Sponsored Search Auctions

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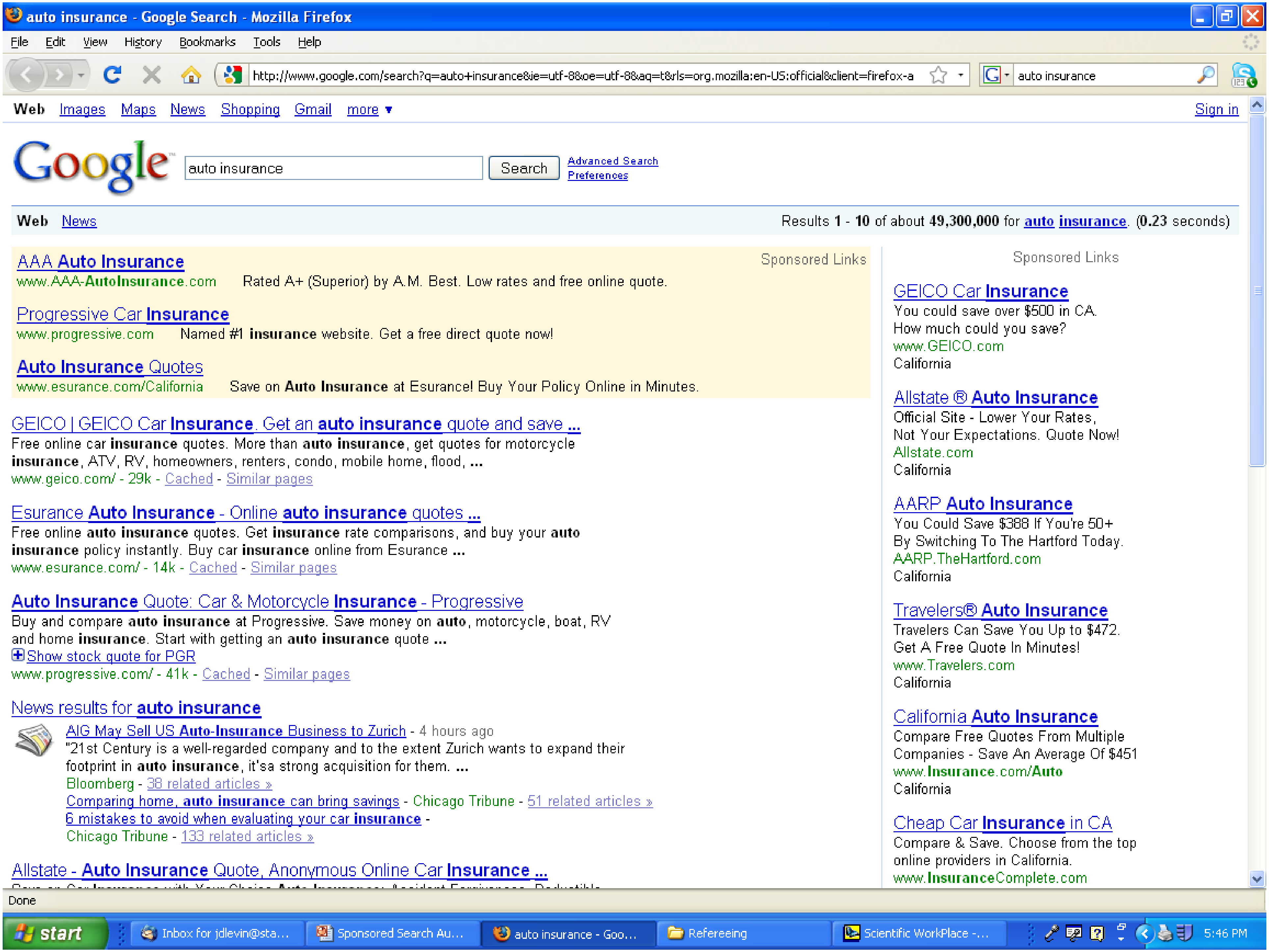
Sponsored Search Auctions

- Google revenue in 2008: $21,795,550,000.
- Hal Varian, Google chief economist:
  - “What most people don’t realize is that all that money comes pennies at a time.”
- Today’s lecture: internet keyword auctions.
  - References: Varian 2008, Edelman et al. 2007.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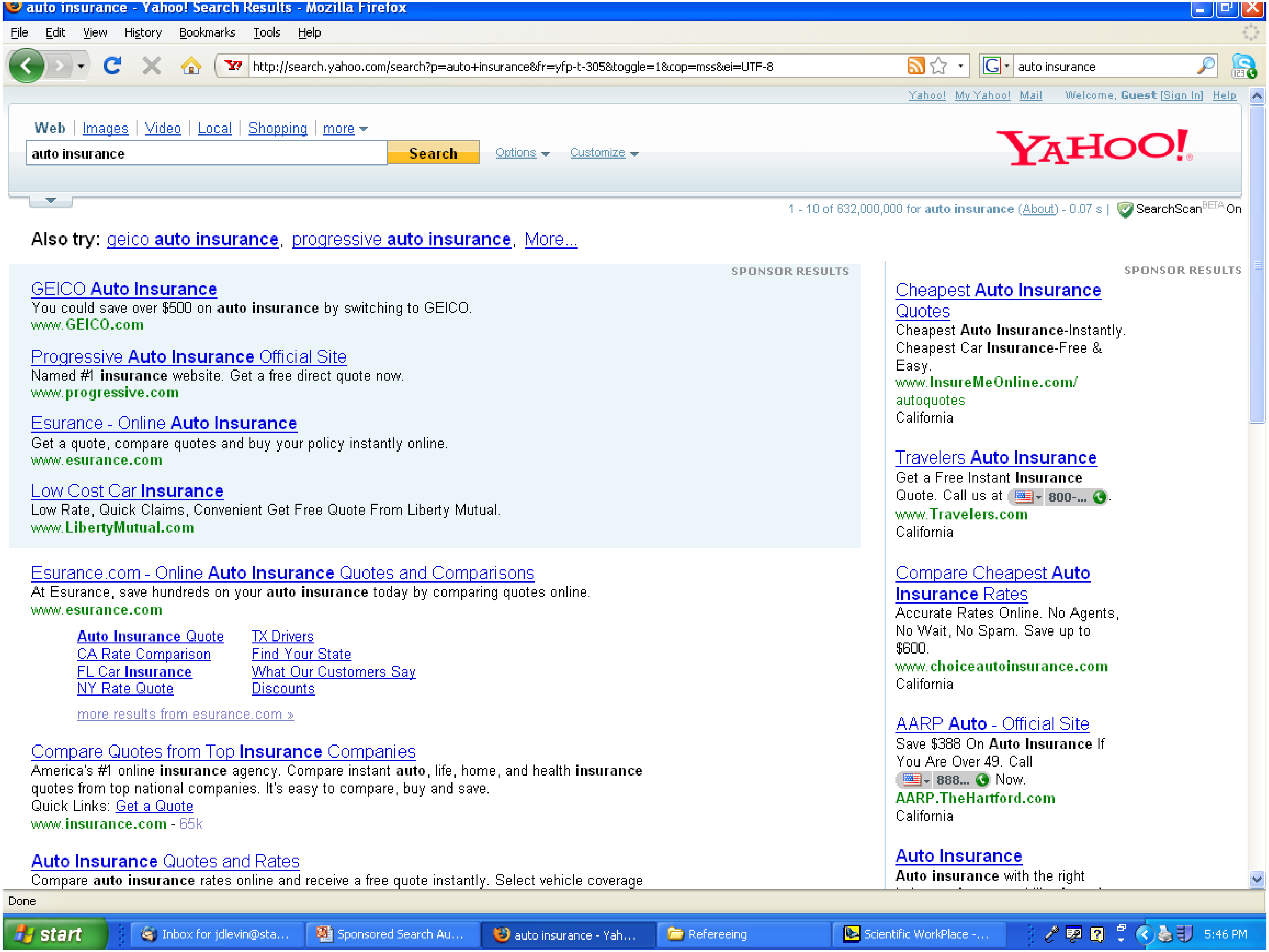
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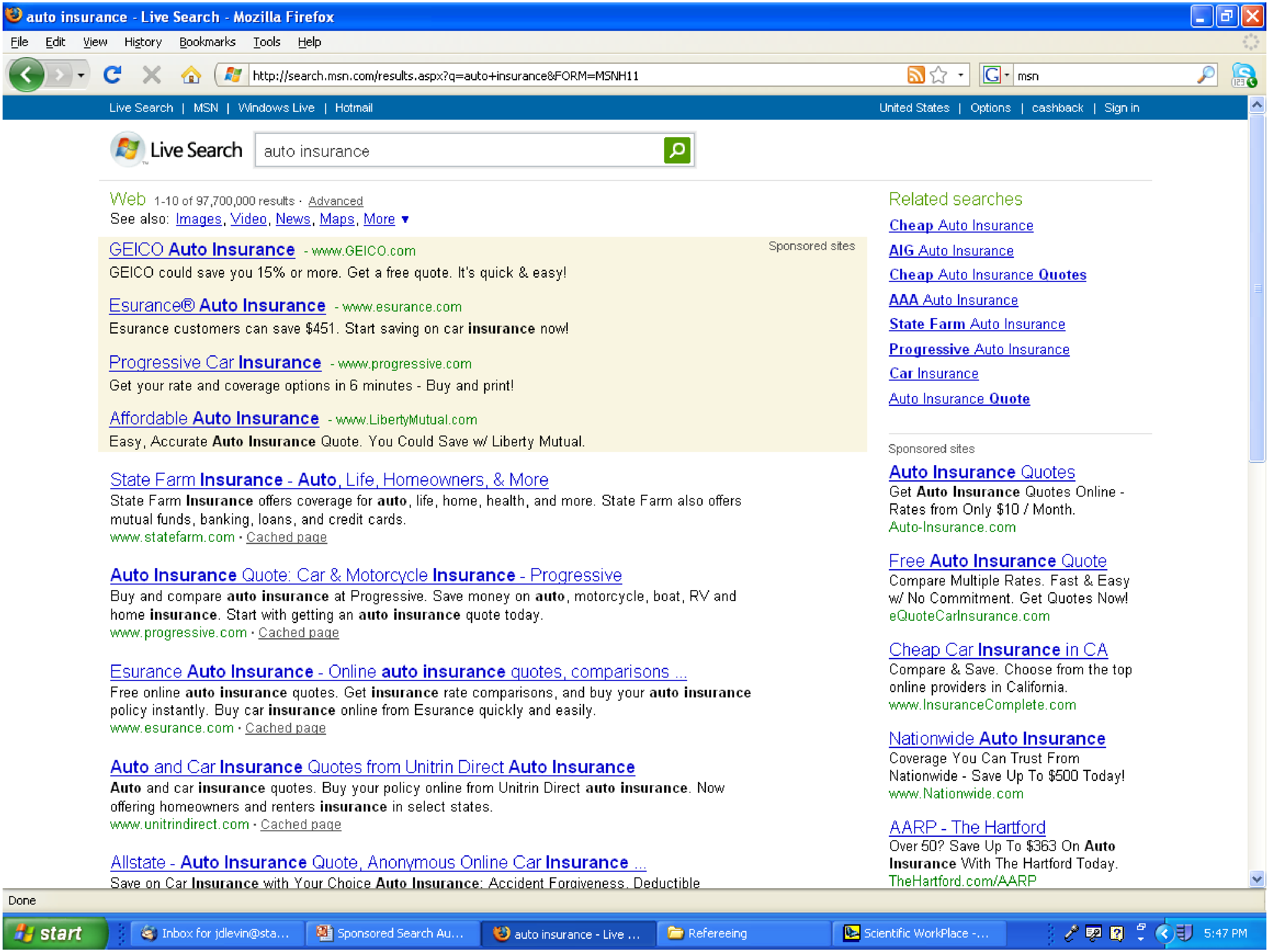
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Keyword Auctions

- Advertiser submit bids for keywords
  - Offer a dollar payment *per click*.
  - Alternatives: price per impression, or per conversion.
- Separate auction for every query
  - Positions awarded in order of bid (more on this later).
  - Advertisers pay bid of the advertiser in the position below.
  - “Generalized second price” auction format.
- Some important features
  - Multiple positions, but advertisers submit only a single bid -- “simplification” (cf Milgrom’s lecture).
  - Search is highly targeted, and transaction oriented.

Brief History of Sponsored Search Auctions

- Pre-1994: advertising sold on a per-impression basis, traditional direct sales to advertisers.
- 1994: Overture (then GoTo) allows advertisers to bid for keywords, offering some amount *per click*. Advertisers pay their bids.
- Late 1990s: Yahoo! and MSN adopt Overture, but mechanism proves unstable - advertisers constantly change bids to avoid paying more than necessary.
- 2002: Google modifies keyword auction to have advertisers pay minimum amount necessary to maintain their position (i.e. GSP)- followed by Yahoo! and MSN.

Example

- Two positions: receive 200 and 100 clicks
- Advertisers 1,2,3 have per-click values $10, $4, $2.

- Overture auction
  - Advertiser 2 has to bid $2.01 to get second slot
  - Advertiser 1 wants to bid $2.02.
  - But then advertiser 2 wants to top this, and so on.

- GSP auction
  - One eqm: truthful bids of $10, $4, $2.
  - Revenue is 200*$4 + 100*$2 = $1000.

Example, continued…

- Consider VCG auction
  - Dominant to bid true value.
  - Advertiser 2 pays $200 (displaces 3) for 100 clicks, or $2 per click.
  - Advertiser 1 pays $600 (displaces 3 *and* 2) for 200 clicks, or $3 per click.
  - Revenue of $800 is lower than GSP…

Model

- K positions $k=1,..,K$
- N bidders $i = 1,\ldots,N$

- Bidder i values position k at $u_{ik} = v_n \bullet x_k$
  - x_k is probability of a click, $x_1>x_2>\ldots>x_K$
  - v_n is value of a click, $v_1>v_2>\ldots>v_K$

- Efficient allocation is assortative.

GSP Auction Rules

- Each agent *i* submits bid b_i
- Positions assigned in order of bids
- Agent i's price per click is bid of agent in the next slot down.
- Let b^k denote *k*th highest value and v^k value.
- Payoff of *k*th highest bidder:

$$v^k \bullet x_k - b^{k+1} \bullet x_k = (v^k - b^{k+1}) \bullet x_k$$

Truthful bidding?

- Not a dominant strategy to bid "truthfully"
  - Two positions, with 200 and 100 clicks.
  - Consider bidder with value 10
  - Faces competing bids of 4 and 8.
    - Bidding 10 wins top slot, pay 8: profit 200 • 2 = 400.
    - Bidding 5 wins next slot, pay 4: profit 100 • 6 = 600.
  - If competing bids are 6 and 8, better to bid 10…

GSP equilibrium Analysis

- Full information Nash equilibrium
  - NE means no gain from changing positions

- A Nash eqm is a profile of bids $b^1, \ldots, b^K$ such that

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m+1}) \cdot x_m \quad \text{for } m>k$$

$$(v^k - b^{k+1}) \cdot x_k \geq (v^k - b^{m}) \cdot x_m \quad \text{for } m<k$$

- Lots of Nash equilibria, including some that are inefficient (try to show this).

Locally Envy-Free

- *Definition:* An equilibrium is *locally envy-free* if no player can improve his payoff by exchanging bids with the player ranked one position above him.
  - Motivation: “squeezing” – if an equilibrium is not LEF, there might be an incentive to squeeze.
  - Add the constraint for all k

$$(v^k - b^{k+1}) \bullet x_k \geq (v^k - b^k) \bullet x_{k-1}$$

Stable Assignments

- Treat positions as players. Coalition value from a position-bidder pair is $v_i x_k$, and price of position is p_k
  - Payoff to agent is $(v_i - p_k)x_k$
  - Payoff to position is $p_k x_k$
- All stable assignments are efficient (assortative), and the relevant blocks are bidders looking to move up or down one position. (think about this).
- Prices that support a stable allocation satisfy:

$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k-1}) \bullet x_{k-1}$$
$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k+1}) \bullet x_{k+1}$$

Equivalence Result

- ***Theorem:***
  - Outcome of any locally envy-free equilibrium is a stable assignment.
  - Provided that |N|>|K|, any stable assignment is an outcome of a locally envy-free equilibrium.

Revenue and Prices

- ***Theorem***
  - There exists a bidder-optimal stable assignment (equivalently, GSP equilibrium) and a seller-optimal one.
  - The bidder optimal stable assignment is payoff-equivalent to the VCG outcome.

- ***Corollary:*** any locally envy free GSP equilibrium generates at least as much revenue as VCG.

Example of LEF Equilibria

- Three positions with 300, 200, 100 clicks
- Four bidders with values \$3, \$2, \$1, \$1
- Efficient assignment is assortative
- Supporting prices
  - Bidder 3 pays \$100 for slot 3, $p_3 = 1.$
  - Bidder 2 pays \$200-300 for slot 2, $p_2 \in [1,3/2].$
  - Bidder 1 pays \$400-600 for slot 3, $p_3 \in [4/3,2].$
- Try solving for bids that generate these prices.
- Relationship between VCG and LEF eqm
  - VCG payments are \$100, \$200, \$400, revenue \$700.
  - LEF payments range from \$700 up to \$1000.

Structure of Clearing Prices

- Supporting prices satisfy

$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k-1}) \bullet x_{k-1}$$
$$(v_k - p_k) \bullet x_k \geq (v_k - p_{k+1}) \bullet x_{k+1}$$

- Re-arranging we get

$$p_{k-1} x_{k-1} \geq p_k x_k + v_k(x_{k-1} - x_k)$$
$$p_{k-1} x_{k-1} \leq p_k x_k + v_{k-1}(x_{k-1} - x_k)$$

- This gives a simple *recursive* way to find the highest and lowest equilibrium payments.

Features of Equilibrium

- Allocation is efficient (assortative)
- Increasing price of marginal clicks
  - Varian points out this is testable.
  - Implies bidders are click-constrained!
  - Pricing should be linear if bidders satiated…
- Bids “reveal” bounds on bidder values.
  - Apparently not so easy to invert in practice.
  - Actual bidding is surprisingly unstable…

Ascending auction

- Incomplete information about values
- Price rises from zero, advertisers can drop out at any time, fixing their bid.

- ***Theorem (Edelman et al.)***.
  - There is a unique perfect equilibrium in which an advertiser with value v_i drops at

$$p_i(n,h,v_i)= (x_n / x_{n-1})(v_i - b_{n+1})$$

  - The equilibrium outcome is the same as VCG
  - The equilibrium is an *ex post* equilibrium.

Optimal auction design

- Suppose each bidder *i* draws values from F_i
  - Define marginal revenue: $MR_i(v_i)=v_i-(1-F_i)/f_i$
  - Seller has total quantity $x= x_1+\ldots+x_K$.
- Optimal auction problem:
  - Choose allocation of clicks $z_1,\ldots,z_N$ to maximize $\sum_i MR_i(v_i)z_i$ subject to the allocation being feasible.
  - Solution: assign slots in order of marginal revenue, so long as it is positive.
- Optimal reserve prices: if the environment is symmetric, optimal to run a position auction with reserve price r^* that satisfies $MR(r^*)=0$.
  - Of course, need to know distribution of per-click values…

Bidder-Specific Click Rates

- Some ads may be more relevant than others.
  - eg if query is “Pottery Barn,” what ad will get clicked?
- Natural to extend model so click rates differ.
  - Suppose $Pr(click) = a_i \cdot x_k$
  - Values: $u_{ik} = v_i \cdot (a_i \cdot x_k) = (v_i a_i) x_k$
  - Bids are still made on a per-click basis
- Value rank: rank bids by expected revenue, by $b_i \cdot a_i$
  - Eqm allocation will maximize total value.
  - Bidder-optimal eqm will be payoff-equivalent to VCG
- Bid rank: rank bids directly by b_i.
  - May not be efficient, but may raise revenue.

“Squashing” Example

- Two positions with 200, 100 “base” clicks
- Three bidders with
  - per-click values $2, $1, $1
  - “click-thru rates”: 2,1,1
- Rank bids by bid*CTR
  - Bidder 2 pays $1 per-click for position 2
  - Bidder 1 pays $0.50 per-click for position 1
- Rank bids by bid (i.e. treat B1 “as if” CTR=1)
  - Bidder 2 pays $1 per-click for position 2
  - Bidder 1 pays $1 per-click for position 1.
- Note: when would latter auction be inefficient?

Further issues

- Each query is a separate game
  - Advertisers really have portfolio of bids & broad match…
  - Ignores budget constraints, diminishing returns, etc
  - Hard to think about eg. competing platforms
- Model doesn't allow for much uncertainty
  - Click rates, effectiveness of advertising are known.
  - Seems to be a lot of experimentation in practice. Why?
- Many aspects of search not captured
  - How do people decide whether/what to click?
  - Is there an interaction with "organic" search?
- "Non-search" internet advertising
  - Google uses same auction to place ads on non-query web pages (AdSense).
  - Other companies use related mechanisms to match ads and eyeballs, and sometimes quite different approaches.