Bargaining and Repeated Games

Jonathan Levin

February 2002

1 Sequential Bargaining

A classic economic question is how people will bargain over a pie of a certain size. One approach, associated with Nash (1950), is to specify a set of axioms that a "reasonable" or "fair" division should satisfy, and identify the division with these properties. For example, if two identical agents need to divide a pie of size one, one might argue that a reasonable division would be $(1/2, 1/2)$.

The other approach, associated with Nash's 1951 paper, is to write down an explicit game involving offers and counter-offers and look for the equilibrium. Even a few minutes' reflection suggests this will be difficult. Bargaining can involve bluffing and posturing, and there is no certainty that an agreement will be reached. The Nash demand game demonstrates that a sensible bargaining protocol might have many equilibria. A remarkable paper by Rubinstein (1982), however, showed that there was a fairly reasonable dynamic specification of bargaining that yielded a *unique* subgame perfect equilibrium. It is this model of sequential bargaining that we now consider.

1.1 The Model

Imagine two players, one and two, who takes turns making offers about how to divide a pie of size one. Time runs from $t = 0, 1, 2, ...$ At time 0, player one can propose a split $(x_0, 1 - x_0)$ (with $x_0 \in [0, 1]$), which player 2 can accept or reject. If player 2 accepts, the game ends and the pie is consumed. If player two rejects, the game continues to time $t = 1$, when she gets to propose a split $(y_1, 1 - y_1)$. Once player two makes a proposal, player one can accept or reject, and so on ad infinitum.

We assume that both players want a larger slice, and also that they both dislike delay. Thus, if agreement to split the pie $(x, 1-x)$ is reached at time

t, the payoff for player one is $\delta_1^t x$ and the payoff for player two is $\delta_2^t(1-x)$, for some $\delta_1, \delta_2 \in (0,1)$.

1.2 A Finite-Horizon Version

To get a flavor for this sort of sequential-offer bargaining, consider a variant where there is some *finite* number of offers N that can be made. This model was first studied by Stahl (1972). To solve for the subgame perfect equilibrium, we can use backward induction, starting from the final offer.

For concreteness, assume $N = 2$. At date 1, player two will be able to make a final take-it-or-leave-it offer. Given that the game is about to end, player one will accept any split, so player two can offer $y = 0$.

What does this imply for date zero? Player two anticipates that if she rejects player one's offer, she can get the whole pie in the next period, for a total payoff of δ_2. Thus, to get her offer accepted, player one must offer player two at least δ_2. It follows that player one will offer a split $(1-\delta_2, \delta_2)$, and player two will accept.

Proposition 1 *In the $N = 2$ offer sequential bargaining game, the unique SPE involves an immediate $(1-\delta_2, \delta_2)$ split.*

1.3 Solving the Rubinstein Model

It is fairly easy to see how a general N-offer bargaining game can be solved by backward induction to yield a unique SPE. But the infinite-horizon version is not so obvious. Suppose player one makes an offer at a given date t. Player two's decision about whether to accept will depend on her belief about what she will get if she rejects. This in turn depends on what sort of offer player one will accept in the next period, and so on. Nevertheless, we will show:

Proposition 2 *There is a unique subgame perfect equilibrium in the sequential bargaining game described as follows. Whenever player one proposes, she suggests a split $(x, 1-x)$ with $x = (1-\delta_2)/(1-\delta_1\delta_2)$. Player two accepts any division giving her at least $1-x$. Whenever player two proposes, she suggests a split $(y, 1-y)$ with $y = \delta_1(1-\delta_2)/(1-\delta_1\delta_2)$. Player one accepts any division giving her at least y. Thus, bargaining ends immediately with a split $(x, 1-x)$.*

Proof. We first show that the proposed equilibrium is actually an SPE. By a classic dynamic programming argument, it suffices to check that no player can make a profitable deviation from her equilibrium strategy in one single period. (This is known as the *one-step deviation principle* — see e.g. Fudenberg and Tirole's book for details.)

Consider a period when player one offers. Player one has no profitable deviation. She cannot make an acceptable offer that will get her more than x. And if makes an offer that will be rejected, she will get $y = \delta_1 x$ the next period, or $\delta_1^2 x$ in present terms, which is worse than x. Player two also has no profitable deviation. If she accepts, she gets $1 - x$. If she rejects, she will get $1 - y$ the next period, or in present terms $\delta_2 (1 - x) = \delta_2(1 - \delta_1 x)$. It is easy to check that $1 - x = \delta_2 - \delta_1\delta_2 x$. A similar argument applies to periods when player two offers.

We now show that the equilibrium is unique. To do this, let $\underline{v}_1, \overline{v}_1$ denote the lowest and highest payoffs that player one could conceivably get in any subgame perfect equilibrium starting at a date where he gets to make an offer.

To begin, consider a date where player *two* makes an offer. Player one will certainly accept any offer greater than $\delta_1\overline{v}_1$ and reject any offer less than $\delta_1\underline{v}_1$. Thus, starting from a period in which she offers, player two can secure *at least* $1 - \delta_1\overline{v}_1$ by proposing a split $(\delta_1\overline{v}_1, 1 - \delta_1\overline{v}_1)$. On the other hand, she can secure *at most* $1 - \delta_1\underline{v}_1$.

Now, consider a period when player one makes an offer. To get player two to accept, he must offer her *at least* $\delta_2 (1 - \delta_1\overline{v}_1)$ to get agreement. Thus:

$$\overline{v}_1 \leq 1 - \delta_2 (1 - \delta_1\overline{v}_1).$$

At the same time, player two will certainly accept if offered more than $\delta_2(1 - \delta_1\underline{v}_1)$. Thus:

$$\underline{v}_1 \geq 1 - \delta_2 (1 - \delta_1\underline{v}_1).$$

It follows that:

$$\underline{v}_1 \geq \frac{1 - \delta_2}{1 - \delta_1\delta_2} \geq \overline{v}_1.$$

Since $\overline{v}_1 \geq \underline{v}_1$ by definition, we know that *in any subgame perfect equilibrium*, player one receives $v_1 = (1 - \delta_2) / (1 - \delta_1\delta_2)$. Making the same argument for player two completes the proof. *Q.E.D.*

A few comments on the Rubinstein model of bargaining.

1. It helps to be patient. Note that player one's payoff, $(1 - \delta_2) / (1 - \delta_1\delta_2)$, is increasing in δ_1 and decreasing in δ_2. The reason is that if you are more patient, you can afford to wait until you have the bargaining power (i.e. get to make the offer).

2. The first player to make an offer has an advantage. With identical discount factors δ , the model predicts a split

$$\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$$

which is better for player one. However, as $\delta \to 1$, this first mover advantage goes away. The limiting split is $(1/2, 1/2)$.

3. There is no delay. Player two accepts player one's first offer.

4. The details of the model depend a lot on there being *no immediate counter-offers*. With immediate counter-offers, it turns out that there are many many equilibria!

2 Finitely Repeated Games

So far, one might have a somewhat misleading impression about subgame perfect equilibria, namely that they do such a good job of eliminating unreasonable equilibria that they typically make a unique prediction. However, in many dynamic games, we still have a very large number of SPE.

2.1 A Simple Example

Consider the following game G:

	A	B	C
A	$4,4$	$0,0$	$0,5$
B	$0,0$	$1,1$	$0,0$
C	$5,0$	$0,0$	$3,3$

This game has three Nash equilibria, (B, B), (C, C) and $(\frac{3}{4}B+\frac{1}{4}C, \frac{3}{4}B+\frac{1}{4}C)$. Note that A is strictly dominated by C.

Suppose that the players play G twice, and observe the first period actions before choosing their second period actions. Suppose that both players discount the future by δ. Thus player i's total payoff is $u_i\left(a^1\right) + \delta u_i\left(a^2\right)$, where $a^t = \left(a_1^t, a_2^t\right)$ is the time t action profile.

In this setting, note that repeating any one of the Nash equilibria twice is an SPE of the two period game. So is playing one of the NE in the first period and another in the seond. Moreover, by making the choice of *which* Nash Equilibrium to play in period two *contingent* on play in period one, we can construct SPE where play in the first period does not correspond to Nash play.

Proposition 3 *If $\delta \geq 1/2$ then there exists an SPE of the two-period game in which (A, A) is played in the first period.*

Proof. Consider the following strategies.

- Play (A, A) in the first period.
- If first period play is (A, A), play (C, C) in the second period. Otherwise, play (B, B) in the second period.

To see that this is an SPE, we again look for profitable one-time deviations. Consider the second period. Following (A, A), playing (C, C) is a NE so neither player benefits from deviating. Similarly, following something else, (B, B) is a NE, so there is no gain to deviating.

Now consider the first period. By following the strategy, a player gets:

$$\begin{aligned} \text{Payoff to Strategy} &: \quad 4 + \delta \cdot 3 \\ \text{Best Deviation Payoff} &: \quad 5 + \delta \cdot 1 \end{aligned}$$

So long as $2\delta \geq 1$, there is no gain to deviating. *Q.E.D.*

2.2 Finitely Repeated Prisoners' Dilemma

The construction we just used requires that the stage game have at least two Nash equilibria, with different payoffs. Of course, not every game has this property. For instance, consider the prisoners' dilemma:

	C	D
C	$1, 1$	$-1, 2$
D	$2, -1$	$0, 0$

Proposition 4 *In a T-period repetition of the prisoners' dilemma, the unique SPE is for both players to defect in every period.*

Proof. We use backward induction. Consider period T. In this period, play must be a Nash equilibrium. But the only equilibrium is (D, D). Now back up to period $T - 1$. In that period, players know that *no matter what happens, they will play (D, D) at date T.* Thus D is strictly dominant at date $T - 1$, and the equilibrium must involve (D, D). Continuing this for periods $T - 2, T - 3$, etc... completes the proof. *Q.E.D.*

3 Infinitely Repeated Games

We now consider infinitely repeated games. In the general formulation, we have I players and a stage game G which is repeated in periods $t = 0, 1, 2, ...$ If a^t is the action profile played at t, player i's payoff for the infinite horizon game is:

$$u_i\left(a^0\right) + \delta u_i\left(a^1\right) + \delta^2 u_i\left(a^2\right) + ... = \sum_{t=0}^{\infty} \delta^t u_i\left(a^t\right).$$

To avoid infinite sums, we assume $\delta < 1$. Sometimes, it is useful to look at *average payoffs*:

$$(1-\delta) \sum_{t=0}^{\infty} \delta^t u_i\left(a^t\right)$$

Note that for player i, maximizing total payoff is the same as maximizing average payoff.

Before we go on to consider some examples, it's worth commenting one two interpretations of the discount factor.

- Time preference. One interpretation of δ is that player's prefer money today to money tomorrow. That is, if the economy has an interest rate r, then $\delta = 1/(1+r)$. If there is a longer gap between stage games, we would think of δ as smaller.

- Uncertain end date. Another interpretation of δ is that it corresponds to the probability that the interaction will continue until the next date. That is, after each stage game, there is a $1-\delta$ probability that the game will end. This leads to the same repeated game situation, but with the advantage that the game will actually end in finite time — just randomly.

A *history* in a repeated game is a list $h^t = \left(a^0, a^1, ..., a^{t-1}\right)$ of what has previously occurred. Let H^t be the set of t-period histories. A *strategy* for player i is a sequence of maps $s_i^t : H^t \to A_i$. A mixed strategy σ_i is a sequence of maps $\sigma_i^t : H^t \to \Delta(A_i)$. A strategy profile is $\sigma = (\sigma_1, ..., \sigma_I)$.

3.1 The Prisoners' Dilemma

Proposition 5 *In the infinitely repeated prisoners' dilemma, if $\delta \geq 1/2$ there is an equilibrium in which (C, C) is played in every period.*

Proof. Consider the following symmetric strategies — called "grim trigger" strategies:

- Play C in the first period and in every following period so long as no one ever plays D.
- Play D if either player has ever played D.

To see that there is no profitable single deviation, suppose that D has already been played. At this point, player i has two choices:

- Play C for a payoff $-1 + \delta \cdot 0 + \delta^2 \cdot 0 + ... = -1$
- Play D for a payoff $0 + \delta \cdot 0 + \delta^2 \cdot 0 + ... = 0$.

So player i should certainly play D. On the other hand, suppose D has not been played. At this point i has two choices:

- Play C for a payoff $1 + \delta + \delta^2 + ... = 1/(1-\delta)$.
- Play D for a payoff $2 + \delta \cdot 0 + \delta^2 \cdot 0 + ... = 2$.

If $\delta \geq 1/2$ it is better to play C so we have an SPE. *Q.E.D.*

It is tempting to think of this proposition as saying that if people interact repeatedly then they **will** cooperate. However, it does not say this. What it says is that cooperation is one **possible** SPE outcome. However, there are many others.

- For any δ, there is a SPE in which players play D in every period.
- For $\delta \geq 1/2$, there is a SPE in which the players play D in the first period and C in every following period.
- For $\delta \geq 1/\sqrt{2}$, there is a SPE in which the players alternate between (C, C) and (D, D).
- For $\delta \geq 1/2$, there is a SPE in which the players alternate between (C, D) and (D, C).

A good exercise is to try to construct these SPE.

3.2 The Folk Theorem

Perhaps the most famous result in the theory of repeated games is the folk theorem. It says that if players are *really* patient and far-sighted (i.e. if $\delta \to 1$), then not only can repeated interaction allow many SPE outcomes, but actually SPE can allow virtually *any* outcome in the sense of average payoffs.

Let G be a simultaneous move game with action sets $A_1, ..., A_I$, and mixed strategy sets $\Sigma_1, ..., \Sigma_I$, and payoff functions $u_1, ..., u_I$.

Definition 6 *A payoff vector $v = (v_1, ..., v_I) \subset \mathbb{R}^I$ is* ***feasible*** *if there exist actions profiles $a^1, ..., a^K \in A$ and non-negative weights $\lambda^1, ..., \lambda^K$ with $\sum_k \lambda^k = 1$ such that for each i,*

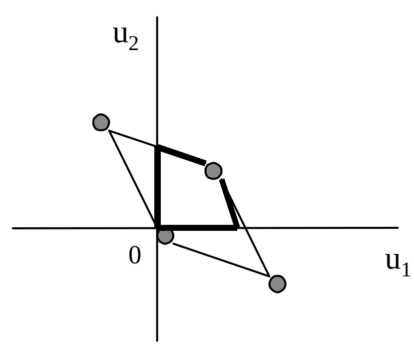
$$v_i = \lambda^1 u_i \left(a^1\right) + ... + \lambda^K u_i \left(a^K\right).$$

Definition 7 *A payoff vector v is* ***strictly individually rational*** *if for all i,*

$$v_i > \min_{\sigma_{-i} \in \Sigma_{-i}} \max_{\sigma_i \in \Sigma_i} u_i \left(\sigma_i, \sigma_{-i}\right) = \underline{v}_i.$$

We can think $\underline{v}_i$ as the lowest payoff a rational player could ever get in equilibrium if he anticipates his opponents' (possibly non-rational) play. We refer to $\underline{v}_i$ as i's *min-max payoff*.

Example In the prisoners' dilemma, the figure below outlines the set of feasible and individually rational payoffs.



Theorem 8 *(Folk Theorem) Suppose that the set of feasible payoffs of G is I-dimensional. Then for any feasible and strictly individually rational payoff vector v, there exists $\underline{\delta} < 1$ such that for any $\delta \geq \underline{\delta}$ there is a SPE σ^* of G such that the average payoff to σ^* is v_i for each player i.*

The Folk Theorem says that *anything* that is feasible and individually rational is possible.

Sketch of Proof. The proof is pretty involved, but here is the outline:

1. Have players on the equilibrium path play an action profile with payoff v (or alternate if necessary).
2. If some player deviates, *punish him* by having the other players for T periods play some σ_{-i} against which i can get at most $\underline{v}_i$.
3. After the punishment period, *reward* all players other than i for carrying out the punishment. To do this, switch to an action profile that gives each player $j \neq i$ some payoff $v_j > \underline{v}_j$.

Q.E.D.

Note that the Folk Theorem does not call for the players to revert to the static Nash equilibrium as a punishment. Instead, they do someting potentially worse — they min-max the deviating player. Of course, in some games (i.e. the prisoners' dilemma), the static Nash equilibrium *is* the min-max point. But in other games (e.g. Cournot), it is not.

4 Applications of Repeated Games

Repeated game models are perhaps the simplest way to capture the idea of ongoing interaction between parties. In particular, they allow us to capture in a fairly simple way the idea that actions taken today will have consequences for tomorrow, and that there is no "last" date from which to unfold the strategic environment via backward induction. In these notes, we will give only a small flavor of the many applications of repeated game models.

4.1 Employment and Efficiency Wages

We consider a simple model of employment. There is a firm and a worker. The firm makes a wage offer w to the worker. The worker then chooses whether to accept, and if he accepts whether to "Work" or "Shirk". Thus, the firms action space is $[0, \infty)$, and the worker chooses $a \in \{\emptyset, W, S\}$.

Payoffs are as follows. If the worker is not employed, he gets $\overline{u} > 0$. If he is employed and works, he gets $w - c$ (here c is the disutility of effort). If he is employed and shirks, he gets w. The firm gets nothing if the worker turns down the offer, $v - w$ if the worker accepts and works, and $-w$ if the worker accepts and shirks. Assume that $v > \overline{u} + c$.

In this environment, it's efficient for the firm to hire the worker if the worker will actually work. However, once hired, the worker would like to slack off. In the one-shot game, if the firm offers a wage $w \geq \overline{u}$ the worker will accept the job and shirk. The worker will reject the offer if $w < \overline{u}$. The firm thus gets $-w$ if the worker accepts and 0 otherwise. So the firm will offer $w < \overline{u}$ and there will be no employment. Is there any way for the parties to strike and efficient deal?

One solution is an incentive contract. Suppose that the worker's behavior is *contractible* in the sense that the firm can pay a contingent wage $w(a)$. If the firm offers a contract $w : A \to \mathbb{R}$ given by $w(W) = \overline{u} + c$ and $w(S) = \overline{u}$, the worket will accept and work. However, this solution requires in essence that a court could come in and verify whether or not a worker performed as mandated — and the employment contract would have to specificy the worker's responsibilities very carefully.

Another possibility arises if the employment relationship is ongoing. Suppose the stage game is repeated at dates $t = 0, 1, \ldots.$

Proposition 9 *If $v \geq \overline{u} + c/\delta$, there is a subgame perfect equilibrium in the firm offers a wage $w \in [\overline{u} + c/\delta, v]$ and the worker works in every period.*

Proof. Consider strategies as follows. In every period, the firm offers a wage $w \in [\overline{u} + c/\delta, v]$ and the worker works. If the worker ever deviates by turning down the offer or shirking, the firm forever makes an offer of $w = 0$ and the worker never accepts (i.e. they repeat the stage game equilibria for every following period. If the firm ever deviates by offering some other w, the worker either rejects the offer — if $w < \overline{u}$ — or accepts and shirks. From then on they repeat the stage game equilibrium.

The firm's best deviation payoff is zero, so it has no incentive to deviate so long as $v \geq w$. Now consider the worker. He has no incentive to reject the offer so long as $w \geq \overline{u} + c$. He has an incentive to work rather than shirk if:

$$\underbrace{w - c}_{\text{Payoff today if } W} + \underbrace{\frac{\delta}{1-\delta}(w - c)}_{\text{Future payoff if employed}} \geq \underbrace{w}_{\text{Payoff today if } S} + \underbrace{\frac{\delta}{1-\delta}\overline{u}}_{\text{Future payoff if fired}}$$

Or in other words if $w \geq \overline{u} + c/\delta$. *Q.E.D.*

The relationship described in the equilibrium is sometimes referred to as an *efficiency wage contract* since the worker receives more than his opportunity cost. That is, since $w - c > \overline{u}$, the worker makes a pure rent due to the incentive problem. However, by paying more, the firm gets more!

4.2 Collusion on Prices

Consider a repeated version of Bertrand competition. The stage game has $N \geq 2$ firms, who each select a price. Customers purchase from the least expensive firm, dividing equally in the case of ties. Quantity purchased is given by $Q(P)$, which is continuous and downward-sloping. Firms have constant marginal cost c. Let

$$\pi(p) = (p - c)\, Q(p)$$

and assume that π is increasing in p on $[c, p^m]$ — where p^m is the monopoly price. This game is repeated at each date $t = 0, 1, 2, ...$ Firms discount at δ.

Note that the static Nash equilibrium involves each firm pricing at marginal cost. The question is whether there is a repeated game equilibrium in which the firms sustain a price above marginal cost.

Proposition 10 *If $\delta < N/(N-1)$ all SPE involve pricing at marginal cost in every period. If $\delta \geq N/(N-1)$, there is a SPE in which the firms all price at $p^* \in [c, p^m]$ in every period.*

Proof. We look for a collusive equilibrium. Note that since the stage nash equilibrium gives each firm it's min-max payoff zero, we can focus on Nash reversion punishments. Suppose the firms try to collude on a price p^*. A firm will prefer not to deviate if and only if:

$$\frac{1}{N}\pi\left(p^*\right) \cdot \frac{1}{1-\delta} \geq \pi\left(p^*\right).$$

This is equivalent to

$$\delta \geq \frac{N}{N-1}.$$

Thus patient firms can sustain a collusive price. *Q.E.D.*

Note that cooperation becomes more difficult with more firms. For $N = 2$, the critical discount factor is $1/2$, but this increases with N. Note also that the equilibrium condition is independent of π — thus either monopoly collusion is possible ($p = p^m$) or nothing at all.

4.3 Collusion on Quantities

Consider a repreated version of the Cournot model. The stage game has $N = 2$ firms, who each select a quantity. The market price is then $P(Q) = 1 - Q$. Firms produce with constant marginal cost c.

Let $Q^m = (1-c)/2$ denote the monopoly quantity, and π^m the monopoly profit. Let $Q^c = (1-c)/3$ denote the Cournot quantities and π^c the Cournot profit for each firm.

Note that since the static Cournot model has a unique equilibrium, we must focus on the infinitely repeated game to get collusion. We consider SPE with Nash reversion.

Proposition 11 *Firms can collude on the monopoly price as part of a SPE if the discount factor is sufficiently high.*

Proof. Let's look for an equilibrium in which the firms set quantities q^m in each period and play static Cournot forever should a firm ever deviate. Provided no one deviates, each firm expects a payoff

$$\text{Collusive Payoff } = \frac{1}{1-\delta}\frac{1}{2}\pi^m$$

On the other hand, by deviating a firm can get:

$$\text{Deviation Payoff} = \max_{q_i} \left(P\left(q^m + q_i\right) - c\right) q_i + \frac{\delta}{1-\delta}\pi^c.$$

The best deviation is to set $q_i = (3/4 - c)/2$. For the case of $c = 0$, there is no incentive to deviate provided that:

$$\frac{1}{1-\delta}\frac{1}{2}\pi^m = \frac{1}{1-\delta}\frac{1}{8} \geq \frac{9}{64} + \frac{\delta}{1-\delta}\frac{1}{9} = \left(1 - \frac{1}{4} - q^{dev}\right) q^{dev} + \frac{\delta}{1-\delta}\pi^c.$$

That is if $\delta \geq 9/17$. *Q.E.D.*

Several comments are in order.

1. First, note that as N increases the monopoly share of each firm becomes smaller. However, the static Cournot profits also become smaller. As an exercise see if you can figure out whether collusion becomes harder or easier to sustain.

2. In the Cournot model, the static Nash equilibrium is not the min-max point. Thus, there are typically "worse" SPE punishments than Nash reversion. In turn, this means that collusion can be sustained for lower discount factors than with Nash reversion. Finding the "optimal" collusive agreement for a given discount factor turns out to be a difficult problem. See Abreu (1986, JET and 1988 EMA) for details.

4.4 Multimarket Contact

A common wisdom in Industrial Organization is that collusion will be easier to sustain if firms compete simultaneously in many markets. The classic intuition for this is that opportunism is likely to be met with retaliation in many markets. This may limit the temptation to compete agressively. We now explore this idea in a framework pioneered by Bernheim and Whinston (1990).

Consider two firms that simultaneously compete in two markets.

- Let G_{ik} denote the gain to firm i from deviating in market k for the current period, for a particular equilibrium.
- Let π^c_{ik} denote the discounted payoff from continuation (next period forward) for firm i in market k, assuming no deviation from the equilibrium in the current period.
- Let π^p_{ik} denote the discounted "punishment" payoff from continuation (next period forward) for firm i in market k, assuming that i deviates from the equilibrium in the current period.

Separate Markets. If the markets are separate, the equilibrium condition is that for each i and each k,

$$\delta\pi^c_{ik} \geq G_{ik} + \delta\pi^p_{ik}.$$

There are $N \times K$ such constraints.

Linked Markets. Now suppose that a deviation in *any* market is met by punishment in all markets. The equilibrium condition is that for each i,

$$\sum_k \delta\pi^c_{ik} \geq \sum_k G_{ik} + \sum_k \delta\pi^p_{ik}.$$

There are now N constraints.

We can make the immediate observation that *multimarket contact pools constraints across markets.* Clearly, if the separate constraints are satisfied, the linked constraints will be as well. The interesting question is whether linking strictly expands the set of collusive outcomes. The answer is typically yes *so long as there is enforcement slack in some market.* If there is slack in one of the individual constraints, with pooling there will be slack in the aggregate constraint — potentially allowing for a more collusive equilibrium.