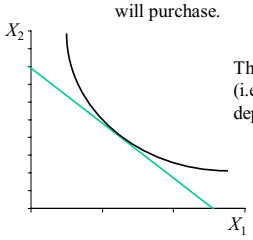
Demand Part I

- Recap: The Consumer's Optimization Problem
  - The budget constraint and the tangency condition determine the amount of each good the consumer will purchase.



The consumer's choice of (X_1,X_2) (i.e. demand for X_1 and X_2) depends upon:
- prices (p_1, p_2)
- income (I)
- preferences—$U(X_1,X_2)$

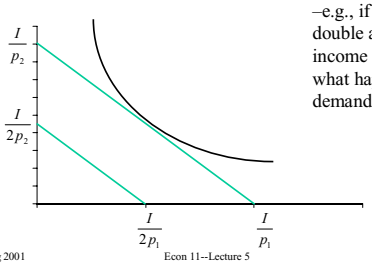
Demand Functions

- A Marshallian demand function relates the quantity demanded of a good to prices and income
- Demand depends on all prices
- Preferences and constraints together determine the shape of demand

$$X_1 = f(p_1, p_2, I)$$
$$X_2 = g(p_1, p_2, I)$$

Comparative Statics

- What happens to demand when prices or income changes?



–e.g., if prices double and income doubles, what happens to demand?

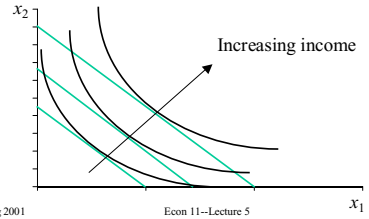
Zero Degree Homogeneity of Demand

- A function $f(x_1,x_2,\ldots x_n)$ is homogenous of degree k if $f(tx_1,tx_n,\ldots tx_n) = t^k f(x_1,x_n,\ldots x_n)$
- Marshallian demand functions are homogenous of degree zero. This fact is consistent with the absence of "money illusion."

$$X_1(p_1, p_2, I) = X_1(2p_1, 2p_2, 2I)$$

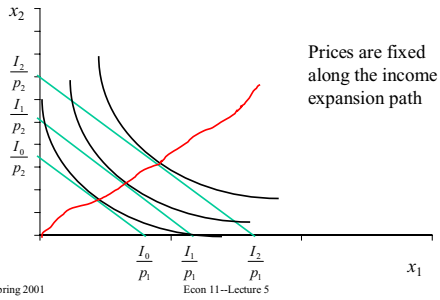
What happens to demand when income changes?

- Budget constraint shifts in/out. Slope of budget constraint does not change.



Income Expansion Path

(Income-Offer Curve)

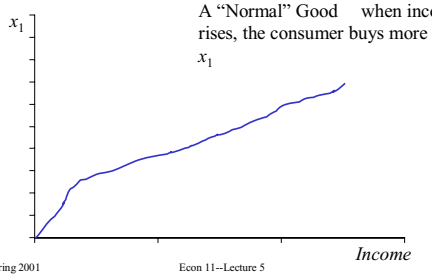


Prices are fixed along the income expansion path

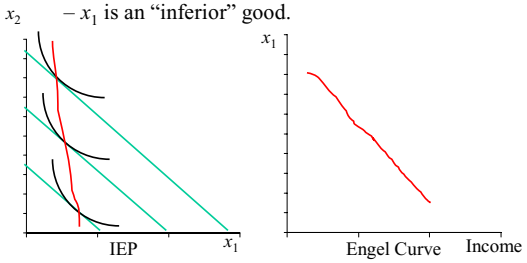
Engel Curves

- Engel curve relates income to quantity demanded.

A "Normal" Good when income rises, the consumer buys more of x_1



- But what if the IEP or Engel curve looks like this? An increase in income leads to more x_2 but less x_1.
  - x_1 is an "inferior" good.



Normal and Inferior Goods

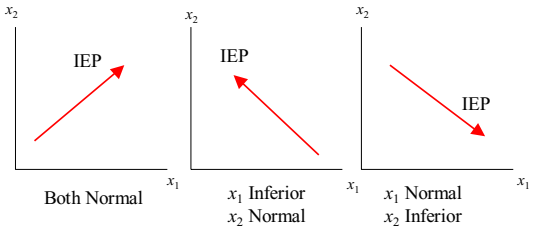
- *Normal Good*: Demand for a good x increases with income
  - This implies that the slope of the Engel curve is positive. $\frac{\partial X}{\partial \mathrm{I}} > 0.$
- *Inferior Good*: Demand for a good x decreases with income
  - This implies that the slope of the Engel curve is negative. $\frac{\partial X}{\partial \mathrm{I}} < 0.$

Examples

Normal Goods	**Inferior Goods**
Beef	Potatoes
Cars	Bus tickets
Haircuts at a salon	Haircuts by your mother

All Goods Can't Be Inferior

- "Proof" #1: If income expands, the IEP cannot point toward the origin.



All Goods Can't Be Inferior

- Proof #2: use budget constraint.

$$p_1 x_1 + p_2 x_2 = \mathrm{I}$$

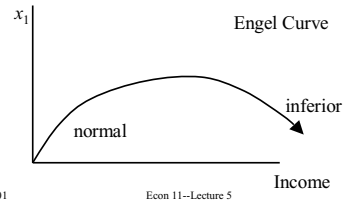
$$p_1 \frac{\partial x_1}{\partial \mathrm{I}} + p_2 \frac{\partial x_2}{\partial \mathrm{I}} = 1$$

Both $\frac{\partial x_1}{\partial \mathrm{I}}$ and $\frac{\partial x_2}{\partial \mathrm{I}}$ can't be negative.

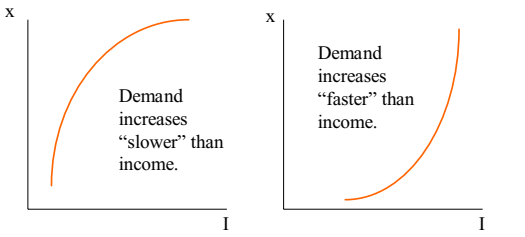
Thus, both x_1 and x_2 can't be inferior goods.

A Good Can't be Inferior at all Income Levels

- Why not? Start with zero income. As income increases, if you ever consume that good, it is normal (at that income level).
- In order for a consumer to purchase <u>less</u> of a good as income increases, he must once have consumed <u>some</u> of it.



Consider 2 Engel Curves



% change in x < % change in I
Necessity

% change in x > % change in I
Luxury

Elasticity

- The elasticity of y with respect to x is defined as the percentage change in y induced by a <u>small</u> percentage change in x.
- Why do we need this concept? It is unit free.
  - e.g. How much coffee are you willing to trade for a bagel? Determined by the slope of indifference curve = MRS.
  - The value of the MRS depends upon the "units" of coffee we are using.

Elasticity

- Elasticity of y with respect to x is defined as the percentage change in y induced by a small percentage change in X.

$$\varepsilon_{y,x} = \frac{dy}{dx}\cdot\frac{x}{y} \approx \frac{\Delta y}{y} / \frac{\Delta x}{x}$$

$$\varepsilon_{Y,X} = \frac{d\ln y}{d\ln x} \quad \text{since} \quad d\ln y = \frac{dy}{y}$$

Income elasticity of demand

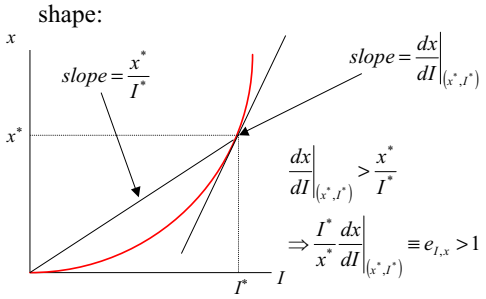
- Elasticity of a good, x, with respect to income.

$$\frac{dx}{dI}\cdot\frac{I}{x} \equiv e_{x,I}$$

<u>Definitions</u>

$e_{x,I} < 0$	Inferior	$e_{x,I} < 1$	Necessity
$e_{x,I} > 0$	Normal	$e_{x,I} > 1$	Luxury

- Engel curves for luxuries have a convex shape:



Engel's Law

- Engel's Law: "Food is a necessity"
- Expenditure on Food / Income
- 1935-1939 35.4%
- 1952 32.2%
- 1963 25.2%
- 1998 19%

- If x is a necessity, then as income increases, the share of income spent on x decreases:
  - Define the share of income spent on x as S_x

$$S_x = \frac{xp_x}{\mathrm{I}}$$

  - I will prove that if x is a necessity:

$$\frac{dS_x}{d\mathrm{I}} < 0$$

$$\log S_x = \log x + \log p_x - \log \mathrm{I}$$

Totally differentiate this log "share" equation:

$$d \log S_x = d \log x + d \log p_x - \Delta \log \mathrm{I}$$

Hold prices constant, i.e., set $d \log p_x = 0$

$$\Rightarrow d \log S_x = d \log x - d \log \mathrm{I}$$

$$\Rightarrow \frac{d \log S_x}{d \log \mathrm{I}} = \frac{d \log x}{d \log \mathrm{I}} - 1$$

$$\Rightarrow \frac{I}{S_x}\frac{dS_x}{dI} = \frac{I}{x}\frac{dx}{dI} - 1$$

$$\frac{I}{S_x}\frac{dS_x}{dI} = \frac{I}{x}\frac{dx}{dI} - 1$$

$$\Rightarrow \frac{I}{S_x}\frac{dS_x}{dI} = e_{I,x} - 1$$

but for necessities, $e_{I,x} < 1$

$$\Rightarrow \frac{I}{S_x}\frac{dS_x}{dI} < 0 \Rightarrow \frac{dS_x}{dI} < 0$$

so the data confirm Engel's Law

- The expenditure weighted sum of income elasticities is equal to 1.
- Thus, all goods cannot be necessities. Nor can all goods be luxuries.

$$S_1 e_{I,1} + S_2 e_{I,2} = 1$$

where

$$S_1 = \frac{x_1 p_1}{\mathrm{I}} \qquad S_2 = \frac{x_2 p_1}{\mathrm{I}}$$

- Proof: Start with the budget constraint

$$p_1 x_1 + p_2 x_2 = \mathrm{I}$$

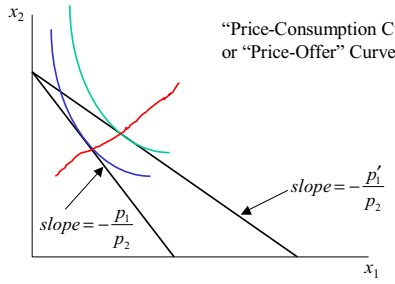
$$\Rightarrow p_1 \frac{dx_1}{d\mathrm{I}} + p_2 \frac{dx_2}{d\mathrm{I}} = 1$$

$$\Rightarrow \frac{x_1}{\mathrm{I}}\frac{\mathrm{I}}{x_1} p_1 \frac{dx_1}{d\mathrm{I}} + \frac{x_2}{\mathrm{I}}\frac{\mathrm{I}}{x_2} p_2 \frac{dx_2}{d\mathrm{I}} = 1$$

$$\Rightarrow \left(\frac{x_1 p_1}{\mathrm{I}}\right)\left(\frac{\mathrm{I}}{x_1}\frac{dx_1}{d\mathrm{I}}\right) + \left(\frac{x_2 p_2}{\mathrm{I}}\right)\left(\frac{\mathrm{I}}{x_2}\frac{dx_2}{d\mathrm{I}}\right) = 1$$

$$\Rightarrow S_1 e_{I,1} + S_2 e_{I,2} = 1$$

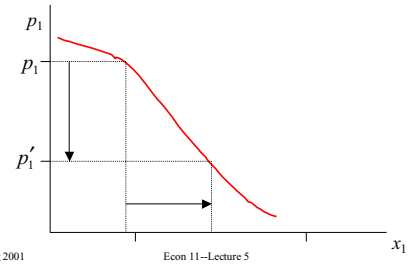
What happens to demand when price changes?



"Marshallian" Demand Curve

(Demand Curve)

- In the graph, we hold constant income and the prices of all other goods.



The Law of Demand

- The 'Marshallian" demand curve slopes downward (usually).
  - The "weak" law of demand.
  - It is theoretically possible for the Marshallian demand curve to slope upward.
- The "Marshallian demand curve is the demand curve that we most often use. Thus, we often just call it the "demand curve."