Empirical Studies of Auctions

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Empirical Studies of Auctions

- How well do theoretical models of auction explain real-world bidding behavior?
- Can we use auction theory models to guide empirical analysis of bidding markets, e.g.
  - Assess market power or level of competition
  - Identify bidder collusion
  - Evaluate affect of merger, or change in auction rules
  - Assess the importance of asymmetric information
- How can the combination of theory and data guide auction design decisions?
Brief History of Empirical Auction Studies

- Early descriptive work in 1960s and 1970s describing features of bidding for treasury bills, oil leases, timber in national forests.
  - Johnson (1979), Hansen (1986) use change in US Forest Service policy to compare revenue in open and sealed bid auctions — results are inconclusive.

  - They show, remarkably, that bidders owning neighboring tracts make much higher expected profit than *de novo* bidders with potentially less information.
  - Athey and Levin (2001) use *ex post* data to test for presence of asymmetric information in timber auctions, and identify gaming of auction rules.
Paarsch’s (1992) Stanford dissertation estimates parametric IPV and CV sealed tender models and tests between them.

Laffont, Ossard and Vuong (1995) show how prices from an ascending auction data can be used to estimate bidder value distributions, apply their idea to French eggplant auctions.

Guerre, Perrigne and Vuong (2000) show how bid data from sealed bid auctions can be “inverted” to recover bidder value distributions, using an old idea from the IO literature.

Dozens of papers follow develop and extend this idea to ascending auction data, multi-unit auctions, studies of collusion, market power, etc...
Bidders 1, ..., $N$ draw independent private values from $F$.

In sealed tender, bidder $i$ solves:

\[
\max_{b_i} (v_i - b_i) \Pr(\max_{j \neq i} \beta(v_j) \leq b_i) = \max_{b_i} (v_i - b_i) F(\beta^{-1}(b_i))^{N-1}
\]

The equilibrium condition (FOC + symmetric eqm $\beta(\bullet)$):

\[
v_i - \beta(s_i) = \frac{1}{\beta'(v_i)} \left( \frac{1}{(N-1)f(v_i)/F(v_i)} \right)
\]

Solving this differential equation:

\[
b_i = \beta(v_i) = \frac{\int_{-\infty}^{v_i} x f(x) F(x)^{N-2} dx}{F(v_i)^{N-1}} = \mathbb{E}[\max_{j \neq i} v_j | v_i \geq \max_{j \neq i} v_j].
\]
Define:

\[ G_i(b) = \Pr(\max_{j \neq i} b_j \leq b_i) = \Pr(b_i \text{ is winning bid}) \]

Rewrite bidder \( i \)'s problem is

\[ \max_{b_i} (v_i - b_i) G_i(b) \]

And in equilibrium, we must have

\[ b_i = v_i - \frac{G_i(b_i)}{g_i(b_i)}. \]

Note the analogy to standard monopoly/monopsony theory: \( G_i(b) \) is a residual supply curve facing bidder \( i \).
Data consist of bids $b_{1t}, \ldots, b_{Nt}$ from $T$ auctions...

Fix a bidder $i$. Use observed bids to construct

$$G_i(b) = \Pr \left( \max_{j \neq i} b_j \leq b_i \right) = \prod_{j \neq i} \Pr (b_j \leq b_i | X_t)$$

The right hand side can be estimated from the data: $\hat{G}_i$.

Use equilibrium condition to recover $v_i$'s!

$$v_{it} = b_{it} + \frac{\hat{G}_i(b_i)}{\hat{g}_i(b_i)}.$$

Does not require bidder symmetry, and can be extended to allow each auction to have different “characteristics” $x_t$, so $\hat{G}_i(b_i | x_t)$, or to allow for correlated bids/values.
Comparing Open and Sealed Bid Auctions

- U.S. Forest Service uses mix of open and sealed bidding
  - 1980s, two distinct regions: Northern, California
  - Tract size and location largely determined format, format was randomized in some sales.
- Under the assumptions of the revenue equivalence theorem:
  - Expect same revenue, participation, allocation on average.
  - Assumptions: symmetry, risk neutrality, independent values, competitive bidding.
- Data: observe: sale format, bids, identities, lots of information about each tract being sold.
Outline of the approach

- Regression analysis shows departures from RET.
- Model to explain departures: relax RET assumptions
  - Heterogeneous bidders: mills v. loggers
  - (Non)-competitive bidding at ascending auctions
- Use GPV method to estimate parameters of model and...
  - Assess whether model can explain RET departures.
  - Assess competitiveness (competition vs. collusion)
  - Welfare analysis of sealed vs open bidding.
Comparing Auction Results

- Estimate OLS regression:

\[ Y_t = \alpha \cdot Sealed_t + X_t \beta + N_t \gamma + \varepsilon_t \]

- \( Y_t \) is outcome (entry, revenue, etc.)
- \( X_t \) auction characteristics, \( N_t \) “potential” entrants.
- \( Sealed_t \) dummy equal to one if sealed bid.

- Interpretation: \( \alpha \) measures “average” difference between running sealed and open auction, holding fixed the sale environment.
Comparing Auction Results

- Each entry interpreted as percentage increase in sealed bid auction relative to open auction

<table>
<thead>
<tr>
<th></th>
<th>( \ln(\text{Logger Entry}) )</th>
<th>( \ln(\text{Mill Entry}) )</th>
<th>( \ln(\text{Wins}) )</th>
<th>( \ln(\text{Revenue}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Northern</strong></td>
<td>0.089 (0.036)</td>
<td>-0.014 (0.030)</td>
<td>0.039 (0.026)</td>
<td>0.094 (0.038)</td>
</tr>
<tr>
<td><strong>California</strong></td>
<td>0.101 (0.045)</td>
<td>-0.026 (0.038)</td>
<td>0.036 (0.036)</td>
<td>0.027 (0.051)</td>
</tr>
</tbody>
</table>
Mills and loggers, all risk-neutral, IPV.

A bidder must pay $K$ to learn his value and enter.

Bidder $i$ has private value $v_i \sim F_M(\cdot), F_L(\cdot)$

Assume $F_M$ is stochastically higher than $F_L$.

Bidders observe who they’re bidding against before submitting bids.

Let $N_M, N_L$ denote number of potential entrants.

Let $n_M, n_L$ denote realized entrants.
Asymmetric IPV Auctions

- **Fixed participation** (*Maskin and Riley, 2000*): In ascending auction, highest-value wins. In first-price auction, mills bid less aggressively than loggers, i.e. $b_M(v) < b_L(v)$ for all $v$. So loggers win more often and make higher profits in first-price auction relative to ascending auction.

- **Endogenous participation**: Under some conditions, there exists unique type-symmetric equilibrium where: (i) Mills enter with probability one in both formats. (ii) Loggers randomize, enter with higher probability in first-price.

- **Basic model predictions**
  - Mill bids are higher than logger bids in both formats.
  - Loggers participate and win more in sealed bid auctions.
  - Ambiguous revenue comparison: depends on parameters.
  - Ascending auction is socially efficient; sealed isn’t.
Applying the model to data

- Use sealed bid data to estimate model primitives
  - the value distributions $F_L, F_M$ and entry cost $K$.
- Ask if calibrated model can explain data from both kinds of auctions, and if not, what change in assumptions will allow it to do so.
- Empirical work proceeds in a series of steps.
(1) Estimate bid distributions

- Each sale is different. Conditional bid distributions $G_L(\cdot | X, N, \xi, n)$, $G_M(\cdot | X, N, \xi, n)$.
  
  - Some differences are observed: $X, N, n$
  - Some differences aren’t observed: $u \sim G_U$.

- Assume $b_{it} = h(X_t) \cdot \phi(u_t) \cdot \eta_{it}$ — implies $G_L, G_M, G_U$ are identified.

- Details: fit Weibull distribution to observed bids:

\[
G_k(b|X, N, u, n) = 1 - \exp \left( -u \cdot \left( \frac{b}{\lambda_k(X, N, n)} \right)^{\rho_k(n)} \right),
\]

- $\ln \lambda_k(X, n) = X \beta_X + N \beta_N + n \beta_n + \beta_k$ and $\ln \rho_k(n) = n \gamma_n + \gamma_k$.
- assume $u$ is Gamma distributed with mean 1, variance $\theta$.
- Estimate parameters $\beta, \gamma, \theta$ by maximum likelihood.
(2) Estimate value distributions

- Equilibrium condition for optimal bidding:

\[ b_{it} = v_{it} + \frac{1}{\sum_{j \neq i} g_j(b_{it} | X_t, N_t, u_t, n_t)} \]

- Due to unobservable \( u \), can’t recover each \( v_{it} \)...

- Instead recover value distributions \( F_L(\cdot | X, N, u), F_M(\cdot | X, N, u) \)

\[ F_i(v | X, N, u) = G_i(\beta_i(v) | X, N, u) \]
(3) Calculate expected outcomes given entry

- Have already estimated $F_L(\bullet|X, N, u)$, $F_M(\bullet|X, N, u)$ and $G_u(u)$.
- Given tract characteristics $(X, N, u)$ and entry $(n_L, n_M)$ can calculate sealed bid equilibrium and (easily) open auction equilibrium. Integrating out $u$ gives expected outcomes:
  - Expected sealed and open prices given $(X, N, n)$
  - Expected sealed and open allocation given $(X, N, n)$
  - Expected logger and mill profits given $(X, N, n)$
(4) Estimate distribution of logger entry

- In unique entry equilibrium, all mills enter and each logger enters with probability $p_t$.
- Specify functional form for $p$ as function of $(X, N)$

$$p(X, N) = \frac{\exp(X\alpha_X + N\alpha_N)}{1 + \exp(X\alpha_X + N\alpha_N)}$$

- Estimate $\alpha$ by maximum likelihood.
(5) Estimate entry costs

- Equilibrium condition for logger entry (randomization):

\[ \sum_{n_L} \Pi_L(X, N, n_L, n_M) \cdot \Pr(n_L, n_M|X, N, i \text{ enters}) = K(X, N) \]

- We’ve estimated \( \Pi_L(X, N, n) \), and know \( n_M = N_M \).

- Distribution of logger entry is binomial:

\[ \Pr(n_L|X, N, i \text{ enters}) = \binom{N_L - 1}{n_L - 1} p(X, N)^{n_L - 1} (1 - p(X, N))^{N_L - n_L}. \]

- Plug in to calculate LHS and infer \( K(X, N) \).
(6) Compute expected auction outcomes

- Each sale tract is characterized by a pair \((X, N)\)
- We have estimated \(F_L(\cdot|X, N, \xi)\), \(F_M(\cdot|X, N, \xi)\), \(G(\cdot)\) and \(K(X, N)\)
- We have computed sealed and open equilibria for all \((X, N, \xi, n)\)
- We have computed expected outcomes given \((X, N, n)\).
- Solve for sealed/open entry equilibria given \((X, N)\).
- Compute expected entry and auction outcomes given \((X, N)\).
(7) Answering economic questions with the model

- How strong are mills compared to loggers?
- How large are profit margins in bids? How large are entry costs?
- How well does the model explain departures from RET?
- Does the assumption of competition or collusion fit the data better?
- How important is endogenous entry?
- What are the welfare consequences of open vs sealed bidding?
Estimation results

- Bids increase with number of competitors.
- Mill bids are 15-25% greater than logger bids on average.
- Mills bid larger margins: mill bid function lies below logger’s.
- Estimated profit margin (cond’l on entry): 10.7% at the median.
- Estimated entry cost: $4695 at the median.
## Predicted versus Actual Outcomes

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Predicted Cond’l on Entry</th>
<th>Predicted w/ Eqm Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sealed Bid Sales</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Sale Price</td>
<td>69.4</td>
<td>69.9 (1.4)</td>
<td>70.4 (1.6)</td>
</tr>
<tr>
<td>Logger Wins (%)</td>
<td>68.1</td>
<td>68.0 (0.9)</td>
<td>65.0 (0.01)</td>
</tr>
<tr>
<td>Logger Entry</td>
<td>3.23</td>
<td></td>
<td>3.23 (0.1)</td>
</tr>
<tr>
<td><strong>Open Sales</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Price <em>(competition)</em></td>
<td>63.3</td>
<td>67.9 (1.8)</td>
<td>67.8 (2.1)</td>
</tr>
<tr>
<td>Avg. Price <em>(collusion)</em></td>
<td>63.3</td>
<td>44.2 (1.3)</td>
<td>44.1 (2.2)</td>
</tr>
<tr>
<td>Logger Wins (%)</td>
<td>60.0</td>
<td>56.0 (0.01)</td>
<td>54.4 (0.02)</td>
</tr>
<tr>
<td>Logger Entry</td>
<td>2.75</td>
<td></td>
<td>2.67 (0.2)</td>
</tr>
</tbody>
</table>

*Sealed Bid Sales* *(in-sample predictions)*

*Open Sales* *(out-of-sample predictions)*
## Welfare Comparison: Sealed vs. Open

<table>
<thead>
<tr>
<th></th>
<th>Sealed vs. Competitive Open Auctions</th>
<th>Sealed vs. Part. Collusive Open Auctions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exogenous Entry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Sale Price</td>
<td>0.03%</td>
<td>3.98%</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>Avg. Sale Surplus</td>
<td>-0.08%</td>
<td>same as comp.</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Logger Wins</td>
<td>0.46%</td>
<td>same as comp.</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td></td>
</tr>
<tr>
<td><strong>Predict Entry &amp; Bidding</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. Sale Price</td>
<td>1.49%</td>
<td>5.57%</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(0.82)</td>
</tr>
<tr>
<td>Avg. Sale Surplus</td>
<td>-0.30%</td>
<td>same as comp.</td>
</tr>
<tr>
<td></td>
<td>(2.70)</td>
<td></td>
</tr>
<tr>
<td>Logger Wins</td>
<td>2.46%</td>
<td>same as comp.</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Logger Entry</td>
<td>0.34%</td>
<td>same as comp.</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td></td>
</tr>
</tbody>
</table>
Other interesting auction work

- Hortacsu (2002) uses T-bill auction data to estimate values of bidders and compare uniform versus discriminatory price auctions.
- Wolak (2003, etc.) uses electricity auction data to estimate cost of generation units and study aspects of electricity market design, including competitiveness.
- Haile and Tamer (2003) show how data from open auctions can be used to provide bounds on bidder values, though not exact estimates without strong assumptions about bidding behavior.
- Hendricks, Pinske and Porter (2000) estimate a common value model of oil lease auction and use it to calculate the relevant information contained in competing bids.