Demand III

- Last lecture we covered:
  - Substitution and Income Effects
  - Slutsky Equation
  - Giffen Goods
  - Price Elasticity of Demand

Substitutes and Complements

- We will now examine the effect of a change in the price of another good on demand.
- Define \( x_1 \) and \( x_2 \) as “Gross Substitutes” if an increase in the price of \( x_2 \) leads to an increase in the demand for \( x_1 \).
  \[
  \frac{dx_1}{dp_2} > 0 \implies \text{Gross Substitutes}
  \]

Substitutes and Complements

- Define \( x_1 \) and \( x_2 \) as “Gross Complements” if an increase in the price of \( x_2 \) leads to a decrease in the demand for \( x_1 \).
  \[
  \frac{dx_1}{dp_2} < 0 \implies \text{Gross complements}
  \]

Cross-Price Elasticity of Demand

\[
\varepsilon_{12} = \frac{dx_1}{dp_2} \frac{p_2}{x_1}
\]

- \( \varepsilon_{12} > 0 \) \implies \text{Gross Substitutes}
- \( \varepsilon_{12} < 0 \) \implies \text{Gross Complements}

Why estimate elasticities rather than just the derivatives?—Elasticities are unitless.

Hicksian Demand Functions

- Recall “Marshallian” Demand Functions
  - \( x_1 = f(p_1, p_2, I) \) hold income constant
- “Hicksian” or “Utility Constant” or “Compensated” Demand Function
  - Hicksian demand functions hold utility constant
    \( x_1 = h(p_1, p_2, U) \)
Hicksian Demand

- Decreasing \( p_1 \)

Hicksian vs. Marshallian Demand

- Hicksian demand curves are steeper for normal goods
- Hicksian demand curves are flatter for inferior goods

Hicksian Demand Functions

- Recall Slutsky Equation
  \[
  \frac{dx_i}{dp_1} = \frac{dx}{dp_1} \text{Compensated} - \frac{dx_i}{dl} x_i^0
  \]
- Hicksian (or Compensated or Utility constant demand functions) yield the amount of good \( x_i \) purchased at prices \( p_1 \) and \( p_2 \) when income is just high enough to get utility level \( u^0 \).

\[
x_i = h(p_1, p_2, u^0)
\]

Law of Demand

- Hicksian Demand Curves must slope down.
  - Why? The substitution effect is negative.

Calculating Hicksian Demand

- For Hicksian demand, utility is held constant.
- The trick to calculating Hicksian demand is to use expenditure minimization subject to a constant level of utility, rather than utility maximization subject to a constant level of income.
- Expenditure minimization is known as the “dual” problem to utility maximization.

Calculating Hicksian Demand (II)

- Suppose \( U_0 = U(x_1, x_2) \) is a utility function at a given utility level \( U_0 \)
- Prices are \( p_1 \) and \( p_2 \)
- Total expenditures are \( p_1 x_1 + p_2 x_2 \)
- The expenditure minimization problem is:

\[
\min_{x_1, x_2} p_1 x_1 + p_2 x_2 \\
s.t. \quad U(x_1, x_2) = U_0
\]
Calculating Hicksian Demand (III)

• We can set up the Lagrangian objective:
\[
max_{x_1, x_2} L = p_1 x_1 + p_2 x_2 - \lambda (U(x_1, x_2) - U_0)
\]

• The solution to this problem will be two Hicksian demand functions:
\[
x^*_1 = h_1(p_1, p_2, U_0)
\]
\[
x^*_2 = h_2(p_1, p_2, U_0)
\]

Net Complements and Net Substitutes

• Assume 3 goods, \(x_1, x_2,\) and \(x_3\)

• Define \(x_1\) and \(x_3\) as “net substitutes” if an increase in the price of good 2 leads to an increase in the compensated demand for good 1.
\[
\frac{dx_1}{dp_2} > 0 \Rightarrow \text{net substitutes}
\]

Net Substitutes

\[\text{Increase in } p_2 \Rightarrow \text{Increase in } x_1\]

Net Complements

\[\text{Define } x_1 \text{ and } x_2 \text{ as “net complements” if an increase in the price of good 2 leads to a decrease in the compensated demand for good 1.}\]
\[
\frac{dx_1}{dp_2} < 0 \Rightarrow \text{Net complements}
\]

Fact: All goods can’t be net complements.

Can’t draw this in a two dimensional graph!

Compensated Price and Income Elasticities Of Demand

\[
\eta_i = \frac{dx_i}{dI} \frac{1}{x_i}
\]
\[
\varepsilon_i' = \frac{p_2}{x_i} \frac{dx_i}{dp_2} \text{compensated}
\]
\[
\varepsilon_{12} = \frac{p_2}{x_1} \frac{dx_1}{dp_2} \text{compensated}
\]
The Slutsky Equation

- The Slutsky Equation can also be written in terms of elasticities.

\[
\frac{dx}{dp} + \frac{dx}{dl} = \frac{dx}{dl} \\
\frac{p}{x} \left( \frac{dx}{dp} + \frac{dx}{dl} \right)
\]

Compensated price elasticity of demand = Price elasticity + Share of expenditure x Income elasticity

The Consumer Price Index (CPI)

- What is the CPI?
  - The CPI is an index which tells us how much it would cost in current prices to buy a fixed bundle of goods. Currently, we use a 1982-1984=100 base for the CPI. This means we use the average bundle purchased in the 1982 to 1984 period as a "representative bundle of goods."
  - In Aug 1997, the CPI was 160.8. This means that it now costs 1.608 times more to purchase "a representative bundle" than it did in the 1982-1984 period.
  - The inflation rate is the rate of change in the CPI
- In Aug 1996 the CPI was 157.3. Thus, the inflation rate over this period was 2.23%

Issues

- Is the CPI a "true" cost-of-living index?
- If the CPI rose by 10% from 1986 to 1987 and your income rose by the same 10%, are you worse off, better off, or just as well off?
- Does the CPI overstate the "true" inflation rate?
  - A few years ago, Alan Greenspan, the Fed Chairman, stated that the CPI overstates the "true" inflation rate by roughly 0.5 to 1.5%.
  - The Boskin Commission found that the CPI overstated inflation by 1.1%. What do they mean?

Suppose I purchase \( (x_1^0, x_2^0) \) in year 0 at prices \( (p_1^0, p_2^0) \) with income \( I \)

Budget constraint: \( x_1^0 p_1^0 + x_2^0 p_2^0 = I \)

Suppose that both prices increase by the inflation rate \( \pi \)

\( p_1^1 = p_1^0 (1 + \pi) \)
\( p_2^1 = p_2^0 (1 + \pi) \)

Q. By how much will income have to increase in order to keep me at my original level of utility?

A. Income needs to increase by \( \frac{\pi}{\frac{I_2}{p_2^0} (1 + \pi)} \)
Now suppose that only the price of good 1 increases by $2\pi$

$$p_1^1 = p_0^1(1 + 2\pi) \quad p_2^1 = p_2^0$$

Furthermore, suppose I spend exactly $\frac{1}{2}$ of my income on good 1 in year 0. Thus

$$p_1^0 x_1^0 = \frac{I^0}{2}$$

The increase in “CPI” is just the increase in the cost of purchasing $(x_1^0, x_2^0)$

$$\left(p_1^1 x_1^0 + p_2^1 x_2^0\right) - \left(p_1^0 x_1^0 + p_2^0 x_2^0\right) = 2\pi p_1^0 x_1^0 = \pi I_0$$

If the CPI is a good indicator of changes in welfare resulting from changes in prices, then restoring my ability to purchase the original consumption bundle should leave me no better off than before the price change. However, the graph demonstrates that I will be better off. Thus CPI overestimates the “true cost of living”.