Abstract

A set of voters consults a set of experts before voting over two alternatives. Agents have private biases over which alternative they prefer ex ante, but may be swayed by information about relative values of the alternatives. Experts observe private signals about the relative values of the alternatives and can choose to either reveal that information or conceal it, but they cannot lie. We examine how disclosure and voting behaviors vary with the intensity and heterogeneity of the preference biases, the informativeness of signals, and the structure of the voting rule. The voting rule that maximizes information disclosure can be a supermajority rule favoring one of the two alternatives, and in some cases unanimity rule can dominate simple majority rule both in terms of information revelation and total utility maximization, even in a fully symmetric society. The voting rule that maximizes information disclosure need not coincide with the voting rule that maximizes total utility. We also show that in a large enough society, full information revelation is approximated via any voting rule, and with sufficient symmetry simple majority rule is approximately ex ante efficient.

Keywords: Voting, Information, Disclosure, Deliberation, Experts, Committees, Sender-Receiver

JEL Classification Codes: D72, D71, D83

1 Introduction

A committee, legislative body, or other group has to choose one of two alternatives, for instance the status quo versus an alternative, or a choice between two candidates or programs.
Often simple majority rule is used in such settings, but there are also many cases where supermajority rules are used, favoring one of the two alternatives (for example in enacting certain types of bills in a legislature or in various committee settings).¹ When should majority rule be used and when should a supermajority rule be used?

Justifications for simple majority rule are evident from the symmetric way in which it treats alternatives, as well as voters. As well-articulated in May’s Theorem (1952), in private value settings when deciding between two alternatives, simple majority rule is the unique voting rule that does not favor either alternative or any voter, and is positively responsive to voters’ preferences.² The justifications in the previous literature for moving away from simple majority rule have generally come from some asymmetries in the underlying environment that may call for favoring some alternative or voters.³

Here we examine the choice of a voting rule in a setting where deliberation takes place before the vote. In such settings, the choice of a voting rule has significant impact on what is revealed in deliberation. Moreover, even in settings with strong underlying symmetries, supermajority rules can lead to more information revelation than a simple majority rule. Thus, it is important to understand how the choice of a voting rule interacts with information revelation, as that can impact outcomes and welfare and applies to many settings. For example, expert testimony is an essential input into judicial and legislative decisions. Moreover, since almost all organizations involve some specialization, managers and boards generally rely on the expertise of others in order to make informed decisions.

In our model, a decision is to be made between two alternatives, say \(x\) and \(y\). Individuals have their own private leanings for one or the other alternative. There is also a common state of the world which indicates which of the two choices is preferable to unbiased decision makers. For example, a decision might be at which of two possible spots to locate a public facility. Agents may prefer to have the public facility closer to their home. In addition, there may be differences in which location is better (perhaps cheaper) from a neutral perspective. Various agents may have different intensities of private bias (e.g., how much closer they are to one facility than the other, or their costs of travel, etc.).

Before a vote is taken, deliberation takes place. A set of non-voting experts receive private information in the form of (binary) signals correlated with the common state of the world: so either a signal indicating that \(x\) is the more likely state, or a signal that \(y\) is the more likely state. Beyond the signals being imperfect, there is also a possibility that an

¹Examples of supermajority rules abound, including votes on budget changes in the California legislature, decision-making in the European Union’s council of ministers and in the security council of the United Nations.

²Justifications have also been given in common value incomplete information settings (e.g., Condorcet (1785)), as well as private value incomplete information settings Rae (1969), Badger (1972), Barbera and Jackson (2004); again with symmetries in the underlying environment.

³Direct asymmetries in preferences or in how representative some voters might be of their constituents can call for supermajority rules (e.g., Barbera and Jackson (2006)), as can more implicit asymmetries, such as an asymmetric agenda setting system (Buchanan and Tullock (1962)), learning aversion (Louis (2009)), or mistrials (Coughlan (2000)).
expert does not receive a signal and thus is uninformed. In the deliberation phase, each expert chooses to either publicly reveal his or her signal (if s/he has seen one) or claim that he or she has not seen one. Voters then cast their votes for either $x$ or $y$. We examine how equilibrium depends on the precision of the signals, the probability that an expert is uninformed, the distribution of biases among voters and experts, and the voting rule for how the decision is made.

It is important to emphasize that experts may only choose either to reveal the (verifiable) signals that they observe or pretend that they have not observed a signal, and cannot lie about their signals. This makes the model more relevant for some applications than “cheap talk” models in which communication is non-verifiable, and also turns out to be easier to analyze. An important aspect of the model is that there is some probability that an expert is uninformed. This makes it possible for experts to conceal information without other agents being sure of whether or not the expert has concealed information.

Our analysis begins with the simplest setting: one expert and one voter (a “sender-receiver” setting). This helps lay out some of the basic incentive issues that arise in the model. We include it mainly for pedagogical purposes, as various versions of sender-receiver models have been studied in the literature following the seminal work of Crawford and Sobel (1982), including studies of optimal selection of biases in advisors (Calvert (1985)) and settings with verifiable evidence (Austen-Smith (1994), Che and Kartik (2009)). Some of the insights in this simple setting are summarized as follows. Without loss of generality, take the voter to be biased in favor of $x$ (or unbiased), and to avoid trivialities suppose that the voter prefers $y$ if he or she knows that a “$y$” signal has been observed (the binary signal indicating that the $y$ state is more likely than the $x$ state), and (weakly) prefers $x$ if either no information or information in favor of $x$ is observed. Note that in this case all the voter needs to know in order to make an optimal decision (from his or her point of view) is whether or not a $y$-signal has been observed, as otherwise the voter (weakly) prefers $x$. If the expert’s bias is sufficiently close to the voter’s (so that the expert wishes the same decisions to be made as a function of each possible information realization as the voter), then all equilibria are equivalent to the expert revealing all signals. The more interesting cases are those where the expert is sufficiently biased one way or the other (relative to the voter) so as to disagree with what the voter would do conditional on some type of signal. Experts who are more $y$-leaning will reveal $y$-signals when they are observed, but experts who are sufficiently biased in favor of $x$ may choose to hide $y$-signals. Thus, even though a voter has

\footnote{Although outright lying is relevant in some applications, there are many settings where the more relevant consideration is that agents selectively filter information but actually present facts and verifiable statements and reports. Indeed, in many legislative and judicial settings, experts offer reports and testimony that can be scrutinized for veracity. Thus, a relevant model in many instances is one where experts cannot lie, but might hide their information. This follows the tradition of “persuasion games,” emanating from the seminal work by Milgrom (1981) and Grossman (1981).}

\footnote{If the voter is so biased as to always want to choose $x$ then deliberation becomes irrelevant.}

\footnote{Some equilibria may involve some signal hiding, but only in cases where revealing that signal would not change the voter’s behavior, and are equivalent to full revelation.}
an $x$-bias, the voter would prefer a strongly $y$-biased expert over a strongly $x$-biased expert, and in fact learns everything that he or she needs to know to make an informed decision from any $y$-biased expert.\footnote{Calvert (1985) obtained a similar conclusion that a decision maker would prefer a biased advisor, but driven by a high cost of obtaining information from an advisor thus making it only worthwhile to consult advisors who are likely to provide more extreme information. Che and Kartik’s (2009) model is much closer to the one-expert and one-voter case of our model. Although our focus is different (they are concerned mainly with information acquisition), and our model for the one-voter, one-expert case is starker than theirs, the idea that having an opposing expert can lead to more information than an expert with a similarly signed but more extreme bias is clear from the Che and Kartik (2009) analysis.}

We then turn to the main topic of this paper: settings with many experts and/or voters. We begin by showing that, in strategic terms, the outcome of the voting stage is determined entirely by the incentives of the pivotal voter, and so characterizing the equilibria amounts to characterizing equilibria with many experts and just one voter. We then show that equilibria are equivalent to a very intuitive class of “monotone” equilibria: experts who are sufficiently more biased towards $y$ than the pivotal voter only reveal $y$ signals, experts who are sufficiently more biased towards $x$ only reveal $x$ signals, and experts with intermediate biases reveal all signals.

We can then explore how information revelation depends on the voting rule. To see why simple majority rule can be dominated by more extreme rules, consider a setting where most experts are biased either towards $x$ or $y$. If simple majority rule is used and the pivotal voter is close to neutral, then sufficiently $x$-biased experts reveal only $x$ signals, and sufficiently $y$ biased experts reveal only $y$ signals. By changing the voting rule to favor one of the alternatives, who the pivotal voter is changes and the new pivotal voter is more biased, say in favor of $x$ (if the new voting rule requires more votes to select $y$), than the pivotal voter under simple majority rule. In that case, the $x$-biased experts are more willing to reveal all of their information because their incentives are more closely aligned with the pivotal voter. More $x$-biased experts feel safe in revealing signals in favor of alternative $y$, since the pivotal voter is similarly biased and will only vote for $y$ if there is strong information supporting $y$. This does not change the incentives of the strongly $y$-biased experts since they will continue to reveal information in favor of $y$ but not in favor of $x$ and do not change strategies. Thus, by biasing the voting rule, the society can become more informed. Whether or not this leads to an increase in total utility depends on particular parameter values and can go either way. In particular, it is possible that the total expected utility maximizing voting rule is a supermajority rule even in a fully symmetric society. Generally, the optimal choice of a voting rule, in terms of either information revelation or ex ante efficiency, depends on the setting, and can lead to simple majority rule in some settings and supermajority rules in other settings.

We conclude our analysis with results on the approximate informational equivalence of all voting rules in societies with many experts, and on the approximate optimality of simple\footnote{We also discuss the case of more than two signals in Section 6, which leads to some similar sorts of conclusions, but with more complicated strategies and sometimes incomplete comparisons.}
majority rule in large symmetric societies, reaffirming that simple majority rule can be the efficient rule in some relevant settings.

In terms of relations to the earlier literature, the fact that voting rules impact deliberation has been well-noted in important papers by Austen-Smith and Feddersen (2005, 2006), building on insights from earlier work by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998). Our work contrasts with that work both in terms of focusing on settings in which information can only be revealed or not rather than on cheap-talk, and in finding that there are nontrivial settings where simple majority rule is dominated. Thus, our analysis differs both in premises and conclusions.

Deliberation models where agents can conceal information but cannot overtly lie are tractable which allows us to characterize how experts’ incentives react to the setting and the voting rule. The contrast between concealment and cheap talk can be seen in the literature on “persuasion games,” as introduced by Milgrom (1981) and Grossman (1981). It is important for our approach that there is some probability that an expert does not observe a signal, as otherwise an expert’s information can be perfectly inferred.

Before proceeding, let us make two additional remarks on the related literature. First, perhaps the closest antecedent to our paper is one by Schulte (2010) who examines a setting where agents may withhold information in a committee decision-making setting. However, Schulte’s main focus is on identifying situations where full revelation is possible and showing that it occurs only with sufficient alignment of preferences. Our results have little overlap with Schulte’s, as we are interested in characterizations of equilibrium properties including situations where preferences are not aligned. Second, most of the literature on voting rules lies either in an “Arrovian” world where everyone has their own private preference and there is simply an aggregation problem, or one where everyone has the same ultimate preference.

There is also work by Gerardi and Yariv (2006, 2007) that shows a certain form of equivalence between voting rules, based on a lack of anticipation of being pivotal in a vote. In our setting that is not an issue, and the equilibrium selection issues that they face are overcome.

Subsequent investigations include important ones by Milgrom and Roberts (1986), Okuno-Fujiwara, Postlewaite and Suzumura (1990), Shin (1994) and Lipman and Seppi (1995), among others (see Milgrom (2008) for some recent discussion). There are also several papers examining evidence or partial verifiability, costly lying, as well as sequential forms of deliberation, and including some on mechanism design in such settings, including work by Green and Laffont (1986), Lagerlof (1997), Glazer and Rubinstein (2001), Krishna and Morgan (2001), Ben Porath and Lipman (2008), Neilson and Winter (2008), Gerardi, McLean and Postlewaite (2009), Kartik (2009), and Kartik and Tercieux (2012).

If an expert is known to have observed a signal, then the expert has an incentive to reveal the signal that is most favorable to him or her, as otherwise any listening agent may infer worse information. Thus the most favorable signals for an expert are revealed. Inducting on this logic, if an expert were known to reveal some signals and not others, then the expert would have an incentive to deviate and reveal the most favorable signal that is not revealed as otherwise it might be interpreted as being less favorable. Thus, the only signal that might be concealed is the least favorable signal, which is then revealed by default. Such an unraveling result relies on a decision maker being certain that the expert has observed a signal (as shown, for example, by Okuno-Fujiwara, Postlewaite and Suzumura (1990), see their Example 3). See also the sanitization strategy discussed in Shin (1994).
over alternatives (as in a Condorcet setting) but may differ in information, even if they may have different tolerances for errors (e.g., as in Austen-Smith and Banks (1996)). Our model is thus in line with Austen-Smith and Feddersen (2006) in that agents’ preferences have both private and common components. Despite their wide applicability, such settings are under-studied.

2 The Model

A society of agents, \( N = \{1, \ldots, n\} \), must decide between two alternatives, \( x \) and \( y \), with \( x \) referred to as the status quo.

2.1 Preferences

Agents’ preferences involve both a private component as well as a common component that depends on a state of the world. A state \( \omega \) in \( \{x, y\} \) captures which is the better alternative from the common perspective.

An agent’s preferences are characterized by a parameter \( \lambda_i \in [-1,1] \) which encapsulates both the direction of the agent’s private preference and the weight on the common state. \( |\lambda_i| \) is the weight that the agent places on the private component and \( 1 - |\lambda_i| \) is the weight that the agent places on the common component.\(^{12}\) The sign of \( \lambda_i \) captures the agent’s private leaning, with \( \lambda_i > 0 \) indicating a preference for \( x \) and \( \lambda_i < 0 \) indicating a preference for \( y \). We say that agent \( i \) is an \( x \)-supporter if \( \lambda_i > 0 \) and a \( y \)-supporter if \( \lambda_i < 0 \).

Given that there are two alternatives, the choice of \( x \) versus \( y \) can be determined by keeping track of the difference in utilities from choosing \( x \) versus \( y \), which is denoted by \( u(\lambda_i, \omega) \). An agent prefers \( x \) to \( y \) if \( u(\lambda_i, \omega) > 0 \), prefers \( y \) to \( x \) if \( u(\lambda_i, \omega) < 0 \), and is indifferent if \( u(\lambda_i, \omega) = 0 \).

Letting \( f(\omega) = 1 \) if \( \omega = x \) and \( f(\omega) = -1 \) if \( \omega = y \), agent \( i \)'s net utility for choosing \( x \) minus the utility from choosing \( y \) is

\[
u(\lambda_i, \omega) = \lambda_i + (1 - |\lambda_i|)f(\omega).
\]

An agent with \( \lambda_i > 1/2 \) prefers \( x \) to \( y \) regardless of the state, and similarly an agent with \( \lambda_i < -1/2 \) prefers \( y \) to \( x \) regardless of the state, while agents with \( -1/2 < \lambda_i < 1/2 \) prefer the alternative indicated by the state, but with varying preference intensity depending on

\(^{12}\)For example, if these are political candidates, then one interpretation is that the state represents which is the “more competent” candidate. Instead, if we think of these as potential projects, then the state might represent which project is the least costly, independent of other potentially idiosyncratic benefits of the projects \( x \) and \( y \). The private preference of an agent captures the value of the project independent of the cost. For instance, we might think of locating a new public facility either in location \( x \) or \( y \). An \( x \)-supporter might be someone who lives at location \( x \), and prefers that the facility be located at \( x \) ignoring cost issues, while the state captures which of the two locations is cheaper. The agent’s weighting factor then encapsulates how the agent trades off proximity to the project versus cost efficiency.
their type and the state. In the special case such that $\lambda_i = 0$, an agent prefers whichever alternative corresponds to the state.

### 2.2 Information and Experts

The subset of the population who may see information is the set of experts and is denoted by $N_E \subset N$. $n_E = \#N_E$ denotes the number of experts.

Information is noisy in two respects. First, there is a probability $\gamma$ that an expert agent sees a signal and a probability $1 - \gamma$ that the agent does not see a signal. Second, even when an expert receives information, it comes in the form of a signal that may be imperfectly correlated with the true state. In particular, if agent $i$ is informed, then information comes in the form of a private signal denoted $z_i \in \{x, y\}$, where $z_i = \omega$ with probability $p \geq 1/2$. Signals are independently and identically distributed conditional on $\omega$ across the experts who observe signals.

We assume that states $x$ and $y$ are equally likely ex ante, as this simplifies the statements of many results (as it symmetrizes the uncertainty structure). The more general case with asymmetries is a straightforward variation, with shifts in various cutoff values and equilibrium strategies, but qualitatively similar insights. Given this, conditional on seeing a signal $y$, the conditional probability that $\omega = y$ is $p$, and similarly for $x$.

### 2.3 Deliberation

Agents meet and communicate signals before voting. Deliberation is in the form of verifiable revelation of a signal. In particular, experts can choose to reveal their signals or to keep them private. This matches a variety of applications where information is in the form of verifiable facts. A committee member can choose to provide evidence, or they can choose to keep it to themselves. Here experts can conceal information, but they cannot “lie”.

More formally, a revelation strategy for an expert agent $i \in N_E$ is a function

$$r_i : \{x, y, \emptyset\} \to \Delta (\{x, y, \emptyset\}).$$  

(2)

The requirement that an expert can conceal information, but cannot lie about it is formalized by requiring that

- $r_i(x) \in \Delta (\{x, \emptyset\})$,
- $r_i(y) \in \Delta (\{y, \emptyset\})$, and
- $r_i(\emptyset) = \emptyset$.

The revelations of the experts are simultaneous and are publicly observed by all agents.

\[13\]We allow for mixed strategies in an expert’s revelation behavior. An expert will only mix when indifferent and so (except in non-generic cases) will not change the pivotal voter’s behavior; and thus mixing does not play a prominent role in our analysis but is important for existence of equilibrium.
2.4 Voting

A set of voters, $N_V \subset N$, votes after the deliberation phase. Let $n_V = \#N_V$ be the number of voters.

A voter’s strategy depends on the information that he or she has observed. We focus on the case where experts and voters are distinct. This case highlights the central issues, while privately informed voters (e.g., an expert voter who conceals a signal) introduce pivotal voting issues that substantially complicate an already complex scenario. We discuss extensions to the case of overlap at the very end of the paper, so until then take $N_V = N \setminus N_E$.

A voting strategy for voter $i \in N_V$, is a mapping $v_i : \{x, y, \emptyset\}^{n_E} \rightarrow \Delta(\{x, y\})$.

2.5 The Game and Equilibrium

In a first stage, experts who have observed a private signal simultaneously choose whether to reveal their signals. Any revealed signals are then public, and so observed by all agents.

In a second stage, voters simultaneously cast their votes.

If at least $q \in \{0, 1, \ldots, n_V \}$ voters vote for $y$ then $y$ is chosen, and otherwise $x$ is chosen.

Simultaneous voting models have some degenerate equilibria. For example, suppose that a voter has such a strong private preference for $x$ that he or she would prefer $x$ regardless of the state. With more than one voter and a quota less than unanimity, there is still a sequential equilibrium where all voters vote for $y$ in the voting stage regardless of any information revelation in the first stage. This is a sequential equilibrium because they correctly forecast that their votes have no chance of affecting the outcome in equilibrium. This involves a weakly dominated strategy. Thus, to rule out such degeneracies it is important to rule out weakly dominated strategies. Such strategies are ruled out via Selten’s (1975) perfection (i.e., trembling hand perfect equilibria of the extensive form game) and so we use that solution concept, which refines sequential equilibria.

An equilibrium is a list of strategies, $r_i$ for each $i \in N_E$ and $v_i$ for each $i \in N_V$, that form a trembling hand perfect equilibrium of the extensive form game where the $\lambda_i$’s and identities of the agents are common knowledge.

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14The simultaneity of the revelation is not important to the model. As will be clear from the analysis, qualitatively similar results would hold if signal revelation was sequential, although the details of the equilibria could depend on the order of revelation, which we avoid here.

15In our games, each player only moves once, and so trembling hand perfect equilibria coincide with van Damme’s (1984) notion of quasi-perfect equilibria, and are thus immune to the criticism of Mertens (1995). It would not matter if we used quasi-perfection instead. Using sequential or perfect Bayesian equilibrium would not rule out the use of some weakly dominated strategies, as we just noted. In addition, perfection also rules out some implausible beliefs.

16Kreps and Wilson (1982) partly motivate their definition of sequential equilibria as being easier to calculate than perfect equilibria and also coinciding with perfect equilibria for many games; but not as necessarily being a more appropriate solution concept. Here, the two do not coincide, and the sequential equilibria that involve the use of dominated strategies are ones that seem problematic and unnatural, with voters going against their own incentives.
In some cases, we also wish to rule out weakly dominated strategies on the part of experts, when they anticipate that voters will not use weakly dominated strategies. Such dominated strategies on the part of experts are not always ruled out by trembling hand perfection, and so we employ the iterative elimination of weakly dominated strategies.\textsuperscript{17,18} See Proposition 1 and its discussion for additional details.

We are explicit whenever we use the iterative elimination of dominated strategies, and otherwise “equilibrium” refers to the definition of trembling hand perfection as given above.

Equilibria always exist as the game is finite.

Note that given the separation between experts and voters, the voters are all symmetrically informed when voting and there is no information gained from being pivotal. In fact, in an equilibrium, a voter’s beliefs about the relative probability of the state being \( x \) versus \( y \) are completely determined by the strategies of the experts and the realized state, and there is no need to refine beliefs (on or off an equilibrium path).

\section{One Expert and One Voter: Sender-Receiver Games}

We start with a standard sender-receiver setting, which highlights some of the incentive effects. As emphasized in the introduction, this setting is well-studied in the literature. We include it because it provides some useful background for our subsequent analysis of many experts and/or voters. Also, as our model differs from others (e.g., Che and Kartik (2009)) in terms of specifications of preferences, the results in this section are not corollaries of other results and so we include these for the sake of completeness.

There are two agents: one expert and one voter and the voting quota is \( q = 1 \). We denote the expert by \( E \) and the voter by \( V \), with corresponding \( \lambda_E \) and \( \lambda_V \). Without loss of generality, consider the case where the voter is an \( x \) supporter, so that \( \lambda_V \geq 0 \), as the other case is analogous.

\subsection{Critical Types and Voter Behavior}

In order to characterize equilibrium behavior, it is useful to define some critical levels of \( \lambda_i \).

Let \( \bar{\lambda}(p) \) be the level of \( \lambda \) above which an agent would prefer \( x \) regardless of whether the agent knew there to be a \( y \) signal, and below which the agent would prefer \( y \) conditional on a \( y \) signal. This varies with the precision of a signal. For example, if signals are very noisy,

\textsuperscript{17}To see the role of the iterative elimination of weakly dominated strategies in establishing uniqueness, consider the following situation. Consider an expert with a slight \( y \) bias and a voter with a slight \( x \) bias. Let the expert reveal \( x \) signals but not \( y \) signals, quite to the contrary of what the expert would like to do, and the voter choose \( x \) conditional on an \( x \) signal and \( y \) otherwise. This is a trembling hand perfect equilibrium, but the expert’s strategy of hiding \( y \) signals would not survive an iterative elimination of dominated strategies: the expert is weakly better off revealing \( y \) signals than concealing them.

\textsuperscript{18}For recent justifications of such an approach, see Brandenburger, Friedenberg, and Keisler (2008) and Keisler and Lee (2011).
then the agent will prefer $x$ even with a very low $\lambda_i$ as there is little information in the signal and the agent’s private leaning will dominate. As signals become more precise ($p$ increases), this threshold increases.

In particular $\bar{\lambda}(p)$ is the unique solution to

$$\bar{\lambda}(p) + (1 - \bar{\lambda}(p))E[f(\omega)|z_i = y] = 0.$$  

This can be rewritten as

$$\bar{\lambda}(p) + (1 - \bar{\lambda}(p))(1 - 2p) = 0$$

or

$$\bar{\lambda}(p) = \frac{2p - 1}{2p}.$$  

At one extreme such that $p = 1$, signals are fully informative and $\bar{\lambda}(p) = 1/2$, while otherwise $\bar{\lambda}(p)$ is less than $1/2$ and non-negative. At the other extreme, such that signals are not informative at all ($p = 1/2$), then $\bar{\lambda}(p) = 0$.

There is another critical level of $\lambda$ that is also useful in determining how a voter acts. The critical question is what are an $x$ supporting voter’s preferences conditional on having an expert claiming to have not seen a signal, but also conditional on knowing that the expert hides certain signals, and in particular might be hiding a $y$ signal. If the voter is biased enough towards $x$, then the fact that there is some possibility that a signal was not observed will be enough to get the voter to vote for $x$. However, if the voter is close enough to being unbiased, then if the voter believes that the expert is hiding $y$ signals (but not $x$ signals), then conditional on having nothing revealed the voter will vote for $y$.

In particular, define $\Lambda(p, \gamma)$ to be the unique solution to

$$\Lambda(p, \gamma) + (1 - \Lambda(p, \gamma))E[f(\omega)|z_i \in \{y, \emptyset\}] = 0,$$

where $z_i \in \{y, \emptyset\}$ indicates that either there is no signal observed or that a $y$ signal was observed (and not revealed). Given that

$$E[f(\omega)|z_i \in \{y, \emptyset\}] = \frac{\gamma}{2 - \gamma}(1 - 2p)$$

it follows that

$$\Lambda(p, \gamma) = \frac{2p - 1}{2p - 2 + \frac{2}{\gamma}}.$$  

As $\gamma$ goes to 1, $\Lambda(p, \gamma)$ converges to $\bar{\lambda}(p)$, and as $\gamma$ goes to 0 so does $\Lambda(p, \gamma)$.

To summarize:

- a voter with $\bar{\lambda}(p) < \lambda_i$ will vote for $x$ regardless of any information;

- a voter with $\Lambda(p, \gamma) < \lambda_i < \bar{\lambda}(p)$ will vote for $y$ if shown a $y$ signal, but otherwise will vote for $x$ even if the expert is following a strategy of hiding $y$ signals; and
• a voter with $0 < \lambda_i < \lambda(p, \gamma)$ will vote for $y$ if she does not see an $x$ signal and the expert is following a strategy of revealing $x$ but hiding $y$ signals.

Analogous statements hold with the roles of $x$ and $y$ reversed for opposite values of the thresholds.

3.2 A Full Characterization of Equilibrium

With critical types defined, we now provide a characterization of equilibrium behavior.

Let us say that an equilibrium is fully-revealing equivalent if the outcome is the same as in an equilibrium in a game where the voter is shown the signal whenever one is observed, and an equilibrium is fully-hiding equivalent if the outcome is the same as in an equilibrium in a game where the voter is never shown the signal. The use of the word “equivalent” reflects the fact that there actions of the experts that are inconsequential to the outcome.

We also define another type of equilibrium that is a special case of a partially revealing equilibrium. Let us say that an equilibrium is $x$-revealing and $y$-hiding equivalent, if it is equivalent to an equilibrium in a game in which the voter is shown all $x$ signals but none of the $y$ signals.

If the voter has a bias $\lambda_V > \lambda(p)$, then the voter always votes for $x$ in any trembling hand perfect equilibrium regardless of observation, and so all equilibria result in $x$ always. So, consider the more interesting case such that $0 < \lambda_V < \lambda(p)$.\(^{19}\)

There is a multiplicity of equilibria whenever $-\lambda(p) < \lambda_E$ that stems from a multiplicity of potential voter beliefs. These are pared down once we iteratively eliminate weakly dominated strategies. For instance, when no signal is revealed to the voter, the voter could have various beliefs about the real signal and thus various voting behaviors. The only undominated behavior, however, is for the voter to vote in alignment with a revealed signal (presuming that we are in the non-trivial case where $-\lambda(p) < \lambda_V < \lambda(p)$). Thus, when we iteratively delete weakly dominated strategies, an expert reveals signals whenever the expert would like that to be the outcome, and so any expert with $\lambda_E < \lambda(p)$ will reveal $y$ signals to make sure that the voter votes for $y$. Similar sorts of reasonings lead to effectively unique equilibrium outcomes.

In the appendix we provide a complete characterization of all of the equilibria, without the refinement used in the following proposition. See the discussion after Proposition 7 for details about equilibria eliminated by the refinement.

Proposition 1 Consider a setting where the voter has an $x$-bias with $0 < \lambda_V < \lambda(p)$ (as the other case with a $y$ bias is analogous). Equilibria that survive the iterative elimination of weakly dominated strategies\(^{20}\) are characterized as follows:

\(^{19}\)We avoid cases of indifference, as they are easy to discern from our analysis, but more complicated to describe given the indifferences that occur and resulting multiplicities of behaviors.

\(^{20}\)Strategies are viewed as ex ante strategies in the corresponding normal form game.
• If $\lambda_E < \bar{\lambda}(p)$ then all equilibria are fully-revealing equivalent.

• If $\bar{\lambda}(p) < \lambda_E$ then all equilibria are $x$-revealing and $y$-hiding equivalent.

The proposition is easy to understand (although the uniqueness claims require some proof). Given the voter’s $x$ bias, the only information that the voter needs in order to make a fully informed decision is whether or not a $y$ signal is observed. Experts who are not too $x$-biased ($\lambda_E < \bar{\lambda}(p)$) wish to reveal $y$ signals whenever they are observed, since they wish to have $y$ chosen in such cases, while experts who are more $x$ biased prefer to hide $y$ signals.

To see the role of the iterative elimination of weakly dominated strategies in establishing uniqueness, consider the following situation. Consider an expert with a slight $y$ bias and a voter with a slight $x$ bias. Let the expert reveal $x$ signals but not $y$ signals, and the voter choose $x$ conditional on an $x$ signal and $y$ otherwise. This is a trembling hand perfect equilibrium, but the expert’s strategy of hiding $y$ signals would not survive an iterative elimination of dominated strategies: the expert is weakly better off revealing $y$ signals than concealing them.

We remark on the extreme case such that $\gamma = 1$, so that the expert observes a signal for sure. In that case, the voter can infer hidden signals based on incentives. In particular, an $x$-revealing and $y$-hiding equilibrium is equivalent to a fully revealing equilibrium because whenever no signal is revealed, the voter can infer that a $y$ was observed. This follows the same logic as the unraveling in a persuasion game in Milgrom (1981). Thus, in this extreme case, all equilibria are fully revealing.\(^{21}\) Thus, the ability of experts to manipulate information comes from the fact that expert can credibly claim to have not observed a signal.

3.3 The Devil’s Advocate

From the characterization of equilibria above we note an interesting feature. A voter with a bias of $\lambda_V > 0$ always does at least as well in terms of expected utility (from either an ex ante or interim perspective) with any expert with $\lambda_E < \bar{\lambda}(p)$ as with any expert with any other $\lambda_E$, and in some cases does strictly better. In fact, when the voter is an $x$-supporter who could potentially be swayed by a $y$ signal, he or she is strictly better off with a strong $y$-supporting expert than a strong $x$-supporting expert. A strongly $y$-biased expert has an incentive to show $y$ signals whenever those might sway the $x$ supporter’s behavior, while a strongly $x$-biased expert will hide $y$ signals.\(^{22}\)

\(^{21}\)In that case, $x$-revealing and $y$-hiding equivalent are effectively fully revealing, as not reporting a signal is inferred to be a $y$ signal.

\(^{22}\)In our analysis we have taken information as given. There are also questions about incentives for experts to gather information as in Che and Kartik (2009). Here, $y$-biased experts would have incentives to gather information and reveal $y$-signals, as otherwise $x$ is the outcome, while strongly $x$ biased experts have no incentive to gather information.
4 Multiple Voters and Experts

We now turn to the main focus of our paper, the case of many experts and/or voters. In particular, consider a case where $N_E$ and/or $N_V$ have more than one agent.

We first show that equilibria in settings with more than one voter are equivalent, in strategic terms, to equilibria in settings with just one voter who has the same bias as the pivotal voter (who has the $q$-th lowest bias) in the many voter case. This means that characterizing equilibria in cases with many voters can be accomplished by analyzing cases with just one voter.

4.1 A Pivotal Voter

For a voting rule $q$ and a profile of voters biases $(\lambda_1, \ldots, \lambda_{n_V})$ let $\lambda(q)$ be the pivotal $\lambda_i$; that is $\lambda$ such that there are no more than $q - 1$ voters with lower $\lambda_i$’s and no more than $n - q$ voters with higher $\lambda_i$’s. If we order voters $\{1, \ldots, n_V\}$ so that $\lambda_i \leq \lambda_{i+1}$ for each $i < n_V$, then it is clear that $\lambda(q) = \lambda_q$.

**Lemma 1** Consider two societies, $S_1$ and $S_2$, that both have experts’ biases given by $(\lambda_1, \ldots, \lambda_{n_E})$. $S_2$ has voters’ biases of $(\lambda_1, \ldots, \lambda_{n_V})$ with $q$-th lowest voter bias $\lambda(q)$, and $S_1$ has a single voter with bias $\lambda(q)$. For any equilibrium under voting rule $q$ in $S_2$, there is an equivalent equilibrium (with a quota of 1) in $S_1$.\(^{23}\) Also, for any equilibrium in $S_1$, there is an equivalent equilibrium under voting rule $q$ in $S_2$.

Lemma 1 shows that it is only the pivotal voter who matters in understanding the strategic structure of equilibrium. Of course, the full set of agents matters in assessing welfare, but focusing on just the incentives of the pivotal voter simplifies the equilibrium analysis. The reasoning behind this is as follows. All voters have the same information when voting (and the same beliefs in a trembling hand perfect equilibrium) and necessarily employ undominated strategies under trembling hand perfection and so all vote for their preferred alternatives given their beliefs. Thus, all voters with biases lower than the $q$-th voter will vote for $y$ whenever the $q$-th voter does, and similarly, higher bias voters vote for $x$ whenever the $q$-th voter does, and so the $q$-th highest bias is always pivotal. So, analyzing equilibrium is equivalent to analyzing the single voter case.\(^{24}\)

Note that we are still interested in the analysis of the multi-voter case, and in particular understanding how changing $q$ can change the equilibrium, but the above result implies that such changes can be analyzed by identifying how it changes the pivotal voter’s bias.

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\(^{23}\)Equivalence means that experts employ the same strategies and the voting outcome is the same under the equilibria for all realizations of signals.

\(^{24}\)In a case where there are multiple voters with bias $\lambda(q)$, it is possible in some cases that they are indifferent in some situations and so employ different strategies, and possibly randomize. The equivalent one-voter equilibrium will then involve mixing. That is why there is no claim in the lemma about the voters’ strategies being equivalent, just that the outcome is equivalent.
4.1.1 One Expert and Multiple Voters

Lemma 1 implies that the case of one expert and many voters is already characterized by our analysis of the one voter case, whose full details are given in Proposition 7 in the appendix.

**Corollary 1** Consider a society with a single expert with a bias $\lambda_E$ and voters’ biases of $($\underline{\lambda_1}, \ldots, \lambda_{n_V} )$ and a positive $^{25}$ $q$-th lowest bias $\lambda(q)$. The equilibria under voting rule $q$ are characterized by Proposition 7 with $\lambda_V$ replaced by $\lambda(q)$.

4.2 Monotone Equilibria

By Lemma 1, the remaining equilibrium characterizations can be achieved by analyzing the case of many experts and effectively one voter. Thus, we now provide results about equilibria with more than one expert and one voter.

We now concentrate on a particular class of equilibria such that an expert $i$ reveals $y$ signals when $\lambda_i \leq \lambda(q)$ and $x$ signals when $\lambda_i \geq \lambda(q)$. Thus, experts who are more $y$ leaning than the pivotal voter always reveal $y$ signals, and experts who are more $x$ leaning always reveal $x$ signals. The question of which experts reveal all signals is then the remaining issue.

Given experts’ biases of $(\lambda_1, \ldots, \lambda_{n_E})$ and voters’ biases of $(\lambda_1, \ldots, \lambda_{n_V})$, it is useful to define what it means for an equilibrium to be monotone: there exist types $\lambda_L \in [-1,1]$ and $\lambda_H \in [-1,1]$, with $\lambda_L \leq \lambda(q) \leq \lambda_H$, such that

- if $\lambda_i < \lambda_L$ then expert $i$ reveals $y$ signals but conceals $x$ signals,
- if $\lambda_i > \lambda_H$ then expert $i$ reveals $x$ signals but conceals $y$ signals,
- if $\lambda_L < \lambda_i < \lambda_H$ then expert $i$ reveals all signals.

The definition of monotone equilibria allows for some mixing of the two nearby strategies at the threshold levels of $\lambda_L$ and $\lambda_H$.

**Proposition 2** For any society and voting rule there exists a monotone equilibrium.

As we saw in the case of just one expert, one has to move beyond trembling hand perfection to identify only monotone equilibria (as discussed following Proposition 1). With just one expert, an iterative elimination of weakly dominated strategies was sufficient to narrow the equilibria down to those where agents follow their basic biases in natural ways. With many experts and voters, however, the problem becomes more intricate, since voters can condition their voting in somewhat peculiar ways as a function of which experts reveal signals and which ones do not, and may anticipate peculiar behavior by other voters, and so it can be that the first round of domination does not have any bite, and nor do other rounds. One would have to resort to other refinements that couple equilibria with iterative

$^{25}$The negative case is clearly an easy variation.
domination arguments in order to narrow things down to what seem to be natural sorts of behaviors. It is more transparent to proceed directly with a focus on equilibria where an expert who is more biased in a given direction than the pivotal voter reveals signals that are in favor of that bias - simply as a direct refinement on behavior. With this refinement in hand, we are led quickly to equilibria that are ‘equivalent’ to monotone equilibria.

Two equilibria are *shuffling-equivalent* if there is a permutation of the labels of experts such that the joint distribution of the permuted experts’ signals and the voting outcomes under the first equilibrium matches the joint distribution of the un-permuted experts’ signals and the voting outcomes under the second equilibrium.

**Proposition 3** For any society and voting rule, any pure strategy equilibrium such that an expert $i$ reveals $y$ signals when $\lambda_i \leq \lambda(q)$ and $x$ signals when $\lambda_i \geq \lambda(q)$ is shuffling-equivalent to a monotone equilibrium.

Not all equilibria are monotone, and so the shuffling-equivalency is needed in the above proposition. To see an easy example of a non-monotone equilibrium, consider a case where $\lambda(q)$ is such that the voter votes for $x$ unless he or she is sure that there are two $y$ signals. Let both experts have $\lambda_i < 0$ so they are $y$ supporters. There is an equilibrium where neither expert reveals anything and the voter always votes for $x$. There is an equilibrium where both experts reveal all signals, and the voter votes for $y$ conditional on seeing two $y$ signals, and there is an equilibrium where one expert reveals all signals and the other hides $x$ signals (and there are various mixed strategy equilibria). In the case where one expert reveals $x$ signals and the other does not, we can assign the strategies to the experts in any way and still have it be an equilibrium. To have it be monotone, we need to assign the strategies in order of the biases.

There is no guarantee that monotone equilibria are unique, as the following example shows.

Let $\emptyset$ represent the observation of no signal from an expert who is following a strategy of revealing all signals, and let $\emptyset(x)$ represent the observation of no signal from an expert who is following a strategy of revealing $y$ signals and hiding $x$ signals.

**Example 1** Multiple monotone equilibria

There is one voter and three experts such that:

- $\lambda_V > 0$ and in particular the voter prefers $y$ when the revealed signals are any permutation of $(x, y, y)$ or $(y, \emptyset, \emptyset(x))$ (or any combination that leads to a higher posterior on $y$ than these);
- $\lambda_1 = -1$, so that expert 1 always prefers $y$;
- $\lambda_2 = -\varepsilon$ (with a small enough $\varepsilon$ as specified below) and expert 2 prefers $x$ when the revealed signals are any permutation of $(x, y, \emptyset(x))$ (or any combination that leads to a higher posterior on $x$).
• $\lambda_3 > \lambda_V$ (with more detail specified below) and expert 3 prefers $y$ when the revealed signals are any permutation of $(x, y, y)$ (or any combination that leads to a higher posterior on $y$), but prefers $x$ when the revealed signals are any permutation of $(y, \emptyset, \emptyset(x))$ (or any combination that leads to a higher posterior on $x$).

At least two monotone equilibria are possible (such that each survives the iterative elimination of weakly dominated strategies). In both equilibria, expert 1 reveals $y$ and hides $x$ signals. In one of the equilibria experts 2 and 3 both fully reveal their signals and in the other equilibrium expert 2 only reveals $y$ signals and expert 3 only reveals $x$ signals.

It is easy to verify that expert 1’s strategy survives the iterative elimination of weakly dominated strategies and is a best response in both cases. Similarly, it is straightforward to check that expert 2 will reveal $y$ signals and expert 3 will reveal $x$ signals. So, let us check the parts of the strategies that change between the two equilibria.

Let us first verify the essential parts of the fully revealing equilibrium. Suppose that all experts follow the equilibrium strategies in which experts 2 and 3 reveal all signals, and that the voter holds beliefs based on these strategies. There are only two scenarios where expert 2’s deviating from the equilibrium by hiding the $x$ signal can change the voting outcome: the revealed signals (as a function of the strategies) by expert 1 and 3 are (a) $(s_1, s_3) = (y, \emptyset)$, or (b) $(s_1, s_3) = (\emptyset(x), y)$. For small enough $\varepsilon$ expert 2 wants to reveal $x$ in (b), and hide $x$ in (a), and as long as $\varepsilon$ is small enough, expert 2 is almost indifferent in (a) but more strongly prefers revealing $x$ in (b) and thus overall. Note here, neither revealing $x$ or not for expert 2 dominates the other (given the iterative deletion of dominated strategies). Similarly, there are also only two scenarios where expert 3’s deviating from the equilibrium by hiding $y$ signal can change the voting outcome: the revealed signals (as a function of the strategies) by expert 1 and 2 are (a) $(s_1, s_2) = (y, x)$, or (b) $(s_1, s_2) = (\emptyset(x), \emptyset)$. Expert 3 wants to reveal $y$ in (a) and hide $y$ in (b) and if expert 3 is close enough to be indifferent between $x$ and $y$ when the signals are any permutation of $(y, \emptyset, \emptyset(x))$, then expert 3 prefers revealing $y$ overall. Note that either revealing $y$ or not for expert 3 is undominated (given the iterative deletion of dominated strategies).

It is even easier to verify that the second is also a valid equilibrium. Suppose that all agents use the second equilibrium strategy. Note that off the equilibrium path when experts reveal the signal that they are not expected to reveal, the voter’s belief over other experts’ strategy will not change (which is consistent with the belief restriction following trembling hand perfection). There is only one scenario where expert 2’s deviating from the equilibrium by revealing $x$ signal might change the voting outcome: $(s_1, s_3) = (y, \emptyset(y))$, and in that case expert 2 prefers to hide the signal $x$; and similarly there is only one scenario where expert 3’s deviating from the equilibrium by revealing $y$ signal might change the voting outcome: $(s_1, s_2) = (\emptyset(x), \emptyset(x))$, and in that case expert 3 prefers to hide the signal $y$.

Although the details are involved, the multiplicity of equilibria comes from the fact that there can exist complementarities in experts’ strategies. Whether or not one expert reveals certain signals can impact whether or not another expert’s revelation of those signals
influences a vote.

5 Comparing Voting Rules

We now investigate one of our central questions concerning the comparison of voting rules. We begin by showing that the optimality of simple majority rule in symmetric settings (with exogenous agendas) depends on the information structure and communication. In particular, there are robust circumstances under which unanimity rule leads to a higher total expected utility for society than simple majority rule, and vice versa. Thus, information revelation considerations can lead to different orderings of voting rules.

This stems from the more general observation that the voting rule can affect deliberation. As the voting quota $q$ is varied, the pivotal voter can change. This can change the incentives for experts to reveal information. Roughly, as the quota increases we get more revelation of $y$ signals and less of $x$ signals. Increasing the threshold makes it more demanding for $y$ to be chosen and can lead to having a pivotal voter who is more $x$-biased than with a lower threshold. This can in turn lead more experts to be willing to reveal $y$ signals and fewer to be willing to reveal $x$ signals. Sometimes different voting rules can be strictly ordered in terms of information revelation, while in other cases they are not comparable (one leading to more signals of one type and fewer of another).

5.1 Unanimity Rule Can Dominate Simple Majority Rule

We begin with one of the more important conclusions of our analysis relative to the previous literature on voting and deliberation: it is possible to have unanimity rule dominate simple majority rule.\(^{26}\)

**Proposition 4** There are (symmetric) settings where the equilibrium outcome under unanimity rule leads to a higher total ex ante expected utility and leads to more information revelation than the equilibrium outcome under simple majority rule.

To get some intuition as to why this might occur, consider the following scenario. A committee must decide whether to approve or appoint a candidate to a post (we may even consider an example of an academic department hiring a faculty member). The committee consists of people with various biases regarding the candidate’s area or views. In particular, suppose that some would approve the candidate only if there are many very positive signals about the candidate’s quality, while others would approve the candidate with very few positive signals. The experts (for instance, outside letter writers) have a similar distribution of biases. If simple majority rule is used, then the opposed experts might choose to hide positive signals. In contrast, if unanimity rule is used for approval, the experts who are initially

\(^{26}\)Although the result is stated for unanimity rule, it should be clear from the analysis that other settings will favor various supermajority rules over simple majority rule.
more opposed are willing to reveal positive signals as they know that the candidate will be approved only if there are many positive signals revealed, in which case they themselves would support approval.

Proposition 4 is shown via the following two examples, which makes it clear that there are many such instances. We begin with a simple example in which unanimity leads to a higher total ex ante expected utility than simple majority rule. This example has some asymmetry in the setting which makes it transparent, but nicely illustrates how the revelation strategies change with the voting rule. We then return to provide a fully symmetric example where the same is true.

Example 2 Unanimity Dominates Simple Majority Rule

There are three voters with \( \lambda_1 = -\varepsilon \) and \( \lambda_2 = \lambda_3 = -\lambda(p) - \varepsilon \). There is one expert with \( \lambda_E = -\varepsilon \).

In this setting, simple majority rule leads to \( y \) always in any equilibrium, since voter 2 and 3 prefer \( y \) regardless of the signal and so the expert’s revelation is irrelevant. Unanimity leads to full revelation in an equilibrium (surviving the iterative elimination of weakly dominated strategies) and \( x \) if the signal is \( x \) and otherwise leads to \( y \). Unanimity rule leads to strictly higher total expected utility and to full revelation of information: For small enough \( \varepsilon \), voters 2 and 3 are relatively indifferent between \( x \) and \( y \) when there is an \( x \) signal, and yet voter 1 and the expert both strongly prefer \( x \) when the signal is \( x \). In this case, the increase in utility from unanimity versus simple majority rule is substantial, and thus the overall total utility is higher for unanimity rule compared to simple majority rule.

In the above example there is an asymmetry in the underlying structure that makes it easy to see why unanimity might dominate simple majority rule. However, unanimity can dominate simple majority rule even in settings that are completely symmetric. The idea is that under simple majority rule all experts can have incentives to hide some signals (faced with a pivotal voter who is unbiased and votes based simply on preponderance of evidence). In contrast, under unanimity it can be that at least some experts are willing to reveal all signals since now the pivotal voter has a similarly biased preference and might only vote for the opposing alternative if there is sufficient evidence.

Example 3 Unanimity Dominates Simple Majority Rule (A Symmetric Case)

Again, let \( \emptyset \) represent the observation of no signal from an expert who is following a strategy of revealing all signals, and let \( \emptyset(x) \) represent the observation of no signal from an expert who is following a strategy of revealing \( y \) signals and hiding \( x \) signals.

Let \( \lambda^* > 0 \) be a type of voter who prefers \( y \) when observing any permutation of \((y, \emptyset, \cdot, \cdot)\) and \((y, y, \emptyset(x), \emptyset(x))\) (or any combination that leads to a higher posterior on \( y \)) where the two \( \cdot \) entries are signals that cancel each other out (so \((\emptyset, \emptyset), (x, y)\) or \((\emptyset(x), \emptyset(y))\)); and prefers \( x \) when observing any permutation of \((y, \emptyset(x), \cdot, \cdot)\) (or any combination that leads to a higher posterior on \( x \)).
There are four experts, two with $\lambda^*$ and two with $-\lambda^*$, and three voters with types $\lambda^*, 0, -\lambda^*$, so that the setting is fully symmetric. We examine the following equilibria (where the voter randomizes the vote with equal probability when indifferent):

- Under simple majority rule, the pivotal voter has $\lambda = 0$ and in a monotone equilibrium the two $x$-biased experts reveal only $x$ signals and two $y$-biased experts reveal only $y$ signals;

- Under unanimity rule, the pivotal voter has $\lambda = \lambda^*$, and in a monotone equilibrium the 2 $y$-biased experts reveal only $y$ signals and two $x$-biased experts reveal all signals.

Thus, it is clear that unanimity rule provides more information revelation. Let us now verify that it also leads to higher overall utility.\textsuperscript{27} Figure 1 details the differences in total utility as a function of the signal realizations. “+” means unanimity increases total expected utility relative to simple majority rule conditional on the signal combination, “−” means unanimity reduces utility and blank means there is no net change. Signal combinations are listed up to a permutation. Moreover, in the two cases in the $A$ region, the plus and minus are of equal magnitude (and probability) and cancel each other out, and the same for the two cases in the $B$ region. If $\gamma > 2/3$ such that $P(\emptyset) < P(x) = P(y)$ ex ante, the combined utility difference is positive in the $C$ region since the plus equals the minus and the plus appears with higher probability. In all, as long as $\gamma > 2/3$, higher total utility is achieved under unanimity.

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Figure 1: A symmetric example: Unanimity leads to a higher total utility than majority rule.

While the previous example shows that the increase in information revelation that accompanies unanimity rule can lead to an increase in total utility, there are also settings where it leads to a decrease in total utility. The following example provides one such setting.

\textsuperscript{27}One can construct examples in simpler settings, e.g., with two experts and three voters, that lead to similar conclusions regarding more information revelation, but we could not find such an example with a higher total utility.
Example 4 More information is revealed under unanimity rule, but less utility is realized

Suppose that an agent with bias $\lambda^* > 0$ prefers $y$ if the (two) signals are $(y, y)$ and otherwise prefers $x$. Consider an example where there are three voters with biases $\lambda^*, 0, -\lambda^*$ and two experts with biases $\lambda_1 = \lambda^*, \lambda_2 = -\lambda^*$. Let a voter randomize his or her vote with equal probability when indifferent.

- Under simple majority rule, the pivotal voter has $\lambda = 0$ and in a monotone equilibrium the $x$-biased expert only reveals $x$ signals and $y$-biased expert only reveals $y$ signals;

- Under unanimity rule, the pivotal voter has $\lambda = \lambda^*$, and in a monotone equilibrium the $y$-biased expert only reveals $y$ signals and $x$-biased expert reveals all signals.

Under simple majority rule, the only scenarios of total utility loss are experts hiding signals: (a) $(s_1, s_2) = (y, \emptyset)$ and (b) $(s_1, s_2) = (\emptyset, x)$, where mistakes are made with probability one half. Under unanimity rule, the pivotal voter is strongly in favor of $x$ (as it takes two $y$ signals to get her choose $y$). So it corrects the half of the mistakes in (b) but makes mistakes in (a) such that the plus and minus cancel each other out. However, unanimity leads to a lower total utility in other cases where simple majority maximizes the utility. For instance, if $(s_1, s_2) = (\emptyset, y)$, unanimity leads to a choice of $x$ where the choice of simple majority is $y$ which is the optimal one. Thus, the total expected utility of society goes down.

The above examples point out that either simple majority or unanimity rule can be optimal, depending on the circumstances.

One conjecture might be that as we shift the types of the agents systematically in one direction or another, we should see a corresponding shift in the voting rule that maximizes total utility. That turns out not to be the case.

Example 5 The optimal voting rule is not well-ordered relative to the agents’ biases

There are three voters and one expert.

First, there are three voters $\lambda_1 = -\lambda(p) + \varepsilon$ and $\lambda_2 = \lambda_3 = -1$. There is one expert with $\lambda_E = -\lambda(p) + \varepsilon$. As argued in the Example 2, simple majority rule leads to $y$ always, while unanimity leads to full revelation of information and hence to $x$ if the signal is $x$ and otherwise leads to $y$. However, unanimity rule leads to strictly lower total utility. For small $\varepsilon$, voter 1 and the expert are relatively indifferent between $x$ and $y$ when there is an $x$ signal, and yet voters 2 and 3 both strongly prefer $y$.

When the $\lambda$s increase from the case above to Example 2 such that $\lambda_1 = -\varepsilon$, $\lambda_2 = \lambda_3 = -\lambda(p) - \varepsilon$ and $\lambda_E = -\varepsilon$, the optimal rules change from simple majority to unanimity.

Then, when the $\lambda$s increase again to $\lambda_1 = \lambda(p) + \varepsilon$, $\lambda_2 = \lambda_E = 0$ and $\lambda_3 = -\lambda(p) - \varepsilon$, simple majority dominates unanimity again. It is optimal to have the outcome match the signal in such a symmetric world. Since voter 1 always votes for $x$ and voter 3 always votes for $y$, unanimity cannot always pick the optimal outcome. Voter 2’s and the expert’s preferences depend on the signal in a manner so that simple majority rule leads the outcome to match the signal.
5.2 Information Revelation Versus Utility Maximization

The previous section shows that changing the voting rule can change revelation strategies. We also see a tension as in some cases, increasing the level of revelation requires a more biased voting rule relative to the population, which then decreases the total utility. It is also worth pointing out another subtlety: increasing revelation even keeping the voting rule fixed is not necessarily a good thing. That is, if we could control the strategies of the experts, and dictate whether they show or hide their signals, would forcing revelation of the information lead to higher total ex ante expected utility than forcing the hiding of information? The following example makes it clear that there are many instances where information revelation and utility maximization are not always aligned and in fact being uninformed can be better than having information revealed. We know from other literatures that information can sometimes be harmful (for example it can reduce possibilities for insurance), but we include the example since the intuitions are different in this environment.

Example 6 Total Utility Maximization can Require Muzzling the Expert

Consider a problem of choosing location at which a bridge (or a stadium, etc.) might be built in a country. There are two candidate locations $x$ and $y$. The state concerns which is the more costly location. A minority of the population lives very close to location $x$ and would benefit from a bridge at location $x$ and would not benefit at all from a bridge built at location $y$. Thus for the minority the cost is a minor part of their preference, and will not sway their preference over locations. A majority of the population lives closer to $x$ than $y$, but would slightly prefer to have the built at location $y$ if it turns out to be less expensive to build there. This is also the case for the expert.\footnote{Let the society be such that the minority of voters have $\lambda_i = 1 - \varepsilon$ and the remaining voters and the expert have $\lambda_i = \bar{\lambda}(p) - \varepsilon$ for some small $\varepsilon$ (small enough so that the total utility maximizing choice is location $x$ regardless of the signal).}

If the voting rule is simple-majority and a signal $y$ is observed, then the voters will choose location $y$ which leads to a lower total expected utility than location $x$. If the expert is not consulted, then location $x$ will be the outcome leading to a higher total expected utility. Thus, a social planner interested in maximizing total utility (whether or not the expert is included) would like to prevent the expert from revealing his or her signal.

This example depends on taking the voting rule as given. The inefficiency in the example stems from the fact that the voters who are pivotal are nearly indifferent when they vote for $y$, yet other voters have a strong preference for $x$. Another approach would be to set $q$ to unanimity which would have the same effect as muzzling the expert. In either case, the equilibrium turns out to be equivalent to one in which the information is not revealed or ignored - one by forcing the experts to be silent and the other by choosing a pivotal voter who does not listen to information.

An asymmetry is important in this example. If the society is symmetric, with each agent balanced by some other agent with an exactly opposite bias, then the optimal decision is to
choose in favor of the signal and a utilitarian social planner will not want to prevent experts from revealing their signals in a symmetric equilibrium, and in that case total expected utility would increase with information revelation.

The example is somewhat degenerate in that the total utility maximizing decision would be the same regardless of any information, and so a fixed decision would do as well as any voting mechanism. That is due to the fact that we have taken underlying preferences in our model to be fixed. The example is easily extended so that voting is optimal, but information revelation is not; by introducing another dimension to the state of the world: have the example either be as stated or else be the reverse where all agents prefer \( y \) (some strongly and some weakly) in the absence of any information. Then having a vote would be needed to discover which alternative maximizes total utility, but one would still wish to suppress information.

5.3 Approximately Full Revelation and Equivalence of Voting Rules with Large Numbers of Experts

Despite the variety of possibilities that can ensue in small societies regarding revelation of information as well as the optimality of different voting rules, we can deduce some systematic conclusions about large societies.

**Proposition 5** Consider any information setting where \( p > 1/2 \). For any \( \varepsilon > 0 \), there exists \( n \) such that if there are at least \( n \) experts, then for any voting quota and monotone equilibrium the probability that a pivotal voter votes as if he or she knew the true state is at least \( 1 - \varepsilon \).

The intuition here is clear: given that each expert reveals some information, the voter gets at least some information from all experts and by the law of large numbers becomes arbitrarily well informed. This happens even if all experts are extremely biased. For example, if all experts reveal \( x \) signals but hide \( y \) signals, then by counting how many out of \( n \) experts end up revealing \( x \) signals, the voter can infer (with high probability) the state.

**Corollary 2** Consider any information setting where \( p > 1/2 \). For any \( \varepsilon > 0 \), there exists \( n \) such that if there are at least \( n \) experts, and there is symmetry in the distribution of both experts’ and voters’ types, then for any voting quota and monotone equilibrium the total ex ante expected utility of the society is no more than \( 1 + \varepsilon \) times that under simple majority rule.

We can also deduce conclusions about information precision under which equilibria are close to fully revealing, regardless of the voting rule.

Let us say that two agents are similar if conditional of knowing the full profile of signals and/or non-observation of signals, they would always make the same choice.
Let us say that an equilibrium is $\varepsilon$-close to fully revealing if on the equilibrium path, with probability at least $1 - \varepsilon$, all voters vote as they would conditional on knowing the actual realized signal distribution.

**Remark 1**

1. If the agents are all similar, then there exists a fully revealing equilibrium for every voting rule.

2. For any $\varepsilon > 0$, if all experts have biases strictly between $-1/2$ and $1/2$, then there exists $p' < 1$ such that for all $p > p'$ and every voting rule there exists a monotone equilibrium that is $\varepsilon$-close to fully revealing.

3. For any $\varepsilon > 0$ there exists $\gamma' < 1$ such that for all $\gamma > \gamma'$ and every voting rule every monotone equilibrium is $\varepsilon$-close to fully revealing.

Remark 1 provides conditions on information under which deliberation will be fully revealing or nearly so, regardless of the voting rule. That does not imply that all voting rules will be equivalent, as changing the pivotal voter’s type can change the outcome. Part 1 addresses a case where all agents agree completely about the decision that should be made for every realization of signals, and so they are happy to share their information. Such a result appears in Schulte (2010) and the analogous proof in the setting here is straightforward and thus omitted. Part 2 addresses a case where signals are nearly perfectly correlated with the state. In that case, based on a single signal an expert is confident enough that the decision should be made in favor of that signal and reveals it in cases where it would substantially influence a voter’s choice. The expert realizes that there is some probability that other experts may have conflicting signals, but that probability is very small, both absolutely and relative to the chance that the other experts may have not observed signals. The details of why this results in almost full revelation, and not necessarily full revelation appear in Example 7. Part 3 addresses a case where voters are almost sure that experts have all seen signals. In that case, given experts’ biases, an observer can infer what the missing signals are when an expert claims not to have seen a signal. That is, every monotone equilibrium is $\varepsilon$-close to fully revealing.

**Example 7 $\varepsilon$-closeness to Full Revelation.**

To see why the conclusion is only $\varepsilon$-close to fully revealing and not fully revealing, consider a setting with one expert who is an $x$-supporter and one who is a $y$-supporter and suppose that the pivotal voter is an $x$-supporter with the same bias as the $x$-supporting expert, and let all agents have biases less than $1/2$ in magnitude. For high enough $p$ the monotone equilibrium is for the $x$-supporting expert to reveal information fully and for the $y$-supporting expert to hide $x$ signals. The only time the $y$-supporting expert’s hiding of $x$ signals will make a difference is in a case where the $x$-supporting expert revealed a $y$ signal. For high $p$,
these conflicting signals occur with a very low probability, and in that particular case, the
y-supporting expert gains by hiding the signal. Note that this means that there does not
exist a fully revealing equilibrium in this setting. Nevertheless, as \( p \) nears 1, since the case
of conflicting signals becomes increasingly rare, the vote will be as if the voter(s) were fully
informed with a probability of approaching 1.

Let us also discuss how the case of many experts compares to having a fixed number of
experts (e.g., one) and making signals become more precise. If an expert is not excessively
biased (\( \lambda_i < 1/2 \)), then once signals are precise enough the expert’s preference for the
common aspect will outweigh the private value, and so the expert will wish to choose in
favor of the signal and be willing to reveal all signals. Thus, with small enough bias, the
equilibrium will become fully revealing in the limit as signals become very precise. However,
if the expert is sufficiently biased (\( \lambda_i > 1/2 \)) then this is no longer the case, and the outcome
contrasts with increasing precision through many experts. Each expert has an incentive to
reveal some information, and with many experts the pattern of revelations reveals the state
with high likelihood regardless of how biased the experts are. Thus, varying signal precision
and varying the number of experts are not always equivalent in terms of their implications.

6 An Extension to Many Signals

Before concluding, we offer a discussion of an extension of the model.

Our analysis has focused on a simple model with just two possible signals, in favor of
one or the other alternative. This is for both tractability and comparability to the previous
literature. Nonetheless, it is important to be sure that the basic intuitions extend to richer
settings. As such, in this section we present an extension to a case of many signals, and
show that some basic properties still hold. We concentrate on the case of one expert and
one voter for simplicity.

Consider the following variation of the model, with one expert and one voter. Signals
come in some set \{\( s_1, \ldots, s_K \)\} with \( K \geq 3 \), and without loss of generality the signals are
labeled so that \( \Pr(\omega = x|s_k) < \Pr(\omega = x|s_{k+1}) \) (as one can consolidate signals that have
equal probabilities and then order the signals). Thus, higher labeled signals lead to higher
posterior probabilities that the state is \( x \). Again, there is a probability \( 1 - \gamma > 0 \) that no
signal is observed.

Let \( 1/2 > \lambda_V > 0 \), and to avoid indifference, suppose that the voter has a strict preference
over the alternatives conditional on any signal in \{\( s_1, \ldots, s_k \)\} \cup \{\emptyset\}. This, of course, holds
for a full measure of \( \lambda_V \).

Let \( \lambda^* > \lambda_V \) be the smallest \( \lambda \) for which there is some signal conditional upon which
the voter would strictly prefer \( y \), and an expert with bias \( \lambda \) would weakly prefer \( x \). For any
\( \lambda_E \geq \lambda^* \), let \( s(\lambda_E) \) and \( \pi(\lambda_E) \) be the lowest and highest labeled signals conditional upon
which the voter would strictly prefer \( y \), and the expert with bias \( \lambda_E \) would weakly prefer \( x \),
respectively.
Proposition 6 Consider the setting described above. There exists an equilibrium (that survives the iterative elimination of weakly dominated strategies) described as follows. If $E < E^*$, then there is a fully revealing equilibrium and all equilibria surviving the iterative elimination of weakly dominated strategies are equivalent to it.\(^{29}\) If $E \geq E^*$ (and both the voter and expert are never indifferent conditional on any information set), then there is an equilibrium (that survives the iterative elimination of weakly dominated strategies) such that the expert reveals signals below $S(E)$ and above $\bar{S}(E)$ when observed, and conceals signals from $S(E)$ to $\bar{S}(E)$ and all equilibria surviving the iterative elimination of weakly dominated strategies are equivalent to it.

This is an extension of Corollary 1.

Proposition 6 shows that the central intuition behind the devil’s-advocate result still holds, as do the basic sorts of incentives to reveal signals or conceal them. In particular, an important conclusion is that voter with a given bias who will sometimes be swayed by signals can be strictly better off with an expert with any opposing bias compared to an expert with a strong bias in the same direction. The result comes from the fact with $E \geq E^*$ either the voter ends up voting for $y$ when no signal is revealed, even in cases when the expert has not really seen any signal. Both the voter and the expert would like to be able to know that the expert has not seen a signal, but such statements are not credible given that the expert has incentives to conceal some $y$ signals. Or, the voter ends up voting for $x$ conditional on no signal being revealed, in which case the voter would have liked to observe some of the hidden signals.

The equilibria are slightly richer than those in the two signal cases, as now it is possible for a more $x$-biased expert to have some very extreme $y$-favoring signals that are so convincing that even he or she would like to have $y$ chosen. Of course, as $E$ increases, the interval of signals from $S(E)$ to $\bar{S}(E)$ widens, and eventually includes all signals conditional upon which voter would choose to vote for $y$.

7 Concluding Remarks

Although we have found that simple majority rule can be justified in terms of symmetry and large numbers of experts, in small societies information revelation and welfare are inexorably tied to the voting rule. Our analysis shows that the equilibria generally exhibit intuitive and simple features. In particular, beyond having experts aligned with voters, information revelation is higher in situations where experts biases are strongly opposed to the pivotal voter as compared to being strongly biased in the direction of the pivotal voter’s leaning. These sorts of considerations can lead non-majority rules to be optimal in (even symmetric) settings.

In our model experts can legitimately claim not to have any information, and this is what makes the analysis non-trivial. As we mentioned, this contrasts with the case such that

\(^{29}\)Again, equivalence is that the same outcomes are reached for every information realization.
experts are sure to have observed information, in which case one can infer their information. This suggests that having information about how informed an expert is can potentially make a big difference in terms of outcomes.

We focused on settings where experts do not vote. This applies to many settings (e.g., legislatures, some committees, courts, etc.), but there are also settings where voters themselves have expert knowledge. Such settings introduce additional complications since then an expert’s vote can convey hidden information, and so voters can potentially infer some hidden information from being pivotal. Learning hidden information from being pivotal can have subtle incentive effects that depend on the voting rule, as we know from Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998), and also deserves further study.30

References


30McLennan’s (1998) analysis shows that if agents’ preferences are aligned, then equilibria that efficiently aggregate information are possible. The pivot considerations are an issue here due to agents’ differing private preferences. Being pivotal reveals an agent’s relative position in voting, which can then reveal how some other agents are voting, which can in turn reveal something about potentially hidden information.


Proposition 7 Consider a setting where the voter has an $x$-bias with $0 < \lambda_V < \lambda(p)$, as the other case with a $y$-bias is analogous. Equilibria are characterized as follows:

- If $\lambda_E < -\lambda(p)$ then all equilibria are fully-revealing equivalent.

- If $-\lambda(p) < \lambda_E < \lambda(p)$ then
  - if $\lambda(p, \gamma) < \lambda_V < \lambda(p)$, all equilibria are fully-revealing equivalent,
  - while if $0 < \lambda_V < \lambda(p, \gamma)$ then there are two types of equilibria: those that are $x$-revealing and $y$-hiding equivalent and those that are fully-revealing equivalent.

- If $\lambda(p) < \lambda_E$ then
  - if $\lambda(p, \gamma) < \lambda_V < \lambda(p)$, all equilibria are $x$-revealing and $y$-hiding equivalent,
  - while if $0 < \lambda_V < \lambda(p, \gamma)$ then there are two types of equilibria: those that are $x$-revealing and $y$-hiding equivalent and those that are fully-hiding equivalent.

The structure of the equilibria in Proposition 7 is pictured in Figure 2.

There are some special features worth pointing out before proceeding to the proof. In the case where $0 < \lambda_E < \lambda(p)$ and there are two types of equilibria, both the expert and the voter prefer the fully revealing equilibrium to the partially revealing one. Thus, there is a strict Pareto ranking over the equilibria in this range. The difficulty is that the voter chooses $y$ sometimes when the signal is actually $\emptyset$. Both would prefer that the voter be fully informed in that case. The problem is that since the expert sometimes hides $y$ signals, then the voter cannot be sure that no signal was observed and the expert’s credibility is ruined. Once this is the expectation of the voter, and since the voter votes for $y$ conditional on not seeing any signal, the expert has no incentive to reveal $y$ signals. Note that the Pareto dominated equilibrium is eliminated by iterative elimination of weakly dominated strategy, which could be seen from Proposition 1.

There is also an interesting phenomenon with respect to the expert. If the equilibrium is $x$-revealing and $y$-hiding equivalent, then the expert is actually better off replacing him
or herself with an expert who is a strong $y$ supporter so that $\lambda_E < -\bar{\lambda}(p)$, who will reveal $y$ signals! In fact, this is true for an expert who actually has exactly the same bias as the voter, and so points to another difference from the cheap-talk literature. The point that a sender would be better off replacing him or herself with a sender with different preferences is something that one also sees in some cheap-talk models. However, there in the cheap-talk models senders would be better off if replaced with a sender who has preferences closer to those of the receiver, while here it is just the opposite. An expert with the same preferences as the voter can be better off being replaced by an expert with a very strong and different bias, and the voter would prefer this too.

**Proof of Proposition 7**: Given that $0 < \lambda_V < \bar{\lambda}(p)$, the voter will vote for $y$ conditional on seeing a $y$ signal and will vote for $x$ conditional on seeing an $x$ signal in any trembling hand perfect equilibrium (regardless of the strategies of the experts). What the voter chooses conditional on seeing no signal depends on the anticipated strategy of the expert and the precise bias of the voter.

There are two cases to consider: $\Delta(p, \gamma) < \lambda_V < \bar{\lambda}(p)$ and $0 < \lambda_V < \Delta(p, \gamma)$.

If $\Delta(p, \gamma) < \lambda_V < \bar{\lambda}(p)$, then regardless of the expert’s strategy, the voter will vote for $x$ conditional on seeing no signal in any trembling hand perfect equilibrium. This follows since by the definition of $\Delta(p, \gamma)$, conditional on not seeing a signal the voter would vote for $x$, even with the most extreme beliefs such that the voter knows the expert to be hiding $y$ signals. Thus, it follows that an expert who has $\lambda_E < \bar{\lambda}(p)$ will strictly prefer to reveal $y$ signals, and with $\lambda_E > \bar{\lambda}(p)$ will strictly prefer to hide $y$ signals against any strategy that the voter could be using in equilibrium. Whether or not the expert reveals $x$ signals is inconsequential.
in this case, since the voter will vote for \( x \) whether such a signal is seen or not. The result follows directly.

The remaining case is where \( 0 < \lambda_V < \lambda(p, \gamma) \).

First, let us find any equilibria where the voter votes for \( x \) with some positive probability when observing no signal. In any such equilibrium, given that the voter will vote for \( y \) conditional on seeing a \( y \) signal, an expert who has \( \lambda_E < \lambda(p) \) will strictly prefer to reveal \( y \) signals. So, any such equilibrium where the expert has \( \lambda_E < \lambda(p) \) is one where the expert reveals \( y \) signals, and then the voter votes for \( x \) (for sure) whenever not seeing a signal and it is equivalent to a fully revealing equilibrium (which is then easily seen to be an equilibrium).

So, consider a case where the voter votes for \( x \) with a positive probability when seeing no signal and faces an expert with \( \lambda_E > \lambda(p) \). Such experts will then strictly prefer to hide \( y \) signals. Given that \( 0 < \lambda_V < \lambda(p, \gamma) \), it must then be that the expert is also hiding \( x \) signals, or else the voter would vote for \( y \) conditional on seeing no signal. For the expert to be willing to hide \( x \) signals, it must that the voter votes for \( x \) with certainty conditional on not seeing a signal. Thus, the equilibrium must be equivalent to one which is fully hiding. This is then easily seen to be an equilibrium.

Finally, let us find all equilibria where the voter votes for \( y \) conditional on seeing no signal. In that case, if \(-\lambda(p) < \lambda_E\), then the expert will strictly prefer to reveal \( x \) signals. So equilibrium would have to be one of revealing \( x \) signals and at least partially hiding \( y \) signals (as otherwise the voter would be voting for \( x \) conditional on seeing no signal). These are all equivalent to \( x \)-revealing and \( y \)-hiding equilibria, which are easily seen to be equilibria. So consider the case where \( \lambda_E < -\lambda(p) \). In that case, the expert would then prefer to hide \( x \) signals, and so conditional on seeing no signal revealed (and regardless of the expert’s strategy with respect to \( y \)), the voter must vote for \( x \), which then contradicts the supposition, and so there are no such equilibria in this case.

**Proof of Proposition 1:** In all cases \( 0 < \lambda_V \). Consider the case \( \lambda_V > \lambda(p, \gamma) \). First, undominated strategies of the voter are clearly such that the voter votes for \( x \) when observing \( x \) signal or no signal and votes for \( y \) when observing \( y \) signal. Then undominated strategies of the expert with \( \lambda_E < \lambda(p) \) involve revealing \( y \) signals and for an expert with \( \lambda_E > \lambda(p) \) involve hiding \( y \) signals. Note that revealing \( x \) signals (or not) won’t change the vote outcome and so the equilibrium under the iterative elimination of weakly dominated strategies is fully-revealing for experts with \( \lambda_E < \lambda(p) \) and is \( x \)-revealing and \( y \)-hiding equivalent for experts with \( \lambda_E > \lambda(p) \).

Next, consider the case such that \( 0 < \lambda_V < \lambda(p, \gamma) \). First, undominated strategies of the voter are clearly such that the voter votes for \( x \) when observing \( x \) signal and votes for \( y \) when observing \( y \) signal. However, voting for \( x \) or \( y \) when observing no signal depends on the strategy of the expert and so both are possible as undominated strategies. For instance, if the expert fully reveals signals, then she would vote for \( x \) when seeing no signal and if the expert only reveals \( x \) signal, then she would vote for \( y \) when seeing on signal. Note that no strategies of the expert are revealed in the first round since the voters strategy can
condition upon the experts revelations in arbitrary ways (e.g., either voting with or against the revealed signal.

On a second iteration of eliminating dominated strategies, some strategies of the expert become weakly dominated by others. In particular, if \(-\lambda(p) < \lambda_E < \lambda(p)\), then when observing an \(x\) signal the expert prefers \(x\) and it is weakly better to reveal \(x\) against all undominated voter strategies and strictly better against some, since in an undominated strategy the voter votes according to the signal when revealed, but may vote in either direction conditional upon not seeing a signal. Similarly, the only undominated strategies at a second iteration for the expert have the expert revealing \(y\) signals. Thus the only equilibrium surviving iterative elimination of weakly dominated strategy is the fully-revealing equilibrium.

If \(\lambda_E > \lambda(p)\), it still follows that \(x\) signals are reveled in an iteratively undominated strategies, but in this case revealing \(y\) signals is (second round) weakly dominated by not revealing them since the voter votes for \(y\) when seeing a \(y\) signal in an undominated strategy, but may not when seeing no signal. So the only equilibrium left is the \(x\)-revealing and \(y\)-hiding equilibrium.

The remaining case is where \(\lambda_E < -\lambda(p)\). By the reasoning above, it follows that the only strategies surviving an iterative elimination of weakly dominated strategies are \(y\)-revealing, and \(x\)-hiding. Note, however that these are equivalent to being fully revealing.

**Proof of Lemma 1:** First, for any equilibrium under voting rule \(q\), we define the equivalent equilibrium in the reduced game with one pivotal voter as follows: the experts employ the same strategy. If there is only one pivotal voter, then she votes as in the original game. If there are multiple pivotal voters in the original game, then the voter employs a strategy that mimics the aggregate random outcome of the vote in the original game conditional on each configuration of signals. All we need to check is that it is indeed an equilibrium of the reduced game.

Note that all voters in the original game observe the same revealed signals and hold the same beliefs regarding the experts’ strategies. In the reduced game, suppose that the pivotal voter with bias \(\lambda(q)\) is indifferent after observing all information. Then the strategy of the voter in the reduced game is a best response. Next, without loss of generality, suppose that the voter strictly prefers \(x\) after observing all information (the case of \(y\) is analogous). It follows that voters with bias greater than or equal to \(\lambda(q)\) will also all strictly prefer \(x\) in the original game, and thus the outcome must be \(x\) in the original game which is consistent with the preference of the pivotal voter in the reduced game. Also, experts in the reduced game won’t want to change their strategies since if such a change would change the pivotal voter’s preference, it would also change the outcome in the original game, which would be a contradiction of the original equilibrium. Thus, there is no change in the strategy that could improve the outcome in the reduced game as well.

Then for any equilibrium for the society with a single pivotal voter, we extend this equilibrium to the original set of voters in \(S_1\). Consider the strategy: voter \(i\) with \(\lambda_i \neq \lambda_q\)
votes for the alternative he/she prefers given the strategy of experts and the revealed signals. (Voter $i$ can use any strategy if he/she is indifferent as he or she will not be pivotal.) If there is only one voter with $\lambda = \lambda_q$, this voter will simply use the strategy of the single voter in the other society. If there are multiple voters with $\lambda = \lambda_q$, the strategy is a bit complex: given the revealed signals

- if the pivotal voter votes for $x$ for sure, all voters with $\lambda = \lambda_q$ vote for $x$;
- if the pivotal voter votes for $y$ for sure, all voters with $\lambda = \lambda_q$ vote for $y$;
- if the pivotal voter uses a mixed strategy, we rank all voters (arbitrarily if there is a tie) and find the voter of rank $q$, all voters with $\lambda = \lambda_q$ vote for $x$ if their ranks are lower than $q$, all voters with $\lambda = \lambda_q$ vote for $y$ if their ranks are higher than $q$ and the voter with rank $q$ uses the mixed strategy of the single voter in the other society.

It is straightforward to check that every voter uses an optimal and interim undominated strategy here and the outcome mimics the equilibrium in the other society and that the experts also find their strategies optimal. So this is an equivalent equilibrium as described. □

**Proof of Proposition 2:**

There are two parts in the proof: first we show there is an equilibrium where experts with $\lambda \leq \lambda(q)$ reveal at least $y$ signals and experts with $\lambda \geq \lambda(q)$ reveal at least $x$ signals, and that if the equilibrium is not monotone then we can switch the strategy to make it monotone, with it remaining as an equilibrium and being “shuffling-equivalent” to the original one.

We start with a restricted problem such that experts with $\lambda \leq \lambda(q)$ are required to reveal $y$ signals, experts with $\lambda \geq \lambda(q)$ are required to reveal $x$ signals. A perfect equilibrium exists in this restricted game by the finiteness of the game.

We note that in any equilibrium there is an independent belief restriction such that voters’ beliefs over the strategy of each expert is independent of their beliefs regarding other experts’ strategies and other revealed signals.\(^{31}\) Note that this independent belief restriction is implied by trembling hand perfection: In each equilibrium with small trembles, the voter has some belief about the experts’ strategy which is independent of any possible revelation of signals. For instance, even if the voter believes the some expert is hiding $y$ signal but in fact that expert reveals $y$, this is viewed as a result of small trembles such that no belief should be changed. As the trembles go to zero, the believes converge to the limit belief which also needs to be independent.

Now we withdraw the above revelation restrictions and show that it is still an equilibrium. Note that under the independent belief restriction, revealing a $y$ signal always leads to a higher posterior probability on $y$ than not revealing a $y$ signal. This follows from the fact that revealing $y$ (by expert $i$ for instance) leads to a weakly higher posterior on $y$ than

\(^{31}\)For example, consider an equilibrium such that no expert is supposed to reveal any $x$ signal. However, consider beliefs off the equilibrium path, so that one of the experts has revealed an $x$ signal. This belief restriction implies that the voter still believes that the other experts are not revealing any of their $x$ signals.
not revealing it if only based on expert $i$’s information, and the other experts’ strategies and beliefs regarding others’ strategies are not affected by the revealed signal (and signals are positively correlated). If expert $i$ with $\lambda_i \leq \lambda(q)$ observes $y$ signal and chooses hiding it instead, the only possible change to the outcome is lowering the probability of $y$, since hiding $y$ would reduce the posterior probability on $y$. However in any scenario such that the voter with $\lambda(q)$ changes to vote in favor of $y$ conditional on expert $i$ revealing the $y$ signal versus voting for $x$ if it is hidden (the only possible change given the direction of change of the posterior), then it would have to be that expert $i$ with $\lambda_i \leq \lambda(q)$ would also prefer $y$ since the bias towards $y$ is at least as high for the voter. Thus, experts with $\lambda \leq \lambda(q)$ weakly prefer to reveal $y$ regardless of other experts’ strategies, and similarly experts with $\lambda \geq \lambda(q)$ weakly prefer to reveal $x$.

Thus, there is an equilibrium where experts with $\lambda \leq \lambda(q)$ reveal at least $y$ signals and experts with $\lambda \geq \lambda(q)$ reveal at least $x$ signals.

Then we need to show if two experts don’t act monotonically, we can switch their strategies and the resulting profile will still be an equilibrium. Consider the case such that in an equilibrium two experts ($\lambda(q) \leq \lambda_1 < \lambda_2$) don’t act monotonically such that expert one reveals only signal $x$ while expert two reveals all signals. We want to prove that switching these two strategies and keeping all others fixed is also an equilibrium.

It is clear that this does not change the incentives of voters or any other experts, so we only need to consider the incentives of the two experts.

In the new strategy, given expert one reveals all signals and the voter expects expert two to reveal only signal $x$, it is optimal for expert two to reveal only signal $x$. Because expert two now faces exactly the same situation as expert one in the original equilibrium except $\lambda_1 < \lambda_2$. Expert one finds it is optimal to only reveal $x$ signal, then expert two with a higher bias on $x$ should find it optimal as well.

On the other hand, given expert two reveals only signal $x$ and the voter expects expert one to reveal all signals, it is optimal for expert one to do so. First, as argued above expert one with $\lambda(q) < \lambda_1$ when observing $x$ signal, weakly prefers to reveal it. Second, expert one won’t strictly prefer to reveal only signal $x$ since in the original equilibrium expert two doesn’t strictly prefer to do so. Thus, expert one weakly prefers to fully revealing both signals.

Similar arguments apply to switching the strategies if $\lambda(q) \geq \lambda_1 > \lambda_2$ and expert one reveals only signal $y$ and expert two reveals all signals. So after switching strategy, the new equilibrium is monotone and by the feature of switching strategy, it is shuffling-equivalent to the original equilibrium. Thus there exists a monotone equilibrium. ■

**Proof of Proposition 3:** This follows from the second part of the proof above. ■

**Proof of Proposition 5:** If $|\lambda_v| \geq \frac{1}{2}$, the pivotal voter will vote for her favorite choice regardless of the signals such that we only need to consider the case where $|\lambda_v| < \frac{1}{2}$. So if the voter knows that true state, the vote will be consistent with the state. All we need to
show is for any ε > 0 there is n_ε such that if there are at least n_ε experts and the true state is x, the voter will vote for x with probability at least 1 − ε.

There are three types of strategies in a monotone equilibrium: full revelation, revealing only x, and revealing only y. At least one of these three strategies is played by at least a third of the experts. Let us argue that a voter who only paid attention to a least informative type of strategy played by a subset of only a third of the experts, would vote correctly with probability at least 1 − ε. For example, consider a set of experts who reveal y signals whenever they have them (other cases are analogous). With m such experts, the expected number who will reveal y signals is pγm if the state is y and (1 − p)γm if the state is x. Moreover, by the law of large numbers, for large enough m, the fraction who reveal y signals is above γm/2 with a probability at least 1 − ε if the state is y and below γm/2 with a probability of at least 1 − ε if the state is x. Depending on the pivotal voter’s preferences, there is a large enough m such that the voter will vote in accordance with the true state with probability at least 1 − ε if only paying attention to this information. Including additional signals only increases the precision of the voter’s information.

Proof of Proposition 6:

Consider a case such that λ_E < λ^*, such that if some signal makes the voter strictly prefer y, the expert also strictly prefers y. In this case, undominated strategies of the voter are such that the voter votes for the preferred alternative when observing a signal; and when observing no signal, there are two possible undominated strategies of the voter depending on the value of λ_V: (i) voting for x is the only undominated strategy; (ii) both voting for x or y are undominated. Then we look at the undominated strategies of the experts: for all signals where the voter prefers y (by assumption the expert also strictly prefers y), the only undominated strategies for the expert are to reveal these signals (as not revealing can face a strategy that votes for x, as just argued, while revealing only faces votes for y). So, now iterate on the undominated strategies of the voter: if the expert does not reveal a signal, there could not have been a signal observed such that the voter would prefer y since those signals would be revealed. Thus, the only undominated strategy (given previous eliminations) is to vote for x when observing no signal since λ_V > 0. Thus, all remaining equilibria are fully-revealing equivalent (and this set is non-empty as equilibria exist).

Now suppose λ_E ≥ λ^* such that the expert strictly prefers x and the voter strictly prefers y when the signal is between γ(λ_E) and ζ(λ_E), and otherwise they both agree in either strictly preferring x or y. The first elimination of dominated strategies of the voter is the same as above. As above, for signals below ζ(λ_E), where both the voter and the expert strictly prefer y, the only undominated strategies for the expert are to reveal these signals. Also, the only undominated strategies for the expert are not to reveal signals between γ(λ_E) and ζ(λ_E), as there is a possibility that the voter will vote for y if not seeing a signal, but would vote for y conditional on seeing one of those signals. Finally, signals above ζ(λ_E) are revealed by the expert if there is any undominated strategy of the voter that votes for y when not seeing a signal, and otherwise all votes by the voter are for x when not seeing a signal and
so equivalent to having these signals revealed. Thus, this equilibrium has the same outcome as the equilibrium where the expert uses a revealing-hiding-revealing strategy. □