Vickrey-Clarke-Groves Mechanisms

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[1] These slides are based on Paul Milgrom's.

Motivation

- We consider the problem of how to implement efficient allocations in an environment where each participant has private information about their preferences.
- Participants may misrepresent their preferences.
    - A bank may overstate its need for a federal bailout, hoping to get taxpayers to absorb its losses.
    - A buyer might understate its value, hoping to get a lower price.
- VCG mechanisms achieve "strategy-proof" implementation of efficient allocations in quasi-linear environments, but can have trouble with budget balance.

The model

- A set of participants N $(S \subset N)$, and the designer 0.
- Types $\mathbf{t} = (t_1, ..., t_N)$ describe participant information.
- Outcome is $(x, \mathbf{p})$, where $x \in X$ is a decision and $\mathbf{p} = (p_1, ..., p_N)$ are participant payments.
- Payoffs: $u_i(x, \mathbf{p}, \mathbf{t}) = v_i(\mathbf{x}, t_i) - p_i$.
- Define:

$$\begin{aligned} V(S, \mathbf{t}) &= \max_{x \in X} \sum_{i \in S} v_i(x, t_i) \\ \hat{x}(S, \mathbf{t}) &\in \arg\max_{x \in X} \sum_{i \in S} v_i(x, t_i) \end{aligned}$$

Generality of the model

- The model covers a broad range of possible situations.
- The decision x can be anything
  - The allocation of goods in an auction
  - The decision to build a public project
  - A decision about government or firm policy
  - The resolution of a dispute between two or more parties.
- There are no a priori restrictions on who pays what as a function of the decision.

Important restrictions of the model

- Each participant knows his or her own values.
- Preferences are over final outcome, not process.
- Preferences must be quasi-linear in money.
- There are no limits to making transfer payments.

The VCG Mechanism

- Look for a mechanism with three characteristics
  - Direct revelation mechanism
  - Efficient (surplus-maximizing) decision
  - Player i's report does not affect the total payoff to all others (w/ designer included).

Deriving the VCG Mechanism

- Base case: i reports that all values are zero.
- Optimal decision satisfies (nb: can ignore i):

$$\hat{x}(N-i, t_{-i}) \in \arg\max_{x \in X} \sum_{j \in N-i} v_j(x, t_j)$$

- Total payoff to players $-i$ is:

$$V(N-i, t_{-i}) + p_i(0, t_{-i}) = V(N-i, t_{-i}) + h_i(t_{-i}).$$

- where

$$h_i(t_{-i}) \triangleq p_i(0, t_{-i}).$$

Deriving the VCG Mechanism, cont.

- Suppose reports are $\mathbf{t} = (t_1, ..., t_N)$.
- Efficient decision is $\hat{x}(N, \mathbf{t}) = \arg\max_{x \in X} \sum_{j \in N} v_j(x, t_j)$.
- For each player i, we have

$$\sum_{j \in N-i} v_j(\hat{x}(N, \mathbf{t}), t_j) + p_i(\mathbf{t}) = V(N-i, t_{-i}) + h_i(t_{-i})$$

- Therefore the VCG payment must satisfy

$$p_i(\mathbf{t}) = V(N-i, t_{-i}) - \sum_{j \in N-i} v_j(\hat{x}(N, \mathbf{t}), t_j) + h_i(t_{-i})$$

Interpreting the VCG Mechanism

- The VCG payment for player i is

$$p_i(\mathbf{t}) = V(N-i, t_{-i}) - \sum_{j \in N-i} v_j(\hat{x}(N, \mathbf{t}), t_j) + h_i(t_{-i})$$

- The VCG payoff for player i is

$$v_i(\hat{x}(N, \mathbf{t}), t_i) - p_i(\mathbf{t}) = V(N, \mathbf{t}) - V(N-i, t_{-i}) - h_i(t_{-i})$$

- So i pays his expected "externality" on others, and receives his marginal contribution to social surplus, up to a constant $h_i(t_{-i})$.

Pivot Mechanisms

- A player is *pivotal* if $\hat{x}(N, \mathbf{t}) \neq \hat{x}(N - i, t_{-i})$
- The VCG mechanism with $h_i(t_{-i}) = 0$ for all i is call the *pivot mechanism* — only pivotal players pay.
- In a pivot mechanism, player i's payoff (valued at his reported type) is just equal to his *marginal contribution to social surplus*:

$$v_i\left(\hat{x}(N, \mathbf{t}), t_i\right) - p_i\left(\mathbf{t}\right) = V\left(N, \mathbf{t}\right) - V\left(N - i, t_{-i}\right)$$

Strategy-Proofness

- A direct mechanism is *strategy-proof* if for all players i, truthful reporting maximizes i's payoff regardless of the other player's choices.
- Sometimes we say that a general mechanism is strategy-proof (or a dominant strategy mechanism) if there is *some strategy* for each player i that is weakly dominant.

VCG Strategy-Proofness

Theorem

The Vickrey-Clarke-Groves mechanism is strategy-proof.

Proof of VCG Strategy-Proofness

- Consider i's payoff from truthful reporting minus its payoff from false reporting:

$$\begin{aligned}
\Delta &= \left\{ v_i\left(\hat{x}(N, t_i, t_{-i}), t_i\right) - p_i\left(t_i, t_{-i}\right)\right\} \\
&\quad - \left\{ v_i\left(\hat{x}(N, s_i, t_{-i}), t_i\right) - p_i\left(s_i, t_{-i}\right)\right\} \\
&= \left\{ \sum_{j \in N} v_j\left(\hat{x}(N, t_i, t_{-i}), t_j\right) - V\left(N - i, t_{-i}\right) - h_i\left(t_{-i}\right) \right\} \\
&\quad - \left\{ \sum_{j \in N} v_j\left(\hat{x}(N, s_i, t_{-i}), t_j\right) - V\left(N - i, t_{-i}\right) - h_i\left(t_{-i}\right) \right\} \\
&= \sum_{j \in N} v_j\left(\hat{x}(N, t_i, t_{-i}), t_j\right) - \sum_{j \in N} v_j\left(\hat{x}(N, s_i, t_{-i}), t_j\right) \geq 0
\end{aligned}$$

- The gain from truthful reporting is just equal to the social benefit caused by the true report.

Second Price (Vickrey) Auction

- Single good, worth t_i each each bidder i.
- Pivot mechanism:
  - Award item to bidder with the highest "report"
  - Losing bidder pays 0
  - Winning bidder pays second highest value
- Derivation of payments:

$$
\begin{aligned}
p_i(\mathbf{t}) &= V\left(N-i, t_{-i}\right) - \sum_{j \in N-i} v_j\left(\hat{x}(N, \mathbf{t}), t_j\right) \\
&= \begin{cases} V\left(N-i, t_{-i}\right) & \text{for winner} \\ 0 & \text{for losers} \end{cases}
\end{aligned}
$$

Balancing the Budget?

- In many situations, the designer might want a mechanisms in which the total transfer payments add up to zero (or to some positive amount).
  - We want to design a platform for bilateral exchange in which seller receive exactly the payment to buyer (perhaps minus a fee).
  - We want to design a system to distribute the cost of a public project, which will be funded exactly by the contributions of participants.

The Problem with Budget Balance

Theorem

A VCG mechanism that results in zero total transfers may not exist.

- Recall there is some flexibility in choosing the $h_i(\cdot)$'s, but it may not be possible to choose them to that total payments sum to zero.
- Example: A single good must be allocated to bidder 1 or 2, where 1's possible values are {1,3} and 2's are {2,4}.
  - If realized types are $(1, 2)$, then 1 pays $p_1 = h_1(2)$ and 2 pays $p_2 = 1 + h_2(1)$. If $p_1 + p_2 = 0$, then $h_1(2) + h_2(1) + 1 = 0$.
  - If realized types are $(3, 4)$, then $h_1(4) + h_2(3) + 3 = 0$.
  - If realized types are $(1, 4)$, then $h_1(4) + h_2(1) + 1 = 0$.
  - If realized types are $(3, 2)$, then $h_1(2) + h_2(3) + 2 = 0$.
- Adding the first two and last two terms, we need to have $3 = 4$!
- Evidently, budget balance cannot be achieved here.

Attractive features of the Vickrey auction

- Can be used in broad class of environments.
- Can be used even if bidders want packages.
- Outcome is efficient given reported values.
- Dominant strategy to report true values, so little incentive to invest in gaming and mechanism is “detail free” (Wilson, 1987).

- So why don’t we see Vickrey auctions in use?

Practical problems with Vickrey auction

- Pushes complexity onto bidders.
- Reveals a lot of information.
- Possible to have very low-revenue outcomes.
- Highly susceptible to collusion.
- Perverse incentives for mergers or de-mergers.
- Requires unlimited budgets, or else problems.

Low Revenue (or "non-core") Outcomes

- Two items A and B.
- Package bidder values A, B together at 10.
- One individual bidder for each item, with value 9.
- Efficient to award items to the individual bidders.
- Each creates surplus of 8, so pays 1.
- So auction revenue is 2, although package bidder would pay 10!

Losing bidders can collude to win

- Two items A and B.
- Package bidder values A, B together at 10.
- One individual bidder for each item, with value 2.
- With honest bidding, package bidder wins.
- Suppose individual bidders both report 9.
- Items are awarded to the individual bidder and each pays 1, so profitable collusion leads to very inefficient outcome.

- Collusion is always a concern in auctions, but in a Vickrey auction collusion by even a small number of parties can have a big effect.

Profitable use of “shill” bidders

- Two items A and B.
- Bidder 1 is willing to pay 10 for the pair.
- Bidder 2 is willing to pay 9 for the pair.
- If honest, bidder 1 wins and pays 9.
- If bidder 2 enters as 2A and 2B, each of which bids 9 for a single item, it wins both and pays 2.

- Note that if it bidders 2A and 2B bid 10 for each item, they each pay zero, so this example also shows that revenue can go down when bids go up!

Budget constraints create problems

- Two items A and B.
- Bidder values A at 200, B at 100, budget of 150.
- Can't bid true values and be assured of staying within the budget.
- A "straightforward" bid might be 150 for A, 100 for B, and 150 for the pair.
- But the mechanism will interpret this as saying that the bidder has *zero* value for B if it is awarded A.
- Example of a more general problem: complex to bid with a budget in a Vickrey auction.

General lessons

- The real game is bigger than you think....
  - Mechanisms that look ideal for a given environment may be susceptible to changes in the environment...
  - Mergers, investments, entry decisions, design or partitioning of items for sale, collusion, etc.
- The Vickrey auction has serious drawbacks
  - These drawbacks have been fully understood only in recent years, but have informed a lot of practical auction design.