Simultaneous Ascending Auctions

Jonathan Levin
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FCC Spectrum Auctions

- Auctions to allocate radio spectrum
  - Pioneered by FCC in 1994, and followed by UK, Germany, Netherlands, Belgium, Mexico, India, etc.
  - Generally perceived as quite successful, raising hundreds of billions of dollars in revenue.

- Structure of typical auction
  - FCC specifies a set of licenses to be sold, with each license conveying the right to use a portion of the spectrum in a certain geographic area.
  - Licenses allocated using SAA format proposed by Milgrom-Wilson-McAfee; some changes over time.
SAA Rules

- Auction consists of multiple rounds. In each round
  - FCC sets minimum acceptable bid for each object, some increment above current high bid.
  - Each bidder can submit bids on any number of items, subject to “activity rules”.
  - If multiple bids on a license, FCC selects one randomly to become standing high bid.
- Auction ends when no new bids are submitted.
- Information revealed:
  - FCC has typically revealed all bids after each round.
  - In recent auctions, FCC has anonymized the bids.
Activity Rules

- FCC uses Milgrom-Wilson “activity rule”
  - Each bidder \( j \) starts with some eligibility \( e_j(1) \) determined by initial deposit, measured in “bid units”.
  - “Activity” in a round consists of new bids and standing high bids from the prior round: must have \( A_j(n) \leq e_j(n) \).
  - A bidder’s “eligibility” evolves as \( e_j(n+1) = \min(e_j(n), \alpha A_j(n)) \), where \( \alpha \) is close to but possibly larger than 1.
- Activity rule keeps the auction moving, but we will see later that it also has strategic consequences.
Why multiple rounds?

- Relative to sealed bidding, information revelation...
  - Allows bidders to identify target licenses “on the fly”
  - Mitigates inefficiency due to the winner’s curse
  - Helps bidders to assess “roaming” opportunities.

- The SAA design has some other virtues.
  - It’s transparent, and easy to check up on the gov’t.
  - Activity rule prevents super-slow bidding.

- Skeptics might argue...
  - Design is vulnerable to demand reduction/collusion.
  - Design does not facilitate new entry or “package” bidders.
Roadmap for Lecture

- Non-strategic theory
  - SAA is conceptually similar to matching theory algorithm (eg Kelso-Crawford).
  - With substitutes demand, “straightforward” bidding leads to approx. competitive equilibrium.

- Strategic bidding
  - Demand reduction, collusion may be a problem.
  - Bidders that want to buy some minimal set of licenses face complex “exposure” problems.
Non-strategic theory

- \{1, \ldots, L\} is a set of *indivisible* licenses with typical subset \(S\).
- Bidders’ payoffs are the value of licenses acquired minus the amount paid \(v_j(S) - m_j\). (assume free disposal).
- Demand “correspondence”

\[
D_j(p) = \arg\max_S v_j(S) - p(S)
\]

- “Personalized price” \(p_{jn}^k\) for bidder \(j\) on item \(k\) at round \(n\) is the lowest price at which \(j\) might conceivably acquire \(k\)
  - the high bid if \(j\) is the *standing high bidder* on \(k\)
  - the high bid plus one increment otherwise
Definitions

- Bidder $j$ demands set $S$ at price vector $p$, if $S \subseteq D_j(p)$.
- Licenses are substitutes (standard definition) if:

  $$\left(k \in D_j(p), p' \geq p, p'_k = p_k\right) \Rightarrow k \in D_j(p')$$

- Examples
  - A bidder who wants just one license.
  - A bidder who wants spectrum in several areas, but has declining marginal value for bandwidth in each area.
- Bidder $j$ bids straightforwardly if in each round she bids on a preferred set of licenses given her current standing high bids and next price increments for other licenses.
Substitutes and “no regret”

- **Theorem:** Assume that licenses are substitutes for $j$. If $j$ bids straightforwardly at every round $n$, $S_j^n \subseteq D_j(p^{jn})$.
  - That is, at every round $j$ demands its preferred licenses at its personalized prices.

- This means $j$ never “gets stuck” with a standing high bid on a license it no longer wants as other prices rise. There is *no regret*.
  - This property depends crucially on substitutes.
The Exposure Problem

- Suppose a bidder has value
  - 10 for either A or B alone
  - 30 for A and B together

- The “exposure problem”
  - If both license prices reach 12, straightforward bidding means bidding on both A and B.
  - If bidding on A stops and B’s price subsequently climbs to 20, the bidder will regret purchasing A.
Market clearing prices?

- Bidder 1 values: 17 for A, 22 for B, 34.5 for both.
- Bidder 2 values: 20 for A, 20 for B, 37.5 for both.

<table>
<thead>
<tr>
<th>Round</th>
<th>A’s price: $p_A$</th>
<th>B’s price: $p_B$</th>
<th>A’s High Bidder</th>
<th>B’s High Bidder</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>11</td>
<td>16</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>12</td>
<td>17</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
<td>13</td>
<td>17</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>14</td>
<td>17</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>29</td>
<td>14</td>
<td>18</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Describing Outcomes

- Auction won’t necessarily find exact market clearing prices, but it gets close…

- In particular, the auction outcome with straightforward bidding will be an exact competitive equilibrium for a nearby set of values.

- The nearby values are constructed as follows:
  - Identify the goods that bidder $j$ wins at the auction.
  - Define $j$’s modified values for any set of goods $T$ to be the original value minus one bid increment for each good in $T$ that $j$ does not win.
Substitutes: Competitive Equilibrium

**Theorem:** Suppose the licenses are substitutes and that all bidders bid straightforwardly. Let \((p^*, S^*)\) be the final standing high bids and license assignment and suppose the minimum bid increment vector is \(q\). Then the final allocation is “nearly efficient” and \((p^*, S^*)\) is a competitive equilibrium for a nearby economy with individual valuations defined by:

\[
\hat{v}_j(T) = v_j(T) - q \cdot 1_{T \backslash S_j^*}
\]
Example, continued...

<table>
<thead>
<tr>
<th>Round</th>
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<td>29</td>
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<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

- Bidder 1 values: 17 for A, 22 for B, 34.5 for both.
- Bidder 2 values: 20 for A, 20 for B, 37.5 for both.
- Nearby values
  - Bidder A: (16, 22, 33.5) and Bidder B (20, 19, 36.5)
  - Final prices (14,18) and allocation clear the market using the nearby values.
Summary of non-strategic SAA theory

- Suppose bidders view licenses are substitutes and bid straightforwardly in the SAA.

  - **Arbitrage:** The final prices for identical items will differ by at most one bid increment.
  - **Efficiency:** If the bid increments are small, the final license allocation will be efficient.
  - **Competitive Equilibrium** The final prices will “close” to competitive equilibrium prices.
Good market design

- UK sale of 3G spectrum (1999)
  - 5 national licenses, 2 larger than the others.
  - 4 incumbent (2G) operators, plus entrants.
  - Each bidder could win at most one license.

- What happened in the auction
  - Straightforward bidding a natural strategy.
  - Outcome widely perceived as efficient.
  - British government raised 22 billion pounds.
Bad market design

- Netherlands 3G auction in 1999.
  - 5 nationwide licenses, pretty similar.
  - 5 incumbent (2G) operators.
  - Prior to auction, major outside telecom firms (Deutsche Telekom, DoKoMo, Hutchinson Whampoa) all reach partnership agreements with an incumbent.
  - Only one additional entrant, startup called Versatel.

- What happened in the auction
  - On day 1, Telfort (owned by BT) sends Versatel a letter saying that it “can’t win” and should drop out immediately!
  - Versatel shortly drops out: total revenue of 3bn euros – at UK prices, auction would have raised 10bn euros.
Strategic Demand Reduction

- **German GSM auction (2000)**
  - 10 nationwide licenses, almost identical.
  - Starting price of zero.
  - Bid increments of DM 10m.
  - Bidders: Mannesman, T-Mobile (large) and small guys.

- **What happened in the auction**
  - Round 1: Mannesman bids 36.6m for each of 5 bands, and reduces eligibility.
  - Round 2: T-Mobile (Deutsche Telekom) bids 40m for the other five bands, reduces eligibility.
  - No bids in round 3!
Complexity and strategy

  - 90 MHz of nationwide spectrum, 1122 licenses
  - Regional licenses (10, 10, 20 MHz), 6 to cover US
  - Smaller licenses (10, 20, 20 MHz), ≥176 to cover
  - Total of 168 bidders, including major incumbents, small firms and two potential national entrants.

- Entrants face a difficult problem
  - Theory doesn’t provide much guidance on how to bid in a way that avoids the exposure problem…
  - This has been a standard concern in spectrum auctions.
Timing problems & opportunities

- Activity rules force bidders to make early commitments
  - Creates exposure problem for entrants (package bidders)
  - Creates difficulties for bidder with budget constraint
  - Not so easy to arbitrage different size licenses: easy to substitute from a big license to smaller licenses, but not so easy to get back!

- *Empirical proposition*: bidding tends to start on large licenses, and these licenses tend to clear first.
Auction 35: bidding activity
Auction 35: time of last bids

Figure 3b: Round of Final Bid by License Size (Auction 35)
The exposure problem

- New entry may require a package of licenses
  - Because markets clear at different times, could easily end up with some very expensive spectrum but not enough for viable entry.

- Fundamentally a problem of uncertainty.
Prices vary enormously!

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>When?</th>
<th>$ Billion</th>
<th>$/MHz-Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>PCS C Block</td>
<td>1996</td>
<td>13.4</td>
<td>1.77</td>
</tr>
<tr>
<td>10</td>
<td>C Block Re-auction</td>
<td>1996</td>
<td>0.7</td>
<td>1.50</td>
</tr>
<tr>
<td>11</td>
<td>PCS DEF Blocks</td>
<td>1997</td>
<td>2.7</td>
<td>0.36</td>
</tr>
<tr>
<td>22</td>
<td>PCS</td>
<td>1999</td>
<td>0.5</td>
<td>0.20</td>
</tr>
<tr>
<td>34</td>
<td>800 MHz</td>
<td>2000</td>
<td>0.3</td>
<td>0.18</td>
</tr>
<tr>
<td>35</td>
<td>PCS C&amp;F</td>
<td>2001</td>
<td>17.6</td>
<td>4.37</td>
</tr>
<tr>
<td>58</td>
<td>Broadband PCS</td>
<td>2005</td>
<td>2.3</td>
<td>1.05</td>
</tr>
</tbody>
</table>
Role of bidder budgets

- Many bidders appear to be limited in their bidding by *budgets*, rather than *values*.
  - This neglected pattern is significant for both bidder strategy and auction design.

- *Empirical proposition*: at an aggregate level, budgets appear to play key role in determining prices.
“Exposure” forecasts prices

Figure 4: Revenue and Exposure in Auction 35
Forecasting in the AWS auction

Figure 5: Revenue and Exposure in Auction 66
Peak/final exposure FCC sales

Figure 8: Budget Forecasting in Major FCC Auctions

Gross Auction Revenue (USD millions)

Auction 35
Auction 66
Auction 58
Auction 33
Auction 22
Auction 30
Auction 34
Auction 37
Auction 53
Auction 44
Auction 30
Exposure by bidders in AWS

Figure 6b: Bidder Exposure in Auction 66
Use of budget forecasts

- To avoid the exposure problem:
  - Allows an entrant to identify if a desired aggregation is achievable at reasonable price.

- To acquire licenses cheaply:
  - Allows a bidder to anticipate price anomalies when individual licenses clear in sequence.
Controlling auction prices

- Simultaneous ascending auction
  - Entrant wants two licenses: value $v_{12} > v_1 + v_2$
  - Individual bidders: values $u_i \sim F_i$

- If the entrant can control the rate of price increases, how should it behave?
Suppose prices are \((p_1, p_2)\) and entrant wins 1

\[
\pi_1(p_1, p_2) = v_1 - p_1 + Q_2(p_1, p_2)
\]

where

\[
Q_2(p_1, p_2) = \mathbb{E}[\max\{v_{12} - v_1 - u_2, 0\} | u_2 \geq p_2]
\]

Given fixed price path, optimal to make initial exit somewhere between blue curves. Conditional on planning to make first drop at \(p\), all paths to \(p\) are equivalent.

**Theorem.** Any path to \(p^*\) is optimal.

If many individual bidders (i.e. \(n_i > 1\)),

\[
v_{12} - v_1 = v_{12} - v_2
\]

A path that results in initial exit at \(p \neq p^*\) cannot be optimal.

Answer: gather as much “free information” as possible.
Pacing and efficiency

- **Theorem.** Any change in pacing that benefits entrant also increases efficiency.

  - Entrant may win too many or too few licenses from efficiency perspective, but always pays social cost.
The AWS auction

- Recall basic structure of licenses:
  - “Large” regional licenses (40 MHz)
  - “Small” EA/CMA licenses (50 MHz)

- Competitive landscape: 168 bidders, major incumbents, and two potential national entrants
  - SpectrumCo: cable TV consortium
  - Wireless DBS: satellite TV consortium

- Prior to auction, appeared there would be room for at most one successful entrant, if any.
Controlling the pace

- Bidding started on large regional licenses.
  - But prices rose uniformly on coasts/interior, creating serious exposure problem…

- In response, SpectrumCo makes maximal ($750m) jump bid, *doubling* prices in Northeast and West.
  - What happens? Wireless DBS takes waivers, then exits
  - FCC eliminates jump bidding in subsequent auctions.
Budget forecasting

- As of round 13, the situation is
  - High bids on REAGs (40 MHz): $ 5.03 bn
  - High bids on EA/CMAs (50 MHz): $ 0.76 bn
  - Level where auction exposure had peaked: $ 14.2 bn

- SpectrumCo - alone among the major bidders - gives up REAGs and switches to smaller licenses.
SpectrumCo’s licenses (20 MHz)
Failure of price arbitrage

Table 1: Prices Paid by the Five largest Buyers in Auction 66

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Total Winning Bids</th>
<th>Per MHz-Pop</th>
<th>SpectrumCo’s Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpectrumCo</td>
<td>$2,377,609,000</td>
<td>$0.45</td>
<td>--</td>
</tr>
<tr>
<td>Cingular</td>
<td>1,334,610,000</td>
<td>$0.55</td>
<td>$511 m</td>
</tr>
<tr>
<td>T-Mobile</td>
<td>4,182,312,000</td>
<td>$0.63</td>
<td>$943 m</td>
</tr>
<tr>
<td>Verizon</td>
<td>2,808,599,000</td>
<td>$0.73</td>
<td>$1,476 m</td>
</tr>
<tr>
<td>MetroPCS</td>
<td>1,391,410,000</td>
<td>$0.96</td>
<td>$2,699 m</td>
</tr>
<tr>
<td>Four incumbents</td>
<td>9,716,931,000</td>
<td>$0.68</td>
<td>$1,191 m</td>
</tr>
</tbody>
</table>