\hat{x}, a)$.*

Proof. Note that

$$S(\hat{x}', a') - S(\hat{x}, a) \geq S(\hat{x}', a') - S(\hat{x}', a) \geq \sum_i [U_i(\hat{x}', a_i, a'_{-i}) - U_i(\hat{x}', a)] \geq 0,$$

where the first inequality is due to $S(\hat{x}', a) \geq S(\hat{x}, a)$, the second is due to $a' \in A^\circ(\hat{x}')$, and the third is due to positive (negative) externalities at \hat{x}' and $a' \geq (\leq) a$. ■

²²This definition makes the optimistic assumption that if a property rights allocation induces multiple Nash equilibria, the agents coordinate on a surplus-maximizing Nash equilibrium. Note that if arbitrary equilibrium selections were allowed, agents may switch between equilibria arbitrarily depending on property rights, even when property rights are not payoff relevant, which could alter the effects of property rights. However, Proposition 4 below could be reformulated under the alternative assumption that the worst equilibrium is always selected – a case examined, e.g., by Mookherjee (1984).

Lemma 1 considers property rights changes that both increase the ex post surplus holding actions fixed, and also lead to surplus-enhancing behavioral changes. Such changes involve no trade-offs. In general, the optimal (second-best) property rights allocation will trade off these two effects. Our next result shows that a second-best property rights allocation is distorted away from what would be efficient ex post given the actions individuals take, and this distortion is in the direction that increases (reduces) incentives for actions with positive (negative) externalities. For simplicity, we formulate the result for the case in which the property rights allocation is one-dimensional. This result will be applied later to cases with multidimensional property rights (e.g., specifying different agents' access to different assets) by varying one of these dimensions at a time.

Proposition 4 *Suppose that $\hat{X} \subset \mathbb{R}$, that $U_i(\hat{x}, a_i, a_{-i})$ has increasing differences in (\hat{x}, a_i) for all i , and that actions are strategic complements for all property rights allocations. Take any second-best outcome $(\hat{x}^{**}, a^{**}) \in O^{**}$, and any ex post efficient allocation given actions a^{**} , $\hat{x}' \in \hat{X}^*(a^{**})$. If actions generate positive (negative) externalities at \hat{x}' , then $\max\{\hat{x}', \hat{x}^{**}\} \in \hat{X}^{**}$ ($\min\{\hat{x}', \hat{x}^{**}\} \in \hat{X}^{**}$).*

Remark 2 *In the simple case in which there is a unique second-best outcome, i.e., $O^{**} = \{(\hat{x}^{**}, a^{**})\}$, and actions generate positive (negative) externalities at all \hat{x} , Proposition 4 implies that \hat{x}^{**} is weakly higher (lower) than any ex post efficient allocation $\hat{x}' \in \hat{X}^*(a^{**})$ given actions a^{**} .*

Proof. Suppose that actions generate positive externalities at \hat{x}' , and let $\hat{x}' > \hat{x}^{**}$ (with the reverse inequality the conclusion is trivial). Under the hypotheses of the proposition, Theorems 5 and 6 in Milgrom and Roberts (1990) imply that given the property rights allocation \hat{x}' there is a largest Nash equilibrium action profile $a' = \max A^\circ(\hat{x}')$, and $a' \geq a^{**}$. Since $\hat{x}' \in \hat{X}^*(a^{**})$ implies that $S(\hat{x}', a^{**}) \geq S(\hat{x}^{**}, a^{**})$, we can apply Lemma 1 to see that $S(\hat{x}', a') \geq S(\hat{x}^{**}, a^{**})$, so $\hat{x}' \in \hat{X}^{**}$. The proof for the case of negative externalities is similar. ■

As an illustration, consider again the Agency Model (Example 1) with $a \in \mathbb{R}$ and $Y(a)$, $Y(a) + V_P(a)$, and $V_A(a)$ all nondecreasing in a . Then externalities are positive for any ownership share \hat{x} , and the agent's payoff has increasing differences in (\hat{x}, a) . Proposition 4 tells us that the second-best property rights allocation in that case will be distorted toward agent ownership of the asset: in particular, he may be the optimal owner even if he values the asset less than the principal given his equilibrium action (e.g., due to being more risk-averse than the principal).

An interesting special case arises when the maximal (minimal) property rights allocation is efficient for all investments. In this case, applying Proposition 4 to $\hat{x}' = \max \hat{X}$ ($\hat{x}' = \min \hat{X}$) yields:

Corollary 1 *Suppose that, in addition to the assumptions of Proposition 4, the maximal (minimal) property rights allocation is in $\hat{X}^*(a)$ for all $a \in A$, and actions generate positive (negative) externalities at this allocation. Then this allocation lies in the set \hat{X}^{**} of second-best optimal property rights allocations provided that the set is nonempty.*

For an illustration, take the Agency Model of Example 1 with $A \subset \mathbb{R}$ and $V'_A(a) > 0$ (so the increasing differences and strategic complementarity assumptions of Proposition 4 hold). Then, if agent ownership is optimal for any given action a [i.e., $V_A(a) \geq V_P(a)$ for any a], and we have positive externalities at that ownership allocation [i.e., $Y'(a) > 0$], by Corollary 1 agent ownership is a second-best optimal property rights allocation (provided that one exists). Similarly, if principal ownership is optimal for any action a [i.e., $V_A(a) \leq V_P(a)$ for all a], and we have negative externalities at that ownership allocation [i.e., $Y'(a) + V'_P(a) \leq 0$, for example agent effort may increase asset value but reduce the principal's current profit $Y(a)$ by more], then by Corollary 1 it is a second-best optimal property rights allocation (provided that one exists).

An important application of the Corollary is to the case in which the allocation of property rights has no direct efficiency consequences given the agents' actions, i.e., $\hat{X}^*(a) = \hat{X}$ for

all a . (Recall that this condition arises for pure cash flow rights and pure control rights. In the dynamic setting considered in the next section, this condition will arise as a natural consequence of efficient renegotiation.) For this case, Corollary 1 says that the maximal (minimal) property rights allocation is optimal when the assumptions of Proposition 4 hold and actions generate positive (negative) externalities at this allocation.

3 Property Rights and Investment Hold-up

In the previous section, property rights affected efficiency directly as well as through incentives to take noncontractible actions. However, in a longer-term context, after noncontractible actions are taken, the parties may be able to renegotiate property rights that are ex post inefficient. For example, while giving an agent property rights over an asset may be optimal to increase his incentives for noncontractible investment, if he is not the efficient ex post owner he may resell the asset to whomever is. While such renegotiation improves ex post efficiency, it also introduces a new source of externalities, which may distort investments. These new “bargaining externalities” arise because an individual’s noncontractible actions may affect the surplus created by renegotiation, which is shared in bargaining with other individuals.

Example 3 *Consider again the Agency Model of Example 1. Now, however, assume that as in Hermalin and Katz (1991) and Demski and Sappington (1991), after the agent chooses action $a \in A \subset \mathbb{R}$ (interpreted now as an “investment”), the parties can renegotiate the allocation of the asset to some other allocation $x \in \hat{X}$ (here we assume that there are no ex ante noncontractible allocations, so $X = \hat{X}$). Suppose that renegotiation is always efficient and that it gives the agent share $\lambda \in [0, 1]$ of the renegotiation surplus. Consider the case in which the principal’s inalienable return $Y(a)$ is independent of the agent’s actions, and normalize $Y(a) \equiv 0$. Assume that the agent’s investment raises asset values $[V'_P(a), V'_A(a) > 0]$, and that the principal is always the efficient owner ex post $[V_P(a) > V_A(a) \text{ for all } a]$.*

Since the agent's investment has a positive externality on the principal at the first-best allocation [since $V'_P(\cdot) > 0$], the first-best cannot be achieved if renegotiation is impossible, and we then have underinvestment under principal ownership ($\hat{x} = 0$) according to Proposition 2. In that case, agent ownership ($\hat{x} = 1$), which eliminates this externality and increases the agent's incentives to invest, may be second-best optimal (by Proposition 4) even though the asset then ends up in the hands of the agent, who values it less than does the principal.

When perfect renegotiation is possible, however, agent ownership will always be renegotiated *ex post* to give the asset to the principal. In that case, it may be possible to achieve the first best. Given this renegotiation, the "post-renegotiation" payoffs of the principal and the agent, $U_P(\hat{x}, a)$ and $U_A(\hat{x}, a)$ respectively, when $\hat{x} = 1$ are

$$\begin{aligned} U_P(1, a) &= (1 - \lambda)[V_P(a) - V_A(a)] \\ U_A(1, a) &= V_A(a) + \lambda[V_P(a) - V_A(a)] - C(a). \end{aligned}$$

While agent ownership eliminates the direct externality of the agent's investment on the principal, renegotiation may create a bargaining externality since the principal gets a share $(1 - \lambda)$ of the renegotiation surplus $[V_P(a) - V_A(a)]$. When either the agent has all of the bargaining power ($\lambda = 1$) or the investment is general in the sense that it raises the principal's and agent's values for the asset equally [$V'_P(a) = V'_A(a)$], there is no bargaining externality, and agent ownership attains the first best. But when the principal has some bargaining power ($\lambda < 1$) and the agent's investment affects the principal's and agent's values for the asset differently, the agent's investment creates a bargaining externality. For example, if $V'_P(a) > V'_A(a)$, so the investment increases the principal's value more than the agent's, the externality from investment is positive, leading to underinvestment by the agent.

The effects of property rights on noncontractible actions/investments when renegotiation is possible after actions have been chosen has been examined in the so-called "hold-up" literature. This literature generalizes the idea from the above example: that agents' incentives

to invest may be reduced when they expect others to “hold them up” and extract some of the gains from the investments in future bargaining. In this section, we survey the hold-up literature. Paralleling our discussion of the static model, we begin by considering when the parties can achieve the first best. We then examine settings in which harmful externalities cannot be eliminated, so that a first-best outcome cannot be achieved. We focus there on the “Property Rights Theory of the Firm” of Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995). Then we discuss how the findings of that theory are modified when some of its assumptions are altered. Finally, we discuss the potential of designing property rights mechanisms and renegotiation procedures that reduce or eliminate investment inefficiencies.

3.1 A Model of Hold-up

We will consider a general hold-up model involving a set \mathcal{N} of N agents whose interaction unfolds according to the timeline described in the introduction and depicted in Figure 1. In the “ex ante” stage, the parties specify property rights $\hat{x} \in \hat{X}$, and then choose actions a . Next, some uncertainty θ is realized. Both uncertainty θ and the chosen actions a are observed by all agents prior to bargaining but are not verifiable. Then, in the “ex post” stage, the parties can renegotiate to some other allocation $x \in X$ (recall that $X \setminus \hat{X}$ represents decisions that are contractible ex post but not ex ante). If bargaining fails, the property rights allocation \hat{x} is unchanged, i.e., $x = \hat{x}$. Agent i ’s payoff given the final allocation x , noncontractible actions a , uncertainty θ , and transfer t_i is $v_i(x, a, \theta) + t_i$.

Let $x^*(a, \theta)$ be efficient ex post decisions given (a, θ) , i.e.,

$$x^*(a, \theta) \in \arg \max_{x \in X} \sum_i v_i(x, a, \theta) \text{ for all } (a, \theta),$$

and denote the resulting maximal ex post surplus by

$$S(a, \theta) = \max_{x \in X} \sum_i v_i(x, a, \theta).$$

The set of first-best investment profiles is $A^* = \arg \max_{a \in A} \mathbb{E}[S(a, \theta)]$.

To formulate results that apply to a variety of settings, we will sometimes avoid a specific model of bargaining and instead make only two weak assumptions on the utilities that the parties obtain by bargaining. We denote by $u_i(\hat{x}, a, \theta)$ party i 's *post-renegotiation utility* given the initial property rights allocation \hat{x} , following investments a and uncertainty realization θ .²³ The first assumption is *feasibility*:

$$\sum_i u_i(\hat{x}, a, \theta) \leq S(a, \theta) \text{ for all } \hat{x}, a, \theta. \quad (4)$$

In particular, if bargaining is efficient, (4) holds with equality for all (\hat{x}, a, θ) . The second is *individual rationality*:

$$u_i(\hat{x}, a, \theta) \geq v_i(\hat{x}, a, \theta) \text{ for all } i, \hat{x}, a, \theta. \quad (5)$$

It says that each agent's payoff in bargaining is at least his payoff from the outcome that follows a bargaining breakdown. The two assumptions allow for a range of bargaining processes. At one extreme we could have no renegotiation at all (bargaining always fails), in which case we are back in the static model of the last section, albeit with some uncertainty in payoffs due to θ . At the other extreme, we could have efficient bargaining, which could in principle take various forms.²⁴

One specific model of individually rational and efficient renegotiation is given by the (generalized) Nash bargaining solution

$$u_i(\hat{x}, a, \theta) = v_i(\hat{x}, a, \theta) + \lambda_i R(\hat{x}, a, \theta), \quad (6)$$

²³Note that this reduced-form utility is assumed independent of any transfer t , since we think of t as a transfer made ex ante before bargaining. For some bargaining solutions (e.g., the Nash solution) an ex post transfer is equivalent to an ex ante transfer, so is irrelevant for bargaining. For other solutions – such as outside option bargaining considered in Section 3.6 below – an ex post transfer differs from an ex ante transfer and affects bargaining. In this case, the ex post transfer would need to be viewed as part of \hat{x} .

²⁴Note that (5) might not hold when $N > 2$ and parties other than i can make agreements to change parts of x without i 's consent and these changes have externalities on party i . We discuss such situations in Section 4.2.

where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$, and $R(\hat{x}, a, \theta) = S(a, \theta) - \sum_j v_j(\hat{x}, a, \theta)$ is the *renegotiation surplus*.

We will say that there is “no renegotiation” at (\hat{x}, a, θ) if (5) holds with equality for each i , that is, if each party’s payoff equals the continuation payoff he would get under the original property rights allocation \hat{x} . (Note that, strictly speaking, the parties might renegotiate their original contract in such a case, but this renegotiation would yield none of them any gain; their payoffs would be the same as if they did not renegotiate.)

We will apply the static analysis of Section 2 to this dynamic setup by defining the agents’ reduced-form payoffs given a property rights allocation \hat{x} and investments a as their expected post-renegotiation payoffs: $U_i(\hat{x}, a) \equiv \mathbb{E}_\theta[u_i(\hat{x}, a, \theta)]$. The set of equilibrium investments is then given by the Nash equilibrium set $A^\circ(\hat{x})$ of the induced static investment game, given by (1).

3.2 Achieving Efficiency

We begin by considering circumstances in which a first-best outcome can be achieved. We first observe that the first best can be achieved provided that it is possible to specify a first-best property rights allocation at which there are no harmful *direct* externalities at the efficient investments; that is, if there would be no harmful externalities in the absence of renegotiation.

Definition 4 *There are no harmful direct externalities at $(\hat{x}, a) \in \hat{X} \times A$ if $v_i(\hat{x}, a, \theta) \leq v_i(\hat{x}, a_i, a'_{-i}, \theta)$ for all i, a'_{-i} , and θ .*

Proposition 5 *Suppose that there exists an efficient investment profile $a^* \in A^*$ and a property rights allocation $\hat{x} \in \hat{X}$ such that (i) \hat{x} is ex post efficient in all states θ given investments a^* , and (ii) there are no harmful direct externalities at (\hat{x}, a^*) . Then, for any bargaining process that satisfies feasibility (4) and individual rationality (5), $a^* \in A^\circ(\hat{x}) \subseteq A^*$, and all investment equilibria yield identical (first-best) payoffs for every agent.*

Proof. Individual rationality (5) and feasibility (4) of bargaining together with (i) imply that $u_i(\hat{x}, a^*, \theta) = v_i(\hat{x}, a^*, \theta)$ for all i and θ , so $U_i(\hat{x}, a^*) = \mathbb{E}_\theta [u_i(\hat{x}, a^*, \theta)] = \mathbb{E}_\theta [v_i(\hat{x}, a^*, \theta)]$ for all i . Using this fact, for all a_{-i} we have

$$U_i(\hat{x}, a_i^*, a_{-i}) \geq \mathbb{E}_\theta [v_i(\hat{x}, a_i^*, a_{-i}, \theta)] \geq \mathbb{E}_\theta [v_i(\hat{x}, a^*, \theta)] = U_i(\hat{x}, a^*),$$

where the first inequality is by individual rationality of bargaining and the second by non-harmful direct externalities at (\hat{x}, a^*) . Thus, the induced static investment game has non-harmful externalities at (\hat{x}, a^*) . Since (\hat{x}, a^*) is a first-best outcome in the induced static game (since $\sum_i U_i(\hat{x}, a^*) = \sum_i \mathbb{E}_\theta [v_i(\hat{x}, a^*, \theta)] = \mathbb{E}_\theta [S(a^*, \theta)]$), applying Proposition 1 to this game yields the result. ■

The intuition behind Proposition 5 is that if a property rights allocation achieves efficiency without renegotiation following equilibrium investments, then any agent's investment deviation, by creating renegotiation surplus, could only have beneficial bargaining externalities on the others. If, in addition, investment deviations don't generate direct harmful externalities, then nobody will deviate from an efficient outcome.

A simple special case in which there are no harmful direct externalities obtains when direct externalities are completely absent:

Definition 5 *Agent i 's investment a_i is a self-investment if agent j 's disagreement payoff $v_j(\hat{x}, a, \theta)$ is independent of a_i for $j \neq i$ at all \hat{x}, a_{-i} , and θ .*

If all agent's investments are self-investments, then each agent's disagreement payoff has the form $v_i(\hat{x}, a_i, \theta)$. This assumption applies, say, to agents' investment in their own human capital. It rules out both investments in physical capital that could benefit other agents and investments in other agents' human capital. If all investments are self-investments and in addition there are no ex ante noncontractible decisions (i.e., $\hat{X} = X$) and no noncontractible uncertainty θ , then by Proposition 5 letting the property rights allocation be a first-best

allocation given efficient investments, $x^*(a^*)$, sustains first-best investments.²⁵

Example 4 Consider again Example 3, but assume that $V_P(a)$ and $Y(a)$ are both independent of a , so the agent's investment is a self-investment, and that the efficient ex post owner may depend on the agent's investment a . In this case, whether renegotiation is possible or not, the first best can be obtained by giving the asset to its efficient owner given first-best investments a^* . If renegotiation were not possible, this would follow from Proposition 1. If renegotiation is possible, the agent's investment deviation may induce renegotiation when it changes the ex post optimal owner. But, as Proposition 5 shows, since such renegotiation could only bestow a positive bargaining externality on the principal, the agent would not want to deviate from the efficient investment a^* .

Consider now a richer application of Proposition 5, which allows for ex ante noncontractible decisions and uncertainty:

Example 5 Grossman and Hart (1986) consider a setting in which two agents make self-investments. However, unlike in our basic model, the ex post disagreement point is given by noncooperative choices of ex ante noncontractible actions $q_1 \in Q_1$ and $q_2 \in Q_2$ by the agents. Furthermore, the role of the property rights allocation \hat{x} is to define control rights over these actions: similarly to the setting in Remark 1, the agent who owns asset $k = 1, 2$ has the right to choose action $q_k \in Q_k$, resulting in payoffs $\nu_i(a_i, \theta, q_1, q_2)$ to each agent i . As a result, the ex post Nash equilibrium choice of q 's yields a default non-cooperative value $q^\circ(\hat{x}, a, \theta) \in Q_1 \times Q_2$. Thus, the disagreement payoffs can be defined as $v_i(\hat{x}, a, \theta) = \nu_i(a_i, \theta, q^\circ(\hat{x}, a, \theta))$. The assumptions of Proposition 5 are satisfied in this setting for any property rights allocation \hat{x} such that (i) $q^\circ(\hat{x}, a, \theta)$ is efficient for all (a, θ) , and (ii) $v_i(\hat{x}, a, \theta) = \nu_i(a_i, \theta, q^\circ(\hat{x}, a, \theta))$ is independent of a_{-i} . Note that both conditions

²⁵This conclusion extends to cases with uncertainty θ that is verifiable, so that a contingent property rights allocation $\hat{x}(\theta) = x^*(a^*, \theta)$ can be specified ex ante. Indeed, Proposition 5 extends to such cases.

are satisfied when property rights \hat{x} eliminate all externalities from ex ante noncontractible actions q . Grossman and Hart's (1998) Proposition 1 identifies some such cases. First, if both decisions q_1, q_2 affect the payoff of only one agent i , giving this agent control over both actions ("agent i integration" in Grossman and Hart's terminology) eliminates all the externalities from q 's, and achieves the first best. Similarly, if the payoff of each agent $i = 1, 2$ depends only on action q_i , then giving each agent i control over action q_i ("nonintegration") achieves the first best.²⁶

The first-best result of Proposition 1 relies on the ability to eliminate renegotiation in equilibrium by specifying a property rights allocation \hat{x} that is ex post efficient in all states θ following first-best investments a^* . This will be impossible to do when the efficient ex post outcome depends on noncontractible uncertainty and/or when ex post efficient allocations following a^* are not contractible ex ante (i.e., are outside \hat{X}). Fortunately, the first-best may still be implementable in some such cases, provided that there is efficient renegotiation, so that property rights have no direct effect on efficiency. For this case, according to Proposition 1, we can sustain efficient investments a^* using any property rights \hat{x} that eliminate harmful externalities at (\hat{x}, a^*) . However, unlike in the static setting, investments now have two sources of externalities: direct externalities and bargaining externalities through renegotiation surplus, and we now need to ensure that both kinds of externalities are nonharmful.

One example of this approach is provided by Edlin and Reichelstein (1996). In their model, all agents' investments are self-investments, hence there are no direct externalities. As for bargaining externalities, it turns out that in some cases there is a property rights allocation that makes them nonharmful:

²⁶Note that some of the assumptions of Grossman and Hart are unnecessary for these results: namely, their assumption that there is no uncertainty, the assumption that payoffs take a separable form that ensures that noncooperative equilibrium values $q^\circ(\hat{x}, a)$ are independent of a , and their restriction to the Nash bargaining solution [instead, any bargaining process satisfying (4) and (5) would work].

Proposition 6 (Edlin-Reichelstein) *Suppose all agents make self-investments, bargaining is given by the Nash bargaining solution, $X \subset \mathbb{R}^m$, and payoffs satisfy the following separability condition.²⁷*

$$v_i(x, a_i, \theta) = \bar{v}_i(a_i) \cdot x + \hat{v}_i(x, \theta) + \tilde{v}_i(a_i, \theta) \text{ for all } i. \quad (7)$$

Then for property rights allocation $\hat{x} = \mathbb{E}_\theta[x^(a^*, \theta)] \in \hat{X}$ (the expected efficient allocation given first-best investments $a^* \in A^*$), we have $a^* \in A^\circ(\hat{x}) \subseteq A^*$, and all equilibria of the investment game yield identical (first-best) payoffs for every agent.*

Proof. Agent i 's payoff in the induced static investment game is

$$U_i(\hat{x}, a) = \mathbb{E}_\theta [u_i(\hat{x}, a, \theta)] = \mathbb{E}_\theta [v_i(\hat{x}, a_i, \theta)] + \lambda_i \mathbb{E}_\theta [R(\hat{x}, a, \theta)].$$

Due to efficient renegotiation, $\hat{X}^*(a) = \hat{X}$ for all a . We will show that there are no harmful externalities (\hat{x}, a^*) where $\hat{x} = \mathbb{E}_\theta[x^*(a^*, \theta)]$, which yields the result by Proposition 1. Given self-investments, it suffices to show that the expected renegotiation surplus $\mathbb{E}_\theta [R(\hat{x}, a, \theta)]$ is minimized at $a = a^*$. To see this, observe that for any $a \in A$ and any $\theta \in \Theta$, $S(a, \theta) \geq \sum_i v_i(x^*(a^*, \theta), a_i, \theta)$, and the inequality holds with equality when $a = a^*$. Therefore,

$$\mathbb{E}_\theta [R(\hat{x}, a, \theta)] \geq \mathbb{E}_\theta \left[\sum_i [v_i(x^*(a^*, \theta), a_i, \theta) - v_i(\hat{x}, a_i, \theta)] \right] = \sum_i \mathbb{E}_\theta [\hat{v}_i(x^*(a^*, \theta), \theta) - \hat{v}_i(\hat{x}, \theta)],$$

and the inequality holds with equality at $a = a^*$, which implies the result. ■

Note that the presence of noncontractible uncertainty θ typically requires equilibrium renegotiation to achieve efficient allocations. However, under the assumptions of Proposition 6, specifying the expected efficient allocation $\hat{x} = \mathbb{E}_\theta[x^*(a^*, \theta)]$ ensures that the expected renegotiation surplus is minimized at first-best investments a^* , and thus eliminates harmful bargaining externalities at (\hat{x}, a^*) .

²⁷Edlin and Reichelstein assume that property rights and investments are one-dimensional ($X, A_i \subset \mathbb{R}$), assumptions that we relax here by allowing arbitrary action sets A_i and multidimensional property rights.

Another way to ensure efficiency is by ruling out bargaining externalities at any property rights allocation \hat{x} , and finding a property rights allocation at which direct externalities are nonharmful. In particular, bargaining externalities are always absent when one agent invests and he has all the bargaining power, hence other agents do not capture any of the renegotiation surplus:

Proposition 7 *Suppose that only agent i invests, that renegotiation is efficient and gives agent i all the bargaining power (i.e., it is the Nash bargaining solution with $\lambda_i = 1$), and that there is an efficient investment $a_i^* \in A_i^*$ and a property rights allocation $\hat{x} \in \hat{X}$ such that there are no harmful direct externalities at (\hat{x}, a_i^*) . Then $a_i^* \in A_i^\circ(\hat{x}) \subseteq A_i^*$, and all equilibria of the investment game yield identical (first-best) payoffs for every agent.*

Proof. The assumptions imply that there are no harmful externalities at (\hat{x}, a_i^*) , and $\hat{X}^*(a_i^*) = \hat{X}$ due to efficient renegotiation, so Proposition 1 yields the result. ■

For an illustration of this result, recall that in Example 3, the first best could be achieved with agent ownership if the agent has all of the bargaining power ($\lambda = 1$); the agent then sells the asset ex post to the principal at price $V_P(a)$ and captures the entire social surplus $V_P(a) - C(a)$, which gives him an incentive to invest efficiently.

Proposition 7 can be extended to settings with many investing agents. Of course, in such settings it is impossible to give more than one agent the full renegotiation surplus. However, if renegotiation is governed by coalitional bargaining (formally defined in Subsection 3.5 below), there may be a set $T \subseteq \mathcal{N}$ of agents each of whom receives his marginal contribution to the total surplus. If only agents from T make investments, we can sustain an efficient investment profile a^* provided that property rights \hat{x} eliminate any harmful direct externalities at (a^*, \hat{x}) (e.g., if each agent has the right to exclude all the others from profiting from his investment). Indeed, since each agent $i \in T$ appropriates his full marginal contribution, he has no aggregate bargaining externality on the others, and so his total externality on their aggregate post-renegotiation payoff $\sum_{j \neq i} u_j(\hat{x}, a, \theta)$ is then nonharmful as well. A part

of Proposition 1 applies to this case, implying that investment profile a^* can arise as a Nash equilibrium (see footnote 16). [For examples of two-sided markets in which all the agents on one side receive their marginal contributions so that their efficient self-investments can be sustained, see Felli and Roberts (2001) and Kranton and Minehart (2001).] However, some inefficient investment equilibria (“coordination failures”) may exist as well. [Coordination failures could be ruled out with a “no complementarities condition,” but it is rather strong – see, e.g., Brandenburger and Stuart (2007).]

Makowski and Ostroy (1995, 2000) note that when each agent has very close substitutes, it may be in fact possible to give *each* agent his marginal contribution. For example, this happens in classical economies when the competitive equilibrium is unique and the core shrinks to this equilibrium as the number of agents grows large. Indeed, when the core is single-valued, it must give each agent his marginal contribution. Examples of two-sided markets in which this occurs so that efficient self-investments by all agents can be sustained are given by Cole et al. (2001a, 2001b).

3.3 Externalities and Distortions

We now shift attention to settings in which the first best cannot be achieved with a property rights allocation. We focus here on cases studied in the Property Rights Theory of Hart and Moore (1990) and Hart (1995). This theory examines situations in which parties make self-investments (such as investments in their own human assets) so that there are no direct externalities. Inefficiencies in this theory are entirely due to harmful bargaining externalities, which arise because investing agents do not fully appropriate their contributions to renegotiation surplus (in contrast to the setting of Proposition 7). In contrast to the settings of Proposition 5 and 6, these bargaining externalities cannot be avoided by eliminating equilibrium renegotiation because of the presence of decisions that are contractible ex post but not ex ante.

Example 6 Consider the setting of Example 4, in which $V_P(a)$ and $Y(a)$ are both independent of a , so that direct externalities are absent. However, suppose that ex ante the parties can only contract on property rights $\hat{x} \in \hat{X} = \{0, \gamma\}$, where $\gamma \in (0, 1)$, while the ex post available outcomes are $X = \{0, \gamma, 1\}$. We can interpret $x = 1$ as the agent's utilization of the asset with "cooperation by the principal," which is not contractible ex ante, while $x = \gamma$ corresponds to the agent's ownership without the principal's cooperation, which is contractible ex ante. Only share γ of the asset can be effectively used by the agent without the principal's cooperation. The payoffs continue to be described as in Example 3.

Suppose that $V_A(a) > V_P$ for all a , which means that agent utilization with full cooperation is always ex post optimal. The parties always renegotiate to this allocation using the Nash bargaining solution, which gives them the following payoffs:

$$\begin{aligned} U_A(\hat{x}, a) &= \hat{x}V_A(a) + \lambda(1 - \hat{x})[V_A(a) - V_P] - C(a), \\ U_P(\hat{x}, a) &= (1 - \hat{x})V_P + (1 - \lambda)(1 - \hat{x})[V_A(a) - V_P] + Y. \end{aligned}$$

Thus, when $\lambda < 1$ and $V'_A(a) > 0$, under both property rights allocations $\hat{x} \in \{0, \gamma\}$ the agent's investment creates a positive bargaining externality, so underinvestment results. Note that if the parties were able to specify agent ownership with full cooperation ($\hat{x} = 1$), the externality would be eliminated and optimal investment would be sustained (according to Proposition 5). The inability to specify cooperation ex ante ($\gamma < 1$) and the agent's lack of full bargaining power ex post ($\lambda < 1$) allow the principal to hold up the agent ex post in return for cooperation, reducing the agent's investment incentives.

Hart and Moore (1990) examine a model of optimal property rights based on the assumption of positive bargaining externalities. Here we discuss their model, focusing for simplicity on the case with $N = 2$ agents whose renegotiation is governed by the Nash bargaining solution (6).²⁸ (We discuss the extension to $N > 2$ agents with coalitional bargaining in

²⁸This analysis applies to $N > 2$ agents provided that any renegotiation requires unanimous agreement.

Section 3.5.) As in Hart and Moore (1990), we assume self-investments, so that we can write each agent i 's disagreement payoff as $v_i(\hat{x}, a_i, \theta)$.²⁹ Thus, any externalities from investments come through the renegotiation surplus $R(\hat{x}, a, \theta)$. We have positive externalities when the renegotiation surplus is increasing in investments, i.e., when the incremental impact of investment on the total surplus $S(a, \theta)$ is greater than on the parties' disagreement payoffs, as was true in Example 6. This condition is defined as follows:

Definition 6 *Investments are relationship-specific at \hat{x} if the renegotiation surplus $R(\hat{x}, a, \theta) = [S(a, \theta) - \sum_i v_i(\hat{x}, a, \theta)]$ is nondecreasing in a for all θ . Investments are relationship-specific if they are relationship-specific at all \hat{x} .*

Note that self-investments are relationship-specific when each agent i 's disagreement payoff $v_i(\hat{x}, a, \theta)$ increases less with his investment than does the efficient surplus $S(a, \theta)$. A simple set of sufficient conditions ensuring this is the following:

Lemma 2 *Suppose that we have self-investments, that X is a subset of a Euclidean space, and that $v_i(x, a_i, \theta)$ has increasing differences in (x, a_i) for all i . Then all investments are relationship-specific at any property rights allocation \hat{x} such that $\hat{x} \leq x^*(a, \theta)$ for all (a, θ) .*

Proof. We then obtain, for any $a'_i > a_i$,

$$S(a'_i, a_{-i}, \theta) - S(a, \theta) \geq v_i(x^*(a, \theta), a'_i, \theta) - v_i(x^*(a, \theta), a_i, \theta) \geq v_i(\hat{x}, a'_i, \theta) - v_i(\hat{x}, a_i, \theta),$$

which implies that investments are relationship-specific at \hat{x} . ■

The lemma shows that investments are relationship-specific if they are complementary to allocations x and the allocations that are contracted ex ante are always below their ex post efficient levels. For example, these assumptions are satisfied for any property rights allocation \hat{x} in Example 6, in which ex post efficiency always requires $x^* = 1 \geq \gamma$, but not

²⁹Hart and Moore (1990) do not explicitly model uncertainty θ , but one can view their payoff functions as expected payoffs over the realizations of some uncertainty.

for the expected-efficient allocation in Proposition 6 above [Edlin and Reichelstein’s (1996) model], which in some states ends up below and in others above the ex post efficient level.

Since relationship-specific investments generate positive bargaining externalities, by Proposition 2 this implies underinvestment when one agent makes a one-dimensional investment. This conclusion extends to multi-agent and/or multidimensional investments, provided that investments are strategic complements (i.e., reduced-form payoffs $U_i(x, a) = \mathbb{E}_\theta[u_i(x, a, \theta)]$ satisfy Definition 3), in which case Proposition 3 implies:

Proposition 8 *Suppose that there is Nash bargaining, and at property rights allocation \hat{x} all agents’ investments are relationship-specific and strategic complements, and the total surplus $S(a, \theta)$ is supermodular in a .³⁰ Then there exist smallest elements of the sets $A^\circ(\hat{x})$ and $A^*(\hat{x})$, denoted \underline{a}° and \underline{a}^* respectively, and $\underline{a}^* \geq \underline{a}^\circ$.*

When there is a unique investment equilibrium, Proposition 8 implies that the equilibrium investments are (weakly) below those in any first-best outcome.

One way to understand this underinvestment problem is by thinking of the parties’ split of the renegotiation surplus $R(\hat{x}, a, \theta)$ in proportion to their fixed bargaining powers as an “output-sharing scheme”. Holmstrom (1982) showed that when output is an increasing function of efforts (as relationship-specificity postulates), then any such output-sharing scheme yields free-riding in effort provision.

3.4 Second-best Property Rights Allocations

3.4.1 The Hart-Moore Theory

In the models of Hart and Moore (1990) and Hart (1995), the ex ante contractible decision $\hat{x} \in \hat{X}$ is a “control structure” over a set K of assets. In contrast to our treatment of the

³⁰Sufficient conditions for strategic complementarity consist of the investment sets A_i being complete lattices, and the functions $S(a, \theta)$ and $v_i(\hat{x}, a_i, \theta)$ for all i being continuous and supermodular in a .

Tragedy of the Commons in Section 2.1, Hart and Moore rule out control structures in which more than one agent can independently access an asset. Thus letting $\hat{x}_{ik} \in \{0, 1\}$ denote whether agent i has access to asset k , the set of feasible control structures is defined as

$$\hat{X} = \left\{ \hat{x} \in \{0, 1\}^{NK} : \sum_i \hat{x}_{ik} \leq 1 \text{ for all } k \in K \right\}. \quad (8)$$

Hart and Moore refer to the case in which no agent has control of an asset k , so that $\hat{x}_{ik} = 0$ for all i , as “joint ownership” of the asset. (This is the opposite of common ownership in the Tragedy of the Commons, in the sense that each agent can exclude the others without himself having the right to access the asset.) Hart and Moore also assume that there are no externalities from access in the following sense:

Definition 7 *There are no externalities from access to assets if, for all agents i , $v_i(\hat{x}, a, \theta)$ does not depend on \hat{x}_{-i} .*

When there are no externalities from access to assets, each agent i 's disagreement payoff depends on his access to assets, but not on other agents' access, and so can be written as $v_i(\hat{x}_i, a, \theta)$. (Note that this assumption would not be natural if common access, as in the Tragedy of the Commons, were allowed). An important implication of this assumption is that, with self-investments and Nash bargaining, it makes each agent's post-renegotiation utility $u_i(\hat{x}, a, \theta)$ additively separable between a_i and \hat{x}_{-i} :

$$u_i(\hat{x}, a, \theta) = (1 - \lambda_i)v_i(\hat{x}_i, a_i, \theta) + \lambda_i S(a, \theta) - \lambda_i \sum_{j \neq i} v_j(\hat{x}_j, a_j, \theta). \quad (9)$$

Hence, an agent's investment incentives are affected only by his own asset ownership.

To study second-best optimal control structures, Hart and Moore also assume that access to assets increases agents' marginal incentives to invest:

Definition 8 *Investments are asset-specific if, for each agent i , $v_i(\hat{x}_i, a, \theta)$ has increasing differences in (\hat{x}_i, a_i) .*

This property implies that the agent’s post-renegotiation utility (9) has increasing differences in (\hat{x}_i, a_i) , so that his investment incentives are increased by shifting assets to him.

It is instructive to compare the predictions of Hart and Moore’s property rights theory to those of the “Simple Allocation Benchmark” described in the Introduction. In the benchmark, there are no investments and no renegotiation, and the optimal property rights allocation simply maximizes the expected surplus $\mathbb{E}[\sum_i v_i(\hat{x}, \tilde{\theta})]$. Thus, in the absence of externalities, it is optimal simply to give assets to agents who value them the most. In contrast, in Hart and Moore’s theory, what determines efficiency is not the expected surplus at the disagreement point *per se* (since it is renegotiated towards full efficiency) but the sensitivity of this surplus to investments. Thus, instead of allocating assets to the highest-value agents, Hart and Moore’s theory allocates assets to agents for whom they create the most beneficial investment incentives. In particular, when only one agent invests, only his incentives matter, and so he should optimally own all the assets:

Proposition 9 (Hart and Moore’s Proposition 2) *If only agent i invests (i.e., A_{-i} is a singleton), his investment is an asset-specific relationship-specific self-investment, there are no externalities from access to assets, and $\hat{X}^{**} \neq \emptyset$, then it is optimal for agent i to own all of the assets, i.e., $(\hat{x}_i, \hat{x}_{-i}) = ((1, \dots, 1), (0, \dots, 0)) \in \hat{X}^{**}$.*

Proof. Note first that since agent i ’s investment is determined by \hat{x}_i , we can set $\hat{x}_{-i} = 0$ without affecting equilibrium investments or the resulting total surplus, which makes the constraint in (8) nonbinding. Next, under the assumptions of the proposition, $U_i(\hat{x}, a_i) = \mathbb{E}_\theta[u_i(\hat{x}, a_i, \theta)]$ has increasing differences in (\hat{x}_i, a_i) . Relationship-specific investment a_i generates positive externalities. Moreover, since ex post bargaining is efficient, the aggregate surplus $\sum_i U_i(\hat{x}, a_i)$ is independent of \hat{x} , or in terms of the notation of Section 2, $\hat{X}^*(a_i) = \hat{X}$. Then applying Corollary 1 sequentially to \hat{x}_{ik} for each $k \in K$ implies that it is optimal to set $\hat{x}_i = (1, \dots, 1)$. ■

To illustrate this result, consider again Example 6, which satisfies all the assumptions of Proposition 9. In this example, the agent underinvests for any ownership structure, since the benefit of his relationship-specific investment is partially captured by the principal through the renegotiation surplus. Since this investment is asset-specific, agent ownership maximizes this investment and the resulting total surplus.

If more than one agent invests, then optimal asset allocation generally involves tradeoffs, since moving an asset from agent i to agent j has the direct effect of raising i 's asset-specific investment incentives (i.e., his best-response curve) and lowering j 's. In some special cases, however, agent j 's incentives may not be affected, in which case, if agents' investments are strategic complements, we may be able to conclude that all agents' equilibrium investments increase, and welfare unambiguously goes up. Hart and Moore (1990) identified a number of such cases, and here we present a few of these results as an illustration. To do so, we first need to state two definitions:

Definition 9 Agent i is indispensable to asset $k \in K$ if, for each agent $j \neq i$, $v_j(\hat{x}_j, a', \theta) - v_j(\hat{x}_j, a, \theta)$ does not depend on \hat{x}_{jk} .³¹

Intuitively, indispensability of agent i to asset k means that the marginal product of other agents' investments on their disagreement payoffs is unaffected by whether they own asset k .

Definition 10 Assets l and m are perfectly complementary if, for each agent i , $v_i(\hat{x}_i, a', \theta) - v_i(\hat{x}_i, a, \theta)$ depends on $(\hat{x}_{il}, \hat{x}_{im}) \in \{0, 1\}^2$ only through the product $\hat{x}_{il}\hat{x}_{im}$.

Intuitively, when two assets are perfectly complementary, the marginal product of each agent's investment when he has only one of the two assets equals the marginal product when he has neither.

³¹Note in particular that this holds trivially when agent i is the only agent who invests, so Proposition 10 below generalizes Proposition 9 to the case of multidimensional investment a_i by a single agent i when the total surplus $S(a_i, \theta)$ is supermodular in a_i .

Proposition 10 (*Hart and Moore's Propositions 4, 6, and 8*) *If investments are asset-specific relationship-specific self-investments that are strategic complements, there are no externalities from access to assets, and $\hat{X}^{**} \neq \emptyset$, then*

- (i) *joint ownership is unnecessary in an optimal control structure; i.e., there exists a second-best ownership structure $\hat{x} \in \hat{X}^{**}$ such that $\sum_i \hat{x}_{ik} = 1$ for all $k \in K$;*
- (ii) *if agent i is indispensable to asset k , then agent i optimally owns asset k , i.e., there exists $\hat{x} \in \hat{X}^{**}$ such that $\hat{x}_{ik} = 1$.*
- (iii) *it is optimal for perfectly complementary assets to be owned together, i.e., there exists $\hat{x} \in \hat{X}^{**}$ such that $\hat{x}_{il} = \hat{x}_{im} = 1$ for some i .*

Proof. Take any $\hat{x} \in \hat{X}^{**}$. For result (i), if $\sum_{ik} \hat{x}_{ik} = 0$ for some $k \in K$, then since $U_i(\hat{x}, a) = \mathbb{E}_\theta[u_i(\hat{x}, a, \theta)]$ has increasing differences in (\hat{x}_i, a_i) and positive externalities from a_{-i} for each agent i , by Corollary 1 changing to $\hat{x}'_{ik} = 1$ for any given i holding the other dimensions of \hat{x} fixed yields another allocation in \hat{X}^{**} . For result (ii), if $\hat{x}_{jk} = 1$ for some $j \neq i$, then since $v_j(\hat{x}_j, a_j, \theta)$, and hence $U_j(\hat{x}, a)$, has (weakly) increasing differences in $(-\hat{x}_{jk}, a_j)$, Corollary 1 implies that changing to $(\hat{x}'_{ik}, \hat{x}'_{jk}) = (1, 0)$ while holding the other dimensions of \hat{x} fixed yields another allocation in \hat{X}^{**} . For result (iii), if $\hat{x}_{il} = \hat{x}_{jm} = 1$ for some agents $i \neq j$, then since $v_j(\hat{x}_j, a_j, \theta)$, and hence $U_j(\hat{x}, a)$, has (weakly) increasing differences in $(-\hat{x}_{jm}, a_j)$, Corollary 1 implies that changing to $(\hat{x}'_{im}, \hat{x}'_{jm}) = (1, 0)$ while holding the other dimensions of \hat{x} fixed yields another allocation in \hat{X}^{**} . ■

Note that while Hart and Moore's concepts of indispensable agents and perfectly complementary assets are formulated in terms of the marginal returns to investments, $v_i(\hat{x}_i, a'_i, \theta) - v_i(\hat{x}_i, a_i, \theta)$ with $a' > a$, it seems natural for the same properties to be also satisfied by the absolute payoffs $v_i(\hat{x}_i, a_i, \theta)$. In that case, the Simple Allocation Benchmark makes exactly the same asset ownership recommendations as Proposition 10, since the proposed asset reallocations (weakly) increase the payoff of each agent (and not just the sensitivity of this

payoff to investment). The same argument applies to the Proposition’s recommendation of no joint ownership. Thus, Proposition 10’s recommendations also hold in settings without investments in which renegotiation is either impossible or breaks down with an exogenous probability. (When the probability of bargaining breakdown is affected by property rights, the conclusion may change, as we analyze in Section 4 below.) Thus, while some of Hart and Moore’s predictions (such as Proposition 9) are distinct from those of the Simple Allocation Benchmark, others are similar, which can make it difficult to test their theory empirically against alternative theories. Another difficulty in testing the theory stems from the sensitivity of its predictions to alternative assumptions about the nature investments, which we now examine.³²

3.4.2 Violation of Self-Investments: Cooperative Investment

We now relax the assumption of self-investments to allow “cooperative” investments, which directly affect *other agents’* disagreement payoffs. Examples include an agent’s training of other agents (i.e., investments others’ human capital) and investments in physical capital owned by others. For simplicity, we assume that these cooperative effects are positive:

Definition 11 Agent i ’s investment is cooperative at \hat{x} if $v_j(\hat{x}, a, \theta)$ is nondecreasing in a_i for $j \neq i$ and all θ . It is cooperative if it is cooperative at every \hat{x} .

In Example 3, the agent’s investment a is cooperative at agent ownership ($\hat{x} = 0$) when $Y'(a) \geq 0$, it is cooperative at principal ownership ($\hat{x} = 1$) when $Y'(a) + V'_P(a) \geq 0$, and it is cooperative when both inequalities hold. Optimal property rights when investments have cooperative components have been examined by Demski and Sappington (1991), Hart et

³²See Whinston (2003) for a discussion of the difficulties in testing Hart and Moore’s theory against the “Transaction Cost Economics” theory of integration [Williamson (1975, 1979, 1985) Klein et al. (1978)], and of the sensitivity of Hart and Moore’s theory to the nature of investments.

al. (1997), Che and Hausch (1999), Edlin and Hermalin (2000), Segal and Whinston (2000, Section 3.2), Guriev (2003), and Roider (2004).

Cooperative investments introduce a direct route by which an agent's investment can generate positive externalities in addition to the bargaining externalities present in Hart and Moore's Property Rights Theory. As a result, the one-sided underinvestment result generalizes:³³

Proposition 11 *If only agent i invests, his investment is a one-dimensional cooperative relationship-specific investment, and there is Nash bargaining, then there is under-investment for all ownership structures: $A^\circ(\hat{x}) \leq A^*(\hat{x})$ in the strong set order for all $\hat{x} \in \hat{X}$.*

Proof. The payoff of agent $j \neq i$ in the induced static game is $U_j(\hat{x}, a) = \mathbb{E}_\theta [v_j(\hat{x}, a_i, \theta) + \lambda_j R(a_i, \theta)]$, which is nondecreasing in a_i . Apply Proposition 2. ■

How do assets affect cooperative investment? To look at this in the cleanest way, we examine the special case of purely cooperative investments, which can be interpreted as investments in the other agents' human capital (e.g., training):

Definition 12 *Cooperative investments are purely cooperative if $v_i(\hat{x}, a, \theta) = \bar{v}_i(\hat{x}, a_{-i}, \theta) + \psi_i(a_i)$ for all i .*

With purely cooperative investments, each agent i 's payoff in the induced static game when there are no externalities from access to assets takes the form:

$$U_i(\hat{x}, a) = \mathbb{E}_\theta [(1 - \lambda_i) v_i(\hat{x}_i, a_{-i}, \theta) - \lambda_i v_{-i}(\hat{x}_{-i}, a_i, \theta) + \lambda_i S(a_i, \theta)] + (1 - \lambda_i) \psi_i(a_i) - \lambda_i \psi_i(a_i).$$

Thus, with purely cooperative investments, agent i 's investment incentives are affected only by the *other* agent's property rights \hat{x}_{-i} , not his own. Moreover, when agent i 's investment

³³The underinvestment result also extends to bilateral cooperative investment when investments are strategic complements. However, strategic complementarity is harder to ensure when agents' disagreement payoffs depend on both their own and others' investments.

is asset-specific, so that $v_{-i}(\hat{x}_{-i}, a_i, \theta)$ has increasing differences in (\hat{x}_{-i}, a_i) , his investment incentives are increased by taking assets away from agent $-i$. Therefore, with purely cooperative investments, all agents' investment incentives are maximized at once with joint ownership, and we have^{34,35}

Proposition 12 *If investments are asset-specific relationship-specific purely cooperative investments that are strategic complements, there are no externalities from access to assets, and $\hat{X}^{**} \neq \emptyset$, then joint ownership is an optimal control structure, i.e., $0 \in \hat{X}^{**}$.*

Intuitively, as first shown by Che and Hausch (1999), an agent's incentive to make a purely cooperative investment is enhanced rather than reduced by bargaining, since the bargaining surplus is sensitive to his investment while his own disagreement payoff is not. It is now desirable to specify property rights that minimize the direct externalities at the disagreement payoffs, which often corresponds to minimizing the disagreement payoffs and maximizing the renegotiation surplus, in contrast to the findings of Propositions 5 and 10. For instance, in Example 3 when $V'_P(a) > V'_A(a) = 0$, it is optimal to specify agent ownership and induce renegotiation, rather than specifying the ex post efficient principal ownership. (One way to think about this is that the investment's direct externality under principal ownership exceeds its bargaining externality under agent ownership.) Note also that the principal does not optimally own the asset in this example even though he is indispensable for it.³⁶

³⁴Note, however, that if only one agent invests, the same optimal outcome as under joint ownership is sustained by making him the owner of all the assets, since this takes access from the other agents. Thus, Proposition 9 does extend to purely cooperative investments. It also extends to cooperative investments that are not purely cooperative: in that case agent i 's investment incentives increase both when assets are taken away from agent $-i$ and when assets are given to agent i .

³⁵Joint ownership also proves to be optimal in some settings in which investments are cooperative but not purely cooperative – see the example in Hart (1995, p. 68).

³⁶Even when there is no physical asset, an “exclusive right” can sometimes be created to enhance efficiency with cooperative investments. For instance, in Example 3, agent ownership could just be the agent's right

3.4.3 Violation of Relationship-Specific Investments: Overinvestment in External Investments

Next, consider the assumption of relationship specificity. Recall that relationship-specific self-investment by agent i increases the efficient surplus $S(a, \theta)$ by more than it increases the agent's disagreement payoff $v_i(\hat{x}, a_i, \theta)$. For an investment that violates this property, take Example 4, which exhibits self-investments, and suppose that principal ownership is always ex post optimal and that $V'_A(a) > 0$. Then under agent ownership the agent's investment increases his disagreement payoff but does not increase the efficient surplus, and so he will overinvest. Since the investment is asset-specific, it is reduced by taking the asset away from the agent, the only investing party, contradicting Proposition 9. In fact, in this example principal ownership induces efficient investment by the agent.

As other examples of this point, Holmstrom and Tirole (1991) consider “negotiated transfer pricing” and show that it may be optimal to ban trade outside the firm to reduce rent-seeking investment, while Bolton and Whinston (1991) show that vertical integration can result in overinvestment in an integrated downstream division. This motivation can also lead to exclusive dealing contracts being adopted to induce retailers not to allocate effort/investment towards other manufacturers; see e.g., Areeda and Kaplow (1988) and Segal and Whinston (2000).³⁷

to exclude the principal from collecting $V_P(a)$ (so the agent may have no direct benefit from the asset: $V_A(a) \equiv 0$). When given this right to exclude, the agent is able to capture some share of his investment return through bargaining, which enhances his investment incentives. Marvel (1982) offers this motive as a justification for exclusive dealing contracts that manufacturers have with retailers. See also Segal and Whinston (2000).

³⁷Other examples of overinvestment in external investments and resulting optimality of taking assets away from investing parties can be found in Rajan and Zingales (1998) and Cai (2003). Schmitz (2006) makes a related point, except that the investment is in *information* about the outside option, and the resulting renegotiation proceeds under asymmetric information and yields inefficiencies (as in the model studied in Section 4.1 below).

3.4.4 Violation of Asset-Specific Investments: Ownership Reduces Investment Incentives

In contrast to the examples considered previously, we can also have examples in which the marginal benefit of investment is higher when an agent does not own an asset than when he does. For instance, in Example 4, we may have $V'_A(a) < 0$ if the investment raises the agent's utility without the asset, thus reducing his willingness to pay for the asset. In such cases, investment is increased by taking the asset away from the agent, i.e., specifying principal ownership ($\hat{x} = 0$). (Recall also that principal ownership sustains efficient investment if it is ex post efficient given this investment.)

3.5 Property Rights with Coalitional Bargaining

Here we enrich the model to include coalitional bargaining with more than 2 agents, to outline the model of Hart and Moore (1990). Now, what each coalition $T \subset \mathcal{N}$ can achieve matters, not just individual agents' disagreement payoffs and the total surplus achievable by the grand coalition \mathcal{N} , as above.³⁸ Our exposition generalizes Hart and Moore's framework in several dimensions.

We begin by defining the value of each coalition T , i.e., how much joint surplus it can achieve through efficient negotiation by itself, without the outsiders. This value is denoted by $S_T(\hat{x}, a, \theta)$, and we assume that $S_{\mathcal{N}}(\hat{x}, a, \theta) = S(a, \theta)$.³⁹ Under standard cooperative game theory, these values are assumed to be independent of what agents outside of T choose to do. For example, this rules out common ownership in the Tragedy of the Commons, where the agents' actions have externalities on each other, but permits joint ownership in which

³⁸We note that in Hart and Moore (1990), x are investments and a are assets, the reverse of our notation here. To maintain comparability to the rest of the chapter, we maintain our earlier notation.

³⁹ For example, these coalitional values could be derived as follows: letting $X_T(\hat{x}) \subseteq X$ denote the set of allocations that can be attained with cooperation of members of coalition T under property rights \hat{x} , the cooperative value of this coalition is $S_T(\hat{x}, a, \theta) = \sup_{x \in X_T(\hat{x})} \sum_{i \in T} v_i(x, a, \theta)$.

neither agent can access the asset without the other’s permission.⁴⁰

Hart and Moore suppose that coalitional bargaining yields Shapley value payoffs. Here we extend their results to general “random-order” bargaining payoffs. To do so, we define $M_T^i(\hat{x}, a, \theta) \equiv S_{T \cup \{i\}}(\hat{x}, a, \theta) - S_T(\hat{x}, a, \theta)$ to be agent i ’s marginal contribution when added to coalition T . Let Π denote the set of orderings of \mathcal{N} and, for each ordering $\pi \in \Pi$ and each agent i , let π^i denote the set of agents who come before agent i in ordering π . Agent i ’s bargaining payoff is assumed to be a weighted average of his marginal contributions to the preceding agents in different orderings, with the weights given by a probability distribution $\alpha \in \Delta(\Pi)$ over orderings:^{41,42}

$$U_i(\hat{x}, a) = \mathbb{E}_\theta \left[\sum_{\pi \in \Pi} \alpha(\pi) M_{\pi^i}^i(\hat{x}, a, \theta) \right].$$

The Shapley value, for example, corresponds to the uniform distribution over orderings: $\alpha(\pi) = 1/N!$ for all π .

We next generalize the notion of relationship-specific investments to the coalitional setting:

Definition 13 *Investments are relationship-specific if, for each $i, j \neq i$, and coalition $T \subseteq \mathcal{N}$, the marginal contribution $M_T^i(\hat{x}, a, \theta)$ is nondecreasing in a_j .*

This definition says that an agent’s investments increase other agents’ marginal contributions. For example, when specialized to the case of self-investments and $N = 2$ it says that an agent’s investment increases surplus more than it increases his disagreement payoff,

⁴⁰For examples of bargaining with externalities, see Section 4.2.

⁴¹This bargaining solution can be characterized either axiomatically (Weber 1988) or with a noncooperative bargaining process in which, for each realized random ordering, agent i makes a take-it-or-leave-it offer to the group of players π^i that precede him.

⁴²Brandenburger and Dekel (2007a, 2007b) study a similar model in which each agent instead expects to receive a weighted average of his highest and lowest payoffs in the core of the cooperative bargaining game (which is nonempty in their settings). Their examples of underinvestment are similar to those of Hart and Moore (1990), but they do not study the effects of property rights.

as in our earlier definition. Definition 13 allows, however, for cooperative investments, e.g., it allows agent j 's investment to increase agent i 's marginal contributions to coalitions that do not include j .

Our next definition generalizes Hart and Moore's assumption A4, allowing for multidimensional investments:

Definition 14 *Investments are strategic complements if, for each i , investment set A_i is a compact lattice, and for each $i, j \neq i$, and each coalition $T \subseteq \mathcal{N}$, the marginal contribution $M_T^i(\hat{x}, a, \theta)$ continuous in a , is supermodular in a_i and has increasing differences in (a_i, a_j) .*

Note that this definition is satisfied, in particular, when only one agent invests and his investment is one-dimensional. We can now state an under-investment result:⁴³

Proposition 13 *If investments are relationship-specific and strategic complements and $S(a, \theta)$ is supermodular in a , then there exist the smallest elements of the sets $A^\circ(\hat{x})$ and $A^*(\hat{x})$, denoted \underline{a}° and \underline{a}^* respectively, and $\underline{a}^* \geq \underline{a}^\circ$.*

Proof. The assumptions imply positive externalities in the induced static investment game, so Proposition 3 yields the result. ■

Note that Proposition 13 generalizes Hart and Moore's under-investment result by not restricting to self-investments (their assumption A3). Indeed, the proposition generalizes the under-investment result we discussed for cooperative investments in Section 3.4.2.

Now consider optimal asset ownership. Following Hart and Moore (1990), we define the set of feasible property rights, which Hart and Moore call "control structures":

Definition 15 *Letting K be the set of assets, a control structure is a vector $\hat{x} \in \{0, 1\}^{2^N \times K}$ satisfying these restrictions:*

⁴³Hart and Moore (1990) also prove uniqueness of equilibrium investments given any property rights allocation. This follows if the coalitional values are concave in investments and bargaining is given by the Shapley value.

- (i) \hat{x}_{Tk} is nondecreasing in the size of coalition T ,
- (ii) $\hat{x}_{Tk} + \hat{x}_{(\mathcal{N}\setminus T)k} \leq 1$,
- (iii) $\hat{x}_{\mathcal{N}k} = 1$ for all k .

Here $\hat{x}_{Tk} = 1$ means that coalition T has access to asset k . Restriction (ii) means that an asset cannot be accessed by both a coalition and its complement, which rules out access externalities. Restriction (iii) means that the grand coalition can access anything. Note that, in general, \hat{x} assigns coalitional rights as well as individual rights. As a special case, we can have $\hat{x}_T^k = \max_{i \in T} \hat{x}_{\{i\}}^k$ for all T , i.e., a coalition can only access assets that could be accessed by one of its individual members. However, we can also have $\hat{x}_T^k > \max_{i \in T} \hat{x}_{\{i\}}^k$. For example, with majority voting a majority coalition can access an asset even when no individual member could do so. [The reverse strict inequality cannot hold by part (i) of the definition.]

We now introduce three definitions that generalize concepts introduced earlier to the coalitional setting:

Definition 16 *There are no externalities from access to assets if, for all coalitions $T \subseteq \mathcal{N}$, $S_T(\hat{x}, a, \theta)$ does not depend on \hat{x}_{-T}*

Definition 17 *Investments are self-investments if, for all coalitions $T \subseteq \mathcal{N}$, $S_T(\hat{x}, a, \theta)$ does not depend on a_{-T} .*

Definition 18 *Investments are asset-specific if, for all coalitions $T \subseteq \mathcal{N}$, $S_T(\hat{x}_T, a, \theta)$ has increasing differences in (\hat{x}_T, a) .*

We can now state:

Proposition 14 *If only agent i invests, his investment is a strategically complementary asset-specific relationship-specific self-investment, there are no externalities from access to assets, and $\hat{X}^{**} \neq \emptyset$, then it is optimal for agent i to own all of the assets, i.e., to set $\hat{x}_T^{**} = 1$ if and only if $i \in T$.*

We omit the proof, which is similar to that of Proposition 9. We could likewise generalize Proposition 10. Note that in deriving these results, Hart and Moore’s concavity assumptions, A1 and A2, are not needed.

3.6 Property Mechanisms

Property rights are simple (“incomplete”) contracts, which are often not sufficient to attain the first-best. This raises the following natural question: is it possible to achieve an improvement with more complicated contracts? As one example, instead of a simple (noncontingent) property right, the parties might specify an option-to-own contract that gives one party the right to buy an asset ex post at a specified price. One could imagine even more complicated contracts, in which all the parties have contractual choices. Formally, a contract then defines a message game in which the parties send messages and the outcome is determined by these messages. Can an improvement be achieved with such message games? If in some cases the answer is no, this can offer a “foundation for incomplete contracts.”

3.6.1 Mechanism Design without Renegotiation

The hold-up model assumes that investments a and uncertainty θ are “observable but not verifiable,” so a contract cannot condition upon them. In particular, it is impossible to stipulate efficient investments in the contract. However, in a contractual message game, the equilibrium outcome can indirectly depend on nonverifiable information such as investments through messages sent in the game. This in turn can improve investment incentives and perhaps even sustain first-best investments.

For the simplest possible example, consider the game in which the agents must report all the information, (a, θ) , and are severely penalized when the reports do not agree. Truthful reporting constitutes a Nash equilibrium of this game. However, this game has two obvious problems. One is that the penalties may be renegotiated by the agents. For now we abstract

from this problem, assuming that renegotiation outside of the contractual framework is impossible (but see the next subsection for the discussion of renegotiation). The other problem is that the game has “bad” equilibria in which the agents coordinate on a reporting a false state. This problem, however, can be avoided: To eliminate bad equilibria, the agents can be made to play a somewhat more sophisticated game, and we can use an equilibrium refinement. In particular, by asking the parties to play an extensive-form game and using the refinement of subgame-perfect equilibrium proposed by Moore and Repullo (1988), the parties can sustain efficient investments in the unique subgame-perfect equilibrium.

For a simple example of such subgame-perfect implementation, consider any setting in which only agent 1 invests and his investment has no harmful direct externalities at (\hat{x}, a^*) for some property rights allocation \hat{x} and some first-best investment $a^* \in A^*$ (e.g., it could just be self-investment). Suppose the initial contract specifies that the agents play the following two-stage game ex post, and that renegotiation of this mechanism is impossible:

Ultimatum Game:

1. Agent 1 names an outcome (x, t) .
2. Agent 2 “accepts” or “rejects.”

If he accepts, the outcome named by agent 1 is implemented, otherwise property right allocation \hat{x} is implemented.

In the unique subgame-perfect equilibrium of this game, a surplus-maximizing outcome results and agent 2 receives his reservation payoff. In effect, this game introduces a contract-specified renegotiation that gives agent 1 all of the bargaining power. Thus, according to Proposition 7, Agent 1 will invest efficiently ex ante.⁴⁴

⁴⁴More generally, with quasi-linear utilities, using the three-stage mechanism of Moore and Repullo (1988), the agents’ payoffs can be made to depend in any manner on the state (a, θ) provided that agents’ preferences vary across states. For example, with self-investments, this always allows the parties to sustain the first best.

A common informal justification of “incomplete contracts” appeals to the parties’ inability to foresee future allocations x or contingencies θ ex ante. However, at the same time parties are still assumed to be rational, foreseeing their future payoffs and choosing investments optimally. One way to reconcile these assumptions, proposed by Maskin and Tirole (1999) is that the parties foresee their payoffs from all possible future contingencies and actions, but cannot “describe” them in an ex ante contract due to not yet having the “names” for them. In this light, note that the Ultimatum Game can be described ex ante without naming any specific ex post decisions or contingencies. Maskin and Tirole generalize this observation, showing that any mechanism can be described ex ante without “naming” actions or contingencies, letting the agents fill in these names in the ex post message game. Thus, the parties’ inability to describe future actions or contingencies ex ante does not impose any additional restrictions on what can be implemented in a contractual message game.⁴⁵

In sum, even if unable to describe future contingencies and actions ex ante, parties that can foresee future payoffs can quite generally achieve efficiency with a sufficiently complicated contract, provided that renegotiation can be prevented.

3.6.2 Mechanism Design with Renegotiation

In reality, the parties may be able to renegotiate the contractual outcome when it is inefficient. This possibility can have dramatic effects on what can be achieved with complicated contracts. As an illustration, note that the Ultimatum Game prescribes an inefficient outcome \hat{x} after agent 2 rejects the offer. Suppose that this outcome is always renegotiated to an efficient outcome, with agent 2 receiving share λ_2 of the renegotiation surplus. Anticipating

⁴⁵However, Segal (1996) and (1999a, Section 4) identifies some settings in which (unlike in the Ultimatum Game) an optimal mechanism requires “filling in the names” of a large number of possible ex post actions. In such settings, with a constraint on the length of ex post communication, the inability to name actions ex ante may indeed constrain contractual mechanisms and prevent any improvement upon an “incomplete contract” such as the null contract or an “authority” contract in which one agent describes the outcome.

such renegotiation, agent 2 will reject any Stage-1 offer leaving him a smaller share of renegotiation surplus than λ_2 . Thus, the Ultimatum Game followed by renegotiation yields exactly the same outcome as does a noncontingent contract specifying property right \hat{x} followed by renegotiation.

Segal and Whinston (2002) examine the power of general two-sided mechanisms (message games) under the assumption of Maskin and Moore (1999) that any inefficient outcome prescribed by the mechanism will be followed by efficient renegotiation. The role of the mechanism is thus to affect the division of surplus between the agents, which in turn affects the parties' ex ante investments. In particular Segal and Whinston (2002) identify conditions under which a mechanism cannot improve upon simple (noncontingent) property rights followed by exogenous renegotiation.

Segal and Whinston's analysis can be explained heuristically as follows: Suppose that there is no uncertainty θ and that only one agent i chooses an investment $a \in A = [\underline{a}, \bar{a}] \subset \mathbb{R}$. Following the investment, the two agents can send messages, and the mechanism prescribes an outcome (x, t_1, t_2) conditional on the messages, but it is always renegotiated according to the Nash bargaining solution. Let $V_i(a)$ denote agent i 's equilibrium utility in the mechanism following any investment choice a . (It is uniquely defined by the Minimax Theorem, since the mechanism defines a constant-sum game due to efficient renegotiation.) Segal and Whinston show that, provided that the agents' utilities satisfy some smoothness conditions and the space \hat{X} of property rights is connected, $V_i(a)$ is differentiable almost everywhere and its derivative takes the form

$$V_i'(a) = \frac{\partial u_i(x(a), a)}{\partial a} \tag{10}$$

for some allocation rule $x(\cdot) : A \rightarrow \hat{X}$. For mechanisms in which one party sends a message ("option mechanisms"), this result follows from Mirrlees's (1971) classical envelope-theorem characterization of incentive compatibility and, in addition, the allocation rule $x(\cdot)$ must then be nondecreasing if $u_i(\cdot)$ has increasing differences. Segal and Whinston show that the

same formula must still hold for two-sided mechanisms, although the allocation rule can now be arbitrary.⁴⁶

To see how (10) implies the optimality of simple contracts, note that if a mechanism sustains investment $a^\circ \in (\underline{a}, \bar{a})$, agent i 's utility must satisfy the first-order condition $V_i'(a^\circ) = 0$. By (10) this first-order condition is preserved if we replace the mechanism with the noncontingent contract specifying allocation $\hat{x} = x(a^\circ)$. Thus, provided that agent i 's payoff is sufficiently concave in a , this noncontingent contract also sustains investment a° . Segal and Whinston show that the optimality of simple (noncontingent) property rights extends to a number of cases with uncertainty θ and multidimensional and/or two-sided investments, provided that the investments affect post-renegotiation payoffs through a one-dimensional state. They also identify additional settings in which noncontingent contracts can be improved upon by more complex contracts.

3.6.3 Irreversible Options and Renegotiation Design

Segal and Whinston's conclusion that mechanisms cannot strengthen incentives over noncontingent contracts hinges on the assumption that renegotiation can always occur *after* the mechanism is played. Noldeke and Schmidt (1995), Watson (2007), and Buzard and Watson (2010) instead allow the mechanism to face one of the parties with a technologically irreversible verifiable choice, and show that this can often be exploited to sustain first-best investments. For example, in the Ultimatum Game, suppose that Agent 2's "acceptance" or "rejection" involves an irreversible choice of allocation x (e.g., there may be a deadline after which the asset is worthless). In that case, effective renegotiation is possible only *before* Agent 2's choice but not after. But note that renegotiation before Agent 2's choice has no

⁴⁶For continuous mechanisms, $x(a)$ is the allocation prescribed by the mechanism in equilibrium in state a . In general, however, $x(a)$ is a limit of off-equilibrium decisions prescribed by the mechanism when agents disagree but send reports close to a .

power, since his equilibrium choice is efficient.⁴⁷

A number of other papers assume that while each agent has an irreversible option, he also has the choice to postpone exercising his option and continue bargaining (i.e., his option can be viewed as an “American” rather than “European” option). Such postponable irreversible options can be used to shift the division of bargaining surplus, mimicking the outcome of the Ultimatum Game. To understand the effect of such options, consider the following noncooperative infinite-horizon bargaining game, which is played ex post following investments a and uncertainty θ . Let $\bar{S} = S(a, \theta)$ denote the the maximum available ex post surplus, and suppose that the parties have “outside option payoffs” (o_1, o_2) , and “disagreement payoffs” (d_1, d_2) , such that $\max\{o_1 + o_2, d_1 + d_2\} \leq \bar{S}$.

Outside Option Bargaining Game: In odd periods, agent 1 is the “proposer” and agent 2 is the “responder.” In even periods, agent 2 is the “proposer” and agent 1 is the “responder.” In each period,

1. The “proposer” offers a new contract, describing payoffs (s_1, s_2) to the two parties, with $s_1 + s_2 \leq \bar{S}$.
2. The “responder” accepts, rejects, or exercises his outside option. If the “responder” accepts, the game ends, and the agents receive payoffs (s_1, s_2) . If the “responder” exercises his outside option, the game ends, and the agents receive payoffs (o_1, o_2) . Otherwise, with probability $\delta \in (0, 1)$ the game proceeds to the next period, and with the complementary probability the game terminates, and the agents receive payoffs (d_1, d_2) .^{48,49}

⁴⁷Rubinstein and Wolinsky (1992) study a model of message games that proceed *concurrently* with renegotiation and find that they could implement the same outcomes as when renegotiation can only precede the game.

⁴⁸Equivalently, we can imagine that the game never terminates but the agents are impatient and discount their future payoffs with discount factor δ , and that they receive flow payoffs $((1 - \delta) d_1, (1 - \delta) d_2)$ in every period before an agreement is reached or an outside option is exercised.

⁴⁹Allowing the proposer to exercise his outside option before proposing would not affect the game’s equi-

Sutton (1986) shows that the unique subgame-perfect equilibrium expected payoffs of this game are

$$(u_1, u_2) = \begin{cases} (e_1, e_2) \equiv (d_1 + \frac{1}{1+\delta} [\bar{S} - d_1 - d_2], d_2 + \frac{\delta}{1+\delta} [\bar{S} - d_1 - d_2]) & \text{if } (o_1, o_2) \leq (e_1, e_2), \\ (o_1, \bar{S} - o_1) & \text{if } o_1 > e_1, \\ (\bar{S} - o_2, o_2) & \text{if } o_2 > e_2. \end{cases}$$

In words, the parties split the renegotiation surplus over the disagreement point in proportion $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$ if neither agent's outside option binds. If one agent's outside option binds, he receives his outside option payoff and the other agent receives the rest of the surplus (both parties' outside options cannot bind at once since $e_1 + e_2 = \bar{S} \geq o_1 + o_2$). This conclusion is known as the *Outside Option Principle*. As $\delta \rightarrow 1$, the first-mover advantage disappears, and the renegotiation surplus is split equally when the outside options do not bind.

Different papers make different assumptions about the nature of the agents' outside options and disagreement payoffs. First note that if both the outside options and the disagreement payoffs are given by the initial property rights, so that $(o_1, o_2) = (d_1, d_2)$, then outside options do not bind and each agent i 's equilibrium payoff is $u_i = \bar{v}_i + \frac{1}{2} [\bar{S} - d_1 - d_2]$ as $\delta \rightarrow 1$. This is the standard noncooperative foundation for the Nash bargaining solution used in our previous model of renegotiation [Stahl (1972), Rubinstein (1982)].

Aghion et al. (1994) assume that each agent's outside option is to enforce the initial property rights \hat{x} (e.g., by irreversibly using their assets), but their disagreement payoffs result from an exogenous "disagreement allocation" x^d that is independent of property rights, and from a contractually stipulated "financial hostage" p^d that agent 2 pays to agent 1. (For example, we can imagine that an asset can only be used once, either unilaterally by his owner

librium outcome (the option would not bind). Allowing the proposer to exercise his outside option after his proposal is rejected would give rise to multiplicity of subgame-perfect equilibria. In that case, by switching between equilibria to punish investment deviations, first-best investments can be sustained, as noted by Evans (2008). Note, however, that this finding relies on the parties being able to play the right continuation equilibrium, much as the Nash equilibrium considered at the start of Section 3.6.1 did.

or by mutual agreement, and that an unused asset becomes worthless with probability $1 - \delta$ in each period, in which case the agents are left with the allocation x^d and agent 1 gets to keep the financial hostage p^d .) In particular, Aghion et al. (1994) observe that by stipulating a sufficiently large financial hostage p^d , and thus ensuring that agent 2's outside option always binds, agent 1 can be given the whole renegotiation payoff $S(a, \theta) - v_2(\hat{x}, a, \theta)$, mimicking the outcome of the Ultimatum Game. By Proposition 7, this induces agent 1 to invest efficiently provided that his investment has no harmful direct externalities at the property rights allocation \hat{x} .⁵⁰ In the special case of self-investments, agent 1's investment incentives do not depend on \hat{x} , and Aghion et al. (1994) show that \hat{x} can be chosen to sustain efficient investment by agent 2 as well.⁵¹

deMeza and Lockwood (1998) and Chiu (1998) follow the same approach to examine the role of property rights \hat{x} when asset use is not verifiable, so the parties cannot stipulate a financial hostage (thus $p^d = 0$). In this case, agent 2's outside option can still be made to bind by giving him the ownership of a sufficiently valuable asset, which induces agent 1 to make an efficient self-investment. This conclusion reverses the key finding of Hart and Moore (1990) that an agent's investment incentives are raised by asset ownership (see Proposition 9 above), even when all of their assumptions hold. Also, in contrast to Hart and Moore (1990), the conclusion does not depend on whether the investment is asset-specific. For

⁵⁰In Hart and Moore (1988) and MacLeod and Malcomson (1993), making one agent's outside bind also induces efficient investment by the other agent, but in their models the disagreement outcome can be stipulated in the initial contract, while the outside option allocation is exogenous and is interpreted as "breach" – e.g., trading with outside partners. (They also allow the parties to stipulate a breach penalty p^b , but do not allow this penalty to depend on which party breached – e.g., because this is not verifiable – so all that matters for incentives is the difference $p^d - p^b$.) MacLeod and Malcomson (1993) postulate the Outside Option Principle, while Hart and Moore (1988) postulate a noncooperative renegotiation game that differs from the one described above but yields the same outcome.

⁵¹The same result is obtained by Chung (1991), who allows the contract to shift bargaining power to one party rather than deriving this shift from the Outside Option Principle.

example, the asset could be fully general (e.g., equivalent to a financial hostage) — what is important is that its ownership gives an attractive unilateral option that forestalls any subsequent renegotiation.

The applicability of these theories depends on the availability of irreversible unilateral decisions. For example, if the use of an asset requires some irreversible modifications that rule out any alternative uses, we can think of this use as an outside option. If, instead, an agent can use the asset without modifying it while continuing to bargain with others regarding its alternative uses, then asset ownership defines the disagreement point, as in Hart and Moore’s (1990) theory. In that case, option contracts are of limited benefit and often cannot improve upon simple property rights.

4 Property Rights and Bargaining Inefficiencies

The allocation of property rights can matter even in the absence of noncontractible actions/investments, when these rights affect the efficiency of agreements reached by the parties when their bargaining falls short of the perfectly efficient Coasian ideal. As noted in the Introduction, in the Simple Allocation Benchmark in which renegotiation is impossible, optimal property rights simply allocate assets to agents who value them the most on expectation. The same conclusion obtains if bargaining completely breaks down with an exogenous probability and is perfectly efficient with the complementary probability. A richer model of property rights emerges when property rights affect the probability and the extent of bargaining inefficiencies.

In this section we discuss the impact of property rights on two sources of bargaining imperfections – asymmetric information and contracting externalities. We can think of the parties bargaining over a final (Stage 4) allocation or over initial (Stage 1) property rights. In the latter interpretation, the parties’ payoffs should be viewed as reduced-form payoffs that account for the effects of property rights on future non-contractible investments and/or

bargaining, and the analysis shows how bargaining outcomes may differ from the optimal property rights identified in Sections 2 and 3. In such cases, legal rules defining ex ante property rights (prior to Stage 1) and/or restricting the parties' abilities to strike bargains may play affect efficiency, in contrast to the assertion of the Coase Theorem.

4.1 Asymmetric Information

One important reason why negotiations may break down is asymmetric information:⁵²

Example 7 *Two agents may each use an asset for production. Ex post efficiency requires that the asset be owned by the agent who can use it most productively. Each agent i 's value from use θ_i is uncertain initially, is independently drawn from a uniform distribution on $[0,1]$, and is known only by agent i .*

The celebrated Myerson-Satterthwaite Theorem [Myerson and Satterthwaite (1983)] states that efficient bargaining is impossible if initially one agent is the owner.⁵³ Formally, their result follows from the finding that no efficient, incentive-compatible, interim individually rational, and budget-balanced mechanism exists in this setting, since any efficient (voluntary) bargaining procedure must have these four properties. Intuitively, efficient revelation of information requires that each agent appropriate the whole gains from trade, which is impossible. When each agent must instead share the gains from trade with the other agent, he has an incentive to understate these gains. Thus, the initial owner – the potential “seller” of the asset – has an incentive to overstate his valuation in negotiations, while the potential buyer of the asset has an incentive to understate his valuation. These incentives for misrepresentation inevitably lead to breakdowns in bargaining, where the asset remains in the hands of the agent with the lower value.

⁵²For another discussion of this issue, see Farrell (1987).

⁵³The result holds more generally: efficient bargaining is impossible whenever each agent i 's value is drawn from a strictly increasing distribution function F_i on some set $[\underline{\theta}_i, \bar{\theta}_i]$ and $(\underline{\theta}_1, \bar{\theta}_1) \cap (\underline{\theta}_2, \bar{\theta}_2) \neq \emptyset$.

On the other hand, efficient bargaining may be possible in this example for other property rights allocations. Cramton, Gibbons, and Klemperer (1987) consider the case in which property rights are divisible – e.g., either the asset is perfectly divisible with agent i 's property right providing him with payoff $\theta_i x_i$ when he owns fraction $x_i \in [0, 1]$, or randomized ownership is possible where x_i is the probability of agent i owning the asset. They show that efficient bargaining is possible (i.e., there is an efficient, incentive-compatible, individually rational, and budget-balanced mechanism) provided that the agents start from an allocation of property rights that is close enough to equal. Intuitively, intermediate ownership levels reduce the incentives for misrepresentation because each agent does not know whether he will ultimately end up as a seller or a buyer of the asset.

4.1.1 A Characterization of when Efficient Bargaining is Possible

We now formulate a model to investigate more generally which property rights permit efficient bargaining [generalizing the Cramton, Gibbons, and Klemperer (1987) insight] and which do not [generalizing the Myerson and Satterthwaite (1983) insight]. The model covers as special cases a large number of settings considered in the literature.

Consider a set $\mathcal{N} \equiv \{1, \dots, N\}$ of expected-utility maximizing agents whose utilities are quasilinear in money. The agents' privately observed types are independently distributed random variables $\tilde{\theta}_1, \dots, \tilde{\theta}_N$ with values in the sets $\Theta_1, \dots, \Theta_N$.⁵⁴ The state space is thus $\Theta = \Pi_i \Theta_i$. The utility of each agent i is given by $u_i(x, \theta_i) + t_i$, where $t_i \in \mathbb{R}$ is the payment to

⁵⁴Riordan (1990) and Schmidt (1996) assume that property rights directly affect the information structure, but here we assume that this structure is independent of property rights. In a hybrid model, Schmitz (2006) endogenizes the effect of property rights on noncontractible efforts to acquire private information.

the agent, $x \in X$ is the final allocation, and $\theta_i \in \Theta_i$ is the agent's type.^{55,56} The total surplus given allocation x and state θ is $s(x, \theta) \equiv \sum_i u_i(x, \theta_i)$. An allocation rule $x^* : \Theta \rightarrow X$ is efficient if $s(x^*(\theta), \theta) = \max_{x \in X} s(x, \theta) \equiv S(\theta)$ for all $\theta \in \Theta$. Throughout we shall assume that an efficient allocation rule $x^*(\cdot)$ exists.

Following Myerson and Satterthwaite, the literature has adopted the following definition:

Definition 19 *The property rights allocation $\hat{x} \in X$ **permits efficient bargaining** if there exists a Bayesian incentive-compatible and interim individually rational mechanism implementing an efficient allocation rule in which the total payment to the agents is zero in all states.*

By the Revelation Principle, to see if efficient bargaining is possible given an initial property rights allocation \hat{x} , we can focus on direct revelation mechanisms $\langle x^*, \tau \rangle$ with an efficient allocation rule $x^*(\cdot)$ and a payment rule $\tau : \Theta \rightarrow \mathbb{R}^N$, for which the Bayesian incentive compatibility and interim individual rationality constraints can be written as

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[u_i(x^*(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i}) \right] \geq \mathbb{E}_{\tilde{\theta}_{-i}} \left[u_i(x^*(\theta'_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta'_i, \tilde{\theta}_{-i}) \right] \quad (\text{IC})$$

for all $\theta_i, \theta'_i \in \Theta$,

$$\mathbb{E}_{\tilde{\theta}_{-i}} \left[u_i(x^*(\theta_i, \tilde{\theta}_{-i}), \theta_i) + \tau_i(\theta_i, \tilde{\theta}_{-i}) \right] \geq u_i(\hat{x}, \theta_i) \text{ for all } \theta_i \in \Theta. \quad (\text{IR})$$

Note that the property rights allocation \hat{x} enters the IR constraints, determining agents' payoffs should bargaining break down (by an agent opting out of the mechanism).

⁵⁵Formally, we assume that the spaces $X, \Theta_1, \dots, \Theta_I$ are measurable, that X is a subset of a convex topological space, and that the functions $u_i : X \times \Theta_i \rightarrow \mathbb{R}$ are measurable and uniformly bounded (i.e., $\sup_{i,x \in X, \theta_i \in \Theta_i} |u_i(x, \theta_i)| < \infty$). The last two assumptions ensure that expectations exist and that the infima below are finite.

⁵⁶For analyses in which types are interdependent, see Fieseler et al. (2003), Jehiel and Palfrey (2006), and Segal and Whinston (2010). See Rahman (2010) for a model with correlated types.

It will prove convenient to focus on mechanisms with payments of the following form:

$$\tau_i(\theta|\hat{x}, \hat{\theta}_i) = \sum_{j \neq i} u_j(x^*(\theta), \theta_j) - K_i(\hat{x}, \hat{\theta}_i) \quad (11)$$

$$\text{where } K_i(\hat{x}, \hat{\theta}_i) = \mathbb{E}_{\tilde{\theta}_{-i}}[S(\hat{\theta}_i, \tilde{\theta}_{-i})] - u_i(\hat{x}, \hat{\theta}_i). \quad (12)$$

Note that these payments describe a Vickrey-Clarke-Groves (“VCG”) mechanism [see Mas-Colell, Whinston, and Green (1995), Chapter 23]. The variable portion of the payment, $\sum_{j \neq i} u_j(x^*(\theta), \theta_j)$, causes each agent i to fully internalize his effect on aggregate surplus, thereby inducing him to announce his true type and implementing the efficient allocation rule $x^*(\cdot)$. The fixed fee K_i , on the other hand, equals type $\hat{\theta}_i$ ’s expected gain from participating in the mechanism absent the fixed charge, so it causes that type’s IR constraint to hold with equality. If we imagine that there is an intermediary in charge of this trading process, its expected profit with this mechanism is given by

$$\pi(\hat{x}, \hat{\theta}) = \mathbb{E} \left[\sum_i \tau_i(\tilde{\theta}|\hat{x}, \hat{\theta}_i) \right] = \sum_i \mathbb{E}_{\tilde{\theta}_{-i}}[S(\hat{\theta}_i, \tilde{\theta}_{-i})] - s(\hat{x}, \hat{\theta}) - (I - 1) \mathbb{E}[S(\tilde{\theta})]. \quad (13)$$

To ensure that all types participate, the participation fee for each agent i can be at most $\inf_{\theta_i \in \Theta} K_i(\hat{x}, \theta_i)$, resulting in an expected profit for the intermediary of

$$\bar{\pi}(\hat{x}) \equiv \inf_{\hat{\theta} \in \Theta} \pi(\hat{x}, \hat{\theta}). \quad (14)$$

The sign of this expected profit determines whether property rights allocation \hat{x} permits efficient bargaining:⁵⁷

⁵⁷Versions of this result appear in Makowski and Mezzetti (1994), Krishna and Perry (1998), Neeman (1999), Williams (1999), Schweizer (2006), Che (2007), Figueroa and Skreta (2008), and Segal and Whinston (2010). Part (i) of the Lemma can be proven by building a budget-balanced mechanism as suggested by Arrow (1979) and d’Aspremont and Gérard-Varet (1979), and satisfying all agents’ participation constraints with appropriate lump-sum transfers. Part (ii) follows from the classical Revenue Equivalence Theorem.

Lemma 3 (i) Any property rights allocation $\hat{x} \in X$ at which $\bar{\pi}(\hat{x}) \geq 0$ permits efficient bargaining. (ii) If, moreover, for each agent i , Θ_i is a smoothly connected subset of a Euclidean space, and $u_i(x, \theta_i)$ is differentiable in θ_i with a bounded gradient on $X \times \Theta$, then property rights allocation $\hat{x} \in X$ permits efficient bargaining only if $\bar{\pi}(\hat{x}) \geq 0$.

4.1.2 Impossibility results

Lemma 3(ii) can be used, for example, to show that efficient bargaining is impossible in some cases. For instance, consider Example 7 when one agent i initially owns the asset. Taking $\hat{\theta} = (\hat{\theta}_i, \hat{\theta}_{-i}) = (1, 0)$ – the types who never find it efficient to trade – we have $S(\hat{\theta}_i, \theta_{-i}) = s(\hat{x}, \hat{\theta}) = 1$ for all θ_{-i} and $S(\theta_i, \hat{\theta}_{-i}) = \theta_i$ for all θ_i . Thus, $\bar{\pi}(\hat{x}) \leq \pi(\hat{x}, \hat{\theta}) = \mathbb{E}[\theta_i - S(\tilde{\theta})] < 0$ whenever trade is efficient with a positive probability.

This impossibility argument can be extended to a number of settings that share the feature that each agent has an “opt-out type” whom it is efficient to keep at his initial property rights allocation regardless of other agents’ types (if $N > 2$, it may still be efficient for the others to trade among themselves). Our exposition here draws on and extends unpublished work by Makowski and Ostroy (1989).

Let $X \subset \prod_i X_i$, where $x_i \in X_i$ represents the property rights allocation of agent i , and assume that there are no externalities at property rights \hat{x} , i.e., that $u_i(\hat{x}_i, x_{-i}, \theta_i)$ does not depend on x_{-i} . As in Section 3.5, define the cooperative value of each coalition $T \subset \mathcal{N}$ given \hat{x} when their types are θ_T as the maximum joint surplus it can attain while holding the outsiders at their status quos:

$$S_T(\hat{x}, \theta_T) = \sup_{x \in X: x_{-T} = \hat{x}_{-T}} \sum_{i \in T} u_i(x, \theta_i).$$

Definition 20 Given property rights $\hat{x} \in X$, type $\theta_i^\circ \in \Theta_i$ of agent i is an **opt-out type** if there is an efficient allocation rule $x^*(\cdot)$ such that $x_i^*(\theta_i^\circ, \theta_{-i}) = \hat{x}_i$ for all $\theta_{-i} \in \Theta_{-i}$.

In words, given property rights $\hat{x} = (\hat{x}_i, \hat{x}_{-i})$, agent i ’s type θ_i° is an opt-out type if regardless of θ_{-i} it is efficient for agent i to get his initial ownership allocation \hat{x}_i . Note that

when an opt-out type θ_i° exists, this is always the type that is least willing to participate in the above mechanism (often called the agent’s “critical type”), i.e., the type achieving the infimum in (14). (Indeed, any other type could get this type’s expected participation surplus, $\mathbb{E}_{\tilde{\theta}_{-i}}[\tau_i(\theta_i^\circ, \tilde{\theta}_{-i})]$, by reporting θ_i° and receiving allocation \hat{x}_i .) Also, in this case we can write $S(\theta_i^\circ, \theta_{-i}) = S_{\mathcal{N}\setminus\{i\}}(\hat{x}, \theta_{-i}) + u_i(\hat{x}, \theta_i^\circ)$. Therefore, when all agents have opt-out types, the expected profits (14) can be rewritten as

$$\bar{\pi}(\hat{x}) = \mathbb{E} \left[S(\tilde{\theta}) - \sum_i \left(S(\tilde{\theta}) - S_{\mathcal{N}\setminus\{i\}}(\hat{x}, \tilde{\theta}_{-i}) \right) \right]. \quad (15)$$

Thus, $\bar{\pi}(\hat{x})$ is exactly the expected profit in a version of the Clarke pivotal mechanism [see Mas-Colell, Whinston, and Green (1995), Chapter 23] that accounts for agents’ property rights: Each agent i receives a payoff equal to his marginal contribution to the total surplus, $S(\theta) - S_{\mathcal{N}\setminus\{i\}}(\hat{x}, \theta_{-i})$, and the intermediary collects the total surplus net of these payoffs. With opt-out types, the possibility of efficient bargaining is determined by the sign of this expression. This profit can be shown to be negative in a number of economic settings:

Definition 21 *Given initial property rights allocation \hat{x} , $w \in \mathbb{R}^N$ is a core payoff vector in state θ if $\sum_{i \in T} w_i \geq S_T(\hat{x}, \theta_T)$ for all $T \subset \mathcal{N}$, with equality when $T = \mathcal{N}$.*

Proposition 15 *If at some property rights allocation $\hat{x} \in X$ there are no externalities, each agent has an opt-out type, and the core is non-empty in all states and contains multiple payoffs with a positive probability, then this property rights allocation does not permit efficient bargaining.*

Proof. If w is a core payoff vector in state θ then using the core conditions for $T = \mathcal{N}$ and $T = \mathcal{N} \setminus \{i\}$ for all i ,

$$\sum_i (S(\theta) - S_{\mathcal{N}\setminus\{i\}}(\hat{x}, \theta_{-i})) \geq \sum_i \left(\sum_j w_j - \sum_{j \neq i} w_j \right) = \sum_i w_i = S(\theta).$$

Furthermore, if there exist two distinct core payoff vectors in state θ , then in at least one of them the core inequality holds strictly for $T = \mathcal{N} \setminus \{i\}$ for some i , so the inequality in the

above display is strict. Hence, under the assumptions of the proposition the expected profit in (15) is negative. ■

The assumptions of Proposition 15 cover many classical economic settings. For one example, consider the double-auction setting of Williams (1999), in which there are N_s sellers with values drawn from a distribution on $[\underline{\theta}_s, \bar{\theta}_s]$ and N_b buyers with values drawn from a distribution on $[\underline{\theta}_b, \bar{\theta}_b]$ with $(\underline{\theta}_b, \bar{\theta}_b) \cap (\underline{\theta}_s, \bar{\theta}_s) \neq \emptyset$. Note that (i) a buyer of type $\underline{\theta}_b$ is an opt-out type if either $\underline{\theta}_b \leq \underline{\theta}_s$ or $N_b > N_s$, and (ii) a seller of type $\bar{\theta}_s$ is an opt-out type if either $\bar{\theta}_s \geq \bar{\theta}_b$ or $N_s > N_b$. Moreover, a competitive equilibrium exists in every state and is not unique with a positive probability. Since a competitive equilibrium is always in the core, Proposition 15 applies when both (i) and (ii) hold.⁵⁸

Proposition 15 also applies to the public good setting of Mailath and Postlewaite (1990), in which each of N consumers' values is drawn from a distribution on $[0, \bar{\theta}]$, the cost of provision is $c > 0$, and the status quo property right is no provision ($\hat{x}_i = 0$ for all i). Letting $x_i \in \{0, 1\}$ denote whether agent i is given access to the public good, and assuming a default of equal cost-sharing among the agents who have access to it, we have $u_i(x, \theta_i) = \theta_i - c / \sum_j x_j$ if $x_i = 1$ and $= 0$ otherwise. Thus, when $\hat{x} = 0$ we have no externalities and each agent's type 0 is an opt-out type. Note that a Lindahl equilibrium exists in every state and is not unique with a positive probability. Since a Lindahl equilibrium is in the core, Proposition 15 applies.⁵⁹

⁵⁸The argument can also be extended to show impossibility whenever $N_b = N_s$. In this case, note that in an efficient allocation any agent of type below $\underline{\theta} \equiv \max\{\underline{\theta}_s, \underline{\theta}_b\}$ receives an object with probability zero, so is therefore indistinguishable from type $\underline{\theta}$, and any agent of type above $\bar{\theta} \equiv \min\{\bar{\theta}_s, \bar{\theta}_b\}$ receives an object with probability one, so is therefore indistinguishable from type $\bar{\theta}$. Therefore, the profit in the mechanism must be the same as if all agents' types were instead distributed on the same interval $[\underline{\theta}, \bar{\theta}]$ (with possible atoms at its endpoints), in which case efficient bargaining is impossible by the argument in the text.

⁵⁹Proposition 15 does not address the extent of bargaining inefficiencies or the form they take. These questions have been studied in a number of papers. Myerson and Satterthwaite (1983) and McKelvey and Page (2002) find that in certain settings the inefficiencies exhibit a "status-quo bias": the final allocation

4.1.3 Efficiency-permitting property rights allocations

We now use Lemma 3 to investigate when efficiency is possible and which property rights allocations permit efficient bargaining. Segal and Whinston (2010) establish the following result:

Proposition 16 *Suppose that the total surplus $s(x, \theta)$ is convex in x for all $\theta \in \Theta$ and that $x^* : \Theta \rightarrow X$ is an efficient allocation rule. Then the initial property rights allocation $\hat{x} = \mathbb{E}[x^*(\tilde{\theta})] \in \hat{X}$ equal to the expected efficient allocation permits efficient bargaining.*

For example, in the Cramton, Gibbons, and Klemperer (1987) model, in which all agents' valuations are drawn from the same distribution and their utilities are linear in the allocation x , the expected efficient allocation involves giving each agent an equal ownership share. But the result applies to cases with asymmetric valuations and to much more general allocation problems. Indeed, the result generalizes a number of previous results in the literature that establish the existence of an efficiency-permitting property rights allocation [Cramton, Gibbons, and Klemperer (1987), Schmitz (2002), Che (2006), Schweizer (2006), Yenmez (2007), Gershkov and Schweitzer (2008)] and also points to a natural property rights allocations that achieves this goal.⁶⁰

To understand the result, recall that – ignoring IR – an efficient ex post allocation can be achieved with a VCG mechanism. Furthermore, as noted by Arrow (1979) and

lies between the initial and efficient allocations. Other papers examine the dependence of inefficiency on the number of agents. In the double-auction setting, Gresik and Satterthwaite (1989) find that the inefficiency in an ex ante optimal mechanism, shrinks to zero as $N_b, N_s \rightarrow \infty$. Intuitively, this relates to the fact that the core converges (in probability) to the unique competitive equilibrium of the continuous limit economy, hence in the limit the agents can fully appropriate their marginal contributions [as in Makowski and Ostroy (1989, 1995, 2001)]. In contrast, in the public good setting of Mailath and Postlewaite (1990), the core grows in relative size as $N \rightarrow \infty$, and inefficiency is exacerbated (in fact, the probability of providing the public good in any mechanism goes to zero).

⁶⁰In some cases, efficient noncontractible actions can also be sustained with this property rights allocation; see the discussion in Segal and Whinston (2010), which extends an insight of Rogerson (1992).

d'Aspremont and Gérard-Varet (1979), the transfers in this mechanism can be modified to yield a balanced-budget mechanism (the “expected externality” mechanism) while preserving agents’ incentives for truth-telling (IC). Observe, however, that one lie agent i could make is to announce randomly each of his possible types with their true probabilities. Doing so results in a random allocation with the same distribution as the efficient allocation. Since the agent never has an incentive to lie, this random outcome must be worse for every type of agent i than what he gets in the mechanism. Now, if agent i ’s payoff is convex in the allocation, the expected efficient allocation must be even worse for him than this random outcome, ensuring IR is satisfied when the initial property rights allocation is the expected efficient allocation. Using Lemma 3(i), this conclusion can be extended to cases in which only the *sum* of the agents’ payoffs, the surplus $s(x, \theta)$, is convex in the allocation x .

Proposition 16 tells us that, in a wide range of circumstances, the expected efficient property rights allocation permits efficient bargaining. Yet, as originally seen in the Cramton, Gibbons, and Klemperer (1987) model, when efficient bargaining is possible there are typically many property rights allocations that allow it. Indeed, this typically follows whenever $\bar{\pi}(\mathbb{E}[x^*(\tilde{\theta})]) > 0$ since $\bar{\pi}(\hat{x})$ is continuous in \hat{x} as long as each $u_i(\cdot)$ is equicontinuous in x .⁶¹ Note, in particular, that the expected efficient allocation typically differs from the property rights allocation that maximizes the intermediary’s profit, $\hat{x}^* = \arg \max_{\hat{x} \in X} \bar{\pi}(\hat{x})$. The latter allocation is studied by Schmitz (2002, proof of Proposition 3), Che (2006), Schweizer (2006), and Figueroa and Skreta (2008).

When the space of feasible property rights X is not convex, it may not contain an expected efficient allocation $\mathbb{E}[x^*(\tilde{\theta})]$, in which case Proposition 16 does not apply and there may not exist a feasible property rights allocation permitting efficient bargaining. For example, in the setting of Example 7, when only extreme property rights allocations are feasible (agent

⁶¹Moreover, whenever the convexity assumptions of Proposition 16 hold, the intermediary’s expected profit function $\bar{\pi}(\hat{x})$ is concave in \hat{x} , implying that the set \hat{X}^* of property rights allocations permitting efficient bargaining is a nonempty convex set.

1 ownership or agent 2 ownership), neither permits efficient bargaining.⁶² Even with a convex allocation space X , the assumption of convexity of the total surplus $s(x, \theta)$ in the allocation x is also crucial. Neeman (1999, p. 685) offers an example of pollution in which the convexity assumption fails and there does not exist an efficiency-permitting property rights allocation.⁶³ On the other hand, Segal and Whinston (2010) offer examples with concave payoffs in which efficient bargaining is still permitted by some property rights allocations (which may or may not include the expected efficient allocation).

Applying the Envelope Theorem to (14) using (13), we can see that $\nabla \bar{\pi}(\hat{x}) = -\nabla_x s(\hat{x}, \hat{\theta})$, so changes in property rights that reduce surplus at the critical types make efficient negotiation [which requires $\bar{\pi}(\hat{x}) \geq 0$] more likely. This highlights an important difference between the role of property rights when we are concerned with bargaining efficiency under asymmetric information and their role when we are concerned with noncontractible investments. In the Hart and Moore (1990) model with noncontractible investments, increased ownership is good in that it encourages investments, which are in general below efficient levels. With asymmetric information bargaining, however, ownership is an impediment (as it was with cooperative investments in Section 3.4.2): efficiency is easiest to achieve when the parties have no claims at all to useful assets. Thus, for example, splitting up the ownership of complementary assets so that no one can use them in the event of a disagreement can enable efficient bargaining.⁶⁴

⁶²For other examples with a nonconvex decision space X in which there is no property rights allocation permitting efficient bargaining, see Ornelas and Turner (2007) and Turner (2007).

⁶³When simple property rights allocations cannot lead to efficiency, more complicated property rights mechanisms might be optimal. The legal literature, for example, has discussed the benefits of liability rules – which can be interpreted as options to own – relative to simple property rights [Ayres and Talley (1995), Ayres (2005), and Kaplow and Shavell (1995-6)].

⁶⁴The benefit of having a default outcome at which the total surplus is low also arises in self-enforcing contracts where the default is used as an off-equilibrium punishment, as discussed in Chapter ?? of this Handbook.

Of course, the optimality of minimizing the disagreement surplus relies on the optimistic assumption that interim bargaining procedures are the best possible from the ex ante viewpoint.⁶⁵ Under the reverse assumption that bargaining is completely impossible (the Simple Allocation Benchmark), it is instead optimal to maximize the expected disagreement surplus. Matouschek (2004) considers a hybrid model in which ex post bargaining is possible over some decisions, but not over others. One agent is initially the owner of an asset x and there are later trade decisions q . (There are no noncontractible actions a .) Ownership of asset x cannot be renegotiated, while trade decisions q can only be negotiated ex post. The question is how ownership of the asset should be allocated ex ante. He finds that, depending on the parameters, the optimal property rights x will either maximize the total surplus at the disagreement point (as if no renegotiation were possible) or minimize it (as if renegotiation were possible over both x and q).

4.2 Bilateral Contracting with Externalities

Even if bargaining over the ex post allocation of property rights occurs under symmetric information, inefficiency may nonetheless arise if there are more than two parties bargaining and they cannot write a single “grand” contract. When agents can write only bilateral contracts (that is, make bilateral trades), they may fail to internalize externalities from their trades on others, yielding inefficient outcomes. In this section, we examine how the initial allocation of property rights affects their final allocation in such settings.

The following example shows one situation in which such externalities can arise.

⁶⁵One sense in which the approach above may be too optimistic is in the IR and IC constraints it assumes. Since an agent may be able to walk away from a deal at the last moment, an ex post IR constraint may make more sense. Galavotti (2008) identifies some cases in which efficiency can be achieved when ex post IR is required. For example, in the setting of Example 7, he shows that equal ownership shares permits efficient bargaining in this stronger (ex post IR) sense. Compte and Jehiel (2006) suggest that an even stronger constraint might be appropriate if agents can lie and then quit.

Example 8 (Common Agency Model) Consider an extension of the Agency Model (Example 1) to the case of $N \geq 2$ principals. Suppose that the asset is divisible, and let $x_i \in [0, 1]$ denote the share of the asset held by each principal i , with the remaining share $1 - \sum_{i=1}^N x_i$ held by the agent. Suppose that the asset's value to the principals and the agent, $V_P \left(1 - \sum_{i=1}^N x_i\right)$ and $V_A \left(1 - \sum_{i=1}^N x_i\right)$ respectively, depend on the agent's share. This dependence captures in reduced form the effects of the entrepreneur's ownership on his noncontractible actions affecting the asset's value. For instance, as in Example 1, holding a larger share of the asset may better incentivize the agent to enhance its value. Alternatively, the agent's control rights stemming from his higher share may enable him to engage in value-reducing activities (such as looting, empire-building, or resisting value-enhancing takeovers). The agent's resulting inalienable costs (or negative benefits) from these activities are represented by $C \left(1 - \sum_{i=1}^N x_i\right)$. As for the principals' inalienable benefits (Y), we assume them to be zero. Therefore, the parties' payoffs exclusive of transfers take the form

$$U_A(x_1, \dots, x_N) = \left(1 - \sum_{i=1}^N x_i\right) V_A \left(1 - \sum_{i=1}^N x_i\right) - C \left(1 - \sum_{i=1}^N x_i\right),$$

$$U_P(x_1, \dots, x_N) = x_i V_P \left(1 - \sum_{i=1}^N x_i\right) \text{ for } i \geq 1.$$

These payoffs exhibit externalities from asset ownership whenever the asset's public value V_P depends on the agent's ownership share. By selling or buying shares from a principal, the agent imposes externalities on all the other principals.

Externalities from bilateral contracts may arise from many underlying forces other than that identified in the example. For example, they may arise when the parties subsequently bargain and the initial bilateral contracts affect the division of subsequent bargaining surplus [see, e.g., Segal (2003a), Gans (2005), and Elliott (2009)]. [Many other examples of externalities are discussed in Segal (1999b).] Here, we incorporate the externalities in reduced-form payoffs, subsuming whatever noncontractible actions or bargaining that generates these externalities.

Many possible bargaining procedures could arise in these settings. Here, we focus on the very simple contracting game considered in Segal (1999b), in which one agent makes simultaneous take-it-or-leave-it bilateral contracting offers to $N \geq 2$ other agents. This game illustrates the role of property rights in these environments, although it lacks some of the richness of outcomes that arise in more dynamic processes of bargaining with externalities that have been studied, e.g., by Jehiel and Moldovanu (1999), Bloch and Gomes (2006), and Gomes and Jehiel (2005).⁶⁶

A key feature of this contracting game turns out to be whether commitment to a set of publicly-observed contract offers is possible – i.e., whether the offers are publicly or privately made.

4.2.1 Public Offers

Suppose that one party, agent 0 (the “proposer”), can contract with N other agents labelled $i = 1, \dots, N$ (the “responders”). Each responder i ’s final property rights allocation is denoted by $x_i \in X_i$, and the allocation $x = (x_1, \dots, x_N) \in X \subset \prod_{i=1}^N X_i$ determines each agent’s nonmonetary utility $U_i(x)$ for $i = 0, 1, \dots, N$ (the proposer’s property rights are the residual available given those of the responders). The initial allocation of property rights, the status quo, is $\hat{x} \in X$. Let X^* denote the set of allocations maximizing the total surplus of the $N + 1$ parties, $\sum_{i=0}^N U_i(x)$.

In this subsection we analyze the following two-stage game: In the first stage, agent 0 publicly proposes a pair (x_i, t_i) to each other agent i , naming the responder’s final property rights allocation and a transfer. We will sometimes refer to these as the proposed “trades.” In the second stage, those agents simultaneously respond, either accepting or rejecting their re-

⁶⁶Much of the literature on bargaining inefficiencies under full information takes a coalitional approach – see Ray (2007) for a survey. This approach allows each agent to enter into one contract (coalition) at a time, while in Example 8 and the model below one of the agents agent could simultaneously write separate contracts with different agents.

spective offers. We study the proposer’s preferred Subgame-Perfect Nash Equilibria (SPNE) of the game.⁶⁷

Since the proposer can always make the status-quo offer $(x_i, t_i) = (\hat{x}_i, 0)$ to agent i , without loss of generality we can restrict attention to equilibria in which all offers are accepted, which occurs if and only if

$$U_i(x) + t_i \geq U_i(\hat{x}_i, x_{-i}) \text{ for all } i \in 1, \dots, N. \quad (16)$$

The right-hand side of (16) is the reservation utility of responder i , i.e., the utility he gets by rejecting his offer, provided that everyone else accepts. In the proposer’s preferred SPNE, all responders’ participation constraints must bind (otherwise she could reduce some payments while preserving the constraints). Expressing transfers from the binding constraints and substituting them in the proposer’s objective function, her profit-maximizing property rights offers, given the initial property rights allocation \hat{x} , can be defined as

$$X_{pub}^o(\hat{x}) \equiv \arg \max_{x \in X} \sum_{i=0}^N U_i(x) - \sum_{i=1}^N U_i(\hat{x}_i, x_{-i}). \quad (17)$$

Note that the proposer’s objective function differs from the total surplus by its last term, the sum of the responders’ reservation utilities, which depends on the initial property rights allocation. As a result, the proposer’s incentive to deviate from efficiency arises precisely when she can harm the other agents at their status quos. This observation leads to the following definition and result:⁶⁸

Definition 22 *There are no harmful externalities on agent $i = 1, \dots, N$ at allocation $x \in X$ if $U_i(x) \leq U_i(x_i, x'_{-i})$ for all $x'_{-i} \in X_{-i}$; these externalities are absent if $U_i(x_i, x'_{-i})$ does not depend on $x'_{-i} \in X_{-i}$.*

⁶⁷Under some conditions, the game may have multiple SPNE, and the equilibrium preferred by the responders may differ from that preferred by the proposer. See Segal (2003b) for conditions in which multiple equilibria arise and analysis of equilibria preferred by the responders.

⁶⁸Propositions 17 and 19 generalize results from Segal (1999) to the case of nonharmful externalities.

Proposition 17 *If $x^* \in X^*$ and there are no harmful externalities on each agent $i \geq 1$ at allocation (\hat{x}_i, x_{-i}^*) , then $x^* \in X_{pub}^\circ(\hat{x}) \subset X^*$.⁶⁹ If externalities on all agents are absent at the status quo \hat{x} , then $X_{pub}^\circ(\hat{x}) = X^*$.*

Proof. For each $x^\circ \in X_{pub}^\circ(\hat{x})$, condition (17) implies that

$$\sum_{i=0}^N U_i(x^\circ) - \sum_{i=1}^N U_i(\hat{x}_i, x_{-i}^\circ) \geq \sum_{i=0}^N U_i(x^*) - \sum_{i=1}^N U_i(\hat{x}_i, x_{-i}^*). \quad (18)$$

By non-harmful externalities, $U_i(\hat{x}_i, x_{-i}^\circ) \leq U_i(\hat{x}_i, x_{-i}^*)$ for all $i \geq 1$, hence (18) implies that $x^\circ \in X^*$. Moreover, for any such $x^* \in X^*$, non-harmful externalities implies that

$$\sum_{i=0}^N U_i(x^*) - \sum_{i=1}^N U_i(\hat{x}_i, x_{-i}^*) \geq \sum_{i=0}^N U_i(x') - \sum_{i=1}^N U_i(\hat{x}_i, x'_{-i}) \text{ for all } x' \in X,$$

so $x^* \in X_{pub}^\circ(\hat{x})$. Finally, if externalities are absent at \hat{x} , then they are non-harmful at (\hat{x}_i, x_{-i}^*) for any $x^* \in X^*$, which implies $X^* \subset X_{pub}^\circ(\hat{x}) \subset X^*$. ■

The result says that when, given initial property rights \hat{x} , the proposer can maximize the total surplus while at the same time minimizing the agents' reservation utilities, this is optimal for her, and any equilibrium outcome must do it. For instance, in the Common Agency Model (Example 8), externalities are absent when the agent is the proposer and initially owns the entire asset ($\hat{x} = 0$).

On the other hand, when externalities are present at any status quo, efficiency cannot be attained since the proposer has an incentive to distort the allocation to reduce the sum of agents' reservation utilities. Segal (1999) shows that the effect of this rent-seeking motivation on the contracting outcome x depends on the sign of these externalities: when efficiency depends only on the aggregate allocation $\sum_i x_i$ as in Example 8 (and a domain restriction holds), positive (resp. negative) externalities on reservation utilities imply that the allocation is too low (high).^{70,71} Note in particular that if the status quo is efficient but has externalities,

⁶⁹It is enough for this result to assume that $\sum_{i=1}^N U_i(\hat{x}_i, x_{-i})$ is minimized at $x = x^*$.

⁷⁰Hence, note that increasing the status quo in Example 8 from $\hat{x} = 0$ to $\hat{x} > 0$ will increase or decrease the final allocation depending on the sign of externalities at \hat{x}

⁷¹This inefficiency is eliminated if the principal can make an offer conditional on the acceptance of all

the final allocation may be inefficient, so bargaining may actually reduce aggregate welfare.

4.2.2 Private Offers

Now consider what happens when the proposer cannot publicly commit. A number of contracting games in which the principal does not have commitment power have been considered in the literature. Here we simply modify the observability in the two-stage game to make the offers private: Each responder $i = 1, \dots, N$ observes the offer (x_i, t_i) made to him but not the offers made to the other agents. [Note that since an equilibrium outcome must still satisfy participation constraints (16), this private observability can only hurt the proposer, agent 0.]

Each responder's acceptance decision in this game depends on his beliefs about offers extended to other agents. In a Perfect Bayesian Equilibrium, arbitrary beliefs can be assigned following the proposer's out-of-equilibrium offers, which gives rise to enormous multiplicity of equilibria. To make a more precise prediction, Segal (1999b) follows Cremer and Riordan (1987), Hart and Tirole (1990), and McAfee and Schwartz (1994) in assuming so-called "passive beliefs": even after observing an unexpected offer from the proposer, a responder continues to believe that other responders have received their equilibrium offers.

Consider the proposer's incentive to deviate from an equilibrium allocation x° . Since she can always make agent i the offer $(x_i, t_i) = (\hat{x}_i, 0)$, without loss we can restrict attention to deviations in which all responders accept their offers. When responder i holds passive beliefs, he accepts an offer (x_i, t_i) if and only if $U_i(x_i, x_{-i}^\circ) + t_i \geq U_i(\hat{x}_i, x_{-i}^\circ)$. The proposer's optimal deviation maximizes her payoff subject to these participation constraints, which always bind at her optimal offers. Expressing transfers from the binding constraints and substituting agents, while implementing maximal punishment following anyone's rejection (as with conditional tender offers in takeovers). However, Segal (1999) shows that when the number of agents is large and their acceptances are subject to noise, the power of such conditional offers to make individual agents pivotal vanishes, and the same inefficiencies obtain as with bilateral contracting.

them in the objective function, we find that x° is an equilibrium allocation x° if and only if

$$x^\circ \in \arg \max_{x \in X} U_0(x) + \sum_{i=1}^N U_i(x_i, x_{-i}^\circ). \quad (19)$$

Letting X_{pr}° denote the set of property rights x° satisfying (19), our first observation is:⁷²

Proposition 18 *The set X_{pr}° of equilibrium property rights allocations with private offers does not depend on the initial property rights allocation \hat{x} .*

While this irrelevance of initial property rights is reminiscent of the Coase Theorem, the contracting outcome with externalities may be inefficient. This inefficiency can be traced to externalities at efficient allocations. When externalities at an efficient allocation are non-harmful, private contracting yields efficient outcomes, regardless of any externalities at other allocations:

Proposition 19 *If there exists $x^* \in X^*$ at which externalities are non-harmful, then $X_{pr}^\circ \subset X^*$.*

Proof. For any $x^\circ \in X_{pr}^\circ$, condition (19) and non-harmful externalities at x^* imply that

$$\sum_{i=0}^N U_i(x^\circ) \geq U_0(x^*) + \sum_{i=1}^N U_i(x_i^*, x_{-i}^\circ) \geq \sum_{i=0}^N U_i(x^*).$$

Therefore, $x^\circ \in X^*$. ■

For an application, consider the Common Agency Model (Example 8) under the assumption that it is efficient to have $x = 0$ (full agent ownership). Then any contracting outcome with private offers must be efficient, regardless of the property rights allocation. Contrast

⁷²The condition implies (but is stronger than) the condition $x_i^\circ \in \arg \max_{x_i \in X_i: (x_i, x_{-i}^\circ) \in X} [U_0(x_i, x_{-i}^\circ) + U_i(x_i, x_{-i}^\circ)]$ for all $i \geq 1$, which is called “contract equilibrium” by Cremer and Rioridan (1987) and “pairwise proofness” by McAfee and Schwartz (1994). The non-emptiness of X_{pr}° is only ensured under additional assumptions [see Segal (1999)’s Appendix B], and all of the subsequent results will be vacuous (but formally correct) when X_{pr}° is empty.

this to the public offers setting where the bargaining outcome may be inefficient due to externalities at the initial property rights allocation.

When externalities *are* present at all efficient allocations, they must distort the contracting outcome. Segal (1999b) shows that when efficiency depends only on the aggregate allocation $\sum_i x_i$ (and a domain restriction holds) this aggregate allocation is too low (resp. high) when there are positive (resp. negative) externalities at any efficient allocation.⁷³ To compare this outcome to that under public offers, note that, with private offers and passive beliefs, the proposer can no longer influence the responders' reservation utilities, but she also ceases to internalize the externalities on responders' equilibrium payoffs from changes in the proposed allocation. So the non-internalized externalities are now those on equilibrium allocations rather than the status-quo allocation. Depending on the comparison of these externalities, one contracting regime may be more or less efficient than the other.

While we have focused on a simple two-stage contracting game, similar results on bargaining inefficiencies have been obtained for infinite-horizon games with frequent recontracting. Thus, in Jehiel and Moldovanu (1999) and Gomes and Jehiel (2005), any allocation can be renegotiated very soon, and due to this lack of commitment the game's equilibrium outcomes are similar to those of the two-stage game with private offers (in particular, they obtain results similar to Propositions 18 and 19). On the other hand, Bloch and Gomes (2006) consider a game in which agents can exercise irreversible outside options during bargaining (as in Section 3.6.3), and the inefficiencies are due to externalities on these outside options, similarly to the two-stage public-offers game.

⁷³Segal and Whinston (2003) show that the proposer may be able to benefit from offering more complicated mechanisms (specifically, menus from which the proposer will later choose) because of their ability to influence responders' beliefs. They show that in many cases in which externalities are present at the efficient property rights allocation inefficiencies must arise in all equilibria when such menus can be used.

5 Conclusion

The importance of property rights for reducing externalities, improving incentives, and achieving more efficient outcomes has been noted for centuries. However, the subject of property rights received renewed attention following the suggestion of Grossman and Hart (1986) and Hart and Moore (1990) that property rights over assets can be used to define boundaries of firms. Since then, much progress has been made in applying game-theoretic tools to study property rights, which has been surveyed in this chapter. Still, much remains to be learned. Notable areas where more work would be useful include further results about their efficient allocation when the first best cannot be achieved (e.g., the comparative statics of second-best property rights in the Hart-Moore model both with self and cooperative investments, optimal property rights when efficient bargaining cannot be achieved), when both ex ante investments and imperfect bargaining are present, and when other instruments such as incentive contracts can also be used [as in Holmstrom and Milgrom (1994)]. In addition, work that examines the determinants of optimal property rights in models with more structure on the actual decisions being made [such as in Alonso et al. (2008) and Rantakari (2008)] would be welcome.

A better understanding of the reasons property rights are used instead of more complete contracts is of great importance. In most existing models of property rights, they can be improved upon by more complicated contracts. The literature on “foundations of incomplete contracts” (discussed in Section 3.6) identified some assumptions under which more complicated mechanisms cannot improve upon simple property rights. However, this literature also finds that the optimal arrangement is sensitive to very fine technological details, such as the degree to which choices are irrevocable or which mechanisms would be enforced by courts. In reality, these details may not be known with great precision to market participants.⁷⁴ One advantage of simple contracts such as property rights over more complicated contracts may

⁷⁴They are even less likely to be known to economists, which would make testing these theories difficult.

be that their performance could be more robust to the features of the environment. This robustness is particularly important for complex environments. On the one hand, in complex environments the agents may have many ways to game a complicated contract, so simple contracts can emerge as being optimal [e.g., Segal (1999a)]. On the other hand, complicated contracts in complex environments may be prohibitively costly to write or execute, because of contracting or communication costs [e.g., Segal (1996)] or cognitive limitations.

Another issue is that property rights over assets are only one of many ways incentives are provided in large firms or other organizations. Other incentive instruments include decision rights that are not tied to assets (e.g., authority over people – see Bolton-Dewatripont chapter), compensation schemes (Mookherjee chapter), relational incentives (Gibbons and Henderson Chapter, Malcomson Chapter), and promotions (Waldman Chapter). One view is that in large firms, property rights do not play a central role. For example, shareholders hold property rights, but effective rights of control seem to really reside with top management. One reason for this may be that managers’s superior information gives them “real authority” over the decisions [Aghion and Tirole (1997)]. On the other hand, changes in the boundaries of large firms – that is, changes in property rights – often seem to lead to changes in many of these other instruments. For example, when one firm acquires another, which firm is the acquirer does appear to matter – e.g., the CEO of the acquired firm, while often given a top job at the time of acquisition, often seems to leave unhappily not long after, and the business of the acquired firm often withers. So somehow property may be mattering, but filtered through the decision rights that are conveyed to management. (On the other hand, the identity of the acquirer may matter simply because its managers are in control.)

Finally, it is important to understand how the predictions of the theories outlined here match with the facts, concerning both the effects of property rights on incentives and the determination of property rights. While different theories do give rise to distinct predictions, the distinctions often hinge on features such as the relative importance of various kinds

of investments that can be difficult to assess empirically [see, for example, the discussion in Whinston (2003)]. In particular, while there exists a substantial literature testing the predictions of “transactions cost economics” [see the Tadelis Chapter], only recently did there emerge papers that try to test some of theories surveyed in this chapter [e.g., Baker and Hubbard (2003, 2004), Elfenbein and Lerner (2003)]. Much more work on this front is needed.

References

- [1] Aghion, P., M. Dewatripont, and P. Rey (1994), “Renegotiation Design with Unverifiable Information,” *Econometrica* 62: 257-82.
- [2] Aghion, P., M. Dewatripont, and P. Rey (2004), “Transferable Control,” *Journal of the European Economic Association* 2: 115-38.
- [3] Aghion, P. and J. Tirole (1997), “Formal and Real Authority in Organizations,” *Journal of Political Economy* 105: 1-29.
- [4] Alonso, R., W. Dessein, and N. Matouschek (2008), “When Does Coordination Require Centralization,” *American Economic Review* 98: 145-79.
- [5] Areeda, P. and L. Kaplow (1988), *Antitrust Analysis, Problems, Texts, Cases, 4th Edition*, Boston: Little Brown.
- [6] Aristotle, Politics. Translated by B. Jowett (1885), *The Politics of Aristotle: Translated into English with Introduction, Marginal Analysis, Essays, Notes, and Indices*, Oxford, UK: Clarendon Press.
- [7] Arrow, K. (1979), “The Property Rights Doctrine and Demand Revelation under Incomplete Information,” In *Economics and Human Welfare*, Academic Press, New York, NY.

- [8] Ayres, I. (2005), *Optional Law: The Structure of Legal Entitlements*, University of Chicago Press.
- [9] Ayres, I. and E. Talley (1995), “Solomonic Bargaining: Dividing a Legal Entitlement To Facilitate Coasean Trade,” *Yale Law Journal* 104: 1027-117.
- [10] Baker, G., R. Gibbons, and K.J. Murphy (2002), “Relational Contracts and the Theory of the Firm,” *Quarterly Journal of Economics* 117: 39-84.
- [11] Baker, G., R. Gibbons, and K.J. Murphy (2008), “Strategic Alliances: Bridges Between ‘Islands of Conscious Power’,” *Journal of the Japanese and International Economies*, 22: 146-63.
- [12] Baker, G. and T. Hubbard (2003), “Make Versus Buy in Trucking: Asset Ownership, Job Design, and Information” *American Economic Review* 3: 551-72.
- [13] Baker, G., and T. Hubbard (2004), “Contractibility and Asset Ownership: On-Board Computers and Governance in U.S. Trucking,” *Quarterly Journal of Economics* 4: 1443-80.
- [14] Baumol, W.J., and W.E. Oates (1988), *The Theory of Environmental Policy (Second Edition)*, Cambridge, UK: Cambridge University Press.
- [15] Buzard, K., and J. Watson (2010), “Contract, Renegotiation, and Hold Up: Results on the Technology of Trade and Investment,” working paper, UC San Diego.
- [16] Bloch, F. and A. Gomes (2006), “Contracting with externalities and outside options,” *Journal of Economic Theory* 127: 172-201.
- [17] Bolton, P. and M.D. Whinston (1993), “Incomplete Contracts, Vertical Integration, and Supply Assurance,” *Review of Economic Studies* 60: 121-48.

- [18] Brandenburger, A., and H. Stuart (2007a), “Biform Games,” *Management Science* 53: 537-49.
- [19] Brandenburger, A., and H. Stuart (2007b), “Creating Monopoly Power,” *International Journal of Industrial Organization* 25: 1011-25.
- [20] Cai, H. (2003) “A Theory of Joint Asset Ownership,” *RAND Journal of Economics* 34: 63-77
- [21] Che, Y.-K., and D. Hausch (1999), “Cooperative Investments and the Value of Contracting,” *American Economic Review* 89: 125-47.
- [22] Che, Y.-K. (2006), “Beyond the Coasian Irrelevance: Asymmetric Information,” unpublished notes.
- [23] Chiu, Y. Stephen, (1998), “Noncooperative Bargaining, Hostages, and Optimal Asset Ownership,” *American Economic Review* 88: 882-901.
- [24] Coase, Ronald H. (1960), “The Problem of Social Cost,” *Journal of Law and Economics* 3: 1-69.
- [25] Chung, T.-Y. (1991), “Incomplete Contracts, Specific Investments, and Risk-Sharing,” *Review of Economic Studies* 58: 1031-42.
- [26] Cramton, P., R. Gibbons, and P. Klemperer (1987), “Dissolving a Partnership Efficiently,” *Econometrica*, 55: 615-32.
- [27] d’Aspremont, C., and L.A. Gérard-Varet (1979), “Incentives and Incomplete Information,” *Journal of Public Economics* 11: 25-45.
- [28] Crémer, J., and M.H. Riordan (1987), “On Governing Multilateral Transactions with Bilateral Contracts,” *RAND Journal of Economics* 18: 436-51.

- [29] De Meza, D., and B. Lockwood (1998), “Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm,” *Quarterly Journal of Economics* 113: 361-86.
- [30] Demski, J.S. and D.A. Sappington (1991), “Resolving Double Moral Hazard Problems with Buyout Agreements,” *RAND Journal of Economics* 22: 232-40.
- [31] Edlin, A. and B. Hermalin (2000), “Contract Renegotiation and Options in Agency Problems,” *Journal of Law, Economics, and Organization* 16: 395-423.
- [32] Edlin, A., and S. Reichelstein (1996), “Holdups, Standard Breach Remedies, and Optimal Investments,” *American Economic Review* 86: 478-501.
- [33] Edlin, A., and C. Shannon (1998), “Strict Monotonicity in Comparative Statics,” *Journal of Economic Theory* 81: 201-19.
- [34] Elfenbein, D. and J. Lerner (2003), “Ownership and Control Rights in Internet Portal Alliances, 1995-1999,” *RAND Journal of Economics* 34: 356-69.
- [35] Elliott, M. (2009), “Inefficiencies in networked markets,” Working Paper, Stanford University.
- [36] Evans, R. (2008), “Simple Efficient Contracts in Complex Environments,” *Econometrica* 76: 459-91.
- [37] Farrell, J. (1987), “Information and the Coase Theorem,” *Journal of Economic Perspectives* 1: 113-29.
- [38] Felli, L., and K. Roberts (2001), “Does Competition Solve the Hold-Up Problem,” SSRN Working Paper 171920.
- [39] Figueroa, N. and V. Skreta (2008), “What to Put on the Table,” SSRN Working Paper 1120987.

- [40] Fieseler, K., T. Kittsteiner, and B. Moldovanu (2003), “Partnerships, Lemons, and Efficient Trade,” *Journal of Economic Theory* 113: 223–34.
- [41] Gans, J.S. (2005), “Markets for Ownership,” *RAND Journal of Economics* 36: 433-55.
- [42] Gershkov, A., and P. Schweinzer (2010), “When Queueing is Better than Push and Shove,” *International Journal of Game Theory* 39: 409–30.
- [43] Gibbons, R. (2005), “Four Formal(izable) Theories of the Firm,” *Journal of Economic Behavior and Organization* 2: 200-45.
- [44] Gomes, A., and P. Jehiel (2005) “Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies,” *Journal of Political Economy* 113: 626-67.
- [45] Gresik, T., and M. Satterthwaite (1989), “The Rate at Which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms,” *Journal of Economic Theory* 48: 304-32.
- [46] Grossman, S.J., and O.D. Hart (1986) “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration,” *Journal of Political Economy* 94: 691-719.
- [47] Guriev, S. (2003), “Incomplete Contracts with Cross-Investments,” *Contributions to Theoretical Economics* 3: Article 5.
- [48] Hart, O. (1995), *Firms, Contracts, and Financial Structure*, Oxford, UK: Oxford University Press.
- [49] Hart, O. and B. Holmstrom (2010), “A Theory of Firm Scope,” *Quarterly Journal of Economics* 125: 483-513.
- [50] Hart, O., and J. Moore (1988), “Incomplete Contracts and Renegotiation,” *Econometrica* 56: 755-85.

- [51] Hart, O., and J. Moore (1990), “Property Rights and the Nature of the Firm,” *Journal of Political Economy* 98: 1119-58.
- [52] Hart, O., A. Shleifer, and R. Vishny (1997), “The Proper Scope of Government: Theory and an Application to Prisons,” *Quarterly Journal of Economics* 112: 127-61.
- [53] Hart, O., and J. Tirole (1990), “Vertical Integration and Market Foreclosure,” *Brookings Papers on Economic Activity*, special issue, 205-76.
- [54] Hermalin, B.E., and M.L. Katz (1991), “Moral Hazard and Verifiability: The Effects of Renegotiation in Agency,” *Econometrica* 59: 1735-53.
- [55] Holmes, O.W. (1881), *The Common Law*. (Reprinted, Boston: Little Brown, 1946.)
- [56] Holmstrom, B. (1982), “Moral Hazard in Teams,” *Bell Journal of Economics* 13: 324-40.
- [57] Holmstrom, B., and P. Milgrom (1991) “Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design,” *Journal of Law, Economics, and Organization* 7: 24-52.
- [58] Holmstrom, B., and P. Milgrom (1994) “The Firm as an Incentive System,” *American Economic Review* 84: 972-91.
- [59] Holmstrom, B. and J. Tirole (1991), “Transfer Pricing and Organizational Form,” *Journal of Law, Economics, and Organization* 2: 201-28.
- [60] Jehiel, P., and A. Pauzner (2006), “Partnership Dissolution with Interdependent Values,” *Rand Journal of Economics* 37: 1–22.
- [61] Jehiel, P., and B. Moldovanu (1999), “Resale Markets and the Assignment of Property Rights.” *Review of Economic Studies* 66: 971–91.

- [62] Kaplow, L. and S. Shavell (1995-6), “Do Liability Rules Facilitate Bargaining: A Reply to Ayres and Talley,” *Yale Law Journal* 105: 221-33.
- [63] Klein, B., R.G. Crawford, and A.A. Alchian (1978), “Vertical Integration, Appropriable Rents, and the Competitive Contracting Process,” *Journal of Law and Economics* 21: 297-326.
- [64] Kranton, R., and D. Minehart (2001), “A Theory of Buyer-Seller Networks,” *American Economic Review* 91: 485-508.
- [65] Krishna, V., and M. Perry (1998), “Efficient Mechanism Design,” SSRN Working Paper 64934.
- [66] Legros, P. and A.F. Newman (1996), “Wealth Effects, Distribution, and the Theory of Organizations,” *Journal of Economic Theory* 70: 312-41.
- [67] Ma, C. (1988), “Unique Implementation of Incentive Contracts with Many Agents,” *Review of Economic Studies*: 55: 555-71.
- [68] MacLeod, W.B., and J.M. Malcomson (1993), “Investments, Holdup, and the Form of Market Contracts,” *American Economic Review* 83: 811-837.
- [69] Mailath, G. and A. Postlewaite (1990), “Asymmetric Information Bargaining Procedures with Many Agents,” *Review of Economic Studies* 57: 351–67.
- [70] Marcet, J.H. (1819), *Conversations in Political Economy (3rd Edition)*, London.
- [71] Marvel, H.P. (1982), “Exclusive Dealing,” *Journal of Law and Economics* 25: 1-26.
- [72] Maskin and Tirole (1990), “The Principal-Agent Relationship with an Informed Principal: The Case of Private Values”
- [73] Maskin, E., and J. Tirole (1999), “Unforeseen Contingencies and Incomplete Contracts,” *Review of Economic Studies* 66: 83-114.

- [74] Makowski, L., and J.M. Ostroy (1989), “Efficient and Individually Rational Bayesian Mechanisms Only Exist on Perfectly Competitive Environments,” UCLA Dept. of Economics Working Paper #566.
- [75] Makowski, L., and C. Mezzetti (1994), “Bayesian and Weakly Robust First Best Mechanisms: Characterizations,” *Journal of Economic Theory* 64: 500–19.
- [76] Makowski, L., and J.M. Ostroy (1995) “Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics,” *American Economic Review* 85: 808-27.
- [77] Makowski, L., and J.M. Ostroy (2001), “Perfect Competition and the Creativity of the Market,” *Journal of the Economic Literature* 39: 479-535.
- [78] Mas-Colell, A., M.D. Whinston, and J.R. Green (1995), *Microeconomic Theory*, New York: Oxford University Press.
- [79] Maskin, E. and J. Moore (1999), “Implementation and Renegotiation,” *Review of Economic Studies* 66: 39-56.
- [80] Maskin, E. and J. Tirole (1999), “Unforeseen Contingencies and Incomplete Contracts,” *Review of Economic Studies* 66: 83-114.
- [81] Matouschek, N. (2004), “Ex Post Inefficiencies in a Property Rights Theory of the Firm,” *Journal of Law, Economics, and Organization* 20: 125-47.
- [82] McAfee, P., and M. Schwartz (1994), “Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity,” *American Economic Review* 84: 210-30.
- [83] McKelvey, R.D. and T. Page (2002), “Status Quo Bias in Bargaining: An Extension of the Myerson–Satterthwaite Theorem with an Application to the Coase Theorem,” *Journal of Economic Theory* 2: 336-55.

- [84] Milgrom, P. and J. Roberts (1990), “Rationalizability and Learning in Games with Strategic Complementarities,” *Econometrica* 58: 1255-78.
- [85] Milgrom, P., and C. Shannon (1994), “Monotone Comparative Statics,” *Econometrica* 62: 157-80.
- [86] Milgrom, P. and I. Segal (2002), “Envelope Theorems for Arbitrary Choice Sets,” *Econometrica* 70: 583-601.
- [87] Mookherjee, D. (1984), “Optimal Incentive Schemes with Many Agents,” *Review of Economic Studies* 51: 433-46.
- [88] Moore, J. and R. Repullo (1988), “Subgame Perfect Implementation,” *Econometrica* 56: 1191-220.
- [89] Myerson, R., and M. Satterthwaite (1983), “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory* 29: 265–81.
- [90] Mylovanov and Troger, “News from the Informed Principal in Private-Value Environments,” 2008, working paper.
- [91] Neeman, Z. (1999), “Property Rights and Efficiency of Voluntary Bargaining Under Asymmetric Information,” *Review of Economic Studies* 66: 679-91.
- [92] Noldeke, G. and K. Schmidt (1995), “Option Contracts and Renegotiation: A Solution to the Hold-up Problem,” *Rand Journal of Economics* 26: 163-79.
- [93] Rahman, D. (2010), “Detecting Profitable Deviations,” Working Paper, University of Minnesota.
- [94] Rajan, R.G., and L. Zingales (1998) “Power in a Theory of Firm,” *Quarterly Journal of Economics* 113: 387-432.

- [95] Rantakari, H. (2008), "Governing Adaptation," *Review of Economic Studies* 75: 1257-85.
- [96] Ray, D. (2007), *A Game-Theoretic Perspective on Coalition Formation*, Oxford University Press.
- [97] Riordan, M. (1990), "What is Vertical Integration?," in M. Aoki, B. Gustafsson, and O. Williamson (Eds.), *The Firm as a Nexus of Treaties*, Sage Publications, London, 1990.
- [98] Roider, A. (2004), "Asset Ownership and Contractibility of Interaction," *RAND Journal of Economics* 35(4): 787-802.
- [99] Rogerson, W. (1992), "Contractual Solutions to the Hold-Up Problem." *Review of Economic Studies*, 59: 774-94.
- [100] Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica* 50: 97-109.
- [101] Rubinstein, A. and A. Wolinsky (1992), "Renegotiation-Proof Implementation and Time Preferences," *American Economic Review* 82: 600-14.
- [102] Schmidt, K.M. (1996), "The Costs and Benefits of Privatization: An Incomplete Contracts Approach," *Journal of Law, Economics, and Organization* 12: 1-24.
- [103] Schmitz, P. (2002), "Simple Contracts, Renegotiation under Asymmetric Information, and the Hold-up Problem," *European Economic Review* 46: 169-88.
- [104] Schmitz, P.W. (2006), "Information Gathering, Transaction Costs, and the Property Rights Approach," *American Economic Review* 96: 422-34.
- [105] Schweizer, U. (2006), "Universal Possibility and Impossibility Results," *Games and Economic Behavior* 57: 73-85.

- [106] Segal, I. (1996), “Communication Complexity and Coordination by Authority,” Working Paper.
- [107] Segal, I. (1999a), “Complexity and Renegotiation: A Foundation for Incomplete Contracts,” *Review of Economic Studies* 66: 57-82.
- [108] Segal, I. (1999b), “Contracting with Externalities,” *Quarterly Journal of Economics* 114: 337-88.
- [109] Segal, I. (2003a), “Collusion, Exclusion, and Inclusion in Random-Order Bargaining,” *Review of Economic Studies* 70: 439-60.
- [110] Segal, I. (2003b), “Coordination and Discrimination in Contracting with Externalities: Divide and Conquer?” *Journal of Economic Theory* 113: 147-81.
- [111] Segal, I. and M.D. Whinston (2000), “Exclusive Contracts and Protection of Investments,” *RAND Journal of Economics* 31: 603-33.
- [112] Segal, I., and M.D. Whinston (2002), “The Mirrlees Approach to Mechanism Design with Renegotiation (with Applications to Hold-Up and Risk Sharing),” *Econometrica* 70: 1-45.
- [113] Segal, I. and M.D. Whinston (2003), “Robust Predictions for Bilateral Contracting with Externalities,” *Econometrica* 71: 757-91.
- [114] Segal, I. and M.D. Whinston (2010), “A Simple Status Quo that Assures Participation (with Application to Efficient Bargaining),” forthcoming in *Theoretical Economics*.
- [115] Simon, H. (1951), “A Formal Theory of the Employment Relation,” *Econometrica* 19: 293-305.
- [116] Stahl, I. (1972) Bargaining theory, Stockholm School of Economics, Stockholm.

- [117] Sutton, John (1986), “Non-Cooperative Bargaining Theory: An Introduction,” *Review of Economic Studies* 53: 709-24.
- [118] Watson, J.C. (2007), “Contract, Mechanism Design, and Technological Detail,” *Econometrica* 75: 55-81.
- [119] Weber, R. (1988), “Probabilistic Values for Games,” in A. Roth, ed., *The Shapley Value*, Cambridge, UK: Cambridge University Press.
- [120] Whinston, M.D. (2003), “On the Transaction Cost Determinants of Vertical Integration,” *Journal of Law, Economics, and Organization* 19:1-23.
- [121] Williams, S.R. (1999), “A Characterization of Efficient Bayesian Incentive Compatible Mechanisms,” *Economic Theory* 14: 155-80.
- [122] Williamson, O.E. (1971), “The Vertical Integration of Production: Market Failure Considerations,” *American Economic Review* 61: 112-23.
- [123] Williamson, O.E. (1975), *Markets and Hierarchies: Analysis and Antitrust Implications*, New York: Free Press.
- [124] Williamson, O.E. (1979), “Transaction Costs Economics: The Governance of Contractual Relations,” *Journal of Law and Economics* 22: 233-62.
- [125] Williamson, O.E. (1985), *The Economic Institutions of Capitalism*, New York: Free Press.
- [126] Yenmez, M. B. (2007), “Dissolving Multi-Partnerships Efficiently,” SSRN Working Paper 1031272.